Competitive Search Equilibrium and Moral Hazard*

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Abstract

Principals seek to enter a productive relationship with agents by posting general incentive contracts. A contract must solve both the ex post moral hazard in production and the ex ante competitive search problem, so it requires rents and introduces a trade off between incentives and participation. To generate rents, principals distort the contingent transfer function, which weakens the incentives and therefore the induced productive action. To mitigate this distortion the optimal contract also includes compensatory transfers to agents who meet a principal but fail to contract. The allocation is Pareto optimal conditional on the action but *neither* output maximizing, precisely because the equilibrium action is distorted, *nor* welfare efficient. A planner is immune to principal competition and so impervious to the interaction between directed search and contracting. Throughout we connect our results to known formulae of the search literature, such as Hosios' condition.

Keywords: moral hazard, asymmetric information, contracts, directed search, search frictions, constrained efficiency. JEL Classification: D82, D83, D86.

1 Introduction

We study optimal contracts and the efficiency of equilibrium allocations in an economy with moral hazard and matching frictions in a competitive search model (Peters, 1991; McAfee 1993; Moen 1997; Shimer, 1996; Julien, Kennes and King, 2000; Burdett Shi and Wright, 2001). Specifically

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we seek to understand *(i)* the effect of market interaction on contract design under moral hazard, *(ii)* the impact of search frictions on the optimal contract and *(iii)* the impact of moral hazard on allocative efficiency under search frictions. This work informs contract design under moral hazard when a (frictional) *market* dictates the agents' outside options. It also complements the search literature by extending it to an economy with moral hazard. In doing so we overturn some known results.

Our model is useful to study several important economic situations. A primary example is the labor market; firms hire workers by posting contracts and output depends stochastically on labor input (action) that is unobservable by the employer. In financial contracting, the success of the borrower's project may depend on her action, which is not observable, and borrowers search for the best terms of trade among competing lenders. This is a natural extension of the classical Holmström-Tirole model (1997).

Principals post the terms of trade non-cooperatively and agents choose to direct their search based on the observed terms of trade. "Searching" means finding the best probability distribution over the set of contract offers. Upon contracting a principal and an agent enter into a productive relationship; the surplus depends on the unobservable effort provided by the agent. This, and directed search, are critical features. This formulation allows us to study a rich set of meeting-contingent contracts that can be posted by principals. In this general framework we obtain novel results. The most striking result is a failure of efficiency that persists across information structures.

In a frictional market a contract must both solve the incentive problem and direct the agents' search by generating rents. Contract design must accommodate this fact. With risk-averse agents, rent-giving is most cheaply accomplished by improving the insurance properties of the contract. Hence the optimal contract includes the usual contingent transfer function but also compensatory transfers paid to the agents meeting, but not contracting with, a principal. These transfers provide insurance but have no direct bearing on the effort decision; they help generate the rents necessary to attract agents while minimizing the distortions of the effort-inducing transfer function. Nonetheless, even with these compensatory transfers incentives are weakened (compared to the standard second-best). That is, a distortion in the output-contingent transfer function is also used to generate rents, which weakens incentives. Principals must trade off incentives and attracting agents, hence the mix of compensatory transfers and incentive transfer function are optimized.

This trade off between incentives and participation leads to our main efficiency result. The equilibrium allocation is *never* (constrained) welfare-efficient. It is inefficient both at the intensive (effort choice) and extensive (entry) margins. Even though we recover a Hosios condition that characterizes (constrained) Pareto optimality, we establish it does not imply (constrained) welfare efficiency. Efficiency fails because the social surplus depends on the agent's action, but principal competition for agents always distorts it. A planner can restore efficiency because he is constrained by the *equilibrium level* of frictions, but not by the *incentives* of directed search. In contrast, Acemoglu and Simsek (2010) show approximate efficiency in a general equilibrium model with moral hazard (so with an endogenous surplus). Crucially however there are no search frictions in their model, so no such trade off between incentive and participation. Here the loss of efficiency is severe in that there exists no corrective instrument that can fully restore efficiency. Restoring efficiency requires rendering the contracting problem independent of the search problem; that is, search cannot be directed. This is exactly what a planner does: he turns the directed search model into a random search model (with the same level of frictions), in which contracting is orthogonal to search.

We explore the trade off between incentives and participation arising from search frictions in details and show it is common to problems with an endogenous surplus and directed search. It is a general property that generating the rents necessary to attract agents is best achieved by using the two available margins: transfer and action. It is the interaction of the endogenous surplus under delegation and directed search that engenders inefficiency in our model, *even under symmetric information.*

Last we study a special case of our general formulation when compensatory transfer are prohibited. The contract is more distorted and the allocation even less efficient than with transfers. In an extension we further discuss the role of these compensatory transfers and confirm they are always helpful, even if they are constrained.

Returning to our examples, in the labor market the contract affects the unobservable hours worked (intensive margin), as well as workers' probability to find a job (extensive margin) via free entry of firms. Our results then suggest that firms may compete too hard for labor, workers may not work enough, there may be too many vacancies or conversely, workers may work too hard, not enough firms enter the market and there may be involuntary unemployment. In financial contracting, the transfer affects the action, hence the probability of repaying the loan, as well as the probability of making an investment in the first place. When the entry cost is high, search frictions induce weaker incentives and therefore a lower action in equilibrium, which implies a lower probability of repayment. In turn principals are less likely to extend a loan; search frictions may lead to credit rationing.

The works closest to ours are Moen and Rosen (2011) and Acemoglu and Shimer (1999). Moen and Rosen (2011) adapt the Laffont-Tirole (1986) model to a search framework. It is a model "false moral hazard" (to use Laffont and Tirole's terminology): effort is first-best conditional on the agent's information rent (no direct effort distortion), so the model reduces to one of adverse selection. Moen and Rosen (2011) find the equilibrium to be constrained efficient, unlike in this work. In their model search is not directed; matching is random instead and so orthogonal to contracts – like our planner. Acemoglu and Shimer (1999) study efficient unemployment insurance in a search and matching framework where firms post wages. Unemployment insurance induces workers to search for high-wage jobs; in response firms improve job attributes (wages and capital/labor ratio). Output is maximized with a measure of unemployment insurance: it induces the best capital/labor ratio, which firms must adjust to generate the rents to overcome workers' outside option. In our model compensatory transfers act like unemployment insurance, but they are set by the principal to attract agents (hence fully internalized). Absent compensatory transfers, attracting agents requires more distortion of the (incentive) transfer function because the lottery over rents is worse. This distortion induces a lower productive action and so is not optimal, analogously to the capital/labor ratio in Acemoglu and Shimer (1999). However the choice of action is left to the agent (not the firm) and is taken ex post. In Acemoglu and Shimer (1999) the action (the capital/labor ratio) is selected ex ante by the firm, which is the residual claimant.

We also contribute to competitive search under informational asymmetries by introducing moral hazard in a search model. Guerrieri (2008) considers adverse selection with frictions, and shows the dynamic equilibrium is inefficient because the agent's outside option changes over time. Guerrieri, Shimer and Wright (2010) study competitive search under adverse selection and find that the competitive equilibrium is not constrained-efficient because the joint surplus is affected by allocative distortions. So informational frictions rather than search frictions, lie at the source of this inefficiency; but thanks to the interaction with search frictions, informational frictions have implications at the extensive margin. Here instead search frictions induce effort distortion (even under symmetric information) because principals trade off transfer and action.

Finally this work contributes to better understand the moral hazard problem in that it endogenizes the relevant outside option. "Participation" really means entering the search process. That decision is made on the basis of the array of contract offers, not any exogenously given outside option. With search frictions the contract must be augmented to include compensatory transfers.

In the next section we specify the model and re-state the benchmark model. Sections 3 and 4 are the heart of the paper. We analyse the search problem and characterize the equilibrium; then we derive welfare implications. We suggest a discussion in Section 5. All proofs and some supplementary material can be found in the Appendix.

2 Model and benchmarks

2.1 The model

The economy is populated by a measure N of homogenous agents and a measure M of homogenous principals, with aggregate market tightness $\Theta = \frac{N}{M} < \infty$. Principals seek to form bilateral relationships with agents subject to moral hazard. An agent's utility is $u(t) - c(a)$, with $u(\cdot)$ increasing and concave; t is the transfer received and $a \in \mathcal{A} \subset \mathbb{R}$ the chosen action at cost $c(a)$ increasing and convex (for example, hours worked). Action a governs the distribution $F(x|a)$ of outcomes $x \in \mathcal{X} \equiv [\underline{x}, \overline{x}] \subset \mathbb{R}$, with density $f(x|a) > 0$. The likelihood ratio f_a/f is increasing, concave in x, hence $F(x|a) < F(x|a')$ for $a' < a$. We also impose that $F_a(F^{-1}(q,a)|a)$ be convex in (q,a), which is sufficient for Concave Local Informativeness (CLI) (see Jewitt, Kadan and Swinkels, 2008 – now JKS). The function $t(x) \in \mathcal{T}$, which is taken to be equicontinuous as in Holmström (1977, 1979).[1] Throughout we suppose that the first-order approach to the agency set up is valid, see Jewitt (1988) for details.

Market interaction. Principals compete by offering terms of trade (contracts) to attract agents, and agents select over principals after observing the terms of trade. Each agent can work for at most one principal, and each principal only needs one agent. Equivalently, principals are capacity

[1]These are, essentially, smooth functions. Examples include Lipschitz continuous functions and C^1 functions. This technical condition guarantee existence of a maximum; details can be availed from the authors upon request.

constrained in their ability to contract. This restriction implies the use of a rationing rule. Under contract posting there is no loss in restricting attention to uniform rationing.[2] Principals can fully commit to these contracts.

Search is directed by the principals' offers, with two effects. First, any one contract affects the others, however only through the search process. Competition directly affects the participation problem only, not the incentive problem (it does indirectly). Bilateral contracting rules out team formation and any incentive provision based on teams. Second, agents formulate a probability distribution over principals but are not ubiquitous; they can only visit one. So the extensive form rules out common agency. This stands in contrast to the works of Attar *et al* (2006, 2007a, 2007b), Martimort (2004), Aubert (2005) or Célérier (2012), for example. In each of these, an agent may be party to more than one contract at once; this decision interferes with both the participation and the moral hazard constraint.

