# Investor Protection and Asset Prices*

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## Abstract

There is substantial empirical evidence that investor protection affects stock returns, volatilities and interest rates. We develop a dynamic asset pricing model to shed light on the empirical regularities and underlying mechanisms at play. Our model features a controlling shareholder who can divert a fraction of the firm's output. The controlling shareholder's power over the firm is endogenous and interacts with investor protection in determining the level of expropriation. In equilibrium, imperfect investor protection implies higher stock holdings by controlling shareholders, lower stock returns, higher stock return volatilities and lower interest rates.

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# 1. Introduction

The protection of minority shareholders against expropriation by controlling shareholders is argued to be an important economic factor affecting asset price dynamics. In particular, the empirical literature provides ample evidence that the level of investor protection has economically significant effects on stock mean-returns, stock return volatilities and interest rates, as elaborated below. However, there is continuing discussion on the direction of these effects and the economic mechanism through which investor protection influences asset prices. The objective of this paper is to shed light on the effects of investor protection on asset prices and to provide theoretical guidance on the direction of these effects. Our paper is the first to incorporate investor protection into a dynamic asset pricing model with endogenous accumulation of control power by controlling shareholders (often taken as exogenous in the extant literature) and expropriation, which allows us to address the empirical regularities in asset return dynamics and to provide new predictions.

We consider a dynamic general equilibrium economy with a representative competitive firm that produces an exogenous stream of output. The firm's stock is owned by two types of shareholders with identical constant relative risk aversion (CRRA) preferences, a minority shareholder and a controlling shareholder who can divert a fraction of the firms output for himself. The diverted fraction is constrained by investor protection in the economy. The investor protection constraint limits the scope of available diversion strategies, and becomes tighter with better protection and looser with increased stock holdings by the controlling shareholder. The diversion of output is further tempered by non-pecuniary costs of stealing imposed by social norms (e.g., Kahneman, Knetsch and Thaler, 1986), which reduce the controlling shareholder's utility.

We provide tractable expressions for the equilibrium processes that preserve the structure of their counterparts in the full protection benchmark economy, familiar from the asset pricing literature. However, these expressions additionally incorporate new terms that depend on the controlling shareholder's stake in the firm and quantify the effects of investor protection. Our expressions are explicit up to the controlling shareholder's stock holding, which solves a fixed point problem. The fixed-point problem arises because, on one hand, the controlling shareholder's stake in the firm affects the equilibrium processes via the fraction of diverted output, and on the other hand, the stock holding is itself influenced by the equilibrium processes. We derive the dynamic equilibrium in terms of the minority shareholder's share in aggregate consumption, which emerges as an endogenous time-varying state variable affecting the relation between investor protection and asset price dynamics by determining whether investor protection constraint binds or not. In general, the effects of investor protection tend to be stronger when the controlling shareholder has a low share of the aggregate consumption and the investor protection constraint binds.

The endogenous accumulation of control power by the controlling shareholder makes the fraction of diverted output a hump-shaped function of the shares held by the controlling shareholder. On one hand, a higher stock holding relaxes the investor protection constraint and allows the controlling shareholder to divert more, but on the other hand decreases his incentive to divert. After some point, the investor protection constraint no longer binds and the equilibrium amount of diverted output decreases with the stock holding.

We find that the controlling shareholder's stock holding is larger in economies with imperfect investor protection than in economies with full protection, consistent with the empirical evidence that ownership concentration is higher in economies with poor protection (e.g., La Porta et al, 1999). Intuitively, poor protection expands the set of diversion strategies and increases the potential gains from higher control power over the firm, which induces buying more shares. We show that the acquisition of shares is financed by leverage, and the leverage-stock price ratio is simply given by the controlling shareholders stock holding over and above his holding in the full protection economy. However, the relationship between investor protection and optimal stock holdings is non-monotonic and depends on whether the investor protection constraint binds or not. This is because investor protection has two opposing effects on the controlling shareholder's optimal portfolio decision. An increase in investor protection reduces the marginal benefit of control and hence reduces the incentive to acquire more shares, while on the other hand, makes the investor protection constraint relevant for a wider range of stock holdings thereby providing an incentive to acquire more shares to relax the constraint. The relative importance of these effects depends on the consumption share of the minority shareholder.

We demonstrate that the stock mean-return decreases with poor investor protection in equilibrium. This is consistent with the empirical evidence on the relation between the realized stock returns and the degree of corporate governance, entrenchment and managerial perks (e.g., Gompers, Ishii and Metrick, 2003; Bebchuk, Cohen and Ferrell, 2009; Yermack, 2006, among others), although there is an ongoing discussion of the robustness of this relation (Core, Guay and Rusticus, 2006; Giroud and Mueller, 2011; Bebchuk, Cohen and Wang, 2013). We contribute to this discussion by providing a theoretical argument in favor of the positive relation between the stock mean-return and investor protection. Our intuition is that, in contrast to the minority shareholder, the controlling shareholder is compensated for holding risky assets not only by the risk premium but also by the fraction of the diverted output. Therefore, the controlling shareholder hoards shares even if the realized risk premium is low, which drives down the stock mean-return in equilibrium.

We also show that the equilibrium stock return volatility is higher with imperfect protection than with full protection and exceeds the volatility of the aggregate output. However, across economies with varying imperfect protection, the relation between volatility and investor protection is non-monotone, and in certain regions of the state-space the volatility is higher in economies with better protection, consistent with the empirical evidence that the volatility is higher in more developed economies with better protection (e.g., Morck, Yeung and Yu, 2000; Jin and Myers, 2006; Bartram, Brown and Stulz, 2012). We find that excess volatility is proportional to the leverage-stock price ratio, and to the best of our knowledge, such a simple characterization of volatility in terms of leverage is new to the literature. Intuitively, leverage finances the acquisition of shares by the controlling shareholder when protection is low, and hence increases the sensitivity of the controlling shareholder's wealth to economic shocks, which translates into higher stock return volatility via the state variable that tracks wealth inequality. The non-monotonicity of volatilities with respect to investor protection is explained by the non-monotonicity of the stock holdings, which determine leverage, as discussed earlier.

Furthermore, we find that the risk-free interest rates decrease with lower protection due to the following two effects in equilibrium. First, because of low risk premium and high volatility, the minority shareholder turns to bond markets and is more willing to provide cheap credit. Second, the acquisition of shares by the controlling shareholder is partially covered by the diverted output, which moderates his demand for credit. These two effects are partially offset by the surge of leverage under poor protection, which increases the demand for borrowing, but the net effect on the equilibrium interest rate is negative. Our result may help explain the empirical evidence that the borrowing rates tend to increase with better protection and governance (Klock, Mansi and Maxwell, 2005; Cremers, Nair and Wei, 2007), however it is important to note that our predictions are about risk-free interest rates.

Finally, we study the effect of the non-pecuniary costs of stealing on equilibrium. Various norms in a society promote fairness, honesty and morality, which may result in disutility from stealing (Kahneman, Knetsch and Thaler, 1986). Consequently, the controlling shareholder may not only be limited by laws and regulations that protect minority shareholders but also by social norms that promote fairness. An additional advantage of this approach is that it does not rely on extreme monetary punishment, which is not observed in real life because of budget constraints and difficulties in implementation (Shleifer and Wolfenzon, 2002). We find that a higher non-pecuniary cost of stealing is associated with higher stock gross return and interest rate, and lower volatility and stock holding of the controlling shareholder. In contrast to the effect of investor protection, the effect of non-pecuniary cost on asset price dynamics is higher when minority investors consumption share is low, i.e., when the controlling shareholder has a high stake in the firm and the investor protection constraint does not bind. There is also a novel interaction between investor protection and non-pecuniary costs of stealing in that a lower non-pecuniary cost of stealing expands the region where investor protection constraint binds.

Our paper makes a methodological contribution by integrating a model with corporate frictions into a general equilibrium asset pricing framework. Solving models with fictions such as non-pecuniary costs and constraints on certain choice variables can be a daunting task. We achieve tractability by allowing investors to optimize two-period CRRA preferences repeated over time, similar to models with myopic investors and overlapping generations. This approach allows us to focus on the effects of investor protection and abstract away from hedging demands for stocks, which are more relevant for the portfolio choice literature. However, we demonstrate that the equilibrium processes in the full protection benchmark economy are similar to those in standard dynamic Lucas-type (1978) economies. The consistency with standard models is achieved by keeping the shareholders risk-averse, which gives rise to familiar expressions for Sharpe ratios and interest rates.

Our paper is related to the scant theoretical literature on equilibrium implications of expropriation by controlling shareholders. Shleifer and Wolfenzon (2002) introduce a model that explains why firms are larger and more valuable with better investor protection. However, the static nature of their model and the risk-neutrality of agents preclude investigating the implications of investor protection on stock returns and volatilities.

Albuquerque and Wang (2008) introduce a production dynamic equilibrium model with buy-and-hold controlling shareholders. They demonstrate that weaker investor protection implies over-investment, higher equity mean-returns and interest rates. The equilibrium equity premia, interest rates and volatilities are constant due to the absence of trading between shareholders. The effects of investor protection on asset prices are introduced by the investment and production decisions of controlling shareholders. Our paper complements their work by focusing on different aspects of investor protection arising due to its effect on asset demands and accumulation of control. Consequently, we uncover new economic forces, absent in Albuquerque and Wang (2008), that decrease equity mean-returns and interest rates when investor protection is low, lead to excess stock return volatility, time-variation of all equilibrium processes and wealth transfers between different categories of shareholders. We also study hitherto unexplored economic implications of leverage and its effect on volatility.

Giannetti and Koskinen (2010) study a model of two countries with different levels of investor protection in a static setting, where investors make portfolio decisions at the initial date and do not rebalance their portfolios. Similar to our paper, they find that stock returns decrease with weaker protection. However, accounting for the dynamic accumulation of control allows us to uncover new effects of investor protection on interest rates, volatilities and the endogenous time-variation of equilibrium processes. We further explain the differences in volatilities across economies with distinct levels of protection and wealth inequality, and shed light on the role of leverage in generating excess volatilities.

Although controlling shareholders are persistent in the short term, there is evidence that they trade in their companies' shares (Anderson, Reeb and Zhao, 2012) and their ownership substantially changes over the longer term (Denis and Sarin, 1999; Franks et al., 2012). The dynamic accumulation of control and the ability of controlling shareholders to trade and rebalance their portfolios are new aspects of our work which play a key role in determining the effects of investor protection on asset holdings and returns. First, endogenizing accumulation of control allows us to explain the high concentration of ownership in economies with poor investor protection. Second, it introduces a new mechanism that links investor protection to expected returns, volatility and interest rates. Finally, the minority shareholder's consumption share emerges as a state variable modulating the importance of this mechanism.

## 2. The Economy with Investor Protection

We consider a pure-exchange continuous-time infinite-horizon economy with a representative firm that produces one consumption good and is owned by two types of shareholders with heterogeneous control power over the firm. In this Section, we discuss the firm, the financial markets, and shareholder optimization problems.

## 2.1. Firms and Financial Markets

The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a Brownian motion $w$. The stochastic processes are adapted to the filtration $\{\mathcal{F}_t, t \in [0, \infty]\}$, generated by $w$. There is one representative firm in the economy which stands for a large number of identical firms. The firm produces an exogenous stream of output $D_t$, which follows a geometric Brownian motion (GBM) process:

$$dD_t = D_t\big[\mu_D dt + \sigma_D dw_t\big], \tag{1}$$

where the output mean-growth rate $\mu_D$ and volatility $\sigma_D$ are constants.

