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THE SIMPLE ARITHMETIC OF CARBON PRICING AND STRANDED ASSETS

Frederick van der Ploeg[¥] *and Armon Rezai*[§]

Climate policy has to combine ethical judgments with projections about future economic, technological, and climatic developments. Integrated assessment models (IAMs) aim to do this, but have been criticized for being highly complex, insufficiently open access, and underestimating the threats of climate change (Stern, 2013; 2016). We present a simple framework that captures the essence of IAMs, makes their underlying assumptions transparent, and opens the discussion of the political and social obstacles to climate policy. Our cost-benefit analysis of climate action yields a simple rule for the optimal global price of carbon in the presence of a backstop renewable energy source that is currently more expensive than fossil fuel. This price is proportional to current world GDP and depends on key ethical considerations, damage flows and geophysical parameters. We also offer rules for the optimal fraction of fossil fuel reserves that should be left in the crust of the earth (cf. Carbon Tracker, 2013; McGlade and Ekins, 2015) and the optimal transition time to the carbon-free era. Our calculations require only a pencil and the back of an envelope, but yield values very close to those obtained from numerically maximizing welfare with a detailed IAM of growth, development, energy and climate change. We hope that our simple arithmetic helps policy makers and climate scientists to gain a better understanding of the ethical, economic and geo-physical drivers of optimal climate policy.[1]

The Cost of Emitting Carbon

The global price of emitting carbon, P, should generally be set to the social cost of carbon, which is the present discounted value of all future economic damages from emitting one ton of carbon today. This price can be levied via either a carbon tax or an emissions permit. To compute future economic damages, we employ a n-box model of the carbon cycle. With the fraction a_i of each emitted ton entering a box i with exponential decay rate b_i, the amount left in the atmosphere after t years equals $\sum_{i=1}^{n} a_i(1-b_i)^t$. The easiest is to have n = 2 with a permanent box to capture that 20% of carbon emissions stays up for thousands of years in the atmosphere (a_1 = 0.2, b_1 = 0) and a transient box

[¥] Department of Economics, University of Oxford, Manor Road Building, Oxford OX1 3UQ, U.K., VU University Amsterdam, and St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034 Russia. Van der Ploeg is grateful for support from FP7-IDEAS-ERC Grant No. 269788. Email: rick.vanderploeg@economics.ox.ac.uk.

[§] Department of Socioeconomics, Vienna University of Economics and Business, Welthandelsplatz 1, 1020 Vienna, and IIASA, Schlossplatz 1, 2361 Laxenburg, Austria. Rezai is grateful for support from the Austrian Science Fund (FWF): J 3633 and OeNB Anniversary Fund (grant no. 15330). Email: armon.rezai@wu.ac.at.

[1] Allen (2016) also offers a simple framework for analyzing the drivers of peak warming and the optimal carbon budget in a consumer-maximizing world, but focuses on the need for carbon capture and sequestration rather than the transition to renewable energy and does not discuss the ethical drivers of climate policy or the optimal timing of the transition to the carbon-free era.

($a_2 = 0.32$ and $b_2 = 0.0023$). This captures well the carbon cycle of the most prominent IAM, i.e., DICE set out in Nordhaus (2008) (Golosov et al, 2014).[2] Next, we assume that the flow damage at time t of an extra ton of atmospheric carbon is proportional to world GDP, i.e., $d\,GDP_t$, where d is estimated to be 5.3% of GDP at roughly 2°C based on DICE and does not vary much with temperature (Golosov et al., 2014).[3] We allow for a mean lag between projected global mean temperature and the damages that result from that and the stock of atmospheric carbon of l years. In our baseline calibration we take l is 70 years.

Ethics and the Discount Rate

Given that climate change resulting from burning carbon today occurs decades or centuries ahead, the flow damages computed above have to be summed and discounted taking account of the slow uptake of atmospheric carbon in biosphere and oceans. Economists use the social discount rate, *SDR*, for this, which is a concept that has been hotly debated in climate economics (Stern, 2007; Nordhaus, 2007; Weitzman, 2007). Most economists use the Keynes-Ramsey rule for distributing consumption optimally over time: $SDR = RTI + IIA \times g - \frac{1}{2} \times RRA \times PRU \times \sigma^2 + RRA \times \sigma^2$ (e.g., Gollier, 2014). The first two terms in the *SDR* trade off the rate of time impatience, *RTI*, which indicates how much less valuable consumption of future generations is simply because it occurs in the future, and the ethical judgement about the permissible levels of income inequality across generations. If living standards grow at a constant rate $g > 0$, future generations will be richer than current generations and the latter should be spared costly investments simply because they are poorer already. This effect is strong if the growth rate, g, and the coefficient of relative intergenerational inequality aversion, *IIA*, are large. The third term in the *SDR* corresponds to the prudence effect, which is large if the variance of expected future consumption growth, σ^2, the coefficient of relative risk aversion, *RRA*, and the coefficient of relative prudence, *PRU* (equal to 1+*IIA* for power utility functions) are high. The fourth term captures risk aversion with respect to uncertainty about future damages which are perfectly correlated with future GDP. We thus get the growth-corrected *SDR*:

$$(1) \qquad SDR - g = RTI + (IIA - 1) \times \tilde{g} \quad \text{with} \quad \tilde{g} \equiv g - \frac{1}{2} \times RRA \times \sigma^2,$$

where $\tilde{g}$ is the trend growth rate corrected for risk aversion with respect to uncertain future consumption growth and consequent damages from global warming.[4] Following Nordhaus (2007), we use baseline values for the RTI of 1.5% per annum and for the IIA and the RRA of 1.45. The baseline standard deviation of global annual consumption growth had a standard deviation of 3.6 percent over