Contracts cannot depend on the identity of agents – this is innocuous since agents are homogeneous. Otherwise they are as general as can be. Denote one such contract by $\mathcal{C}^j = \left(\mathbf{t}^j(x), \mathbf{a}^j, \mathbf{h}^j\right)$, $j \in M$, with $\mathbf{t}^j(x) = (t_1^j(x), ..., t_N^j(x)), \mathbf{a}^j = (a_1^j, ..., a_N^j), \mathbf{h}^j = (h_1^j, ..., h_N^j)$. $\mathcal{C}^j$ offers arrays of transfers $\mathbf{t}_n^j(x), \mathbf{h}^j$ and prescribes an array of actions $\mathbf{a}^j$ that may depend on the number n of agents showing up; these are *meeting contingent contracts.* The transfer functions $\mathbf{t}^j(x)$ depend on the outcome x and $\mathbf{h}^j$ are transfers paid to (from) agents approaching j but unable to contract with him. It is evident they must be independent of x.

We adopt the competitive search version of submarkets akin to the one in Moen (1997) and Mortensen and Wright (2002)). Principals post the terms of trade in any submarkets. The timing is as follows:

1. principals posting similar contracts form a submarket;
2. agents observe all contracts (all submarkets) and select a submarket to participate in;
3. in a submarket principals and agents meet according to some meeting technology;
4. upon meeting, if an agent accepts a contract, she selects an action;
5. payoffs are realized. Agents not meeting anyone receive u_0; agents not contracting receives

[2]Details of this claim available upon request.

the payoff $u(h_n^j + y)$ where $y = u^{-1}(u_0)$ and u_0 is an exogenous outside option; principals not matching receive 0.

Meeting technology. We make a distinction between *meeting* and *matching* (contracting). We can restrict attention to non-rival meeting without loss of generality because contracting is rival (bilateral).[3]

Let $\mu_N(C)$ and $\mu_M(C)$ be positive measures of agents and principals active in a submarket, and let $\theta = \mu_N(C)/\mu_M(C)$ be the local market tightness prevailing in that submarket under contract $\mathcal{C}$. Let $p_n(\theta)$ be the probability that n agents meet a principal in submarket k. Similarly let $q_n(\theta),\ n \geq 1$ be the probability an agent meets a principal with n agents, including herself; for example, $q_1(\theta)$ is the probability an agent is in a pairwise meeting with a principal. Because search is directed $q_0(\theta)$ is not defined: all agents meet a principal if they participate for any strategy they follow (they may not match). By definition $\sum_{n=0}^{\infty} p_n(\theta) = \sum_{n=1}^{\infty} q_n(\theta) = 1$.

We follow the standard assumption that $p_0'(\theta) < 0$: more agents reduces the probability for principals to not meet anyone, and $p_0''(\theta) > 0$. We also suppose that $\sum_{n=1}^{\infty} p_n'(\theta) \geq 0$: in the aggregate, increasing the queue length increases the chance a principal meets any number of agents. Similarly, $\forall n \geq 1, q_n'(\theta) < 0$: more agents reduces the probability for exactly n agent(s) to meet a principal, and so $\sum_{n=1}^{\infty} q_n'(\theta) \geq 0$. The meeting rate for a principal is $\sum_{n=1}^{\infty} p_n(\theta)$: the probability to meet at least one agent. For an agent the meeting and matching rate is $\sum_{n=1}^{\infty} q_n(\theta)\frac{1}{n}$, including the agent. The meeting technology is homogenous of degree one so a consistency requirement links $p_n(\theta)$ and $q_n(\theta)$. Given N agents and M principals in a submarket

$$\forall\ n > 0,\quad \mu_M p_n(\theta) = \frac{\mu_N q_n(\theta)}{n} \Rightarrow n p_n(\theta) = \theta q_n(\theta),$$

which implies

$$\sum_{n=1}^{\infty} p_n(\theta) = \theta \sum_{n=1}^{\infty} q_n(\theta)\frac{1}{n}.$$

Thus, the meeting and matching rate for an agent is

$$\sum_{n=1}^{\infty} q_n(\theta)\frac{1}{n} = \frac{\sum_{n=1}^{\infty} p_n(\theta)}{\theta} = \frac{1 - p_0(\theta)}{\theta}.$$

[3]A rival meeting technology induces a special case of our results; we discuss this point further below.

A common example of this construction is the Poisson distribution:

$$p_n(\theta) = \frac{\theta^n e^{-\theta}}{n!} \text{ and } q_n(\theta) = \frac{\theta^{n-1} e^{-\theta}}{(n-1)!}$$

Payoffs and equilibrium. Upon contracting with one of the n agents he meet, principal j receives

$$\pi(t_n^j, a_n^j) \equiv \int_{\mathcal{X}} \left[z - t_n^j(z)\right] dF(z|a_n^j).$$

For an agent, upon meeting principal j with $n-1$ other agents, and contracting with that principal,

$$U(t_n^j, a_n^j) \equiv \int_{\mathcal{X}} u(t_n^j(z)) dF(z|a_n^j) - c(a_n^j).$$

Prior to the meeting process, the ex ante payoff for principals is given by

$$\Pi(\mathcal{C}^j) \equiv \sum_{n=1}^{\infty} p_n(\theta) \left[\pi(t_n^j, a_n^j) - h_n^j\right], \quad h_1^j = 0.$$

An agent selecting principal j faces a probability distribution over the number of other agents visiting the same principal. Given bilateral agency formations and uniform rationing, with probability $1/n$ the agent matches (gets the contract); otherwise she receives $u(h_n^j + y)$, hence

$$V(\mathcal{C}^j) \equiv \sum_{n=1}^{\infty} q_n(\theta) \left[\frac{1}{n} U(t_n^j, a_n^j) + \left(1 - \frac{1}{n}\right) u(h_n^j + y)\right],$$

Definition 1 *An equilibrium is a tuple* $\left(\widehat{\mathcal{C}}, \widehat{\theta}(\mathcal{C}), \widehat{\mathbf{a}}\right)$ *of contracts, market tightness induced by these contracts and actions, such that*

- *for each principal in submarket* j, $\widehat{\mathcal{C}^j} \in \arg\max_{\mathcal{C}^j} \Pi(\mathcal{C}^j)$*;*
- *agents optimally select submarkets:* $j \succeq k \Leftrightarrow V(\mathcal{C}^j) \geq V(\mathcal{C}^k)$*; and*
- *for each agent contracting with a principal,* $a \in \arg\max_{a'} \; U(t, a')$ *given contract* $\mathcal{C}^j$

Without loss we look for symmetric, subgame-perfect equilibria of this game – as is implicitly assumed in the definition of $\Pi(\mathcal{C}^j)$. This is discussed further Appendix B, where we provide game-theoretic foundations to our large-market model.

2.2 Two benchmarks

Throughout we will refer to the canonical model, which features a single agent and a single principal.

2.2.1 The standard agency problem.

The payoff to the principal and the agent are respectively:

$$\begin{aligned}\pi(t,a) &\equiv \int_{\mathcal{X}} [z - t(z)]\, dF(z|a) \\ U(t,a) &\equiv \int_{\mathcal{X}} u(t(z))dF(z|a) - c(a).\end{aligned}$$

The principal maximizes $\pi(t,a)$ by choice of the contract $(t(x),a)$, subject to $U(t,a) \geq u_0$ and $U_a = 0$ and with u_0 known. The solution $(t^{SB}(x), a^{SB})$ is characterized by the conditions

$$\frac{1}{u'} = \lambda + \mu \frac{f_a}{f} \tag{2.1}$$

and

$$\pi_a + \mu U_{aa} = 0 \tag{2.2}$$

where $\lambda, \mu > 0$ are Lagrange multipliers, together with the two aforementioned constraints (see Holmström, 1979; Jewitt, 1988). We note that $\lambda \equiv \lambda(u_0) > 0$ means that the participation cost is determined in terms of the agent's outside option only.

2.2.2 The planner's solution.

A planner maximizes

$$W(t,a) \equiv \pi(t,a) + U(t,a)$$

and is subject to the same constraints. A solution $(t^W(x), a^W)$ is characterized by the conditions

$$\frac{1}{u'} - 1 = \lambda + \mu \frac{f_a}{f} \tag{2.3}$$

and

$$\pi_a + \mu U_{aa} = 0 \tag{2.4}$$

where the Lagrange multiplier λ^W needs not be positive. Because $\mathbb{E}\left[\frac{1}{u'}\right] = 1 + \lambda \geq 1$ the planner always engages in some redistribution when the agent is risk-averse.[4] Finally we note that the principal's problem is one of *output* maximization, which we return in Section 4.

[4]Simply suppose that $u_0 = 0$ and that $u(0) = 0,\ u'(0) = \infty$.

3 Competitive search and moral hazard

In a large economy, when principals deviate in a submarket the deviation does not affect the maximum expected utility agents receive by participating in any contracts offered by non-deviating principals. This is the *market utility property* (MUP), as used by McAfee (1993), Shimer (1996) and Moen (1997).[5] Let $\mathcal{C}$ be the symmetric contract posted in all other submarkets and yielding expected utility $\widetilde{V}$ defined as

$$\widetilde{V} = \max_{\mathcal{C}} V(\mathcal{C}).$$

With this, and the restriction to symmetric equilibria, we can formulate the problem as one of constrained optimization. Since all principals j in a submarket post the same contract, we now refer to submarket j. The principals in submarket j then solve:

Problem 1

$$\max_{\mathcal{C}^j,\theta} \ \Pi(\mathcal{C}^j) \ s.t.$$

$$U(t_n^j, a_n^j) \geq u(h_n^j + y), \quad \forall n \tag{3.1}$$

$$U_{a_n^j}(t_n^j, a_n^j) = 0, \quad \forall n \tag{3.2}$$

$$V(\mathcal{C}^j) \geq \widetilde{V} \tag{3.3}$$

$$h_n^j \geq 0, \quad \forall n. \tag{3.4}$$

Constraints (3.1) and (3.4) together imply $U(t_n^j, a_n^j) \geq u_0$; (3.1) requires the agent should prefer accepting the contract rather than just receiving the compensatory transfer. Condition (3.3) is the MUP; it is also a participation constraint, where participation is meant as participating in the search process. (3.2) is the moral hazard constraint.[6] Even with this simplification, Problem 1 is cumbersome because the transfers $\mathbf{t}$ and $\mathbf{h}$ depend on the number n of meeting agents. Thankfully,

Proposition 1 *All principals use a unique set of transfers that induce a unique effort level; that is,* $\forall\ j\ , t_n^j(x) = t^j(x), h_n^j = h^j$ *and* $a_n^j = a^j$.

[5] In the Appendix we provide the finite market set up along with the limit equilibria as a foundation for the large market construct. See also Peters (2000), Burdett, Shi and Wright (2001), Julien, Kennes and King (2000), Galenianos and Kircher (2012) and Norman (2015) for more details.

[6] It is technically easier to solve the problem with constraint (3.4), which we show to be slack.