There are two types of shareholders, a controlling $C$ and a representative minority $M$ shareholders. The representative minority shareholder stands for a group of identical shareholders. The shareholders trade continuously in two securities, a riskless bond in zero net supply with an instantaneous interest rate $r_t$ and a stock in positive net supply, normalized to one unit. The stock is a claim to the stream of dividends, which are paid each date $t$ out of output $D_t$. The dividend payout is determined by the controlling shareholder, as discussed below. We focus on Markovian equilibria in which bond price, $B_t$, and stock price, $S_t$, follow processes

$$dB_t = B_t r_t dt, \tag{2}$$

$$dS_t = S_t\big[\mu_t dt + \sigma_t dw_t\big], \tag{3}$$

where the interest rate $r_t$, stock mean-return $\mu_t$, and volatility $\sigma_t$ are endogenously determined in equilibrium, and the bond price at time 0 is normalized so that $B_0 = 1$.

## 2.2. Investor Protection and Shareholder Objectives

The minority shareholder does not have control power, and cannot influence the dividend payout policy. The controlling shareholder can divert a fraction $x_t$ of the firm's output for himself. The remaining non-diverted output $(1 - x_t)D_t$ is paid as a time-$t$ dividend. The diverted fraction $x_t$ is constrained by investor protection in the economy, so that $x_t \leq (1 - p)q(n_t)$, where $p \in [0, 1]$ is interpreted as the *protection of minority shareholders*, with higher $p$ indicating better investor protection, and $q(n) \in [0, 1]$ indicating the controlling shareholder's power over the firm. Consequently, the above *investor protection constraint* on fraction $x_t$ is determined jointly by investor protection $p$ and the controlling shareholder's power $q(n)$ on the firm. To simplify the analysis, we assume that $q(n) = n$, so that the power in the firm is linearly increasing in the the number of shares.[1]

The investor protection constraint captures the fact that better protection restricts the set of available expropriation strategies, while a higher stake in the firm expands it. Controlling shareholders may divert output by employing a wide range of complex strategies rather than outright theft. For example, cash flows can be tunneled through intra-group activities which can be economically large (Bertrand, Mehta, and Mullainathan, 2002; Cheung, Rau, and Stouraitis, 2006; Khanna and Yafeh, 2007; Jiang, Lee and Yue, 2010). Companies can give each other, or to controlling shareholders directly, high (or low) interest loans (Bertrand, Mehta, and Mullainathan, 2002), engage in abnormal sales (Jian and Wong, 2010; Lo, Wong, and Firth, 2010), sell assets below or above their market values (Cheung et al., 2009) and provide loan guarantees (Berkman, Cole and Fu, 2009). One could also include jobs given to relatives, large bonuses, perquisites etc. that are enjoyed by controlling shareholders as a form of wealth transfer from minority shareholders. It is reasonable to assume that as the controlling shareholders power over the firm increases it would be easier to orchestrate such wealth transfer transactions.

The controlling shareholder also incurs a non-pecuniary cost from diverting output, which does not result in the destruction of wealth within the economy. By incorporating this non-pecuniary cost we intend to capture differences in social norms and its effects on controlling shareholders' incentive to expropriate. Various norms in a society promote fairness, honesty and morality, which may result in disutility from diverting output. In particular, Kahneman, Knetsch and Thaler (1986) argue that people have a preference for fairness and provide examples when economic agents commonly allocate resources fairly to others even when they are free to do otherwise. Consequently, the controlling shareholder may not only be limited by laws and regulations that protect minority shareholders but also by social norms that promote fairness.

Individuals may conform with social norms for various reasons. These norms could be self enforced because perhaps they are self promoting, enforced by personal emotions or

[1] We acknowledge that there could be a region where $n$ is sufficiently small and the controlling shareholder may have no power in the firm. Indeed, empirical papers that attempt to identify the existence of controlling shareholders generally require a shareholder with voting block of at least 10% (La Porta, Lopez-de-Silanes, and Shleifer, 2002; Morck, Yavuz, Yeung, 2011). Alternatively, there could be a region where $q(n) = 1$ beyond some $n < 1$ such that the one share-one vote rule is violated and the controlling shareholder has ultimate power even if he does not own all shares. We can easily incorporate such regions in our model. However, the predictions in these regions can readily be inferred from our simpler model as below and are not that illuminating.

enforced by others through disapproval, ridicule or ostracism (Posner, 1997).[2] We capture the effect of norms in controlling shareholders decision to steal simply by incorporating a disutility from stealing. We further assume the disutility of violating social norms, i.e. stealing, is less costly for wealthier agents. This assumption is in line with the findings of the sociology literature that high status agents (in our case wealthy) are less likely to conform with norms (Phillips and Zuckerman, 2001) perhaps because they are less likely to be punished by the society. Moreover, an additional advantage of our framework is that it does not rely on extreme monetary costs associated with stealing to achieve equilibrium. Such extreme monetary punishments are not commonly observed because of budget constraints and difficulties in implementation (e.g., Shleifer and Wolfenzon, 2002). Posner (2000) argues that law enforcement does not explain why most Americans pay their taxes, because the penalty for ordinary tax convictions is low and the probability of detection is very small. This opens up the possibility that social norms may play a role in explaining conformity with regulations. The cost of diverting output also captures various additional aspects of investor protection, other than limiting the set of diversion strategies, that make the misuse of control power more costly in terms of the required effort.

All shareholders have standard constant relative risk aversion (CRRA) preferences

$$u_i(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \tag{4}$$

with the risk aversion parameter $\gamma > 0$. For tractability, we assume that investors are guided

[2]See also Elster (1989) for a review of the literature explaining why norms exist. Although, incorporating social norms into self maximizing agents behavior is encouraged across many fields (Kahneman, Knetsch and Thaler, 1986; Coleman 1988; Elster, 1989; Guiso, Sapienza and Zingales, 2006), arbitrary incorporation of social norms to explain economic puzzles could be a slippery slope. Our approach is different in that we are taking an ex-ante approach of incorporating social norms into the utility function of a self maximizing economic agent to obtain predictions of how social norms and regulations that protect investors interact and affect the equilibrium outcomes. Our approach is consistent with Posner's (1997) conjecture that laws and regulations may both complement and substitute for social norms in shaping the behavior of economic agents.

by myopic preferences over current consumption $c$ and wealth $W$, given by:[3]

$$V_i(c_t, W_t, W_{t+dt}) = \rho_i^\gamma u_i(c_t)dt + (1-\rho_i^\gamma dt)\mathbb{E}_t\Big[u_i(W_{t+dt})\Big] - 1_{\{i=C\}} f(x_t, D_t) u_i'(W_t)dt, \quad (5)$$

where utility function $u_i(\cdot)$ is given by (4), $i = \{C, M\}$, $1_{\{i=C\}}$ is an indicator function, and $\rho_i$ is investor $i$'s time discount parameter. The objective function (5) weighs the utility of current consumption and future expected wealth with weights $\rho_i^\gamma dt$ and $1-\rho_i^\gamma dt$, respectively. The weighing parameter is adjusted for risk aversion $\gamma$ so that shareholder $i$'s consumption-wealth ratio equals $\rho_i$, as demonstrated below. The last term in equation (5) is the controlling shareholder's non-pecuniary cost of diverting a fraction $x_t$ of the firm's output. The cost function $f(x, D)$ is an increasing function of the diverted fraction $x_t$ and output $D_t$, and is weighted by the controlling shareholder's marginal utility of wealth $u_i'(W_t)$, as discussed above, so that the investor loses marginal benefit of wealth. Throughout the paper, we assume for tractability that the cost function $f(x_t, D_t)$ is quadratic and given by

$$f(x_t, D_t) = \frac{k\, x_t^2 D_t}{2}, \quad (6)$$

where the parameter $k$ captures the magnitude of the cost. We observe that objective function (5) can be extended to arbitrary von Neumann-Morgenstern utility function with $u_i'(\cdot) > 0$ and $u_i''(\cdot) < 0$ and increasing cost function $f(x, D)$.

The shareholder protection constraint $x_t \leq (1-p)n_t$ and the cost function $f(x_t, D_t)$ capture different barriers to expropriation in the economy. The former constraint proxies for legal protection of minority shareholders that limits wealth transfer strategies. In contrast to the legal protection, cost function $f(x_t, D_t)$ quantifies non-pecuniary costs of stealing due to disutility arising from violating social norms.[4] The parameters $p$ and $k$ quantify the extents of the quality of investor protection and non-pecuniary costs in the economy, respectively.

[3] Myopic preferences have been widely adopted in the economic literature (e.g., DeLong, Shleifer, Summers and Waldmann, 1990; Pastor, 2000; Acharya and Pedersen, 2004). Such preferences may also naturally arise in an OLG-type framework. Furthermore, in many models logarithmic preferences give rise to investor myopia, similar to that in the objective function (5) (e.g., Detemple and Murthy, 1997; Basak and Croitoru, 2000). In particular, it can be shown that the value function of a dynamic infinite-horizon consumption choice problem with logarithmic preferences has the following structure: $J(W_t, z_t) = (1/\rho_i)\ln(W_t) + \tilde{J}(z_t, t)$, where $z_t$ is a certain state variable. Then, from the Hamilton-Jacobi-Bellman equation for the dynamic problem it is immediate to observe that solving the dynamic problem is equivalent to solving a myopic problem with an objective function $\rho_i \ln(c_t)dt + (1-\rho_i dt)\mathbb{E}_t[\ln(W_{t+dt})]$. The objective function (5) retains the latter structure of problems with logarithmic preferences but has the additional benefit of accounting for risk aversion $\gamma$, which is important for determining stock risk premia.

[4] In reality, there can be certain pecuniary costs of diverting output, for example if the diversion involves paying bribes. These types of costs would be directly accounted for in the controlling shareholder's budget constraint rather than the utility function. The effect of such costs in controlling shareholder's optimal stealing decision have been extensively analyzed (e.g., Sheleifer and Wolfenzon, 2002; Albuquerque and Wang, 2008).

There is substantial evidence that investor protection and social norms about stealing and law abiding could vary across countries and cultures (e.g., Posner, 2000; La Porta et al., 2002).

## 2.3. Shareholders' budget constraints and optimizations

Each shareholder chooses consumption $c_{it}$, the number of shares $n_{it}$, and the controlling shareholder additionally chooses the fraction $x_t$ of diverted output for private consumption. Each shareholder's wealth at time $t$ is given by $W_{it} \equiv b_{it}B_t + n_{it}S_t$, where $b_{it}$ is the number of units of bonds in the shareholder's portfolio, and satisfies the following self-financing budget constraint:

$$dW_{it} = \Big(W_{it}r_t + n_{it}\Big(S_t(\mu_t - r_t) + (1 - x_t)D_t\Big) + 1_{\{i=C\}}x_tD_t - c_{it}\Big)dt + n_{it}S_t\sigma_t dw_t. \tag{7}$$

In this budget constraint, $S_t(\mu_t - r_t) + (1 - x_t)D_t$ is the gross dollar return on the stock in absolute terms, where $(1 - x_t)D_t$ is the dividend per share, and $x_tD_t$ is the diverted output.

The minority shareholder maximizes the following objective function over current consumption $c$ and next period's wealth $W$:

$$\max_{n_t, c_t} V_M(c_t, W_t, W_{t+dt}), \tag{8}$$

where the function $V_M(\cdot)$ is given by equation (5) for $i = M$, subject to self-financing budget constraint (7). The controlling shareholder maximizes his objective function

$$\max_{x_t, n_t, c_t} V_C(c_t, W_t, W_{t+dt}), \tag{9}$$

where the function $V_C(\cdot)$ is as given in (5) for $i = C$, subject to the budget constraint (7), the investor protection constraint $x_t \leq (1-p)n_t$ and the maximum share constraint $n_t \leq 1$. The tractability of the objective function (5) yields closed-form solutions for the optimal portfolio choice and allows us to focus on the economic effects of wealth redistribution, abstracting away from technical complications of fully dynamic models, which are more relevant for portfolio managers rather than shareholders with control power. We note that the controlling and minority shareholders, in general, differ in their time-preference parameters $\rho_C$ and $\rho_M$. This additional source of shareholder heterogeneity generates trade among investors.[5] Finally, we note that the controlling shareholders in our setting act as price takers and do not manipulate their firm's stock price.[6]

[5] The difference in time-discounts has been widely employed in the literature to generate trading between different groups of economic agents (e.g., Kiyotaki and Moore, 1997; Longstaff, 2009).