[2] A 3-box model leads to a slightly better short-run temperature response (Gerlagh and Liski, 2015).
[3] Most IAMs assume damages as a fraction of GDP to be convex in temperature while temperature is usually concave (logarithmic) in atmospheric concentrations so that the overall effect gives a near linear relationship.
[4] Gollier (2013) uses $SDR = RTI + IIA \times g - 0.5\ RRA \times PRU \times \sigma^2$, where $PRU = 1+RRA$.

much of the past century and our baseline thus sets $\sigma = 0.036$, hence the growth correction to allow for prudence is only 0.1% per annum which depresses the SDR by a mere 0.14% per annum.

Simple Rule for the Global Price of Carbon

Taking the present value of the flow damages of what is left in the atmosphere at each future point of time and using (1) for the growth-corrected *SDR* gives our rule for the optimal price of carbon, *P*:

$$(2) \qquad P = \Omega \, d \, GDP \quad \text{with} \quad \Omega \cong \left(\frac{1}{1+(SDR-g)l} \right) \sum_{i=1}^{n} \left(\frac{a_i(1-b_i)}{SDR-g+b_i} \right).$$

This rule offers the following insights. First, the optimal global price of carbon is proportional to and rises with world GDP, about 66 trillion US dollars in 2010, and to the flow cost of global warming per ton of emitted carbon, *d*. Second, the global carbon price is high if the *SDR* is low, which from (1) is the case if welfare of future generations is not discounted much (low *RTI*) and, given trend GDP growth, intergenerational inequality aversion is weak (low *IIA*). Third, if *IIA* > 1, the ethical positive effect of higher trend growth (higher *g*) on the *SDR* via richer future generations dominates the negative effect on *SDR*–*g* due to faster growing damages of global warming. This depresses the optimal price of carbon. In contrast, more uncertainty about future consumption growth, especially if risk aversion is substantial (high *RRA)*, curbs the growth-corrected discount rate and boosts the price of carbon. Fourth, lower decay rates of atmospheric carbon (higher b_i) and a shorter temperature lag (low *l*) increase the carbon price due to longer-lasting and more immediate damages.

The Carbon Budget and Stranded Fossil Fuel Reserves

Recent studies quantify the amount of fossil fuel which must be abandoned in the crust of the earth for global warming to stay below 2ºC (McGlade and Ekins, 2015).[5] Underlying such estimates is the basic economic idea that fossil fuels are used as long as they are cheaper than renewable energy. The carbon price increases the cost of fossil energy and ensures that renewable energy is phased in earlier. To estimate the optimal carbon budget, we suppose that the cost of extracting one ton of carbon falls with time due to directed technical progress at the rate r_E (e.g., due to the recent innovation of horizontal drilling in fracking shale gas) and becomes more expensive over time as less accessible fields have to be explored, hence $E(t) = E_0(1-r_E)^t (S_0 / S(t))^{e_1}$ with $S(t)$ denoting reserves at time *t*. We calibrate the initial cost of fossil fuel extraction so that the energy sector is 5% of GDP, hence E_0 = 0.35 T\$/GtC, and set S_0 = 10,000 GtC, e_1 = 0.5 and r_E = 0. The cost of renewable energy *R* (including cost of infrastructure and feed-in subsidies to foster learning by doing) falls at the rate r_R due to directed technical progress, where we suppose that producing from carbon-free alternatives costs 5.6% of GDP today and 2.7% of GDP at the end of the century, hence R_0 = 0.8T\$/GtCe and r_R = 1% per annum until

[5] This study follows Meinshausen et al. (2009) by focusing on cumulative emissions up to 2050. This is misleading as peak global warming depends on cumulative emissions forever into the future (e.g., Allen et al., 2009; Allen, 2016).

the price of renewable energy reaches its lower floor of 0.4T$/GtCe near the end of the century (Nordhaus, 2014). At time T, $E(T)+P(T)=R_0(1-r_R)^T$ so from then onwards fossil fuel is too expensive and is priced out of the market. This transition condition gives the optimal cumulative emissions or the carbon budget for short, $B \equiv S_0 - S(T)$:

$$(3) \qquad B = S_0\left[1-\left(\frac{E_0(1-r_E)^T}{R_0(1-r)^T - \Omega\big(SDR,g\big)d(1+g)^T GDP_0}\right)^{\frac{1}{e_1}}\right].$$