Proposition 1 is substantive and simplifying. It is substantive in that it claims the optimal contract is independent of the actual number n of agents meeting a principal. Selcuk (2012) shows that using n-contingent transfers amounts to exposing agents (buyers) to a lottery over payoffs, which is costly with risk-averse agents. We show that the *principals* prefer a single contract because it minimizes the cost of implementing their preferred action. That cost is convex and increasing in the action (equivalently, the principals' payoffs are concave decreasing), so principals are better off avoiding a lottery over actions. This result does not invalidate Selcuk's, which may well apply here; rather it complements it. Proposition 1 is also simplifying: it is sufficient to limit attention to n-invariant contracts. With a unique transfer function and uniform rationing, the only relevant events are whether the agent matches with any principal. Recalling

$$\sum_{n=1}^{\infty} p_n(\theta) = 1 - p_0(\theta) \text{ and } \sum_{n=1}^{\infty} \theta \frac{q_n(\theta)}{n} = 1 - p_0(\theta)$$

thanks to the consistency requirement, an agent's expected utility is now independent of n and reads

$$V^j(\mathcal{C}^j) = \frac{1-p_0(\theta)}{\theta} U(t^j, a^j) + \left(1 - \frac{1-p_0(\theta)}{\theta}\right) u(h^j + y), \tag{3.5}$$

and the number of agents receiving h^j (do not contract is)[7]

$$\begin{aligned} \sum_{n=2}^{\infty} p_{n-1}(n-1) &= \sum_{n=1}^{\infty} p_n \theta \\ &= \theta. \end{aligned}$$

Since we focus on symmetric equilibria, we drop the superscript j; the program is

Problem 2

$$\max_{t(x),h,a} [1 - p_0(\theta)] \int [x - t(x)] dF(x|a) - \theta h$$

s.t.

$$\int_{\mathcal{X}} u(t(z)) dF(z|a) - c(a) \geq u(h + y) \tag{3.6}$$

$$\frac{1-p_0(\theta)}{\theta} U + \left[1 - \frac{1-p_0(\theta)}{\theta}\right] u(h+y) \geq \widetilde{V} \tag{3.7}$$

$$\int_{\mathcal{X}} u(t(z)) dF_a(z|a) - c'(a) = 0 \tag{3.8}$$

$$h \geq 0. \tag{3.9}$$

[7]This property is used in large Poisson game: with a large population it does not matter when the observing agent is in or out. In the Poisson example, $\sum_{n=2}^{\infty} P_{n-1}(n-1) = \sum_{n=2}^{\infty} \frac{\theta^{n-1} e^{-\theta}}{(n-1)!}(n-1) = \sum_{n=2}^{\infty} \frac{\theta \theta^{n-2} e^{-\theta}}{(n-2)!} = \theta$.

Proposition 2 *The solution* $(t^S(x), a^S, h^S)$ *to Problem 2 is characterized by the necessary and sufficient first-order conditions*[8]

$$\int_{\mathcal{X}} [z - t(z)] dF_a(z|a) + \widehat{\mu} \left[\int_{\mathcal{X}} [z - t(z)] dF_{aa}(z|a) - c''(a) \right] = 0 \tag{3.10}$$

$$\frac{1}{u'(t)} = \frac{1}{u'(h+y)} \frac{1}{1 - \frac{1-p_0}{\theta}} + \hat{\mu} \frac{f_a}{f} \tag{3.11}$$

$$h = -p_0' \pi - \frac{1 - p_0 + \theta p_0'}{1 - \frac{1-p_0}{\theta}} \frac{U - u(h+y)}{u'(h+y)} > 0 \tag{3.12}$$

with $\hat{\mu} = \frac{\mu}{1-p_0(\theta)} > 0$ *and* $\theta = \Theta$*. The constraints (3.6) and (3.9) are slack but (3.7) binds.*

Expression (3.10) is standard in a moral hazard problem. It results from subgame perfection: for any transfer $t(x)$ a principal offers, the agent chooses the action that is optimal for her – after contracting and knowing the transfer function $t(x)$. Thus search does not (directly) enter this equation. Condition (3.11) shows that the slope of the transfer is related to the likelihood ratio f_a/f, as we know from Holmström (1977, 1979); this is what generates the incentives for effort. From (3.11) and (3.12) we see $t(x)$ and h are jointly determined; hence $t(x)$ is always distorted.

Constraint (3.6) is not active in Problem 2 (therefore $U > u(h+y) > u_0$) because the principal has to contend with a different problem than in the standard model. Here, meeting at least one agent occurs with probability $1 - p_0(\theta)$ only. Hence principals face a trade-off between incentives and participation probability.[9]

Proposition 2 shows that principal competition affords the agent some form of effective bargaining power, i.e. $U > u(h+y) > u_0$ in equilibrium; that bargaining power attracts compensatory transfers h. We explore the role of these transfers h in more detail in Section 5. For now we relate this characterization to known results in the search literature. Denote by

$$\eta(\theta) = \frac{\partial(1 - p_0(\theta))}{\partial\theta} \frac{\theta}{1 - p_0(\theta)}$$

the principals' match elasticity, and integrate Condition (3.11) over $\mathcal{X}$

$$\mathbb{E}\left[\frac{1}{u'(t)}\right] = \frac{1}{u'(h+y)} \frac{1}{1 - \frac{1-p_0(\theta)}{\theta}}.$$

[8]The characterization differs under a rival meeting technology; we show it in Lemma 3 in Section 4.2, where $h \equiv 0$.

[9]A slack participation constraint (3.6) still does matter for the equilibrium outcome: while the net rent $U - u_0$ is determined by market tightness and the severity of the moral hazard problem, it is anchored by u_0.

Finally re-arrange (3.12) and substitute to find

$$\theta h^S = [1 - p_0(\theta)] \left(\eta(\theta)\pi^S - (1 - \eta(\theta))[U - u(h + y)]\mathbb{E}\left[\frac{1}{u'(t)}\right] \right). \tag{3.13}$$

Equation (3.13) is a decomposition of the expected surplus from a match in three parts. The LHS is the total payment to agents who match but do not contract. That total payment is financed by the difference between the principals' share and the agents' share of that surplus. Since $h^S > 0$, this also says principals make no losses (on average), even though we impose no such constraint on the problem (h may, in principle, be arbitrarily large). Next we can re-arrange (3.13) into

$$1 - \eta(\theta) = \frac{[1 - p_0(\theta)]\pi^S - \theta h^S}{[1 - p_0(\theta)] \left(\pi^S + [U^S - u(h^S + y)]\mathbb{E}\left[\frac{1}{u'(t^S)}\right] \right)} \tag{3.14}$$

which is the well-known Hosios sharing rule, however *conditional* on the equilibrium action a^S. The principals' match elasticity is equal to their net profit over the total surplus generated by a match. If writing the principals' free entry condition

$$[1 - p_0(\theta)]\pi^S - \theta h^S(\theta) = k, \quad \theta^S = \Theta,$$

which determines Θ for some entry cost $k > 0$, Condition (3.14) re-arranges as

$$[1 - p_0(\theta^S)][1 - \eta(\theta^S)] \left(\pi^S + [U^S - u(h^S + y)]\mathbb{E}\left[\frac{1}{u'(t^S)}\right] \right) = k. \tag{3.15}$$

The principals' share of the expected surplus from a match is dissipated by the entry cost k.[10]

We return to these conditions when exploring the efficiency properties of the equilibrium, and for now we conclude this Section with a comparative statics result.

Proposition 3 *Principals' equilibrium profit increase with market tightness* Θ : $d\Pi^S/d\Theta > 0$.

As intuition suggests, when there are more agents, each principal is more likely to contract and so can decrease the rents offered to attract them. They do so by making the incentives steeper, which spurs the effort expended by agents.

[10] Rewrite $\left[1 - p_0(\theta^S)\right]\pi^S(\theta^S) = k + \theta^S h^S$: the terms θh adds to the entry cost, and so curbs entry. That is why principals also trade off h and $t(x)$.

4 Efficiency properties

That the agents hold bargaining power determined by the market tightness has consequences for efficiency in this model. This is the issue we now turn to; we also explore the root cause of this inefficiency. Bargaining power matters because the productive surplus is governed by the action, and rents are generated by distorting both the contingent transfer and the action. As established in Proposition 3, these rents, and therefore the equilibrium action, depend on the bargaining power bestowed on agents by market conditions. We begin with some intermediary results.

Lemma 1 *Under search with friction, the optimal action a^S solving (3.10) is lower than the standard optimal action a^{SB} solving (2.2).*

Principals compete to attract agents. Even if using compensatory transfers, attracting risk-averse agents is most efficiently achieved by reducing the variability of the transfer $t(x)$: offering better insurance. As a result contracting agents face weaker incentives to exert effort. Our second intermediary result is

Lemma 2 *Consider contracts inducing the competitive search equilibrium. Social welfare increases with the agents' action.*

A higher action shifts the distribution $F(x|a)$ of the output in a first order sense, so it is obvious that the social surplus of a dyad is increasing in the agent's action − all things otherwise equal. It is also true in equilibrium: although a higher action is more expensive, it remains preferred by principals.

4.1 The equilibrium is not constrained-efficient

This result is wide-ranging; it holds under either criterion of (a) output maximization and (b) welfare maximization. An output-maximizing planner does not care about the welfare effects of surplus distribution and only seeks to maximize output in each match. Under welfare maximization, the distribution of surplus is not neutral because agents are risk averse. Regardless of the criterion, the competitive search equilibrium is never efficient at the *intensive* margin (action), which is either too high or too low. That is, the planner prescribes a different contract than the competitive search equilibrium selects, even if constrained by moral hazard and search frictions. Furthermore, and as a consequence of this inefficiency at the intensive margin, the competitive search equilibrium is also never efficient at the *extensive* margin (entry). Whenever the equilibrium induces too low an action entry is excessive; conversely when the induced action is inefficiently high.[11]

4.1.1 Output maximization

Proposition 4 *Suppose the planner maximizes net output* $\mathbb{E}[x - t(x)|a]$*. The competitive search equilibrium under moral hazard* does not *implement the planner's allocation. Directed search induces*

- *less effort than the planner's solution, which is the standard second-best solution characterized by (2.1) and (2.2); and*
- *excessive entry of principals.*

Here the problem of the planner mirrors that of the principals because their surplus is the net output. The difference is that principals are constrained by their own competition. The planner is constrained by search frictions too but not subject to the MUP because he is agnostic as to which principal fails to contract. So he needs not attract agents by giving away rents; instead a planner directs principals to post contracts and lets principals and agents match randomly. The negative externality on principals that stems from their competing for the agents and distorts the transfers completely disappears, and principals become "efficient monoposonists" (as in Diamond, 1971). To maximize net output subject to moral hazard, the planner prescribes a contract that induces a^{SB} and results in $U = u_0$ and $h = 0$. This is the standard second-best solution, which leaves no distortionary rent to the agents.

