Optimal Incentive Contract with Costly and Flexible Monitoring

Anqi Li [1] Ming Yang [2]

[1]Department of Economics, Washington University in St. Louis

[2]Fuqua School of Business, Duke University

January 2016

Motivation

Choice of monitoring technology has significant impact on employee productivity.

Standard agency models take the monitoring technology as exogenously given.

Need strong assumptions to justify

1. Simple and intuitive contracts;
2. Heterogeneity in managerial practices.

Preview

A principal-agent model with flexible and costly monitoring:

- **Flexibility**: specify the qualitative and quantitative natures of the monitoring technology;
- **Cost**: increasing in the entropy of the agent's compensation.

Endogenize the choice of monitoring technology as part of the contract design problem.

Use factors that affect the monitoring cost to explain

- Simple and intuitive contracts;
- Heterogeneity in human resource practices.

Agenda

1. Baseline model
2. Extensions
3. Conclusion

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Setup

A risk-neutral principal and a risk-averse agent.

Agent payoff $u(w) - c(a)$:

- Consumption $w \geq 0$, $u(0) = 0, u' > 0, u'' < 0$;
- Effort $a \in \{0, 1\}$, $c(1) = c > c(0) = 0$.

Each effort level a generates a probability space (Ω, Σ, P_a).

Principal's goal: elicit high effort from the agent.

Incentive Contract

A pair of monitoring technology $\mathcal{P}$ and wage scheme $w(\cdot)$:

1. $\mathcal{P}$: a partition of Ω whose elements belong to Σ;
2. $w : \mathcal{P} \to \mathbb{R}_+$.

Timeline:

- Parties commit to $\langle \mathcal{P}, w(\cdot) \rangle$;
- The agent privately exerts $a \in \{0, 1\}$;
- Nature draws $\omega \in \Omega$ according to P_a;
- $A(\omega) \in \mathcal{P}$ is publicly realized;
- The principal pays the promised wage $w(A(\omega))$.

Incentive Contract (Cont.)

The contract defines a signal X and a random wage W.

For each effort level a and $A \in \mathcal{P}$:

- X takes value A with prob. $P_a(\omega \in A)$;
- W equals $w(A)$ with prob. $P_a(\omega \in A)$.

Monitoring Cost and Total Cost

Monitoring cost for each given a:

$$\mu \cdot H_a(W)$$

1. $H_a(W)$: entropy of the random wage.
2. $\mu > 0$: cost and benefit of monitoring the agent.

Total cost for each given a:

$$\underbrace{\mathbb{E}_a[W]}_{\text{incentive cost}} + \underbrace{\mu \cdot H_a(W)}_{\text{monitoring cost}}$$

Detect Deviation

For each $A \in \Sigma$, define

$$z(A) = 1 - \underbrace{\frac{dP_0}{dP_1}(A)}_{\text{likelihood ratio}}$$

A contract is incentive compatible for the agent if

$$\int_{A \in \mathcal{P}} u(w(A))z(A)dP_1 \geq c$$

Optimal Incentive Contract

The optimal incentive contract $\langle \mathcal{P}^*, w^*(\cdot) \rangle$ solves

$$\min_{\langle \mathcal{P}, w(\cdot) \rangle} \mathbb{E}_1[W] + \mu \cdot H_1(W)$$
$$\text{s.t.} \quad \text{(IC) and (LL)}$$

Benchmark: Exogenous Monitoring Technology

Standard agency models take $\mathcal{P}$ as exogenously given and solve for

$$\min_{w:\mathcal{P}\to\mathbb{R}_+} \mathbb{E}_1[W], \text{ s.t. (IC) and (LL)}$$

Denote the solution by $w^*(\cdot;\mathcal{P})$.

Lemma 1.

For any given $\mathcal{P}$, there exists $\lambda > 0$ such that for each $A \in \mathcal{P}$, $u'(w^*(A;\mathcal{P})) = \frac{1}{\lambda z(A)}$ if and only if $w^*(A;\mathcal{P}) > 0$.

Increasing Wage Scheme and MLRP

Definition 1.

Suppose $\mathcal{P}$ is totally ordered under $\preceq$. Then the distributions of the signal induced by $\mathcal{P}$ satisfy the monotone likelihood ratio property if any $A, A' \in \mathcal{P}$ such that $A \preceq A'$, we have $z(A) < z(A')$.

Lemma 2.

Suppose $\mathcal{P}$ is totally ordered under $\preceq$. Then $w^*(\cdot;\mathcal{P})$ is increasing if and only if the distributions of the signal induced by $\mathcal{P}$ satisfy MLRP.

Why May MLRP Fail?

For an arbitrary monitoring technology,

1. $\mathcal{P}$ may not be totally ordered, e.g., multi-source feedback;
2. Even if $\mathcal{P}$ is totally ordered, MLRP is still a strong property.

Optimal Contract with Costly and Flexible Monitoring

Theorem 1.

For any $\mu > 0$,

(i) $\mathcal{P}^* = \{A_1, A_2, \cdots, A_n\}$ *for some* $n \in \mathbb{N}$;

(ii) $z(A_1) < z(A_2) < \cdots < z(A_n)$;

(iii) $w^*(A_1) = 0 < w^*(A_2) < \cdots < w^*(A_n)$.

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    - Multi-task
    - Multi-agent
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Multiple Tasks

A risk-neutral principal and a risk-averse agent.

The agent can exert $a_i \in \{0, 1\}$ in each of two tasks $i = 1, 2$.