The carbon budget is low if fossil fuel is expensive to extract and if renewable energy is cheap to produce (high E_0 and low R_0). The carbon budget is curbed by the ethical, economic and geo-physical factors that drive up the price of carbon (high $SDR-g$ or high Ω from (1) and (2), high d and high GDP_0). If fossil fuel extraction is expected to experience rapid directed technical progress, the optimal carbon budget will be higher. If renewable energy is expected to experience rapid technical progress (as is the case now for solar energy), the optimal carbon budget ends up lower as climate policy has become cheaper. The fraction of stranded fossil fuel assets, $S(T)/S_0 = (S_0 - B)/S_0$, is then high.

End of Carbon Era and Peak Warming

Empirical evidence suggests limited substitutability between energy and capital and labour (Hassler et al., 2013), so that fossil fuel use is $\gamma\, GDP$ with γ calibrated at 0.15 GtC per T$ of GDP.[6] The carbon era ends when the total carbon emitted, $B = \sum_{t=1}^{T}(1+g)^t \gamma GDP_0 = \left[(1+g)^{T+1}-1\right]\gamma GDP_0 / g,$ equals the carbon budget, B. This yields the optimal transition time to the carbon-free era:

$$(4) \qquad T = \frac{1}{g}\ln\left(1+g\frac{B}{\gamma GDP_0}\right)-1.$$

The transition thus occurs more quickly if the carbon budget B is small and expected economic growth g is high. Finally, there is a robust relationship between cumulative emissions and peak warming as the pathway of carbon emissions matters less than the cumulative emissions B (Allen et al., 2009):

$$(5) \qquad PW = \Phi(B), \quad \Phi' > 0, \quad \Phi'' \le 0.$$

Although a linear approximation to (5) works reasonable well (Allen, 2016), we use a quadratic which is slightly more accurate.[7] The carbon budget from pre-industrial times onwards corresponding to a maximum of 2°C is 1 TtC, implying a carbon budget for cumulative emissions of 350 GtC or 1283 $GtCO_2$ from 2010 onwards.

[6] It is easy to allow for a negative trend in the energy and carbon intensities of output.
[7] Allen (2016) suggests PW = 0.9 + 2 B/1000, which gives B equal to 550 GtC or 2017 $GtCO_2$. See Appendix A.

Putting It Together

Figure 1 solves (3) through (5) for the carbon budget B, the time T that the carbon-free era starts, and peak warming, both for the social optimum if carbon is priced (in blue) according to (1) and (2) and for the business as usual scenario (BAU, in brown) if climate policy is absent ($P = 0$).[8] In the bottom panel of figure 1, the carbon budget B falls through (3) as the price of carbon is imposed. This implies a quicker transition to renewable energy and lower peak warming (left axis). The top panel illustrates the underlying transitional dynamics in the energy market. As cumulative emissions increase, the cost of extracting the remaining stock fossil fuel increases. Simultaneously, the cost of renewable energy decreases. The top panel shows that the transition to the carbon-free era occurs when the cost of fossil fuel has just exceeded that of renewable energy. The difference between the climate policy (the blue line) and BAU (the dashed brown line) scenarios is the rising carbon tax as determined by rule (2).

From (2) we know that a smaller *SDR* (due to more patience, less intergenerational inequality aversion and, if *IIA* > 1, lower trend GDP growth and more risk aversion), a lower decay rate, and a shorter temperature lag push up the carbon price and shift up the blue line in the top panel of Figure 1. Renewable energy then becomes competitive earlier when energy cost is relatively higher, leaving a bigger fraction of fossil fuel reserves stranded, and cutting cumulative emissions and peak warming.

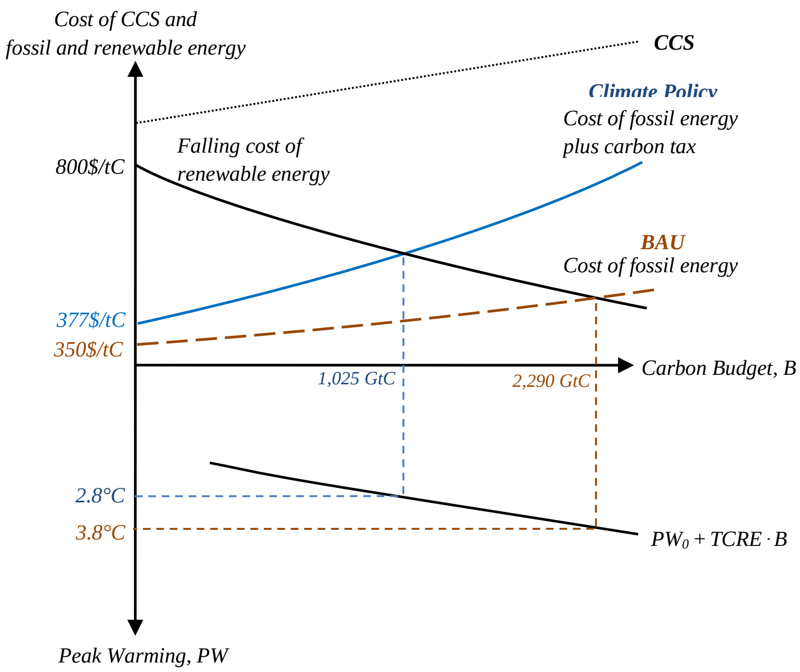

Figure 1: The rules (1)-(2) give the carbon budget and end of carbon era from (3)-(4) and peak warming from (5) in the bottom panel. The top panel illustrates the dynamics as fossil fuel extraction costs and the carbon price rise and the cost of renewable energy falls.