Given that the planner's contract delivers a higher surplus for each principal, more principals may enter. More precisely, in the search equilibrium, the entry of principals generates a negative "within group" externality (congestion), which is offset by a positive "across group" externality on agents (more matching opportunities). These effects are standard in competitive search, and exactly cancel each other when the surplus is fixed. Here there is a third effect: directed search dilutes principals' surplus through ex post competition for agents (Lemma 1). The planner is free of that latter externality. On balance it promotes principal entry.

[11] Up to a set of measure zero; we clarify this exactly in the proof.

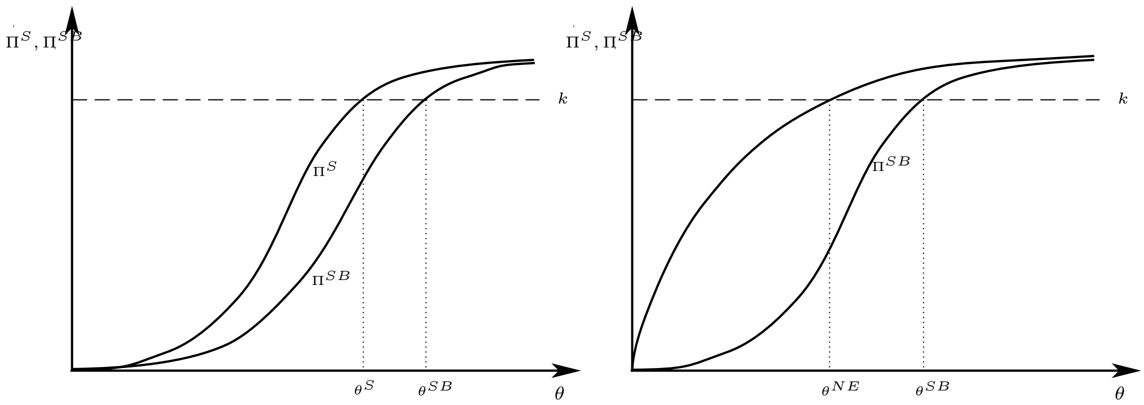

Figure 1. Social planner and competitive equilibrium entry (left); the planner adds a correction term to entry (right)

The planner also adjusts principal entry, which is defined by the condition

$$\left[1 - p_0(\theta) + \theta p_0'(\theta)\right] \pi = k,$$

where the term $\theta p_0'(\theta) = -\theta \frac{\partial}{\partial \theta}[1 - p_0(\theta)] < 0$ is the negative within-group externality that principals exert on each other by entering. From the principals' free entry condition it is clear they ignore the within-group externality.

A depiction of the entry conditions is shown on Figure 1, where the left panel compares the search equilibrium to the planner's solution, and the right panel highlights the planner's correction term.

This result departs from Acemoglu and Shimer (1999) in the sense that their planner implements a positive unemployment insurance, which induces both higher wages and a higher capital-labor ratio. Our planner selects a lower insurance ($h = 0$) and a lower expected transfer to the agents. We solve a different problem on two accounts: first, there is moral hazard in production, not in search; second, their principal can adjust the technology, not ours.

4.1.2 Welfare maximization

Matters are different for a utilitarian planner because of redistributive concerns of a welfare maximizer. Whether the competitive search equilibrium induces excessive or insufficient entry depends on how the marginal private benefit (the principals' surplus) and the marginal social benefit (the social benefit of an additional entrant) rank. We show these results in Figure 2 and now explain how they arise. Construct social welfare from the principals' and agents' payoffs

$$W(t,a) \equiv M\Pi(t,a) + NV(t,a),$$

so welfare per agent reads

$$\frac{W(a)}{N} \equiv \frac{1-p_0(\theta)}{\theta}\left[\pi(t,a) + U(t,a)\right] + \left[1 - \frac{1-p_0(\theta)}{\theta}\right] u(h+y) - h,$$

where the planner is also constrained by moral hazard and the agents' participation. The first term is the aggregate matching surplus of a contracting relationship. The second one is the meeting surplus – the total utility of paying h to the unsuccessful agents.

The details of the planner's solution depends on whether the agents' outside option is constraining as well as on the entry cost. Let $(t_1^W(x), a_1^W, h_1^W)$ denote a solution to the planner's problem where the agents' participation constrain it slack at u_0; that is, $U(t_1^W, a_1^W) > u_0$. Intuitively, the planner is then free to set h_1^W to maximize the last term of the welfare function; redistribution is inexpensive and has no bearing on the optimal action. Then in the knife-edge case where $\Theta = 1$, which corresponds to a unique entry cost k^*, the planner's allocation and the competitive equilibrium agree. For *all* other values of the entry cost k (all other market tightness Θ) the equilibrium is inefficient. If $k < k^*$ ($\Theta < 1$) there is excessive entry of principals, and conversely when $k > k^*$ ($\Theta > 1$).

When the planner is constrained by the agents' exogenous outside option we need another (constrained) solution $(t_2^W(x), a_2^W, h_2^W)$. In that case redistribution is costly in the sense that it impedes on the optimal action through the binding participation constraint: to satisfy participation a planner demands a lower action than if unconstrained.[12] Then the competitive search equilibrium is never efficient; there is always excessive entry of principals.

Proposition 5 *Consider a utilitarian planner; the competitive search equilibrium* does not *implement the planner's allocation.*

1. *If the planner is unconstrained by the agents' outside option u_0, except for a unique entry cost entry k^* inducing $\Theta = 1$, directed search*

   (a) induces too little effort and excess entry for $k < k^*$ ($\Theta < 1$) and

[12] We formalize these statements in the Appendix.

(b) *induces too much effort and insufficient entry for* $k > k^*$ ($\Theta > 1$)

than is socially optimal.

2. *If the planner is constrained by the outside option* u_0*, directed search always generates insufficient effort and excessive entry.*

In the first instance we distinguish between two cases. Below $\Theta = 1$ agents face good matching prospects and principals have to compete hard to attract them. Rents are high and are generated by high compensatory transfers h^S, weak incentives $t^S(x)$ and low action a^S. The surplus is appropriated by the agents through principal competition. Many principals enter because the entry cost is low; they are extracted ex post.

Things are reversed above k^* – when Θ exceeds 1. Matching prospects are poor for agents and principals can extract rents through low compensatory transfers h^S, steep incentives $t^S(x)$ and high action a^S. The surplus is appropriated by the principal but exhausted by a high entry cost k. That entry cost is so high that entry is insufficient. The resulting lack of principal competition then induces a low welfare because agents face excessively harsh contracts.

Exactly at $k = k^*$ (inducing $\Theta = 1$) these two effects offset each other; of all possible $k \in \mathbb{R}_{++}$ one could conceive off, $k = k^*$ has measure zero. Hence the equilibrium is never efficient.

In the second instance the planner cannot implement whatever allocation he desires. Then the competitive equilibrium is uniformly worse. Because the planner is constrained by the agents' outside option, it is exactly as if he were maximizing output. The competitive equilibrium and the planner's solution never converge – for any value of k.

For emphasis, the crucial difference between the planner and the competitive search equilibrium is that competing principals always face the *incentive* to distort the transfer to direct the agents' search, but the planner does not. Instead the planner is constrained only by the *equilibrium* level of frictions. That is, matching is random – not directed, and therefore orthogonal to contracts, in the planner's problem. It neutralizes the interaction between action and search.

At the heart of these results is this. Under directed search, it is always optimal for principals to use the two margins of transfers (t, h) and action a to generate the rents necessary to attract agents – or to extract said rents. So the productive action is always distorted – in either direction. Whether the action is too low or too high, it *always* shifts the Pareto frontier inward – that is, welfare

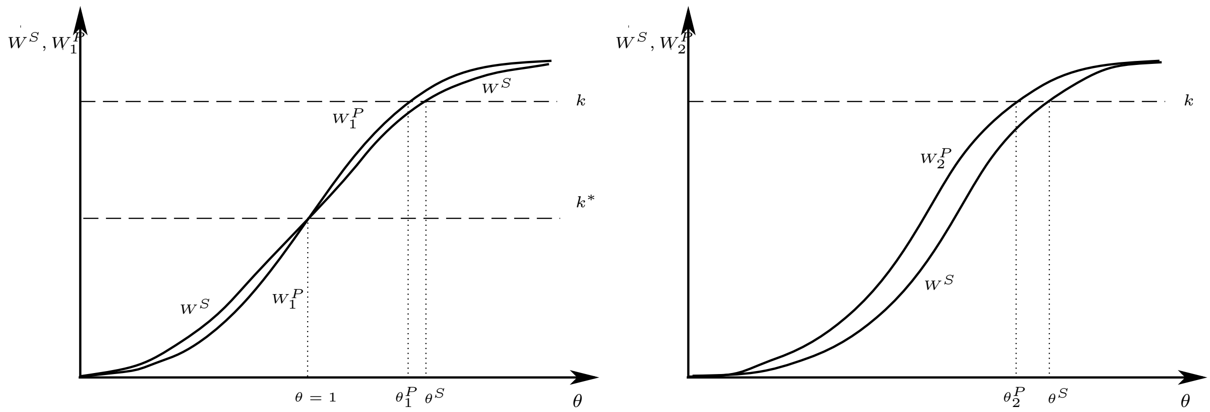

Figure 2. Unconstrained planner and competitive equilibrium entry (left); constrained planner and competitive equilibrium entry (right)

decreases and the allocation is not socially efficient. Hence, while the Hosios Condition (3.14) affirms *Pareto optimality*, as is known in many search problems, it is not sufficient to characterize welfare efficiency. The link between the Hosios condition and welfare efficiency is broken in this model because the social surplus is endogenous to the action, in contrast to many competitive search models where the surplus is fixed. Indeed the Hosios Condition (3.14) is conditional on the action. Under competitive search that action is always suboptimal.

The loss of efficiency is severe in this model in the sense that there exists no instrument (such as taxes) that can be used to *fully* correct a market allocation.[13]

Proposition 6 *Whether maximizing welfare or output, there exists no corrective instrument to fully restore constrained efficiency and satisfy budget balance.*

To build some intuition, consider the problem of excessive entry; a planner may curb it by imposing an entry tax on principals (this is one of many instruments that may be available). Suppose efficient entry may be achieved with this tax; now the receipts must be returned to market participants – agents only. Upon receiving any transfer, any agent re-optimizes and moves away from the allocation that was previously optimal. Efficient entry cannot be sustained. The same reasoning applies to any fiscal instrument.

Efficiency can only be restored if the interaction between directed search and the provision of incentives is neutralized – as is the case in the planner's problem. That plainly means that search cannot be directed for the allocation to be efficient. From a social standpoint there is no loss

[13]We do not explore a partial correction, the extent of which may depend on the selected instrument(s).

in preventing search to be directed (to the benefit of random matching, for example) because all agents and all principals are homogenous, hence matching is not assortative.