[6] This is due to the following reasons. First, their trading, consumption and stealing decisions do not

# 3. Equilibrium with Investor Protection

In this section, we first solve for investors' optimal strategies in a partial equilibrium setting, in which asset price dynamics are taken as given. Then, by substituting the optimal strategies into the market clearing conditions, we obtain the dynamics of asset prices in equilibrium.

## 3.1. Shareholders' Optimal Strategies

We now solve for the optimal stock holdings and consumptions of controlling and minority shareholders. We first note that the maximization of objective functions (8) and (9) turns out to be equivalent to separate optimization problems for consumption $c_t$ and stock holding $n_t$.[7] In particular, investor $i$'s optimal consumption maximizes the following objective function:

$$\max_{c_t} \rho_i^{\gamma} \frac{c_t^{1-\gamma} - 1}{1-\gamma} - W_{it}^{-\gamma} c_t, \tag{10}$$

whereas the optimal stock holding $n_{Ct}^*$ of the controlling shareholder and diverted fraction $x_t^*$ solve a quadratic optimization problem

$$\max_{n_t,\ x_t} J_C(n_t; x_t) = \frac{n_t S_t}{W_{Ct}}\Big((\mu_t - r_t) + (1 - x_t)\frac{D_t}{S_t}\Big) + x_t \frac{D_t}{W_{Ct}} - \frac{k\, x_t^2}{2}\frac{D_t}{W_{Ct}} - \frac{\gamma}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2, \tag{11}$$

subject to constraints $x_t \le (1-p)n_t$ and $n_t \le 1$, and the optimal stock holding $n_{Mt}^*$ of the minority shareholder solves the optimization problem

$$\max_{n_t} J_M(n_t) = \frac{n_t S_t}{W_{it}}\Big((\mu_t - r_t) + (1 - x_t)\frac{D_t}{S_t}\Big) - \frac{\gamma}{2}\Big(\frac{n_t S_t}{W_{Mt}}\sigma_t\Big)^2. \tag{12}$$

Solving for shareholders' optimal consumptions, we find that consumptions $c_i^*$ are given by:

$$c_{it}^* = \rho_i W_{it}. \tag{13}$$

affect Sharpe ratios of their own stocks because all firms are small and have identical outputs $D_t$ driven by the same risk factor represented by Brownian motion $w_t$. In such an economy, any deviation of a firm's Sharpe ratio from that of the market portfolio leads to an arbitrage opportunity (e.g., Basak and Croitoru, 2000), which can be easily eliminated by minority shareholders who do not face any trading frictions. Second, because the controlling shareholders cannot also affect the interest rates, they cannot manipulate the state price density in the economy either, which follows dynamics $d\xi_t = -\xi_t[r_t dt + \kappa_t dw_t]$, where $r_t$ and $\kappa_t$ denote the interest rate and the Sharpe ratio of the market portfolio (e.g., Duffie, 2001).Therefore, all stocks are priced by the same state price denisty and satisfy the valuation formula $S_t = \mathbb{E}_t[\int_t^{+\infty} \xi_\tau (1 - x_\tau) D_\tau d\tau]/\xi_t$. This demonstrates that stock price levels can only be directly affected by fractions $x_t$ of diverted output. Because the controlling shareholders are myopic and look only a single period ahead, sitting at time $t$ they only account for the effect of their stealing on time $t + dt$ dividend $(1 - x_t)D_t dt$, and take the dividends beyond their immediate investment horizon as given.

[7] To demonstrate this, we rewrite the second term in equation (5) for investor preferences as $\mathbb{E}_t[u_i(W_{i\,t+dt})] = u_i(W_{it}) + \mathbb{E}_t[du_i(W_{it})]$, apply Itô's Lemma to $u_i(W_{it})$, where $u_i(\cdot)$ is given by (7), and then, after some algebra, we find that the optimal consumption and the number of shares solve two separate optimization problems.

The optimal consumptions in (13) reveal that the consumption-wealth ratios are constant, and for $\gamma = 1$ are the same as in dynamic portfolio choice problems with logarithmic preferences.

Solving the portfolio choice problem of the controlling shareholder is complicated by the presence of constraints on stock holding $n$ and diverted fraction of output $x$, which renders the value function (11) non-concave function of stock holding $n$. The optimization problem in (11) is solved in two steps. First, we solve for the optimal fraction of diverted output $x_t^*(n_t)$. Second, we substitute fraction $x_t^*(n_t)$ into the objective function in (11), and then maximize it with respect to $n_t$ to obtain the optimal stock holding $n_{Ct}^*$. Proposition 1 below summarizes our results in partial equilibrium.

**Proposition 1 (Partial equilibrium).** *The fraction of diverted output $x_t^*$ and optimal stock holdings $n_{it}^*$ are given by:*

$$x^*(n_t) = \min\Big((1-n_t)/k; (1-p)n_t\Big); \tag{14}$$

$$n_{Ct}^* = \begin{cases} n_{Ct,1}^* = \dfrac{\mu_t - r_t + (2-p)\frac{D_t}{S_t}}{\gamma\sigma_t^2\frac{S_t}{W_{Ct}} + \Big(2(1-p)+(1-p)^2k\Big)\frac{D_t}{S_t}}, & \text{if } J_C^* = J_C(n_{Ct,1}^*; x_t^*) \text{ (region 1)},\\ n_{Ct,2}^* = \dfrac{\mu_t - r_t + \frac{k-1}{k}\frac{D_t}{S_t}}{\gamma\sigma_t^2\frac{S_t}{W_{Ct}} - \frac{1}{k}\frac{D_t}{S_t}}, & \text{if } J_C^* = J_C(n_{Ct,2}^*; x_t^*) \text{ (region 2)},\\ n_{Ct,3}^* = \dfrac{1}{1+(1-p)k}, & \text{if } J_C^* = J_C(n_{Ct,3}^*; x_t^*) \text{ (region 3)},\\ n_{Ct,4}^* = 1, & \text{if } J_C^* = J_C(n_{Ct,4}^*; x_t^*) \text{ (region 4)}; \end{cases} \tag{15}$$

$$n_{Mt}^* = \frac{\mu_t - r_t + (1-x_t)\frac{D_t}{S_t}}{\gamma\sigma_t^2\frac{S_t}{W_{Mt}}}, \tag{16}$$

*where $J_C^* = \max\Big(J_C(n_{Ct,1}^*; x_t^*), J_C(n_{Ct,2}^*; x_t^*), J_C(n_{Ct,3}^*; x_t^*), J_C(n_{Ct,4}^*; x_t^*)\Big)$ and $J_C(n; x)$ is as in* (11). *Moreover, the shareholders' optimal consumptions are given by Equation* (13).

The optimal fraction of diverted output $x^*(n_t)$ is a hump-shaped function of the number of shares $n_t$, and is depicted on Figure 1. After the kink, as the controlling shareholders ownership increases, the amount of stealing decreases since the marginal utility of stealing decreases due to larger cash flow rights. This is intuitive and consistent with the previous literature (e.g., Shleifer and Wolfenzon, 2002; La Porta et al, 2002) that incorporates investor protection as a monetary cost from diversion. This literature explicitly assumes that the level of investor protection and the controlling shareholder's ability to steal do not depend on the ownership structure of the firm (Shleifer and Wolfenzon, 2002). The controlling shareholder has a higher incentive to expropriate when he does not have much firm ownership and

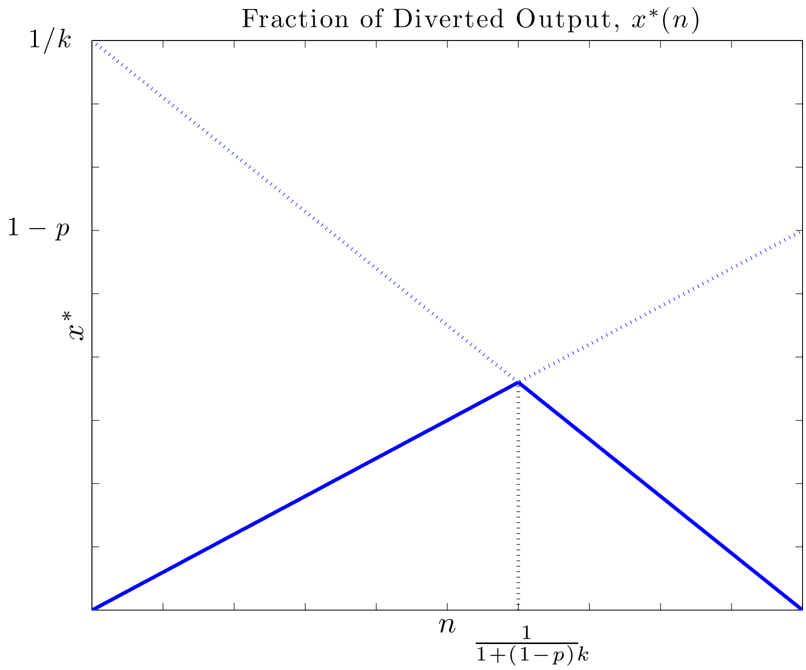


**Figure 1**
**Optimal Fraction of Diverted Output** $x^*(n)$
This Figure shows the tent-shaped optimal fraction of diverted output $x^*$ as a function of the controlling shareholder's stake $n$ in the firm.

given that his ability to steal does not vary with ownership, the controlling shareholder ends up stealing more when his ownership is low. On the contrary, in our model, low ownership implies low power over the firm, which limits the amount that can be diverted. This distinction is especially important given that we internalize the optimal ownership decision of the controlling shareholder.

The figure also shows that, when the investor protection constraint binds, the effect of a change in ownership $n_t$ on the level of diverted output is positive but smaller in countries with better investor protection. On the other hand, when the investor protection constraint is not binding, a higher pecuniary cost (i.e., higher cost parameter $k$) makes the effect of a change in ownership on the level of diverted output negative but smaller in magnitude. Therefore the sensitivity of stealing to controlling shareholder's ownership decreases both with better investor protection and higher non-pecuniary costs of stealing.

The controlling shareholder's optimal stock holding $n^*_{Ct}$ in Proposition 1 captures the tradeoff between the benefits and costs of diverting the output. The expression for stock holding $n^*_{Ct}$ in Equation (15) differs across four regions in the space of the state variables.

Region 1 is such that the fraction of the diverted output is given by $x^*(n^*_{Ct}) = (1-p)n^*_{Ct}$, that is, the investor chooses to divert the maximum possible fraction of output. In this region, laws and regulations that protect minority investor rights is the binding constraint on stealing. This is because marginal benefit from stealing is high with low ownership rights, the constraint on stealing is tight because controlling shareholder's power is low and disutility from stealing is low at low levels of stealing.

Region 2 is such that $x^*(n^*_{Ct}) = (1-n^*_{Ct})/k$, that is, the disutility of diverting output kicks in. In this region, the controlling shareholder has higher power over the firm which makes the constraint imposed by investor protection relatively relaxed. On the other hand, high stake in the firm reduces incentive to expropriate. Consequently, in this region, disutility from stealing rather than investor protection determines the optimal amount of stealing. We observe that, after simple algebra, the stock holding $n^*_C$ in region 2 can be rewritten in the following equivalent way:

$$n^*_{Ct} = \frac{\mu_t - r_t + (1-x^*_t)\frac{D_t}{S_t}}{\gamma\sigma_t^2 \frac{S_t}{W_{Ct}}}, \tag{17}$$

where $x^*(n^*_{Ct}) = (1-n^*_{Ct})/k$. Therefore, interestingly, both types of shareholders invest the same fraction of wealth $n^*_i S/W_i$ in stocks when the economy is in region 2 where the investor protection constraint is not binding.[8]

Finally, we discuss regions 3 and 4. Region 3 is a point where $(1-p)n^*_{Ct} = (1-n^*_{Ct})/k$, and hence the two forces of diverting the maximum and the cost of stealing equate. Region 4 is where $n^*_{Ct} = 1$, and hence the controlling shareholder has full control over the firm.