If *IIA* > 1, faster trend GDP growth makes future generations richer and decreases the ethical onus on current generations to curb emissions, especially if *IIA* is high, so the *SDR* rises and climate policy

[8] We need to solve the simultaneous equations (3) and (4) for B and T, which is done in the Excel sheet accompanying this paper. Only with no technical change whatsoever, $g = r_E = r_R = 0$, is the system recursive.

becomes less ambitious and temperature rises. A lower and falling cost of renewable energy shifts down the cost curve in the top panel (the solid black line), which curbs the carbon budget, brings forward the carbon-free era and cuts peak warming. The energy cost at the time of transition is now relatively lower.

Ethical
Rate of time impatience: RTI = 1.5%/year
Intergenerational inequality aversion and risk aversion: $IIA = RRA = 1.45$
Economic
World economy: $GDP0$ = 66 T\$, g = 2%/year, $\sigma = 0.036$,
Fossil fuel use per unit of world GDP: $\gamma = 1.5$ GtC/T\$,
Fossil fuel cost: $E_0 = 0.35$ T\$/GtC, $r_E = 0$, $e_1 = 0.5$,
Renewable energy cost: $R_0 = 0.8$ T\$/GtC, r_R = 1%/year
Flow damage as fraction of world GDP: $d = 0.053$ \$/tC
Geophysical
Initial stock of fossil fuel reserves: S_0 = 10,000 GtC
Coefficients permanent and transient box of carbon cycle: $a_1 = 0.2$, $b_1 = 0$, $a_2 = 0.32$, $b_2 = 0.0023$
Average lag between temperature/damages and carbon stock: l = 70 years
Transient climate response to cumulative emissions: $TCRE$ = 2 ºC/TtC

Table 1: Baseline calibration of the back-on-the-envelope IAM

Table 2 shows the results of our back-of-the-envelope calculation following the logic of Figure 1. Using our baseline calibration summarized in Table 1, 1,046 GtC is burnt and temperature peaks at 2.8°C. Table 2 also illustrates the impact of *ethical* and *economic* drivers on the optimal carbon budget and peak warming. Ethical considerations influence the transition to renewable energy through the optimal carbon price. As society's aversion to intergenerational inequality falls from 1.45 to 0.5, peak warming falls from 2.8ºC to 1.8ºC and the carbon budget from 1,046 to 264 GtC. Similarly, not discounting welfare of future generations at all, i.e., RTI = 0, cuts peak warming to 2.1°C and the carbon budget to 465 GtC.

Scenario	*PW*	*Carbon Budget*
Baseline	2.8°C	1,046 GtC
Ethical		
Lower inequality aversion (IIA = 0.5)	1.8°C	264 GtC
Lower discount rate (RTI = 0%/yr)	2.1°C	465 GtC
Economic		
Lower economic growth rate (g = 1%/yr)	2.6°C	836 GtC
High Damage scenario (d = 0.08)	2.6°C	885 GtC
Lower initial cost of renewable energy (R_0 = 0.64 \$T/GtC)	2.3°C	642 GtC
Lower initial cost of fossil energy (E_0 = 0.28 \$T/GtC)	3.2°C	1,419 GtC
Faster reductions in cost of renewable energy (r_r = 2%/yr)	2.1°C	474 GtC

Table 2: Sensitivity of peak warming and carbon budget to ethical and economic drivers

If global economic growth slows from 2% to 1% per annum, future material affluence will be lower and the initial carbon price rises in order to shield future generations from climate-related damages to their weakened economy. Peak warming declines to 2.6°C corresponding to a fall of the carbon budget to 836 GtC. Higher vulnerability of the economy to climate change in the form of higher damages has a similar effect on peak warming and the carbon budget. A 20% reduction in today's cost of renewable (fossil) energy significantly expedites (delays) the transition to the carbon-free era and decreases (increases) peak warming to 2.3°C (3.2°C) and the carbon budget to 642 GtC (1,419 GtC). An acceleration of directed technical change in the renewable sector from 1 to 2% per annum, brings forward the energy transition and cuts peak warming to 2.1°C and the carbon budget to 473 GtC. The model can be used to calculate the impacts of shifts in the *geophysical* components in a similar fashion.