Remark 1 *Under output maximization or when the planner is constrained by the outside option u_0, the result is even stronger: there is no corrective instrument irrespective of budget balancing. See the proof for detail.*

4.1.3 Action, directed search and social surplus

Propositions 4 to 6 stand in contrast to much of the directed search literature, where the competitive search equilibrium is constrained-efficient. It turns out they hold *regardless of the information structure* and production technology: inefficiency persists under symmetric information (strictly speaking, except on a set of measure zero). The reason is that the principals have incentives to distort action and transfer remain even under symmetric information.

Search frictions alone induce the effort choice to decrease because a principal must delegate to an agent who is not the sole residual claimant of her action. Even under symmetric information, and regardless of the agents' exogenous outside options u_0, principals use their two margins (t, h) and a to generate the rents they must give to agents to attract them. In fact this is even true even with a deterministic output, and owes to the curvature of $u(\cdot)$ and $c(\cdot)$. Of course this problem does not arise with a fixed surplus, for when the surplus is fixed the only problem on hand is that of distribution. It does not arise either when the agent is the residual claimant of her action (no delegation).

The table below represents a taxonomy of information structure and technology. When social surplus depends on the agent's action (and search is directed) the equilibrium under competitive search (CS) is never efficient. We omit proofs and the supporting analysis, which mirrors that of Section 4.1. The details may be availed from the authors upon request.[14]

Technology/Information	Symmetric	Asymmetric
Deterministic	CS inefficient	n/a
Stochastic	CS inefficient	CS inefficient

Our model, even under symmetric information, differs from a problem with heterogenous agents (but no action a), as in Guerrieri, Shimer and Wright (2010) for example. With agent heterogeneity,

[14] "n/a" because with a deterministic technology the moral hazard problem is moot.

under full information there is no distortion: principals have no second margin a to use. Hence, when agents are heterogenous, inefficiency arises solely because of the interaction between adverse selection and search frictions. Indeed, under adverse selection, incentive compatibility requires distorting the allocation, and principal competition requires further rent giving – that is, further distortions. In contrast, when the action is endogenous, directed search is sufficient to distort it.

4.2 Compensatory transfers.

Depending on the entry cost and on the agents' exogenous outside option u_0, restricting the contract space may worsen or improve the outcome. It also implies that the choice of meeting technology also impacts welfare. More precisely, Proposition 2 establishes that optimal compensatory transfers h^S are always positive. These transfers alleviate the search problem, hence they also matter for efficiency. Suppose that $h \equiv 0$ exogenously so $u(h+y) = u_0$; constraint (3.6) disappears and the program becomes.

Problem 3

$$\max_{t(\cdot),h} [1 - p_0(\theta)] \int_{\mathcal{X}} [z - t(z)] dF(z|a)$$

s.t. (3.7), (3.8) and

$$\int_{\mathcal{X}} u(t(z)) dF(z|a) - c(a) \geq u_0 \tag{4.1}$$

where (3.7) is made to reflect $u(h+y) = u_0$. The characterization below departs from that of Proposition 2.

Lemma 3 *The solution $(t^N(x), a^N)$ to Problem 3 is characterized by the necessary and sufficient first-order conditions*

$$\int_{\mathcal{X}} [z - t(z)] dF_a(z|a) + \widehat{\mu} \left[\int_{\mathcal{X}} [z - t(z)] dF_{aa}(z|a) - c''(a) \right] = 0 \tag{4.2}$$

and

$$\frac{1}{u'(t)} = -\frac{\theta p_0'(\theta)}{1 - p_0(\theta) + \theta p_0'(\theta)} \frac{\pi}{U - u_0} + \widehat{\mu} \frac{f_a}{f} \tag{4.3}$$

with $-p_0' > 0,\ U > u_0,\ \widehat{\mu} = \frac{\mu}{1-p_0(\theta)} > 0$ *and* $\theta = \Theta$.

As before Equation (4.2) is an envelop condition that arises from subgame perfection. Condition (4.3) is a functional equation that defines a fixed-point problem in the space of transfer functions $\mathcal{T}$. It is simpler than it first appears as soon as one notices that π and U are expected values.

That is, for a given function t, $\pi \in \mathbb{R}$ and $U \in \mathbb{R}$. Thus (3.11) rewrites

$$\frac{1}{u'} = \alpha + \widehat{\mu}\frac{f_a}{f},\ \alpha \in \mathbb{R}_+,$$

which mimics Homlström's standard condition (1979). Hence the first-order conditions (4.2), (4.3) and the constraints (3.8) and (3.7) completely identify the solution (for details of this approach see Roger, 2015). As before there is a trade-off between participation and incentives. This trade-off is now handled by a single instrument: the transfer function $t(x)$, which must therefore be suitably distorted.

Remark 2 *Restricting $h \equiv 0$ follows immediately from using a rival meeting technology, so Lemma 3 characterizes the contract when meeting is purely rival. An example of rival meeting technology is pairwise meeting, as in the standard Diamond-Mortensen-Pissarides model. Rivalry in meeting is thus nested in our formulation.*

Remark 3 *Condition (4.3) obtain almost directly from Conditions (3.12) and (3.11) by simply forcing h to zero. So Problem 3 is really a special case of Problem 2, and its solution is the limit of the solution to Problem 2 when $h \to 0$.*

Proposition 7 *The optimal action a^N solving (4.2) (when $h \equiv 0$) is always lower than a^S solving (3.10) (using compensatory transfers).*

In the unrestricted contract of Section 3 compensatory transfers are used to attract agents; they are a form of insurance for the agents. With this insurance the rent U is not the only instrument the principals can use to attract agent, so they can limit the rent U bestowed on the contracting agent. Reducing U allows the principal to expose the contracting agent to more risk, which increases her action. These instruments are not available here. Combining Proposition 7 with Lemma 1,

Corollary 1 *In the competitive search equilibrium, output is higher when using compensatory transfers.*

In summary we can rank the equilibrium action for a given information structure. Under moral hazard

$$a^{SB} > a^S > a^N$$

and welfare in the competitive search equilibrium follows the same ranking thanks to Lemma 2. Proposition 4 then immediately extends.

As before the analysis is more subtle when it comes to welfare maximization. Here too it depends on whether the outside option u_0 is binding for a planner, as well as on the entry cost k. Proposition 7 should help preview some intuition: where the action was too low it is now worse, and conversely.

Corollary 2 *Suppose compensatory transfers h are prohibited. When the planner is*

1. *unconstrained at u_0, there is always less entry, so the equilibrium is*
    - *even less efficient for $k > k^*$; and*
    - *more efficient for $k < k^*$.*
2. *constrained at u_0, there is less entry and the equilibrium is even less efficient.*

Constrained-efficiency fails for the same reasons as before: the action is distorted away, through the transfer function offered to the agent, because of the search problem. It is worse here because principals are further constrained by the absence of transfers h. Whether this implies that entry is more or less efficient than in the search equilibrium with compensatory transfers depends on whether entry is excessive or insufficient.[15]

5 Discussion and applications

In light of our results thus far we raise two items for discussion; we also layout two applications.

5.1 Discussion of results

First we investigate the effect of a cap on compensatory transfers h. Then we discuss the well-known CARA-linear-normal specification.

[15] It is also interesting to note that the adoption of a rival meeting technology, necessarily implying $h \equiv 0$, has welfare consequences.

Constraint on transfers. The analysis of the optimal contract takes transfers h to be unconstrained. While the optimality condition (3.12) characterizing h^S makes it plain it is always bounded, the total value $\theta \cdot h$ may be large. Here we seek to understand what happens when the sum of the transfers are bounded. Such a cap may arise from a binding cash constraint or a liquidity constraint. The answer, not surprisingly, lies between $h \equiv 0$ and unconstrained transfers. What is more surprising is that, even when capped, compensatory transfers go a long way in unwinding distortions in the transfer function $t(x)$.

Proposition 8 *Insert the constraint $\theta h \leq H \in \mathbb{R}_{++}$ in Problem 2 and attach multiplier $\phi \geq 0$. Whenever $\phi > 0$ in equilibrium, the solution is characterized by*

$$h = \frac{H}{\Theta} \tag{5.1}$$

$$\int_{\mathcal{X}} [z - t(z)] dF_a(z|a) + \widehat{\mu} \left[\int_{\mathcal{X}} [z - t(z)] dF_{aa}(z|a) - c''(a) \right] = 0 \tag{5.2}$$

$$\frac{1}{u'(t)} = \frac{1+\phi}{u'(h+y)} \frac{\theta}{\theta - 1 + p_0} + \hat{\mu} \frac{f_a}{f} \tag{5.3}$$

with $\hat{\mu} = \frac{\mu}{1-p_0(\theta)} > 0$ and $\theta = \Theta$. Furthermore,

$$\nu(h^S) < \nu(H/\Theta) < \nu(h \equiv 0)$$

So we see from (5.3) that capping transfers reintroduce some distortion, as measured by the multiplier ϕ. But that distortion is limited and the transfer function resembles that characterized by (3.11). In consequence the multiplier ν is larger than with unconstrained transfer – and lower than without. By extension of our earlier result, the optimal action is also intermediate, and therefore so is welfare. So compensatory transfers always help.

The special CARA-linear-normal case. The CARA-linear-normal framework of Holmström and Milgrom (1987) has become a workhorse of applied research in incentive problems. In our framework this carefully-chosen specification offers inaccurate insights into the effects of principal competition for the agent. To make the point one can dispense with the compensatory transfer h.