## 3.2. Asset Price Dynamics in Equilibrium

In this subsection, we derive the equilibrium mean-return $\mu_t$, volatility $\sigma_t$, riskless rate $r_t$ and shareholder stock holdings $n^*_{it}$. The definition of equilibrium in our pure-exchange economy is standard: the *equilibrium* is a set of processes $r_t$, $\mu_t$ and $\sigma_t$, optimal stock and bond

[8]This result follows from the envelope condition and is due to the fact that the derivative of objective function $J_C(n;x_t)$ in (11) with respect to $x_t$ is zero in region 2 because fraction of diverted output $x_t$ is chosen precisely to satisfy the first order condition with respect to $x_t$, that is, $\partial J_C(n;x_t)/\partial x_t = 0$. Therefore, the first order condition with respect to stock holding $n_C$ is given by $\partial J_C(n;x_t)/\partial n_t = 0$, and hence, the optimal stock holdings of controlling and minority shareholders are similar.

holdings, $n^*_{it}$ and $b^*_{it}$, and consumptions $c^*_{it}$ that satisfy the market clearing conditions

$$\begin{aligned} n^*_{Ct} + n^*_{Mt} &= 1, & (18)\\ b^*_{Ct} + b^*_{Mt} &= 0, & (19)\\ c^*_{Ct} + c^*_{Mt} &= D_t. & (20)\end{aligned}$$

All equilibrium processes are derived as functions of minority shareholder's *share in the aggregate consumption*, defined as $y_t = c^*_{Mt}/D_t$. Similarly to the literature on equilibrium with heterogeneous investors (e.g., Chabakauri, 2015) the consumption share $y_t$ of one of the investors emerges as a crucial state variable that determines the dynamics of asset prices in the economy. Following the literature, we conjecture and then verify that the consumption share follows a Markovian process

$$dy_t = y_t[\mu_{yt}dt + \sigma_{yt}dw_t], \tag{21}$$

where processes $\mu_{yt}$ and $\sigma_{yt}$ are determined in equilibrium as functions of $y_t$.

To facilitate the intuition, we provide first the equilibrium in the benchmark economy with full investment protection $p = 1$, and then compare it with the equilibrium with imperfect protection. In the benchmark economy, the difference in time discount rates $\rho_C$ and $\rho_M$ is the only source of heterogeneity between investors $C$ and $M$, and hence the equilibrium is available in closed form. Lemma 1 below provides the equilibrium processes in the benchmark economy.

**Lemma 1 (Benchmark equilibrium with full protection $p = 1$).** *In the economy with full investor protection $p = 1$ the shareholders' optimal consumptions are given by* (13), and *the equilibrium stock mean-return, interest rate, volatility, and the Sharpe ratio are given by:*

$$\begin{aligned} \mu_t &= \mu_D + \rho_M y_t + \rho_C(1-y_t) - \frac{1}{y_t/\rho_M + (1-y_t)/\rho_C}, & (22)\\ r_t &= \mu_D + \rho_C(1-y_t) + \rho_M y_t - \sigma_D\kappa_t, & (23)\\ \sigma_t &= \sigma_D, & (24)\\ \kappa_t &= \gamma\sigma_D, & (25)\end{aligned}$$

*and the minority shareholder's consumption share mean growth and volatility are*

$$\begin{aligned} \mu_{yt} &= (1-y_t)(\rho_C - \rho_M), & (26)\\ \sigma_{yt} &= 0. & (27)\end{aligned}$$

*The controlling and minority shareholders' optimal stock holdings are given by:*

$$\bar{n}_{Ct} = \frac{(1-y_t)/\rho_C}{(1-y_t)/\rho_C + y_t/\rho_M}, \quad \bar{n}_{Mt} = \frac{y_t/\rho_M}{(1-y_t)/\rho_C + y_t/\rho_M}. \tag{28}$$

The results in Lemma 1 reveal that the equilibrium processes in the benchmark economy are deterministic because the volatility of the state variable is equal to zero, $\sigma_{yt} = 0$. Consequently, the volatility of stock returns coincides with the volatility of output $\sigma_D$. Furthermore, it can be easily demonstrated that the equilibrium processes (22)–(28) are the same as in a Lucas (1978)-type economy populated by Epstein-Zin investors with elasticity of intertemporal substitution equal to unity and risk aversion $\gamma$. Therefore, our specification of myopic CRRA preferences gives rise to equilibrium processes consistent with baseline asset pricing models. The optimal stock holdings of shareholders are driven by consumption share $y$, which changes deterministically over time.

Next, we consider the economy with imperfect protection $p < 1$, in which controlling shareholders can divert part of the firm output. In such an economy, the controlling shareholder's ability to divert output emerges as an additional source of investor heterogeneity in addition to the differences in time discount parameters $\rho_i$. Proposition 2 below reports the equilibrium processes.

**Proposition 2 (Equilibrium with imperfect protection).** *In the equilibrium with imperfect protection $p < 1$ shareholders' optimal consumptions are given by* (13), *and stock mean-return, interest rate, volatility and the Sharpe ratio are given by:*

$$\begin{aligned}
\mu_t &= r_t + \sigma_t\kappa_t - \Big(1 - x^*(n^*_{Ct})\Big)\frac{1}{y_t/\rho_M + (1-y_t)/\rho_C}, &(29)\\
r_t &= \mu_D + \rho_C(1-y_t) + \rho_M y_t - \sigma_D\kappa_t - \rho_C x^*(n^*_{Ct}), &(30)\\
\sigma_t &= \sigma_D\frac{1}{\rho_C n^*_{Ct} + \rho_M(1-n^*_{Ct})}\frac{1}{y_t/\rho_M + (1-y_t)/\rho_C}, &(31)\\
\kappa_t &= \gamma\sigma_D\frac{\rho_M(1-n^*_{Ct})}{\rho_C n^*_{Ct} + \rho_M(1-n^*_{Ct})}\frac{1}{y_t}, &(32)
\end{aligned}$$

*and the minority shareholder's consumption share mean growth and volatility are*

$$\begin{aligned}
\mu_{yt} &= (1-y_t)(\rho_C - \rho_M) + (\kappa_t - \sigma_D)\sigma_{yt} - \rho_C x^*(n^*_{Ct}), &(33)\\
\sigma_{yt} &= \frac{\kappa_t}{\gamma} - \sigma_D, &(34)
\end{aligned}$$

*The controlling shareholder's optimal stock holding* $n^*_{Ct}$ *solves the fixed-point equation*

$$\begin{aligned} n^*_{Ct} &= \underset{n_t,\, n_t\le 1}{\operatorname{argmax}}\Big\{\frac{n_t S_t}{W_{Ct}}\Big(\mu_t - r_t + (1-x^*(n_t))\frac{D_t}{S_t}\Big) \\ &\quad + x^*(n_t)\frac{D_t}{W_{Ct}} - \frac{k\, x^*(n_t)^2}{2}\frac{D_t}{W_{Ct}} - \frac{\gamma}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2\Big\}, \end{aligned} \tag{35}$$

*where* $x^*(n)$ *is given by equation* (14), $\mu_t$, $r_t$ *and* $\sigma_t$ *are given by equations* (29)–(31) *and themselves depend on* $n^*_{Ct}$ *in equilibrium, and ratios* $D_t/W_{Ct}$ *and* $S_t/W_{Ct}$ *are given by:*

$$\frac{D_t}{S_t} = \frac{1}{y_t/\rho_M + (1-y_t)/\rho_C}, \quad \frac{D_t}{W_{Ct}} = \frac{\rho_C}{1-y_t}, \quad \frac{S_t}{W_{Ct}} = \frac{y_t/\rho_M + (1-y_t)/\rho_C}{(1-y_t)/\rho_C}. \tag{36}$$

Proposition 2 derives the equilibrium processes as functions of minority shareholder's consumption share $y_t$ and demonstrates that processes (29)–(32) preserve the structure of processes (22)–(25) in the economy with full protection, but additionally incorporate certain adjustment terms due to imperfect protection. In particular, equations (29) and (30) for stock return $\mu_t$ and interest rate $r_t$ now account for the diverted fraction of output $x^*(n^*_{Ct})$. Intuitively, return $\mu_t$ is affected because smaller dividend is paid out to shareholders. Furthermore, diverting a fraction of output makes the controlling shareholder wealthier, which decreases this shareholder's need for borrowing, and hence, decreases the interest rate, as captured by the last term in equation (30).

To facilitate the analysis of equation (31) for the stock return volatility $\sigma$, we observe that after some algebra it can be rewritten in the following equivalent way:

$$\sigma_t = \sigma_D \frac{\rho_M + (\rho_C - \rho_M)\bar{n}_{Ct}}{\rho_M + (\rho_C - \rho_M)n^*_{Ct}}, \tag{37}$$

where $\bar{n}_{Ct}$ denotes the controlling shareholder's optimal stock holding in the benchmark economy with full protection, given in equation (28). Therefore, with full protection the adjustment term in (37) vanishes, and hence, the stock return volatility equals the output volatility: $\sigma_t = \sigma_D$. In the economy with imperfect protection, equation (37) reveals that volatility $\sigma_t$ is determined by the deviation of stock holding $n^*_{Ct}$ from its benchmark value $\bar{n}_{Ct}$. In particular, stock is more (less) volatile than output, that is, $\sigma_t > \sigma_D$ ($\sigma_t < \sigma_D$), when the controlling shareholder's stock holding is higher than in the benchmark economy, $n^*_{Ct} > \bar{n}_{Ct}$ ($n^*_{Ct} < \bar{n}_{Ct}$), provided that the minority shareholder is more impatient than the controlling shareholder, that is, $\rho_M > \rho_C$, and vise-versa when $\rho_M < \rho_C$. Intuitively, this is the case because the stock price in equilibrium is equal to the aggregate wealth, so that

$S_t = W_{Ct} + W_{Mt}$, as in baseline pure-exchange economies with heterogeneous investors.[9] Therefore, stock volatility $\sigma_t$ depends on volatilities of wealths, which are determined by stock holdings $n^*_{it}$, and on time discounts $\rho_i$, which determine investors' consumptions (13), and hence, the rate of the accumulation of wealth. Equation (32) for the Sharpe ratio can be analyzed similarly (skipped for brevity).

The expression (33) for the drift $\mu_y$ of the consumption share $y$ now incorporates two additional terms relative to its counterpart (26) in the full protection economy. In particular, the last term in (33) reveals that stealing reduces drift $\mu_y$ because it transfers wealth from minority to controlling shareholders, and hence, reduces the rate of growth of consumption share $y$ of the former. In contrast to the benchmark economy, the volatility of consumption share is non-zero, that is, $\sigma_{yt} \neq 0$, and hence, the imperfect investor protection makes the equilibrium processes stochastic through its effect on the redistribution of wealth and consumption in the economy.

In contrast to the full protection benchmark economy, simple closed-form expressions for stock holdings $n^*_{it}$ are no longer available. Consequently, imperfect investor protection gives rise to complex dynamics of equilibrium processes via the effects of protection on the stock holding of the controlling shareholder $n^*_{Ct}$. In particular, stock holding $n^*_{Ct}$ now solves a fixed-point problem in equation (35) in which the equilibrium processes on the right-hand side of this equation are functions of stock holding $n^*_{Ct}$ itself. We solve and analyze the effects of protection on stock holding $n^*_{Ct}$ and other equilibrium processes in the next section.