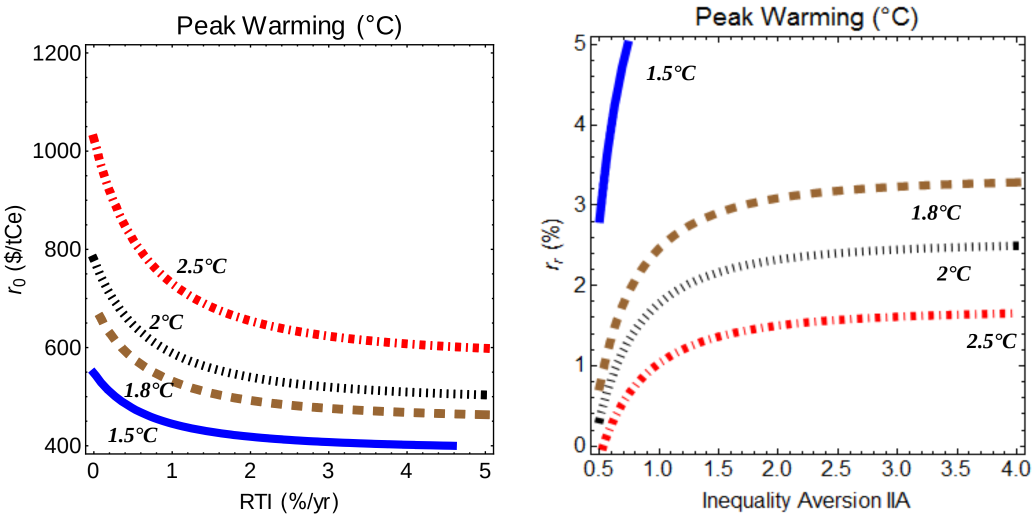

Figure 2: Economic vs. ethical drivers of peak global warming

To illustrate the trade-offs between ethical and economic drivers, Figure 2 shows contours of given peak warming levels for different economic and ethical parameters. The left panel of Figure 2 shows how the discount rate and the initial cost of renewable energy affect the optimal degree of peak global warming. Lower peak warming requires shifting closer to the origin, i.e., lower levels of discounting and initial costs. It also illustrates how given an ethical parameter, peak warming can be brought down by subsidies to renewable energy. The right panel of Figure 2 shows how optimal degrees of peak global warming can be achieved by different levels of *IIA* and the rate of directed technical change. Peak warming can be lowered by moving to the upper-left corner: lower preparedness to sacrifice utility to cut future global warming (higher *IIA*) requires more technical progress (higher r_R) through policies stimulating innovation and R&D. Both panels illustrate how big the challenge and how ambitious policies must be to stay well below 2°C peak warming as agreed in Paris.

Implementing the 2°C Target

At the 2015 Paris Summit it has been agreed to limit global warming well below 2°C and to drive efforts to limit the temperature increase to even further 1.5°C. This is lower than justified by the damages of global warming estimated by economists and used by us. An extra safety margin may be justified to curb risks of tipping points and run-away global warming. A 2°C upper bound tells us from (5) or the bottom panel of figure 1 that the associated carbon budget should be on average 411 GtC instead of 1000 GtC. This requires from (3) a higher carbon price: at the transition to the carbon-free era it should be 230$/tC for most scenarios. We back out from (4) that the transition to the carbon-free era is further brought forward from 56 to 30 years. These results are confirmed by simulations for the optimal baseline and the 2°C upper bound scenarios in our full-scale IAM as shown in Figure 3.

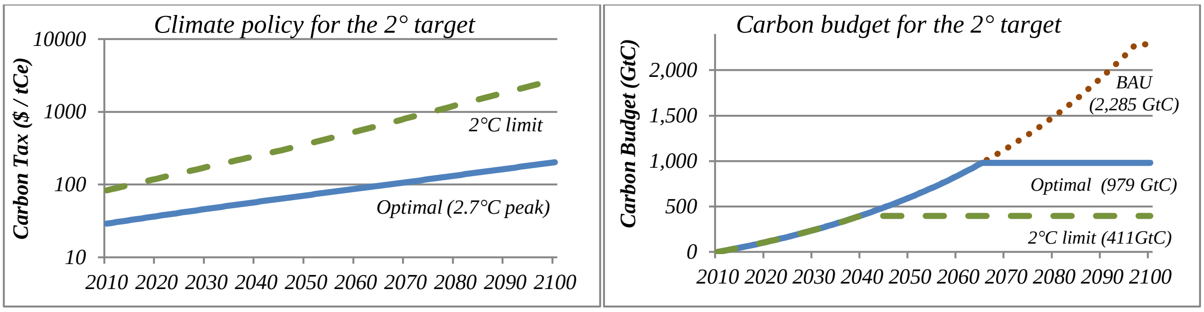

Figure 3: The 2°C upper bound pushes up the required carbon price and requires steeper growth as the unconstrained price yields too high temperature rises *(left)*. Fossil fuel emissions grow at the rate of economic growth until renewable energy is cheaper. The carbon tax brings forward the carbon-free era and leaves a bigger fraction of stranded assets *(right)*.