Let $t(x) = \alpha + \beta x$ be the tariff offered, $c(a) = (c/2)a^2$ and $u = -e^{-r(t-c(a))}$, where r is the coefficient of risk aversion. Let also $x \equiv a + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. Upon accepting the contract the agent's problem is unchanged and described by (3.8); in this framework the agent thus maximizes

$$\alpha + \beta x - \frac{1}{2} c a^2 - \frac{r}{2} \beta^2 \sigma^2$$

whence $a^* = \beta/c$ – see Bolton and Dewatripont (2004), for example. The principal solves

$$\max_{\alpha,\beta,\theta}[1-p_0(\theta)]\left[\frac{\beta}{c}-\left(\alpha+\frac{\beta^2}{c}\right)\right]$$

subject to the constraints

$$\begin{aligned} \alpha+\beta^2/2c-(r/2)\beta^2\sigma^2 &\geq y && (5.4)\\ \frac{1-p_0(\theta)}{\theta}\left[\alpha+\beta^2/2c-(r/2)\beta^2\sigma^2\right]+\left(1-\frac{1-p_0(\theta)}{\theta}\right)y &\geq \widetilde{Y} && (5.5)\end{aligned}$$

where $\widetilde{Y}$ is the consumption level inducing utility $\widetilde{V}$ and $y = u^{-1}(u_0)$. The optimality conditions are standard and confirm that the MUP binds, so we can write

$$\alpha+\frac{\beta^2}{c}=\frac{\beta^2}{2c}-\frac{r}{2}\beta^2\sigma^2+\frac{\theta}{1-p_0(\theta)}\left[\widetilde{Y}-\left(1-\frac{1-p_0(\theta)}{\theta}\right)y\right],$$

substitute in the principal's objective and optimize over β to find $\beta = 1/(1 + rc\sigma^2)$. The fact that principals compete for the agents has no consequences on the incentives they offer them. This outcome owes precisely to that specification, which neutralizes wealth effects. The agent's optimal action defined as $a = \beta/c$ is independent of level of utility; therefore the equilibrium slope parameter β is independent of any rent. In particular it is independent of the level of $\widetilde{V}$.[16] Therefore the equilibrium level of action a^* is as in Milgrom and Holmstrom (1987); it is not distorted. Because there is no distortion at the intensive margin, there are also no distortion at the extensive margin. The allocation and the level of entry are both constrained efficient.

The FOCs (3.11) and (3.12) of our problem clearly point to a different outcome, which is driven by wealth effects. By neutralizing these wealth effects the CARA specification understates the importance of the participation problem. Here it goes as far as *ignoring* the impact of principal competition. Therefore we suggest caution when using the CARA-linear-normal specification, which clearly implies a loss of generality when studying a competitive problem.

5.2 Applications

Labor market. We construct a static application that emulates a steady state version of a continuous time dynamic model. Consider a one period model with a fixed measure L of agents in the labor force and a measure of M firms (to be determined endogenously by free entry). Given

[16] For completeness, the participation constraint (5.5) is then satisfied using the intercept α.

the meeting and matching friction, a proportion of L and F does not match during the period. Let E be the number of employed and N the number of unemployed at the beginning of the period, so that $L = E + N$ and define $e = E/L$ and $u = N/L$ as the employment and unemployment rates, respectively. Similarly, let $v = M/L$ be the vacancy rate. These are endogenously determined during the period; Let the end of period rates be $\bar{u}$ and $\bar{v}$. To remain close to standard search models of the labor market with equilibrium unemployment, we assume that a match can be destroyed exogenously with probability q (separation rate). Let b be the value for unemployed (e.g. home production).

For simplicity we also restrict the contract space so that the transfer $h = 0$. A labor contract is a wage scheme $w(x)$ contingent on the realized observable output x, and suggested effort intensity a.[17] Effort intensity is unobservable by firms upon employment, and thus, subject to moral hazard.

To map our model to standard labor market applications, let $m(\Theta) = (1 - p_0(\Theta))$ be the meeting rate for firms and from constant returns to scale, $m(\theta)/\theta$ the meeting rate for unemployed searching for jobs where $\Theta = N/M$ is the aggregate market tightness in the labor market. The sequence of events is similar to the general model:

1. Firms post one vacancy each along with a contract $(w(x), a, \theta)$ where $\theta \neq \Theta$ off-equilibrium path.

2. Job seekers observe all wages and choose one wage to apply for.[18]

3. Meetings occur, matches are formed and production occurs.

4. With probability q all matches separate and the period ends.

At the beginning of the period, the expected payoffs are as follows – again, in keeping with the notation that has become standard in labor search models. The value of being unemployed and searching is $U(w, a) = \frac{m(\theta)}{\theta} \left(\int_{\mathcal{X}} u(w(x)) dF(x|a) - c(a) \right) + (1 - \frac{m(\theta)}{\theta})b$; likewise the value of posting a vacancy for firms is $V(w, a) = -k + \max_{w(x),a,\theta} m(\theta) \left(\int_{\mathcal{X}} [z - w(z)] dF(z|a) \right)$. After matching, an

[17] Effort could be taken as labor hours. However, employment often specify hours and hours spent at work are often observable. One can then think of ah where a is a labor-augmenting effort that is unobservable.

[18] Although applying for one job at a time seem restrictive, in the model formulation, unemployed send their application to all firms offering the same wage w forming a submarket. In this interpretation, workers make multiple applications.

employed worker receives value $W(w^S, a^S) = \int_{\mathcal{X}} u(w(z))dF(z|a^S) - c(a^S) + qb$, where a^S is the equilibrium level of effort. Firms with filled vacancies get $J(w^S, a^S) = \int_{\mathcal{X}} [z - w^S(z)]dF(z|a^S) + q\underline{y}$, where $\underline{y}$ is the minimum production that a firm can get without the worker; it captures the firms' value upon separation (and without loss could be set at 0).

Following the competitive search construct, firms posting the same wage scheme $w(x)$ form a submarket with an expected queue length θ. As per the definition $\theta = \Theta$ in equilibrium. In the competitive search, firms solves:

$$\max_{(w(x),a,\theta)} V(w, a) \quad s.t.$$

$$\text{(i)} \int u(w(x))dF(x|a) - c(a) \geq b, \text{ (ii)} \int u(w(x))dF_a(x|a) - c'(a) = 0, \text{ (iii) } U(w, a) \geq \widetilde{U}.$$

The solution does not replicate the planner's solution. The posting of vacancies (entry) may be insufficient or excessive, depending on whether $k > (<) k^*$. In equilibrium we find $\theta^S \neq \theta^P$. The moral hazard impact on unemployment is assessed via the end-of-period Beveridge curve: $(1 - \bar{u})q = \bar{u}\frac{m(\Theta)}{\Theta}$ or

$$\bar{u} = \frac{q}{q + \frac{m(\Theta)}{\Theta}}$$

Since $m(\Theta)/\Theta$ is increasing, $\bar{u}^S \neq \bar{u}^P$: the unemployment rate is lower or higher in the competitive search equilibrium than what the planner would implement, depending on whether $k < (>) k^*$.

Much of the literature on unemployment focuses on moral hazard associated with the search effort of unemployed workers (e.g. Hopenhayn and Nicolini, 1997), or on the benefit and costs of unemployment insurance (e.g. Acemoglu and Shimer, 1999). Our model generates outcomes that resemble those of a business cycle: simultaneously we may have (i) employed workers receiving rents, low output per firm and excessive vacancies; or (ii) workers being extracted and excessive (involuntary) unemployment. However the policy implications are very different. Our model suggests that altering the search process is the necessary condition to restore efficiency, and so should be the object of policy intervention. We note that altering the search process has become integral part of the "Active Labor Market policies" now prevalent in European countries (see Martin, 2014).

Financial contracting. Consider a financial contracting model as in the books of Tirole (2006) or Freixas and Rochet (2008) but with a continuous action and probability of success, say $\rho(a)$, continuous, increasing and concave. Financial institutions post contracts and borrowers search over them; let $h \equiv 0$ for ease of exposition. The moral hazard constraint is

$$a \in \arg\max_{a' \in \mathcal{A}} U = \rho(a')R_b - c(a')$$

where $R_b \geq 0$ is the borrower's rent to be determined in equilibrium; let also I be the investment required and R the gross return of the project. Absent the search problem, a lender solves

$$\max_{R_b, a'} \rho(a')(R - R_b) - I$$

subject to the aforementioned constraint and $U \geq u_0$ as well as $\rho(a)(R - R_b) - I \geq 0$. With directed search, the problem becomes

$$\max_{\theta, R_b, a'} (1 - p_0(\theta)) \left[\rho(a')(R - R_b) - I\right]$$

subject to the same constraints and

$$\frac{1 - p_0(\theta)}{\theta} U + \left(1 - \frac{1 - p_0(\theta)}{\theta}\right) u_0 \geq \widetilde{V}$$

It is easy to verify from the first-order conditions that the solution induces a lower action a, which clearly decreases the social surplus $\rho(a)R - I$. There are more borrower failures in equilibrium ($\rho(a^S) < \rho(a^{SB})$), and the condition $\rho(a)(R - R_b) - I \geq 0$ may fail (because of the search frictions); then there is credit rationing because of search.

6 Conclusion

In this paper we show that the interaction of competitive search and moral hazard has implications for the nature of the optimal contract, for the equilibrium level of effort and for efficiency at both the intensive and the extensive margin. We characterize the optimal contract in a general setting and explore thoroughly the source of inefficiency of the competitive search equilibrium.

We allow for directed search and moral hazard where the social surplus depends on the action of the agent(s). Precisely because search is directed, search frictions affect the social surplus to be shared through the level of action, and not just the sharing rule. The reason is that principals compete to attract agents by increasing their rents. They do so using the two margins available to them: the transfers and the action. So markets and frictions matter a great deal, even in bilateral contracting decisions. In this paper search frictions restore some bargaining power on the side of the agents – at a cost, because the social surplus is endogenous to the agents' actions.

We believe that combining agency with competitive search creates a natural environment for a model of competitive agency. First solving the inefficiency problem requires a different policy intervention. Indeed we show that correcting inefficiency requires search to not be directed. Second, search models of monetary policy may benefit of this innovation. It is already known that paying with debt is not the same as paying with cash, not because of record-keeping problems but because of ex post moral hazard (see DeMarzo, Kremer and Skrzypacz (2005) in the context of auctions). That is, there may be reasons that traders have to hold money balances beyond the standard explanation of unavailability of credit. This question is left to future work.

Appendix

This Appendix has two parts. The first one contains the proofs. In the second one the reader one can find additional material that gives game-theoretic foundations to our large-market approach.