We note that the endogenous accumulation of control and the participation of the controlling shareholder in asset markets are key ingredients for explaining certain empirical regularities. In particular, endogenizing the stock holdings allows us to explain the higher ownership concentration in countries with low protection and shed light on the tradeoff between higher control rights and under-diversification. Furthermore, dynamic asset holdings generate endogenous wealth transfers between controlling and minority shareholders which give rise to the stochastic time-variation in asset returns and excess volatility. Finally, these asset holdings also give rise to leverage and help us shed light on the role of leverage in the accumulation of control and its effect on the stock return volatility.

[9] Using market clearing conditions (18) and (20) for the stock and bond markets, we obtain: $W_{Ct} + W_{Mt} = (n^*_{Ct} S_t + b^*_{Ct} B_t) + (n^*_{Mt} S_t + b^*_{Mt} B_t) = S_t$.

# 4. Economic Implications of Investor Protection

In this Section, we present our results with plots as depicted on Figures 2-4 for a plausible set of baseline parameters as functions of consumption share $y$ of the minority shareholders in the economy.[10] The comparative statics results are reported holding consumption share $y$ fixed. Panels (a)–(c) of Figure 2 present the controlling shareholder's equilibrium stock holding $n_C^*$, the fraction of diverted output $x^*$, and the controlling shareholder's leverage-stock price ratio for different values of investor protection $p$ in the economy, holding stealing cost function fixed. Then, we use the results on Figure 2 for the analysis of equilibrium expected gross returns, interest rates, and volatilities depicted on Panels (a)-(c) of Figure 3 for the same protection $p$ and the cost function. Panels (a)-(c) of Figure 4 explore the interaction between the investor protection and the non-pecuniary cost function, and investigate the same processes as on Figure 3 but for different cost function parameters $k$, holding the level of protection $p$ fixed. The numerical approach for deriving the equilibrium processes is explained in the Appendix.

## 4.1. Stock Holdings and Diverted Output

We start our analysis with Figure 2 presenting the controlling shareholder's stock holding, fraction of diverted output and leverage. Panel (a) of Figure 2 demonstrates that lower protection tends to increase the controlling shareholder's stock holding $n_C^*$ relative to the full protection benchmark, i.e., $n_{Ct}^* \geq \bar{n}_{Ct}$. This is because when investor protection is imperfect the controlling shareholder can divert a larger fraction of output when he owns more shares, which increases control over the firm and relaxes the investor protection constraint. This gives the controlling shareholder an incentive to acquire more shares in equilibrium when his consumption share is low. However, when the controlling shareholders consumption share is high, the stock holding is the same as in the benchmark economy, i.e., $n_{Ct}^* = \bar{n}_{Ct}$, because the investor protection constraint is no longer binding.

However, we note that controlling shareholder's stock holding $n_{Ct}^*$ is non-monotone in protection $p$. In particular, in panel (c) we observe that whether stock holding $n_{Ct}^*$ is higher in the economy with $p = 0.9$ or $p = 0.6$ critically depends not only on the protection $p$ but also on the consumption share $y$. This is because a decrease in $p$ has two opposing effects on stock holding $n_C^*$. On one hand, stock holding $n_C^*$ increases because the controlling

[10]We set $\mu_D = 1.7\%$ and $\sigma_D = 3.6\%$, consistent with the estimates in Campbell (2003), and set $\gamma = 5$, $\rho_C = 0.02$, $\rho_M = 0.03$, and $k = 0.3$.

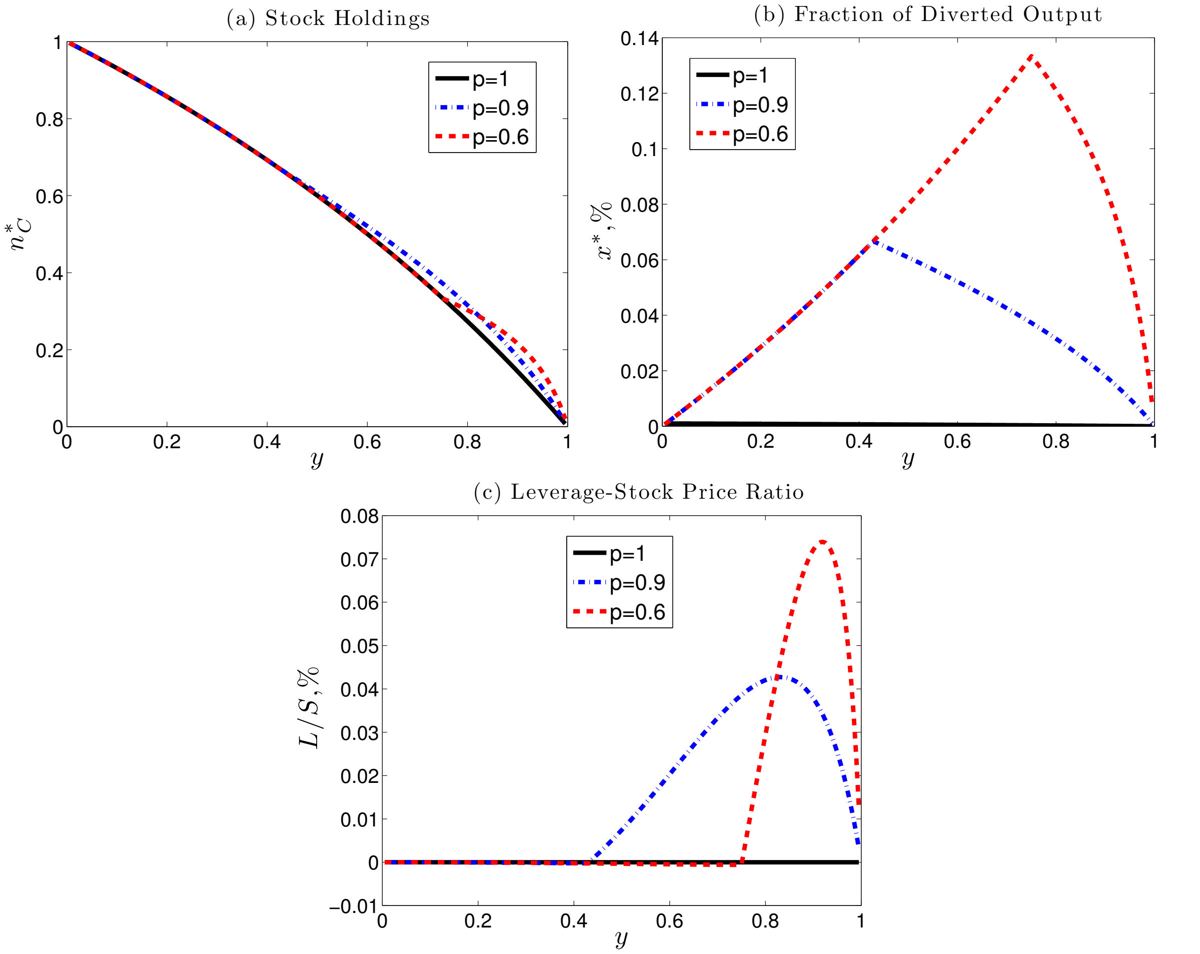


**Figure 2**
**The effect of investor protection on controlling shareholder's stock holding, fraction of diverted output and leverage**
This Figure shows the equilibrium stock holdings $n_C^*$, fraction of diverted output $x^*$ and leverage-stock price ratio $L_C/S$ as functions of consumption share $y$ for fixed stealing cost parameter $k = 5$ and different levels of investor protection $p$, for calibrated parameters.

shareholder can earn extra return by diverting the output. On the other hand, it expands region 2 in equation (15) for stock holding $n_C^*$, in which the disutility of stealing comes into play. The latter effect can be seen on Figure 1, from which we observe that the area in which stealing is a decreasing function of stock holdings becomes larger as protection $p$ goes down.

Panel (b) of Figure 2 shows the fraction of diverted output $x^*$ and how it is affected by investor protection. As would be expected, the fraction of diverted output is considerably reduced in economies with better protection. Panel (c) of Figure 2 shows the controlling shareholder's leverage-stock price ratio. The leverage is given by the wealth invested in stocks in excess of total wealth, $L_t = n_{Ct}^* S_t - W_{Ct}$. Then, taking into account the equation for $S_t/W_{Ct}$ in (36), we obtain the following leverage-stock price ratio:

$$\frac{L_t}{S_t} = n^*_{Ct} - \bar{n}_{Ct}. \tag{38}$$

Panel (a) of Figure 2 demonstrates that the wedge between the controlling shareholder's stock holdings in the economy with imperfect protection and the full protection benchmark is positive, i.e., $n^*_{Ct} - \bar{n}_{Ct} \geq 0$, where the stock holding in the benchmark economy, $\bar{n}_{Ct}$, is shown by the black solid line. Therefore, our analysis reveals that the acquisition of additional shares in economies with poor protection is financed by leverage because, as demonstrated below, the borrowing is cheap in such economies.

## 4.2. Stock Return, Volatility, and Interest Rate

Panel (a) of Figure 3 shows that a higher investor protection $p$ leads to a higher gross stock return $\mu + (1-x^*)D/S$, where $\mu$ is the mean capital gain and $(1-x^*)D/S$ is dividend yield. This finding is consistent with the empirical literature documenting a positive relationship between corporate governance and realized returns. Future returns are positively correlated with a governance index of shareholder rights (Gompers, Ishii and Metrick, 2003), a lower entrenchment index (Bebchuk, Cohen and Ferrell, 2009), a governance index (AGR) from Audit Integrity (Daines, Gow and Larcker, 2010) and lower managerial perks (Yermack, 2006). Similar relations are documented in other countries and cross country studies. Firms with higher governance scores in Germany (Drobetz et al., 2004), firms that do not engage in tunneling using inter-corporate loans in China (Jiang, Lee and Yue, 2010) and countries with better legal institutions (Lombardo and Pagano, 1999) have higher returns. Despite the supporting empirical evidence, it is not immediately clear why such a relationship between expected returns and investor protection exists in equilibrium. For example, taking away a constant fraction of dividends reduces the value of the firm but does not affect the expected return in equilibrium. There is also an ongoing discussion about whether the empirical relationship is robust (Core, Guay and Rusticus, 2006; Giroud and Mueller, 2011; Bebchuk, Cohen and Wang, 2013). Therefore, further guidance from theory, as a contribution to this debate, would be helpful.