Performance of the Simple Rules

Most integrated assessment modellers make many additional, and perhaps more realistic, assumptions about economic damages of climate change or dynamics of specific energy sources, and crunch out the optimal price of carbon (and sometimes also the stock of stranded fossil fuel assets) by numerically maximizing welfare subject to the constraints of their large-scale IAM. The question is how well our simple rules (1) through (5) perform and compare with the optimal time path of carbon prices, carbon budgets and transition times derived from more complex optimizing IAMs. Table 3 therefore compares them with the globally optimal discretionary outcomes and the BAU outcomes for our IAM with DICE damages and Oxford carbon and temperature dynamics.[9]

	Parameters				*Optimal carbon budget*		*Peak temperature*		*Initial price*	
Scenario	*RTI*	*IIA*	*g*	*SDR*	full IAM	toy IAM	full IAM	toy IAM	full IAM	Toy IAM
Conventional (2x2x2)	2%	2	2%	6 %	1,307GtC	1,293 GtC	3.06°C	3.07°C	20 $/tC	12 $/tC
Baseline (Nordhaus)	1.5%	1.45	2%	4.4 %	979 GtC	1,025 GtC	2.73°C	2.78°C	29 $/tC	27 $/tC
Lower discounting	0.1%	1.45	2%	3 %	569 GtC	525 GtC	2.24°C	2.16°C	81 $/tC	95 $/tC
Lower IIA	1.5%	1	2%	3.5 %	759 GtC	748 GtC	2.47°C	2.45°C	51 $/tC	55 $/tC
Lower trend growth	1.5%	1.45	1%	3 %	805 GtC	820 GtC	2.53°C	2.54°C	37 $/tC	38 $/tC
Business-As-Usual	full IAM Cumulative emissions: 2250 - 2300 GtC Peak temperature: 3.89°C toy IAM Cumulative emissions: 2230 - 2275 GtC Peak temperature: 3.84°C									

Table 3: The simple rules predict the fully-fledged IAM outcomes in terms of cumulative emissions and peak temperature well. They also predict the deleterious effects of policy inaction.

[9] With uncertainty about future consumption growth, the *SDR* in the baseline is cut by 0.14% per annum. This curbs the carbon budget and peak global warming by only 36 GtC and 0.04°C, respectively.

Our toy IAM performs remarkably well, despite being based on a simple 2-box carbon cycle, and adapts accurately to changes in ethical judgement and technological progress. Cumulative fossil use differs by at most 8%, peak temperature by 0.08°C, and the initial optimal carbon price by 14 \$/tC.[10] The transition times are also predicted well by the simple rules (e.g., for the baseline 55 and 56 years for the IAM and for the simple rule, respectively. The social optimum avoids the peak temperatures of around 3.9°C by locking up much more fossil fuel than the average 2,250 GtC burnt under BAU and by transitioning to the carbon-free era in 56 years instead of the end of the century.

Carbon Capture and Sequestration as Backstop

Allen et al. (2009) have argued that there should be a mandate that ensures all carbon emissions above the budget compatible with 2°C global warming should be captured and sequestrated. Allen (2016) further argued that considerations for the near-term mitigation efforts induced by pricing carbon should be disregarded for long-term impacts of the carbon price for sequestration efforts once the optimal carbon budget will have been reached. Although this plea resonates, a simple cap is not necessarily an efficient strategy. It is more efficient to price carbon as this offers a direct incentive to capture and sequester carbon (as well as to make renewable energy more attractive to use and develop and to phase out fossil fuel more quickly). Furthermore, a price for carbon allows trading to promote the least costly cuts in carbon emissions. It avoids the government "picking winners" and instead promotes development of a wide variety of renewable energy sources including carbon capture and sequestration (CCS). This is important as CCS will, like many other potential new sources of renewable energy, be at most a partial solution to the climate challenge. But CCS faces particular challenges: huge capital investments, environmental hazards and ugly NIMBY politics. Also, as CCS requires lots of space it is difficult to scale up as costs rise as space is used up. Abatement with CCS (e.g., new coal, coal retrofit, industrial) is still one of the more expensive forms of abatement. Analytically, the cost of fossil fuel with CCS equals $E(t) = E_0(1-r_E)^t \left(S_0 / S(t)\right)^{e_1}$ per ton of carbon plus the marginal cost of abating one ton of carbon, say *A*. This is indicated by the upward-sloping dotted black line in Figure 1. It follows that, like fossil fuel on its own, the cost of CCS rises as reserves are depleted. It thus only becomes attractive for the market when the marginal cost of abatement falls below the carbon price. This happens once GDP and the carbon price have risen far enough or when new technology has diminished the cost of CCS sufficiently, but working against this is that once CCS is scaled up space becomes ever more costly. Given these cost developments, CCS is likely to be dominated by various forms of renewable energy in the market. Forcing it on the market by mandating it is thus an inefficient way to achieve climate objectives and one would hope one does not have to resort to this once it is too late to rely on conventional, more cost-effective climate policies to curb emissions.