A Proofs

Proof of Proposition 1: By inspection of $V(\mathcal{C})$ the concavity of $u(\cdot)$ implies that $h_n^j = h^j$ for any n. Suppose the contract $\mathcal{C}$ is offered in the other submarkets. Attach multipliers γ, μ, ν and ϵ, with $\gamma, \nu, \epsilon \geq 0$, to each of the constraints of Problem 1 and fix the action a_n^j for each n; we solve the cost minimisation by selecting $t_n^j(x), h^j$ for each n. The FOC are

$$\begin{aligned}
\forall n \geq 1, \quad -p_n(\theta)f(x|a_n^j) + \gamma u' f(x|a_n^j) + \mu u' f_a(x|a_n^j) + \nu q_n(\theta) u' f(x|a_n^j) &= 0 \quad \text{(A.1)} \\
\forall n \geq 1, \quad -p_n(\theta) - \gamma u'(h+y) + \epsilon + \nu \frac{q_n(\theta)}{n}\left[1 - \frac{1}{n}\right] u'(h+y) &= 0 \quad \text{(A.2)} \\
\sum_n p_n'(\theta)\left[\int [z - t_n^j(z)] dF(z|a_n^j) - h\right] + \nu \left[\sum_n q'(\theta)\left(\frac{U_n^j + (n-1)u(h+y)}{n}\right)\right] &= 0 \quad \text{(A.3)}
\end{aligned}$$

For each n the complementary slackness conditions are

$$\begin{aligned}
\gamma[U_n^j - u(h+y)] &= 0 \\
\nu[V(\mathcal{C}^j) - \widetilde{V}] &= 0 \\
\epsilon h &= 0.
\end{aligned}$$

Given assumptions on $\sum_n p_n'(\theta)$ and $\sum_n q_n'(\theta)$ Condition (A.3) immediately implies $\nu > 0$. Next suppose $\gamma > 0$; re-arrange (A.1) as

$$\gamma = \frac{p_n(\theta)}{u'(t_n^j(x))} - \left[\mu \frac{f_a(x|a_n^j)}{f(x|a_n^j)} + \nu \frac{q_n(\theta)}{n}\right]$$

and (A.2)

$$\gamma = -\frac{p_n(\theta) + \epsilon}{u'(h+y)} + \nu q_n(\theta)\left(1 - \frac{1}{n}\right)$$

and so $\forall\ x \in \mathcal{X}$ we must have

$$\frac{p_n(\theta)}{u'(t_n^j(x))} - \left[\mu \frac{f_a(x|a_n^j)}{f(x|a_n^j)} + \nu \frac{q_n(\theta)}{n}\right] = -\frac{p_n(\theta) + \epsilon}{u'(h+y)} + \nu q_n(\theta)\left(1 - \frac{1}{n}\right) \in \mathbb{R}_{++}$$

which is impossible; hence $\gamma = 0$ and the first-order condition (A.1) simplifies to

$$\frac{1}{u'(t_n^j(x))} = \nu \frac{q_n(\theta)}{np_n(\theta)} + \tilde{\mu}\frac{f_a(x|a_n^j)}{f(x|a_n^j)},\ n = 1, 2, ..., N \tag{A.4}$$

where $\tilde{\mu} = \frac{\mu}{p_n(\theta)}$. Last suppose $\epsilon > 0$, (A.2) implies

$$\nu q_n(\theta)\left[1 - \frac{1}{n}\right] u'(h+y)|_{h=0} = p_n(\theta) - \epsilon < \nu q_n(\theta)\left[1 - \frac{1}{n}\right] u'(h+y)|_{h>0} = p_n(\theta),$$

which contradicts the fact that $u(\cdot)$ is concave. From Jewitt, Kadan and Swinkels (2008, now JKS) we know that fixing action a_n^j the solution t_n^j to this equation is unique for each n. To see why here, fix a_n^j, then $\widetilde{\mu}$ is fixed, so by (A.4) and monotonicity of u, t_n^j must be unique. By the first-order condition $\frac{\partial U_n^j}{\partial a_n^j} = 0$ (under the conditions of the FOA), we also know that the (agent-) optimal action a^* is unique for a given transfer function t_n^j.

To show the equilibrium contract is a fixed transfer and action independent of the matching states, let $\tilde{\nu} = \nu\frac{q_n(\theta)}{p_n(\theta)}$ and $L(\cdot)$ denote the Lagrangian of the cost-minimization problem. The effective cost of implementing action a_n^j defined as

$$\begin{aligned} C(a_n^j) &= \max_{\tilde{\nu},\tilde{\mu}} \min_{u(t)} L(u(t)) \\ &= \max_{\tilde{\nu},\tilde{\mu}} \tilde{\nu}\left[V(\mathcal{C}^j)\right] + \tilde{\mu}c'(a_n^j) - \int \rho\left(\tilde{\nu} + \tilde{\mu}\frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right) dF(z|a_n^j) \end{aligned}$$

is convex in a_n^j under CLI (see JKS). The first line is an application of the Lagrange duality theorem. In the second line $\rho\left(\tilde{\nu} + \tilde{\mu}\frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right) \equiv \max_{u(t)}\left(\tilde{\nu} + \tilde{\mu}\frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right) u(t) - u^{-1}(u(t))$ is a convex function for any argument $\left(\tilde{\nu} + \tilde{\mu}\frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right)$, $c''(a_n^j) > 0$ by assumption and we make use of the agent's first-order condition $U_{a_n} = 0$ in the $V(\cdot)$ term, which therefore disappears. Take any $a_1 \leq a_2 \leq ... \leq a_N$ (w.l.o.g.) induced by $t_1 \neq t_2 \neq ... \neq t_N$ and define

$$\mathbb{E}[C(a_n^j)] \equiv \sum_{n=1}^{N} \Pr(a_n^j) C(a_n^j).$$

This is a convex function for it is necessarily bounded (below and above). Furthermore, there also exists a convex function

$$C\left(\mathbb{E}[a]\right) \equiv C\left(\sum_{n=1}^{N} \Pr(a_n^j) a_n^j\right).$$

For each n, let a_n^{j*} denote the optimal action; then $\mathbb{E}[a^{j*}] \equiv \sum_{n=1}^{N} \Pr(a_n^j) a_n^{j*}$ and

$$\mathbb{E}[C(a_n^{j*})] \geq C\left(\mathbb{E}[a^{j*}]\right)$$

which contradicts the premise that $t_1 \neq t_2 \neq ... \neq t_N$ are optimal given $\mathbb{E}[a^{j*}]$. ■

Proof of Proposition 2: Form the Lagrangean of Problem 2 with the same multipliers. The FOC w.r.t. $t(x), h, \theta$ are

$$\begin{aligned} -[1-p_0(\theta)]f + \gamma u' f + \mu u' f_a + \nu \frac{1-p_0(\theta)}{\theta} u' f &= 0 \quad \text{(A.5)} \\ -\theta - \gamma u'(h+y) + \epsilon + \nu \left[1 - \frac{1-p_0(\theta)}{\theta}\right] u'(h+y) &= 0 \quad \text{(A.6)} \\ -p_0'(\theta) \int [x - t(x)] dF(x|a) - h - \nu \left[\frac{1-p_0+\theta p_0'}{\theta^2}\right] [U - u(h+y)] &= 0. \quad \text{(A.7)} \end{aligned}$$

Recall that $\pi = \int [z - t(z)] dF(z|a)$. Suppose first that $\nu = 0$, Condition (A.7) implies

$$h = -p_0' \pi > 0 \Rightarrow \epsilon = 0$$

and Condition (A.6) becomes a contradiction since $\theta > 0$, $\gamma \geq 0$ and $u'(h+y) > 0$. Second, suppose $\gamma > 0$, (A.7) yields the same condition for h and so implies $\epsilon = 0$; then equate Conditions (A.5) and (A.6): $\forall\ t(x)$, $\forall\ x$ and any given $h > 0$,

$$\frac{1-p_0}{u'(t(x))} - \left(\mu \frac{f_a}{f} + \nu \frac{1-p_0}{\theta}\right) = \gamma = \nu \left[1 - \frac{1-p_0}{\theta}\right] - \frac{\theta}{u'(h+y)} \in \mathbb{R}_{++}$$

which is impossible. Therefore $\gamma = 0$ too. Last $\epsilon = 0$ by the same argument as in the proof of Proposition 1. Hence the FOC (A.5) always reads

$$\frac{1}{u'} = \frac{\nu}{\theta} + \hat{\mu} \frac{f_a}{f} \quad \text{where } \hat{\mu} = \frac{\mu}{1-p_0}.$$

From Condition (A.6) again one has

$$\nu = \frac{\theta}{u'(h+y)} \frac{\theta}{\theta - 1 + p_0}.$$

To compute h re-arrange Condition (A.7) and substitute:

$$h = -p_0' \pi - \frac{1-p_0+\theta p_0'}{\theta - 1 + p_0} \frac{U - u(h+y)}{u'(h+y)}.$$

Finally we have

$$\frac{1}{u'(t)} = \frac{1}{u'(h+y)} \frac{\theta}{\theta - 1 + p_0} + \hat{\mu} \frac{f_a}{f}$$

as claimed and indeed by integrating over $\mathcal{X}$

$$\mathbb{E}\left[\frac{1}{u'(t)}\right] = \frac{1}{u'(h+y)} \frac{\theta}{\theta - 1 + p_0}$$

because $\int f_a dF(z|a) = 0$. ■

Proof of Proposition 3: We prove the Proposition in two Claims. First,

Claim 1 *The equilibrium expected profit increases with the action:* $d\Pi(a^S)/da^S$*.*

Proof: Write equilibrium profit

$$[1-p_0(\theta)]\left(\int [z-t(z)]dF(z|a^S)\right)-\theta h$$

where $\theta=\Theta$, and the MUP, the participation and moral hazard constraints are satisfied. From Conlon (2008) we know that

$$T(a)\equiv\int_{\mathcal{X}} t(z)dF(z|a)$$

is an increasing, concave function for $t(x)$ is concave in x as it track the likelihood ratio f_a/f. Differentiate with respect to a^S:

$$\frac{d\Pi(a^S)}{da^S}=[1-p_0(\theta)]\left(\int zdF_a(z|a^S)-T'(a^S)\right)$$

since $\theta=\Theta$. This term is necessarily positive because the principal is constrained in his choice of action. The transfer h does not change since the moral hazard constraint $U_a=0$ is an envelop condition on the agent's utility if contracting (i.e. $\theta=\Theta$ in equilibrium). ■

Claim 2 *The equilibrium action increases in the market tightness:* $da^S/d\Theta>0$*.*

Proof: In Problem 2 principals take market tightness as exogenous and their contract offers must satisfy the constraint $U_a=0$. The identity $\frac{d}{d\Theta}U_a\equiv 0$ stemming from this constraint rewrites

$$\int u'\Delta t dF_a+U_{aa}\frac{da}{d\Theta}\equiv 0$$

where Δt denotes a variation in t with respect to $\Theta:$ $\Delta t=\lim_{\Theta_2\to\Theta_1}\frac{t[\Theta_2](q)-t[\Theta_1](q)}{\Theta_2-\Theta_1}$ and $U_{aa}<0$. (Continuity and smoothness of t in Θ follows from the first-order conditions (3.11) and (3.12)). So the sign of $da/d\Theta$ follows that of the first term. In that first term the action a remains constant. Because $\int f_a(z|a)dz=0$, for some $\tilde{x}$,

$$f_a\begin{cases}<0, & q<\tilde{x};\\ =0, & q=\tilde{x};\\ >0, & q>\tilde{x}.\end{cases}$$

Take any $\Theta_2\downarrow\Theta_1$, the corresponding transfers $t[\Theta_2],t[\Theta_1]$ parametrized by market tightness must take the form characterized in Proposition 2, that is, $1/u'=\beta+\tilde{\mu}f_a/f$. By continuity in t and a, if