To understand the intuition, we first consider the benchmark economy with full protection $p = 1$. In this economy, gross stock returns are determined by investors' risk aversions and are sufficiently high to compensate investors for risk taking. Lower investor protection $p < 1$ opens up an opportunity to divert firm cash flows to benefit the controlling shareholders. Therefore, the controlling shareholders are compensated for excessive risk taking not only via

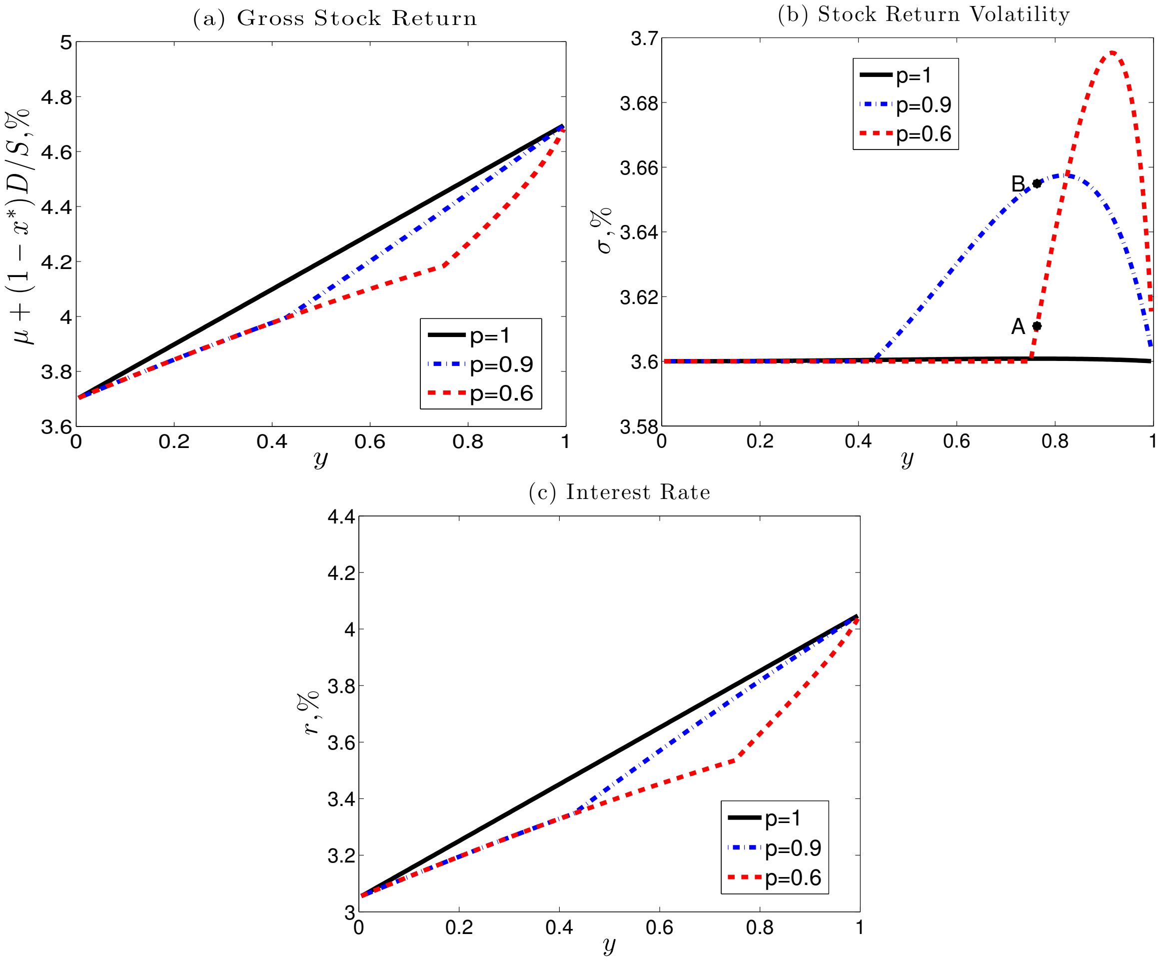


**Figure 3**
**The effect of investor protection on stock gross returns, volatilities and interest rates**
This Figure shows the equilibrium gross stock returns $\mu + (1 - x^*)D/S$, stock return volatilities $\sigma$ and interest rates $r$ as functions of consumption share $y$ for fixed stealing cost parameter $k = 5$ for different levels of investor protection $p$, for calibrated parameters.

risk premia but also via stealing. Consequently, their *effective* risk premia appear to be higher than the risk premia implied by the stock price dynamics. More formally, from the budget constraint (7) of the controlling shareholder $C$ it is immediate to observe that his effective risk premium for holding stocks is given by $\mu - r + (1 - x^*)D/S + x^*D/n_C^*$, and hence, is higher than the risk premium $\mu - r + (1 - x^*)D/S$ for the minority shareholder by the diverted output per share $x^*D/n_C^*$. Therefore, a low risk premium $\mu - r + (1 - x^*)D/S$ is indeed consistent with our equilibrium because the stock market clears due to high demand for stocks by the controlling shareholders. Consistent with our intuition, it has been documented that demand by controlling shareholders for voting shares increases with poor investor protection, which then affects prices.[11]

Panel (b) depicts the stock return volatility $\sigma$ and demonstrates that in our calibration of the model it is higher than in the benchmark economy with full protection, i.e., $\sigma \geq \sigma_D$, and hence, the stock is more volatile than output, consistent with empirical regularities (e.g., Shiller, 1981). The volatility increases because with imperfect protection $p < 1$ the controlling shareholder's stock holding $n^*_{Ct}$ and the minority shareholder's consumption share $y$ become time-varying due to the time-variation in the leverage. Therefore, the time-variation of $y$ adds to the time-variation of output, making stocks more volatile than in our benchmark where $y$ is time-deterministic.

Furthermore, stock return volatility $\sigma$ is a concave function of minority shareholder's consumption share $y$, consistent with the empirical evidence provided by Gul, Kim and Qui (2010), who show that relationship between controlling shareholders ownership and stock volatility is concave. Finally, similar to the controlling shareholder's stock holding $n^*_C$, we observe that volatility $\sigma$ is non-monotone in protection $p$. This is because the interaction between volatility $\sigma$ and protection $p$ depends on the minority shareholder's consumption share $y$. Clearly, the income inequality or distribution of wealth could have a direct effect on portfolio holdings of controlling versus minority shareholders and our model implies that this would interact with the effect of investor protection on equilibrium volatility.

Empirical evidence indicates that the idiosyncratic and total volatilities are higher in more developed countries such as U.S. (Morck, Yeung and Yu, 2000; Bartram, Brown and Stulz, 2012), where investor protection is also relatively higher. Indeed, minority investor protection, property rights protection and opaqueness are suggested as likely culprits (Morck, Yeung and Yu, 2000; Jin and Myers, 2006; Bartram, Brown and Stulz, 2012). Our model can potentially shed some light on this empirical relation between investor protection and volatility. For example, in economies with high consumption shares of minority shareholders stock price volatility can be higher in an economy with high level of investor protection (e.g., point B in Panel (b) of Figure 3) than in an economy with low level of protection (e.g., point A in Panel (b) of Figure 3).

Furthermore, we note an important link between volatility and leverage. In particular, combining equations (37) and (38) for the volatility and leverage, after simple algebra, we

[11] Nenova (2003) documents that the valuation difference between shares that have voting rights versus not is negatively related with investor protection, and Dyck and Zingales (2004) show that the premium paid on block transactions is higher when the buyer comes from a country that protects investors less.

obtain the following expression for the excess volatility $\sigma_t - \sigma_D$ in terms of leverage:

$$\sigma_t - \sigma_D = \frac{L_t}{S_t} \frac{\sigma_D(\rho_M - \rho_C)}{\rho_M + (\rho_C - \rho_M) n^*_{Ct}}. \tag{39}$$

Equation (39) explains close resemblance of volatility $\sigma$ and leverage-stock price ratio $L/S$. Moreover, in our calibration, the latter equation implies a positive relationship between the volatility and the leverage.

Panel (c) of Figure 3 shows that the interest rate $r$ is lower in the economy with poor protection. Intuitively, as elaborated in Section 3.2, imperfect protection $p < 1$ leads to wealth and consumption transfers from minority to controlling shareholders. Therefore, to acquire $n$ shares the latter need to borrow less than in the economy with full protection, which depresses the interest rates. This intuition is also reflected in the last term of equation (30) for interest rates $r$, which demonstrates that higher fraction of diverted output $x^*$ decreases interest rates. Furthermore, in equilibrium, there is a second channel that decreases interest rates. Because the risk premium faced by the minority shareholders is low, investment in stocks is less attractive for them than in the economy with full protection. Therefore, the minority shareholders run to the bond market, and hence, are willing to provide cheap credit, which further decreases the interest rates. This prediction may contribute towards explaing related empirical evidence. For example Klock, Mansi and Maxwell (2005) find that as the governance index decreases the cost of borrowing also decreases. Cremers, Nair and Wei (2007) find that shareholder control is associated with lower yields if the governance index is low.

We remark that all the equilibrium processes have kinks, which arise via the dependence of the latter processes on the fraction of diverted output $x^*(n^*_{Ct})$ on Figure 1. As discussed in Section 3.1, fraction $x^*(n^*_{Ct})$ has a kink at the separation point of regions 1 and 2 in equation (15) for stock holding $n^*_{Ct}$, which correspond to situations when the investor protection constraint $x_t \leq (1-p)n_t$ is binding or not, respectively. The latter constraint is loose when $n$ is sufficiently large, so that the controlling shareholder does not want to steal from himself. Therefore, because stock holding $n^*_{Ct}$ is a decreasing function of consumption share $y$ [see panel (c) of Figure 2], the economy is in region 1 (region 2) when $y$ lies to the right-hand (left-hand) side of the kink. Regions 3 and 4 in equation (15) correspond to the kink itself and to $n^*_{Ct} = 1$, respectively.

When the investor protection constraint does not bind, i.e. the economy is in region 2, the equilibrium stock holdings $n^*_{Ct}$ and volatility $\sigma_t$ coincide with those in the benchmark economy with $p = 1$, as illustrated on panels (c) and (d) of Figure 2. After some algebra, it can be demonstrated that in region 2 the stock holdings of shareholders, given by equations (15) and (16), have the same structure as in the benchmark economy with full protection and are given by Merton's formula $n^*_{it} = \kappa/(\gamma\sigma_t S_t/W_{it})$, where $\kappa$ stands for the market price of risk. Therefore, in region 2 $n^*_C$ and $\sigma$ are the same as in the benchmark.

From the results on Figures 2 and 3, we observe that the effects of investor protection are more conspicuous when either type of shareholders has a large consumption share in the economy. Intuitively, when $y \approx 1$ the controlling shareholder accounts for a tiny fraction of aggregate wealth and consumption, and hence, the effect of stealing is small. Furthermore, when $y \approx 0$ the economy is dominated by the controlling shareholder. As a result, the controlling shareholder holds almost all shares, i.e., $n^*_{Ct} \approx 1$, and hence, the diverted fraction $x^*_t = \min\Big((1 - n^*_{Ct})/k; (1-p)n^*_{Ct}\Big)$ is small because, given the cost of diverting the output, the controlling shareholder finds it sub-optimal to divert the output of a firm in which he is entitled to almost 100% of cash flows. Therefore, there is no diversion of output when $y = 0$, and hence, the effect of investor protection vanishes. Finally, we note that the consumption share drift $\mu_{yt}$ and volatility $\sigma_{yt}$ are non-zero in equilibrium, in contrast to the benchmark economy. However, we do not plot them for brevity because their shapes resemble those of the stock gross return and volatility in panels (a) and (b) of Figure 3, respectively.

## 4.3. Effect of Stealing Costs

Finally, Figure 4 shows the effect of a change in the cost function parameter $k$ on equilibrium processes when the level of investor protection is fixed. In particular, it demonstrates that higher cost of stealing (i.e., high cost parameter $k$) is associated with higher stock gross return and interest rate, and lower volatility and stock holding of the controlling shareholder. Overall, the effect of higher cost of stealing due to social norms on equilibrium is consistent with the effect of better protection $p$.

A comparison of the results in Figures 3 and 4 reveals an important difference between the economic effects of protection $p$ and cost parameter $k$ on stock gross returns. In particular, the change in protection $p$ has stronger effects when consumption share $y$ is high (i.e., the economy is in regime 1) whereas the change in parameter $k$ has stronger effects when consumption share $y$ is relatively low (i.e., the economy is in regime 2). Intuitively, as can be formally seen from expression (14) for diverted fraction of output $x^*$, this is due to the fact that lowering cost of stealing $k$ increases the diverted output fraction $x^*$ only when the investor protection constraint does not bind, that is, when the economy is in region 2.