[10] Allen (2016) proposes an initial carbon tax of 91 \$/tC (25 \$/tCO_2) in a framework with $SDR - g$ = 1.5%/yr and where marginal damages and the carbon tax (2) are linear in GDP.

Conclusion

Our assessment of how the optimal carbon price and stranded assets interact with economic growth, renewable energy technology, fossil fuel scarcity, ethical considerations, and fundamental geophysical parameters is transparent and gives easy-to-understand simple rules that perform well in large-scale IAMs. These rules are transparent and robust and in this sense more useful than a discretionary time path of optimal climate policies usually obtained from IAMs. We hope that our back-on-the-envelope framework allows climate scientists not actively engaged in economic modelling to understand the critical assumptions driving the social cost and price of carbon, untapped fossil fuel and the time to reach the carbon-free era in terms of ethical considerations and expected economic growth and cost reductions in renewable energy. Our results suggest that the global warming damages estimated and ethical assumptions chosen by economists are likely to lead to global warming that exceeds the 2°C target. To ensure that global warming always stays below 2°C, the carbon price must be raised above what conventional economic damages tell us to do and more fossil fuel must be locked up. Recent estimates of the non-climate related health benefits of abandoning fossil fuels (e.g. Parry et al., 2014; Thompson et al., 2014; West et al., 2013), the effects of uncertainty about the steepness of climate damages (Crost and Traeger, 2014) and the potential of multiple abrupt disruptions in the climate system (Cai et al., 2016; Lemoine and Traeger, 2016) provide ample reasons for raising the carbon price.

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Cumulative Emissions and Peak Warming

Allen et al. (2009) present a reduced-form model of the climate and argue that cumulative emissions in all future years – rather than the actual emissions profile – matters for peak warming. Figure A.1 presents the relationship between peak temperature and this carbon budget in TtC for over 500 simulations in the optimizing variant of our IAM. The quadratic approximation used in our toy IAM is robust and very well determined. The DICE model displays a similar relationship between peak warming and cumulative emissions.

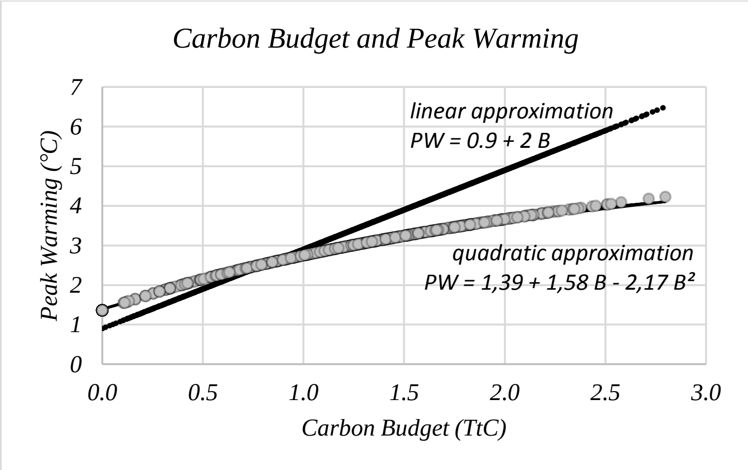

Figure A.1: A robust relationship between the carbon budget and peak warming for the Oxford system based on 500+ simulations in our optimizing IAM.

Allen (2016) states that a global temperature increase in line with cumulative CO_2 emissions suggests the following simple expression for the temperature response to a pulse emission of an additional ton of CO_2 at time *t*: $\delta Temp_{t+t'} = TCRE \times (1 - e^{-KS \times t'})$, where $1/KS$ is the initial pulse-adjustment time scale of the climate system (of order decade or less) and *TCRE* is the approximately constant transient climate response to cumulative emissions. This is valid for cumulative emissions up to 5,000 GtC and gives the linear approximation $PW \cong 0.9^0\mathrm{C} + TCRE \times B,$ where Allen (2016) uses an initial temperature of and a mid-range value of $TCRE = 2°\mathrm{C/TtC}.$ Although this linear approximation performs reasonable well for low levels of peak warming (see plot in Figure A.1), we use a quadratic approximation which is calibrated to the carbon cycle of the full IAM both in terms of the initial temperature intercept and the slope.

Parameter Combinations for 2°C

Working backward form a peak warming target of 2°C, we can use (5) to obtain a carbon budget of 411 GtC. We can then use our rules for the carbon tax (1) and (2), and the carbon budget (3) to find out which parameter combinations would make the 2°C target feasible. Table A.2 collects the pairs of (*g*, *SDR*) consistent with 2°C of warming. These can be achieved through combinations of the rate of time impatience (*RTI*) and the coefficient of relative intergenerational inequality aversion (*IIA*). Higher levels of growth permit a higher *SDR*. Since (ignoring the prudence term) $SDR = RTI + IIA \times g,$ the permissible values for *RTI* are higher and the values for *IIA* lower.