Each effort profile $\vec{a} \in \{0, 1\}^2$ generates $(\Omega, \Sigma, P_{\vec{a}})$.

Principal's goal: elicit high effort in both tasks.

Detect Deviation

For each $A \in \Sigma$ and each $\vec{a} \in \{10, 01, 00\}$, define

$$z_{\vec{a}}(A) = 1 - \frac{dP_{\vec{a}}(A)}{dP_{11}(A)}$$

A contract is incentive compatible for the agent if for each $\vec{a} \in \{10, 01, 00\}$,

$$\int_{A \in \mathcal{P}} u(w(A)) z_{\vec{a}}(A) dP_{11} \geq c(11) - c(\vec{a})$$

Optimal Multi-Task Contract with Costly and Flexible Monitoring

Theorem 2.

For each $\mu > 0$,

(i) $\mathcal{P}^* = \{A_1, \cdots, A_n\}$;

(ii) $w^*(A_1) = 0 < w^*(A_2) < \cdots < w^*(A_n)$;

(iii) *There exist* $\lambda_{\vec{a}}$, $\vec{a} \in \{10, 01, 00\}$, *such that for all* $k = 2, \cdots, n$,

$$u'(w^*(A_k)) = \frac{1}{\sum_{\vec{a}} \lambda_{\vec{a}} z_{\vec{a}}(A_k)}$$

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Multiple Agents

A risk-neutral principal and two risk-averse agents $i = 1, 2$.

Each agent i exerts $a_i \in \{0, 1\}$.

Each a_i independently generates $(\Omega, \Sigma, P_{a_i})$, where

- $\Omega = \{0, 1\}$, $\Sigma = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$;
- $P_1(1) = p \in (0, 1)$ and $1 - \frac{dP_0(1)}{dP_1(1)} = z \in (0, 1)$.

Each $\vec{a} = (a_1, a_2)$ generates $(\Omega \times \Omega, \Sigma \otimes \Sigma, P_{a_1} \times P_{a_2})$.

Incentive Contract

Principal's goal: elicit high effort from both agents.

A monitoring technology $\mathcal{P}$ and a wage scheme $\vec{w}(\cdot)$:

1. $\mathcal{P}$: a partition of $\Omega \times \Omega$ whose elements belong to $\Sigma \otimes \Sigma$;
2. $\vec{w} : \mathcal{P} \to \mathbb{R}^2_+$.

Individual Reward

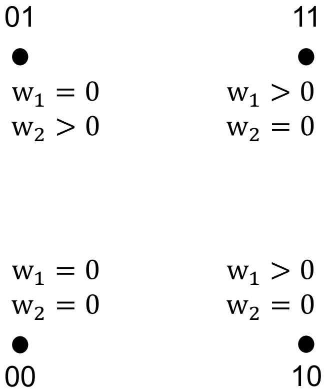

Figure: Γ_4

Tournament

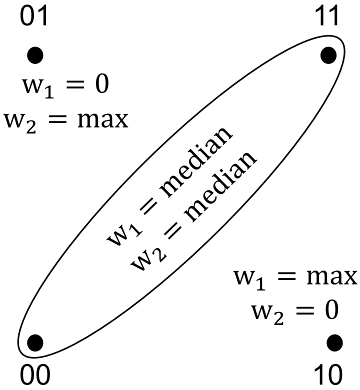

Figure: Γ_{3b}

Group Compensation

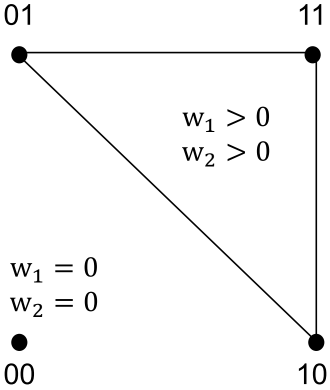

Figure: Γ_{2a}

Group Compensation

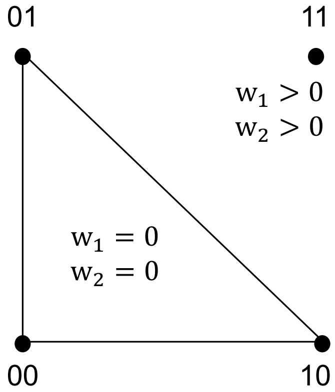

Figure: Γ_{2b}

Optimal Multi-Agent Contract

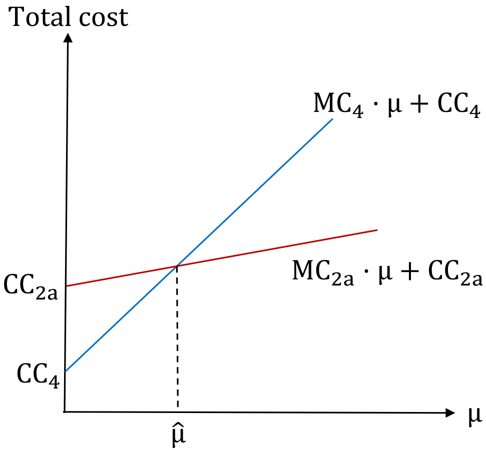

Figure: Individual reward vs. group compensation

Result

1. Difference in μ yields various kinds of incentive schemes.
2. Lack of individual performance appraisal when μ is big.

Explain variation in managerial practices by factors that affect μ:

- **Cost**: information technology, labor market regulation, tacit knowledge transfer;
- **Benefit**: human capital share, product market competition.

Conclusion

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Endogenize the choice of monitoring technology.

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