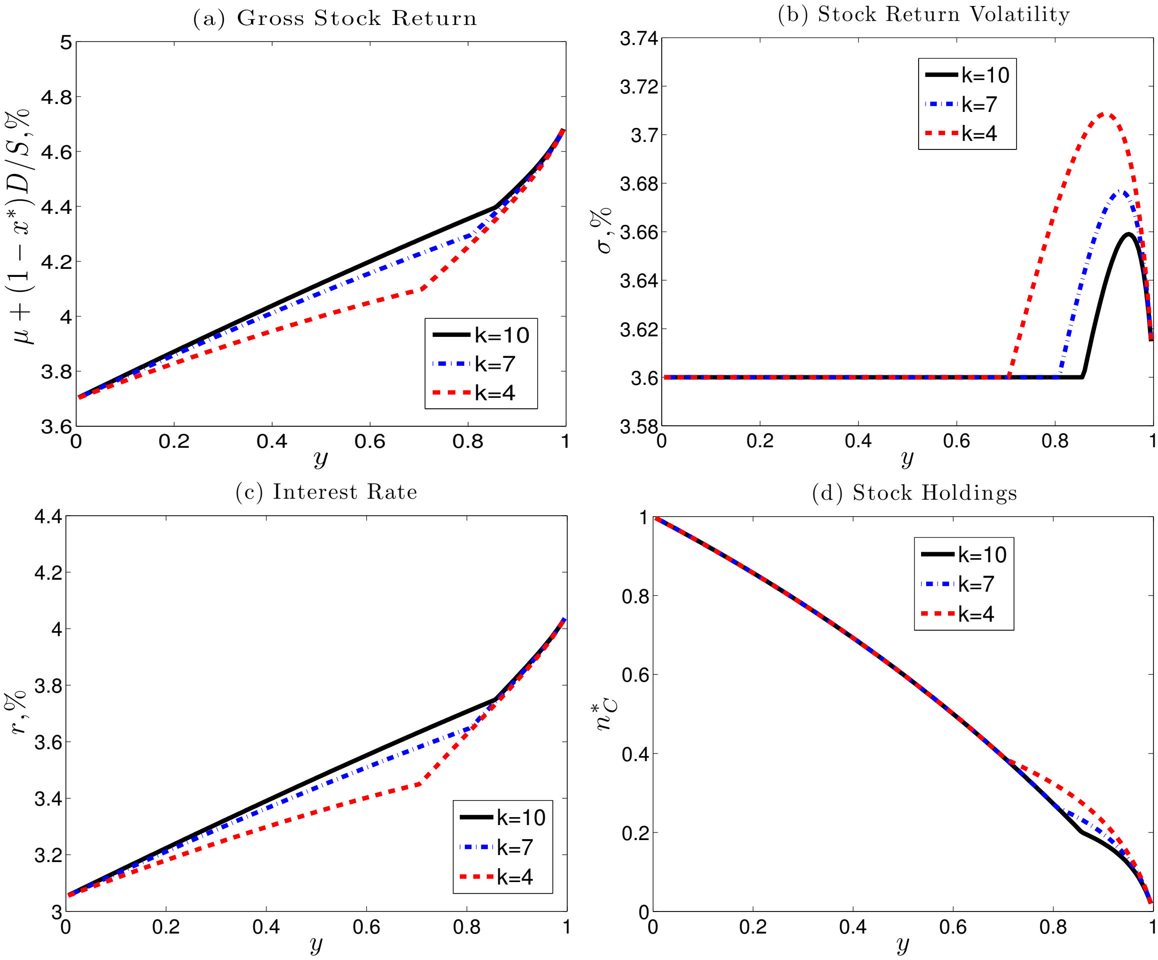


**Figure 4**
**The effect of stealing costs due to social norms on stock gross returns, volatilities, interest rates, and stock holdings**
This Figure shows the equilibrium gross returns $\mu+(1-x^*)D/S$, stock return volatilities $\sigma$, interest rates $r$ and stock holdings $n_C^*$ as functions of consumption share $y$ for fixed investor protection $p = 0.6$ and different levels of cost parameter $k$, for calibrated parameters.

Interestingly, the effect of the cost parameter $k$ on volatilities and stock holdings is present only in economies with sufficiently large minority share $y$, in contrast to the effect of $k$ on gross stock return and interest rate. The intuition for such an asymmetry is as follows. When consumption share $y$ is low, the economy is dominated by the controlling shareholder, and hence, the stock holding of the controlling shareholder $n_C^*$ is high. Therefore, the investor protection constraint $x_t \leq (1-p)n_t$ is not binding, and hence, the economy is in region 2 where the stock holding is the same as in the benchmark economy (i.e., $n_{Ct}^* = \bar{n}_t$) because the controlling shareholder does not have an incentive to steal from himself. Consequently, the leverage given by equation (38) is zero, and hence, equation (39) implies that the volatility is the same as in the benchmark economy because the leverage is the main source of excess volatility in the model.

Furthermore, despite the fact that the stock holding is the same as in the benchmark economy when consumption share $y$ is low, cost parameter $k$ has significant effect on stock return $\mu_t$ and interest rate $r_t$ in region 2 via the last terms in equations (29) and (30), which depend on the fraction of diverted output $x_t^* = (1 - n_{Ct}^*)/k$. The intuition is that despite the fact that $n_{Ct}^* = \bar{n}_t$ when consumption share $y$ is low, the wealth and consumption of the controlling shareholder continue to increase at a higher rate than the wealth and consumption of the minority shareholder due to the diverted cash flow $x_t^* D_t$ in the budget constraint (7) of the controlling shareholder. Therefore, the controlling shareholder extracts higher effective returns from holding stocks, which affects the valuation of stocks and their returns.

Figure 4 reveals an interesting interaction between protection $p$ and cost parameter $k$. In particular, lower $k$ expands region 1 in which the protection constraint is binding. The latter effect can be also observed on Figure 1 and is due to the fact that the controlling shareholder increases stock holding $n_C^*$ beyond the benchmark stock holding $\bar{n}_C$ in region 1 when the cost of stealing is low, which in turn relaxes the protection constraint $x_t \leq (1 - p)n_t$ and makes region 1 wider. Moreover, because the increase in stock holding $n_C^*$ is financed by leverage, the volatility also increases in accordance with equation (39).

# 5. Conclusion

We develop a dynamic asset pricing model where a controlling shareholder can divert a firm's output but is constrained by investor protection and non-pecuniary costs. We demonstrate that in equilibrium the controlling shareholder's asset concentration in the firm is larger with imperfect investor protection. We also find that the stock mean-return and interest rates decrease, while the stock return volatility increases with imperfect protection in equilibrium. Our findings provide support for the empirical evidence on asset prices. We demonstrate that the key to the underlying mechanism of the asset price effects of investor protection are the endogenous dynamic accumulation of control and trading by the controlling shareholder, which are new aspects of our work.

# Appendix: Proofs

**Proof of Proposition 1.** We observe that the controlling shareholder's objective function (11) is a quadratic function of the share of diverted output $x_t$. Maximizing this function with respect to $x_t$ subject to the constraint $x_t \leq (1-p)n_t$, we obtain the optimal fraction of diverted output as $x^*(n_t) = \min\left((1-n_t)/k; (1-p)n_t\right)$. Substituting $x^*(n_t)$ back into the objective function (11), we find that the objective function $J_C(n_t)$ is given by:

$$J_C(n_t) = \begin{cases} J_{C1}(n_t), \; n_t & \leq \; \frac{1}{1+(1-p)k}, \\ J_{C2}(n_t), \; n_t & > \; \frac{1}{1+(1-p)k}, \end{cases} \tag{A.1}$$

where $J_{C1}(n_t)$ and $J_{C2}(n_t)$ are quadratic functions of $n_t$ defined as follows:

$$\begin{aligned} J_{C1}(n_t) \;&=\; \frac{n_t S_t}{W_{Ct}}\Big((\mu_t - r_t) + \Big(1-(1-p)n_t\Big)\frac{D_t}{S_t}\Big) + (1-p)n_t\frac{D_t}{W_{Ct}} \\ &\quad -\frac{k\,(1-p)^2 n_t^2}{2}\frac{D_t}{W_{Ct}} - \frac{\gamma}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2, \end{aligned} \tag{A.2}$$

$$\begin{aligned} J_{C2}(n_t) \;&=\; \frac{n_t S_t}{W_{Ct}}\left((\mu_t - r_t) + \left(1-\frac{1-n_t}{k}\right)\frac{D_t}{S_t}\right) + \frac{1-n_t}{k}\frac{D_t}{W_{Ct}} \\ &\quad -\frac{(1-n_t)^2}{2k}\frac{D_t}{W_{Ct}} - \frac{\gamma}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2. \end{aligned} \tag{A.3}$$

It is immediate to observe that the function $J_{C1}(n_t)$ is a concave function of $n_t$ and achieves a unique global maximum given by $n^*_{Ct,1}$ in equation (15). In contrast, the function $J_{C2}(n_t)$ can be either convex or concave, depending on the cost parameter $k$. It achieves a global maximum or minimum (depending on $k$) at point $n^*_{Ct,2}$ in equation (15). Two other potential points of global maximum are $n^*_{Ct,3} = 1/(1+(1-p)k)$, where $J_{C1}(n_t) = J_{C2}(n_t)$, and $n^*_{Ct,4} = 1$ at which the constraint $n_t \leq 1$ becomes binding. We then determine the global maximum by direct search over points $n^*_{Ct,1}$, $n^*_{Ct,2}$, $n^*_{Ct,3}$, $n^*_{Ct,4}$ to find the point at which the function achieves global maximum, which gives rise to equation (15). The minority shareholder's optimal portfolio (16) is easily obtained by maximizing the quadratic convex objective function (12). ■

**Proof of Lemma 1.** We consider the benchmark economy with full protection. Because there is no stealing in this economy, and hence $x_t = 0$, both investors have the same objective function (12) and their portfolios are given by

$$n^*_{Ct} = \frac{\mu_t - r_t + \frac{D_t}{S_t}}{\gamma\sigma_t^2\frac{S_t}{W_{Ct}}}, \quad n^*_{Mt} = \frac{\mu_t - r_t + \frac{D_t}{S_t}}{\gamma\sigma_t^2\frac{S_t}{W_{Mt}}}. \tag{A.4}$$

Substituting portfolios (A.4) and optimal consumptions $c_i^* = \rho_i W_{it}$ into the shareholders' self-financing budget constraints (7) with $x_t = 0$, we obtain that their wealths under the optimal strategies follow the dynamics:

$$dW_{it} = W_{it}\left[\left(r_t + \frac{\kappa_t^2}{\gamma} - \rho_i\right)dt + \frac{\kappa_t}{\gamma}dw_t\right], \tag{A.5}$$

where $\kappa = (\mu_t - r_t + D_t/S_t)/\sigma_t$. Substituting the shareholders' optimal consumptions $c_i^* = \rho_i W_i$ into the consumption clearing condition (20), we obtain equation $\rho_C W_{Ct} + \rho_M W_{Mt} = D_t$. Applying Itô's Lemma to both sides of the latter equation, matching the $dt$ and $dw$ terms and then dividing both sides of the resulting equations by output $D_t$, we obtain the following system of equations for the interest rate $r$ and Sharpe ratio $\kappa$:

$$(1-y_t)\left(r_t + \frac{\kappa_t^2}{\gamma} - \rho_C\right) + y_t\left(r_t + \frac{\kappa_t^2}{\gamma} - \rho_C\right) = \mu_D, \tag{A.6}$$

$$\frac{\kappa_t}{\gamma} = \sigma_D. \tag{A.7}$$

Solving equations (A.6) and (A.7), we obtain the equilibrium interest rate (23) and Sharpe ratio (25).