For example, with an economic growth rate of 2% per annum, the *RTI* would have to be between 0%–0.8% with a corresponding aversion parameter of 1.4–1. If the trend growth rate rises to 3 percent per annum, the *RTI* has to be between 0%–0.9% with a corresponding aversion parameter of 1.3–1. The reason for limiting *IIA* is that as economic growth rises, future generations are deemed rich enough to handle slightly higher temperature increases (in the absence of abrupt climate change). To offset this effect and maintain the 2°C target, the permissible parameter range for *IIA* is lowered as the trend economic growth rate increases.

RTI	*IIA*	*g*	*SDR*
0 %	1.7	1%	1.7 %
0.25 %	1.45	1%	1.7 %
0.7 %	1	1%	1.7 %
0 %	1.4	2%	2.8 %
0.8 %	1	2%	2.8 %
0 %	1.3	3%	3.9 %
0.9 %	1	3%	3.9 %

Table A.2: Parameter combinations that achieve the target of 2°C ignoring prudence. Higher trend growth requires less patience and a lower rate of intergenerational inequality aversion.

At the time of the switch to the carbon-free era, 2038, the carbon price has to be 250 $/tC in the 2°C scenario. The baseline carbon price is then 147.5 $/tC. Lowering the utility discount rate, keeping *IIA* at 1.45, to zero only raises the carbon price to 108 $/tC so that global warming still exceeds 2°C. If the *RTI* is cut to zero and the *IIA* to 1.4 or the *RTI* is cut to 0.8% and *IIA* to 1, the target of 2°C is compatible with DICE damages.

Description of the optimizing IAM

The economic core of our optimizing IAM is presented in Rezai and van der Ploeg (2016). However, here we use the Oxford carbon cycle (e.g., Allen, et al., 2009) instead of the carbon cycle of DICE or of Golosov et al. (2014). We use our IAM instead of DICE, since we have scarcity rents on fossil fuel and allow extraction costs of fossil fuel to rise as reserves get depleted in order to solve for the optimal amount of stranded assets. The economic part of our IAM is calibrated to data for 2010: world GDP is 66 trillion US $, the initial capital stock is 150 trillion US $ and initial energy use is 9.94 GtCe. The world population is 7 billion initially and is assumed to stabilize at 11 billion at the end of the century. We assume a depreciation rate for capital of 10% per annum and a Cobb-Douglas technology with 30% and 70% as the shares of capital and labor, respectively. We assume that for each trillion of output that is produced $\gamma = 0.150$ GtC of fossil fuel is needed, which is in line with a Leontief technology. The initial cost of renewable energy $b(0)$ is initially $800/tCe. The rate of technical progress in renewable energy is 1% per annum until the price of renewable energy reaches its lower floor of $400/tCe near the end of the century. The cost function for oil extraction has $350/tC $(e_0 = 0.35)$ which gives the share of energy in output of about 5%. Extraction costs evolve with $e_1 = 0.5$ and the initial stock of fossil fuel reserves is 10,000 GtC. This means that initially renewable energy is more than twice as expensive as fossil energy initially but renewable energy has the potential to reach current energy prices after all learning has happened.

Our optimizing IAM has an initial phase where only fossil fuel is used in the production process, an intermediate phase where fossil fuel and renewable energy are used alongside each other, and a final phase where only renewable energy is used. The economic block of our IAM consists of a capital accumulation equation with an associated Euler equation for the optimal expected growth in consumption per capita and a depletion equation for fossil fuel reserves with an associated modified Hotelling rule describing the scarcity rent as the present value of all future reductions in marginal extraction costs of fossil fuel resulting from burning one ton of carbon today. The Oxford carbon cycle consists relative to the DICE and the 2-box carbon cycle of Golosov et al. (2014) of a relatively large number of dynamic equations (i.e., 7) describing the stocks of carbon in the atmosphere and the oceans as well as global temperature. These carbon cycle difference equations have an associated number of difference equations for the co-states, which generate the social cost of carbon. Our full optimizing IAM thus consists of 9 difference equations for the states and another 9 for the co-states. Our solution algorithm for this 18-dimensional two-point-boundary-value-problem solves our IAM in a forward-looking manner such that the transversality conditions are satisfied.

Finally, it should be acknowledged that recently IAMs have been criticized for their lack of proper underpinnings (e.g., Pindyck, 2013). In particular, it is difficult to assess the costs and benefits of cutting global warming many decades and centuries ahead, especially when it comes to costs of global warming, the climate sensitivity and key ethical parameters. In defence, it has been argued that using an IAM is better than no IAM and that there is no reliable alternative to calculate the social cost of carbon (e.g., Metcalf and Stock, 2015). Although IAMs are useful in policy debates, we believe their results are difficult to communicate and comprehend. This is the main reason why we advocate the use of transparent and simple framework and an easy-to-understand rule. This ensures that the communication to policy makers is easier whilst not suggesting more scientific precision than is warranted (cf. Pindyck, 2015).

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