Furthermore, the market clearing in the stock and bond markets implies that the aggregate wealth in the economy is equal to the value of the stock market, that is, $W_{Ct} + W_{Mt} = S_t$. Applying Itô's Lemma to both sides of the latter equation and then dividing both sides by $S_t$, we obtain that the stock return volatility is equal to the volatility of the aggregate output: $\sigma_t = \sigma_D$. To derive the stock drift $\mu_t$, from the definition of the Sharpe ratio $\kappa_t$ and the fact that $\kappa_t = \gamma\sigma_D$ and $\sigma_t = \sigma_D$, we find that $\mu_t = \gamma\sigma_D^2 + r_t - D_t/S_t$. The ratio $D_t/S_t$ is found using the market clearing condition $W_{Ct} + W_{Mt} = S_t$ and optimal consumptions $c_i^* = \rho_i W_{it}$ as follows:

$$\begin{aligned} \frac{D_t}{S_t} &= \frac{D_t}{W_{Ct} + W_{Mt}} = \frac{D_t}{c_{Ct}/\rho_C + c_{Mt}/\rho_M} \\ &= \frac{1}{(1-y_t)/\rho_C + y_t/\rho_M}. \end{aligned} \tag{A.8}$$

Substituting the ratio (A.8) and interest rate (23) into expression $\mu_t = \gamma\sigma_D^2 + r_t - D_t/S_t$, we obtain equation (22) for the drift process $\mu_t$.

We then obtain the optimal portfolios (28) by substituting the equilibrium processes into equations (A.4). Processes $\mu_t$, $r_t$, and $\sigma_t$ are substituted from (22)–(24), ratio $D_t/S_t$ is substituted from (A.8), and the ratios $S_t/W_{Ct}$ and $S_t/W_{Mt}$ are determined analogously to

$D_t/S_t$ as follows:

$$\begin{aligned}\frac{S_t}{W_{Ct}} &= \frac{W_{Ct}+W_{Mt}}{W_{Ct}} = \frac{c_{Ct}/\rho_C + c_{Mt}/\rho_M}{c_{Ct}/\rho_C} \\ &= \frac{(1-y_t)/\rho_C + y_t/\rho_M}{(1-y_t)/\rho_C},\end{aligned} \tag{A.9}$$

$$\begin{aligned}\frac{S_t}{W_{Mt}} &= \frac{W_{Ct}+W_{Mt}}{W_{Mt}} = \frac{c_{Ct}/\rho_C + c_{Mt}/\rho_M}{c_{Mt}/\rho_C} \\ &= \frac{(1-y_t)/\rho_C + y_t/\rho_M}{y_t/\rho_M}.\end{aligned} \tag{A.10}$$

Finally, we obtain the drift $\mu_{yt}$ and volatility $\mu_{yt}$ of consumption share $y_t$. By its definition, the consumption share is given by $y_t = c_{Mt}/D_t$. Because $c_{Mt} = \rho_M W_{Mt}$, the consumption share can be rewritten as $y_t = \rho_M W_{Mt}/D_t$, where $W_{Mt}$ follows the process (A.5). Applying Itô's Lemma to both sides of the equation for consumption share $y_t$ and matching $dt$ and $dw$ terms, we obtain the expressions (26) and (27) for $\mu_y$ and $\sigma_y$. ■

**Proof of Proposition 2.** Equation (29) for the mean-stock return $\mu_t$ follows readily from the definition of the Sharpe ratio $\kappa_t = (\mu_t - r_t + (1-x_t^*)D_t/S_t)/\sigma_t$ and the expression (A.8) for the dividend-stock price ratio $D_t/S_t$. Next, we derive the interest rate $r_t$. The wealths of the controlling and minority shareholders satisfy the budget constraints (7). Applying Itô's Lemma to both sides of the consumption clearing condition $\rho_C W_{Ct} + \rho_M W_{Mt} = D_t$, matching $dt$ and $dw$ terms, and dividing both sides by $D_t$, we obtain the equations:

$$(1-y_t)\left(r_t - \rho_C + \frac{n_{Ct}^* S_t \sigma_t \kappa_t}{W_{Ct}} + x_t^* \frac{D_t}{W_{Ct}}\right) + y_t\left(r_t - \rho_M + \frac{n_{Mt}^* S_t \sigma_t \kappa_t}{W_{Mt}}\right) = \mu_D, \tag{A.11}$$

$$(1-y_t)\frac{n_{Ct}^* S_t \sigma_t}{W_{Ct}} + y_t \frac{n_{Mt}^* S_t \sigma_t}{W_{Mt}} = \sigma_D. \tag{A.12}$$

Using equation (A.12), we simplify equation (A.11) and obtain the interest rate $r_t = \mu_D + \rho_C(1-y_t) + \rho_M y_t - \sigma_D \kappa_t - (1-y_t)x_t^* D_t/W_{Ct}$. The ratio $D_t/W_{Ct}$ is given by $D_t/W_{Ct} = \rho_C D_t/(\rho_C W_{Ct}) = \rho_C D_t/c_{Ct} = \rho_C/(1-y_t)$. Substituting $D_t/W_{Ct}$ into the latter expression for $r_t$, we obtain expression (30). Furthermore, from equation (A.12), we obtain the following equation for volatility $\sigma_t$:

$$\sigma_t = \frac{\sigma_D}{(1-y_t)\dfrac{n_{Ct}^* S_t}{W_{Ct}} + y_t \dfrac{n_{Mt}^* S_t}{W_{Mt}}}. \tag{A.13}$$

Substituting the ratios $S_t/W_{Ct}$ and $S_t/W_{Mt}$ from (A.9) and (A.10), respectively, and $n_{Mt}^* = 1 - n_{Ct}^*$ into equation (A.13), we obtain equation (31) for volatility $\sigma_t$.

Next, we find the Sharpe ratio $\kappa_t$. From equation (16) for the optimal portfolio of the minority shareholder, we obtain:

$$\kappa_t = \gamma\sigma_t n^*_{Mt}\frac{S_t}{W_{Mt}}.$$

Substituting $n^*_{Mt} = 1 - n^*_{Ct}$ and ratio $S_t/W_{Mt}$ from equation (A.10) into the above equation, after some algebra, we obtain equation (32) for the Sharpe ratio.

To obtain the drift $\mu_y$ and volatility $\sigma_y$ of the consumption share $y_t$, we rewrite consumption share as $y_t = c_M/D = \rho_M W_{Mt}/D_t$. Then, applying Itô's Lemma to both sides of equation $y_t D_t = \rho_M W_{Mt}$, where wealth $W_{Mt}$ follows dynamics (A.5) with $i = M$, and matching $dt$ and $dw$ terms, we obtain the following system of equations for the processes $\mu_y$ and $\sigma_y$:

$$\mu_{yt} + \mu_D + \sigma_D\sigma_{yt} = r_t + \frac{\kappa_t^2}{\gamma} - \rho_M, \tag{A.14}$$

$$\sigma_{yt} = \frac{\kappa_t}{\gamma} - \sigma_D. \tag{A.15}$$

Equation (A.15) immediately gives the volatility $\sigma_y$ in (34). Substituting $r_t$ from (30) and $\kappa_t/\gamma$ from (A.15) into equation (A.14), after simple algebra, we obtain equation (33) for the drift $\mu_{yt}$.

The controlling shareholder's optimization problem (11) implies the fixed-point equation (35) for the optimal stock holding $n^*_C$, in which the equilibrium processes depend on $n^*_C$ itself. The ratios $D_t/S_t$ and $S_t/W_{Ct}$ in (36) are derived in the same way as in equations (A.8)–(A.10). The ratio $D_t/W_{Ct}$ in (36) is obtained by multiplying ratios $D_t/S_t$ and $S_t/W_{Ct}$. ■

**Lemma A.1 (Equilibrium stock holding).** *The optimal stock holding $n^*_{C,t}$ of the controlling shareholder in equilibrium has the following representation:*

$$n^*_{Ct} = \begin{cases} n^*_{Ct,1} = n^*_{Ct,2} + \dfrac{(1-p)}{\gamma\sigma_D^2}\left(1 - \dfrac{n^*_{Ct,1}}{n^*_{Ct,3}}\right)\big(\rho_C n^*_{Ct,1} + \rho_M(1-n^*_{Ct,1})\big)^2, & \text{(region 1)}, \\ n^*_{Ct,2} = \dfrac{\rho_M/y_t}{\rho_M/y_t + \rho_C/(1-y_t)}, & \text{(region 2)}, \\ n^*_{Ct,3} = \dfrac{1}{1+(1-p)k}, & \text{(region 3)}, \\ n^*_{Ct,4} = 1; & \text{(region 4)}, \end{cases} \tag{A.16}$$

*Furthermore, for all consumption shares $y \in [0,1]$ there always exists $n^*_{Ct,1} \in [0,1]$, which solves a third degree polynomial equation.*

**Proof of Lemma A.1.** To obtain stock holding $n^*_{Ct,1}$ in region 1, we observe that by the definition of region 1 $n^*_{Ct,1}$ satisfies the following F.O.C. from the objective function $J_{C1}(n_t)$ in equation (A.2):

$$n^*_{Ct,1} = \frac{\kappa_t}{\gamma(S_t/W_{Ct}\sigma_t)} + \frac{(1-p)D_t/W_{Ct}(1-n^*_{Ct,1}-k(1-p)n^*_{Ct,1})}{\gamma\left(S_t/W_{Ct}\sigma_t\right)^2}. \tag{A.17}$$

Substituting the expressions for $\kappa_t$, $\sigma_t$, $D_t/W_{Ct}$ and $S_t/W_{Ct}$ from Proposition 2 into equation (A.17), after straightforward algebra, we obtain:

$$n^*_{Ct,1} = \frac{\rho_M/y_t}{\rho_M/y_t+\rho_C/(1-y_t)} + \frac{(1-p)}{\gamma\sigma_D^2}\left(1-n^*_{Ct,1}(1+k(1-p))\right)\left(\rho_C n^*_{Ct,1}+\rho_M(1-n^*_{Ct,1})\right)^2. \tag{A.18}$$

It can be easily verified that the left-hand side of equation (A.18) is lower than its right-hand side for $n^*_{Ct,1}=0$, and vise versa for $n^*_{Ct,1}=1$. Therefore, by the intermediate value theorem, equation (A.18) has a solution $n^*_{Ct,1}\in[0,1]$, which can be found by a method of bisection.

Similarly, we find the stock holding $n^*_{Ct,2}$ in region 2 by substituting $\kappa_t$, $\sigma_t$, $D_t/W_{Ct}$ and $S_t/W_{Ct}$ from Proposition 2 into the expression for $n^*_{Ct,2}$ in (15):

$$n^*_{Ct,2} = \frac{\rho_M/y_t}{\rho_M/y_t+\rho_C/(1-y_t)}. \tag{A.19}$$

We note that $n^*_{Ct,3}$ and $n^*_{Ct,4}$ remain the same as in (15). ■

**The Numerical Method.** First, we derive the optimal stock holding $n_{Ct}$ as a function of the consumption share $y$ in each of the regions 1, 2, 3, and 4 in equation (15). Lemma A.1 above demonstrates the existence of the stock holding $n^*_{Ct}$ and derives it as an implicit function of the consumption share $y_t$, which can easily be computed by solving a third-degree polynomial equation.[12] Then, for each value of the consumption share $y$, we substitute $n^*_{Ct,1}$, $n^*_{Ct,2}$, $n^*_{Ct,3}$, and $n^*_{Ct,4}$ in turn into the objective function on the right-hand side of (35) and find the element that maximizes the objective function. This way, we obtain the optimal stock holding $n^*_{Ct}$. Substituting $n^*_{Ct}$ into the expressions for equilibrium processes in Proposition 2, we obtain all equilibrium processes as functions of the consumption share $y$.

[12] As demonstrated in Lemma A.1, this polynomial always has a solution in the interval $[0,1]$. Although in general this solution may not be unique, we only obtain unique solutions in our calibration of the model. Furthermore, because the objective function (A.1) may have regions of non-concavity, the optimal portfolio holding $n^*_{Ct}$ may appear to be a discontinuous function of $y$. However, such a discontinuity requires extreme values of exogenous model parameters and does not occur in our calibration of the model.

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