$t[\Theta_1]$ passes through the point $\tilde{x}$, $t[\Theta_2]$ passes arbitrarily close to it. Then, if the contract $t[\Theta_2](x)$ is steeper,

$$t[\Theta_2](x)\begin{cases} < t[\Theta_1](x), & x < \tilde{x} \text{ and} \\ > t[\Theta_1](x), & x > \tilde{x}. \end{cases}$$

so that

$$\Delta t \begin{cases} < 0, & x < \tilde{x} \text{ and} \\ > 0, & x > \tilde{x}. \end{cases}$$

and $\int u'\Delta t dF_a > 0$ necessarily. The converse holds when $t[\Theta_2](x)$ is shallower. To complete the argument we show that indeed a steeper transfer $t[\Theta_2](x)$ strictly satisfies the moral hazard constraint that is binding under $t[\Theta_1](x)$. Take $t[\Theta_2](x)$ steeper than $t[\Theta_1](x)$ (so $t[\Theta_2](x)$ single-crosses $t[\Theta_1](x)$ from below arbitrarily close to $\tilde{x}$). Then

$$\begin{aligned}
& \int_{\underline{x}}^{\tilde{x}} u(t[\Theta_2](x))dF_a(x|a) + \int_{\tilde{x}}^{\overline{x}} u(t[\Theta_2](x))dF_a(x|a) \\
= & \int_{\underline{x}}^{\tilde{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\underline{x}}^{\tilde{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] dF_a(x|a) \\
+ & \int_{\tilde{x}}^{\overline{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\tilde{x}}^{\overline{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] dF_a(x|a) \\
= & \int_{\underline{x}}^{\tilde{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\underline{x}}^{\tilde{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) \\
+ & \int_{\tilde{x}}^{\overline{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\tilde{x}}^{\overline{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) \\
= & c'(a) - \left(\int_{\underline{x}}^{\tilde{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) + \int_{\tilde{x}}^{\overline{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) \right) \\
> & c'(a)
\end{aligned}$$

where the penultimate line comes from the moral hazard constraint under Θ_1. The last line uses the fact that $u(t[\Theta_1](x)) - u(t[\Theta_2](x)) > 0$ and $\frac{f_a}{f} < 0$ to the left of $\tilde{x}$, and conversely to its right. So under $t[\Theta_2](x)$ the agent would rather pick a higher action. Hence $\int u'\Delta t dF_a > 0$ so that $da/d\Theta > 0$. ■

Collecting Claims 1 and 2, it follows that $d\Pi(a^S)/d\Theta > 0$, as claimed. ■

Proof of Lemma 1: In the competitive search equilibrium the equilibrium rent U^S satisfies the relation

$$[[1 - p_0(\theta)]/\theta]U^S + [1 - [1 - p_0(\theta)]/\theta]u_0 = \widetilde{V}.$$

From Proposition 2 we know that the standard second-best rent $U^{SB} = u_0 < U^S$. The statement we want to prove is this: consider any rent level $U \in [u_0, U^S)$; to the rent level U must correspond a higher action. To do so we use the same argument as in Claim 2 of the proof of Proposition 3.

Fix the action a^S (the solution to Problem 2) so that the distribution $F(\cdot|a^S)$ is fixed. For any $U < U^S$, we know that we can construct an alternative transfer scheme t that single-crosses $t^S(\cdot)$ from below at the point $\tilde{x}$ and clears the moral hazard constraint (3.8). Furthermore,

$$\begin{aligned} \int u(t(z))dF(z|a^S) - c(a^S) &< \int u(t^S(z))dF(z|a^S) - c(a^S) = U^S \\ \int u(t(z))dF(z|a^S) &< \int u(t^S(z))dF(z|a^S) \\ \Leftrightarrow \int t(z)dF(z|a^S) &< \int t^S(z)dF(z|a^S). \end{aligned}$$

So the transfer $t(x)$ is cheaper to the principal. This procedure may be carried out for any rent level U and corresponding action a and transfer t, and stops when $U = U^{SB}, a = a^{SB}$ and $t = t^{SB}$. ■

Proof of Lemma 2: Social welfare per agent reads

$$\frac{W(a)}{N} \equiv \frac{1-p_0(\theta)}{\theta}\left[\int_{\mathcal{X}} zdF(z|a) - T(a) + U(t,a)\right] + \left[1 - \frac{(1-p_0(\theta))}{\theta}\right] u(h+y) - h$$

where $\theta = \Theta$ is fixed in equilibrium and with $U(t,a)$ satisfying the standard participation and moral hazard constraint, and where $T(a^S) \equiv \int t(z)dF(z|a^S)$ is known to be increasing concave. Differentiate with respect to the action

$$\frac{dW}{da} = \frac{1-p_0(\theta)}{\theta}\left[\int_{\mathcal{X}} zdF_a(z|a) - T_a + U_a\right],$$

again making use of the envelop condition implied by the moral hazard constraint $U_a = 0$. Because the multiplier μ is known to be positive, the first-order condition (3.10) immediately tells us that $\int xF_a(x|a) - T_a > 0$, hence $\frac{dW}{da} > 0$. ■

Proof of Proposition 4: Fix M, N. In light of Proposition 1 one can use a unique transfer function $t(x)$ and a unique h. The output-maximizing planner solves

Problem 4

$$\max_{t,h,a} O(t,a) = M[1-p_0(\theta)]\pi(t,a) + N\left[\left(1 - \frac{1-p_0(\theta)}{\theta}\right) u(h+y) - h\right]$$

s.t. $U(t,a) \geq u(h+y),\ U_a = 0,\ h \geq 0.$

The MUP (3.7) does not enter the planner's problem because the he can dictate the terms of trade to the principals; more precisely, the planner can impose a contract. Divide by N and observe this a scaled replication of the principals' problem. Then, as a direct consequence of Lemma 2, the planner selects action $a = a^{SB}$ characterized by (2.2) – the standard second-best action. Because $u(\cdot)$ is monotone increasing in t and $t(\cdot)$ must be non-decreasing, the condition $U_{a=a^{SB}} = 0$ guarantees there is a single transfer $t(x)$ that implements the second-best action a^{SB}, namely the standard second-best transfer function t^{SB} characterized by (2.1). It then follows that $U(t, a) = u_0$, and it is enough to also set $h = 0$ – the agent is not exposed to a lottery and always receives her outside option u_0. Hence the planner's preferred solution is indeed the second-best solution, given M, N.

Now let k denote the principals' entry cost; given the standard second-best contract $t^{SB}(x), a^{SB}$ solving (2.1) and (2.2) with $h = 0$ the planner maximizes

$$\frac{O(\theta)}{N} - \frac{M}{N}k = \frac{O(\theta)}{N} - \frac{k}{\theta}$$

by choice of the market tightness θ, with condition

$$-\frac{p_0'(\theta) + 1 - p_0(\theta)}{\theta^2}\left[\pi^{SB} + U^{SB} - u_0\right] + \frac{k}{\theta^2} = 0, \tag{A.8}$$

where $U^{SB)-u_0=0}$, and throughout $\pi \equiv \pi(a, \theta)$.. Using the definition of elasticity $\eta(\theta)$ this rewrites

$$[1 - p_0(\theta)][1 - \eta(\theta)]\pi^{SB} = k. \tag{A.9}$$

This expression must be compared to the free-entry condition of principals under the competitive search equilibrium:

$$[1 - p_0(\theta)]\,\pi^S - \theta h = k, \tag{A.10}$$

which re-arranges as

$$[1 - p_0(\theta)][1 - \eta(\theta)]\left[\pi^S + \frac{U(t^S, a^S) - u(h^S + y)}{\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u'(h^S + y)}\right] = k. \tag{A.11}$$

Unfortunately $\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u'(h^S + y) < 1$, which prevents us from readily concluding even though $\pi^S < \pi^{SB}$ because $a^S < a^{SB}$. To complete the proof we make two claims

Claim 3 *Under competitive search $\theta^S \neq \theta^{SB}$.*

Proof: Suppose $\theta^S = \theta^{SB}$, then using (A.8) and (A.10), at the point $\theta^S = \theta^{SB} = \theta$,

$$\left[1 - p_0(\theta) + \theta p_0'(\theta)\right] \pi^{SB} = \left[1 - p_0(\theta)\right] \pi^S - \theta h.$$

Re-arranging and using the definition $\eta(\theta) = \frac{\partial(1-p_0(\theta))}{\theta} \frac{\theta}{1-p_0(\theta)} = -p_0'(\theta)\frac{\theta}{1-p_0(\theta)}$:

$$\begin{aligned} [1 - p_0(\theta)]\left[\pi^{SB} - \pi^S\right] &= -\theta p_0'(\theta)\pi^{SB} - \theta h \\ &= \eta(\theta)(1 - p_0(\theta))\pi^{SB} - \theta h \\ &= (1 - p_0(\theta))(1 - \eta(\theta))\left[U^S - u(h^S + y)\right] \mathbb{E}\left[\frac{1}{u'(t^S)}\right] \end{aligned}$$

where the last equality uses expression (3.13). The last line can only hold if $\theta = 0$ or if $\eta(\theta) = 0$, that is, only if $-\frac{\theta p_0'(\theta)}{1-p_0(\theta)} = 0$, which can only be true when search is not directed (i.e. $p_0'(\theta) = 0$). ■

Claim 4 $\lim_{\theta\to 0} \frac{d}{d\theta}\left\{[1 - p_0(\theta)]\pi^S - \theta h^S\right\} > 0$ *and* $\lim_{\theta\to 0} \frac{d}{d\theta}\left\{[1 - p_0(\theta) + \theta p'_(\theta)]\pi^{SB}\right\} = 0.$

Proof: The first term computes as $-p_0'(\theta)\pi^S + [1 - p_0(\theta)]\frac{d\pi^S}{d\theta} - \left(h^S + \theta\frac{dh^S}{d\theta}\right)$. Evaluated at $\theta = 0$ we have

$$\begin{aligned} -p_0'(\theta)\pi^S + [1 - p_0(\theta)]\frac{d\pi^S}{d\theta} - \left(h^S + \theta\frac{dh^S}{d\theta}\right) &= -p_0'(\theta)\pi^S - h^S \\ &= \frac{1 - p_0(\theta)}{\theta}\eta(\theta)\pi^S - h \\ &= -\frac{p_0'(\theta)}{1}\eta(\theta)\pi^S - h > 0. \end{aligned}$$

The first line uses $p_0(\theta = 0) = 1$ and the second one the definition of $\eta(\theta)$ again. The last line applies L'Hopital rule to the ratio as $\theta \to 0$. The second expression yield $\theta p_0''(\theta)\pi^{SB} + [1 - p_0(\theta) + \theta p_0'(\theta)]\frac{d\pi^{SB}}{d\theta}$, which is clearly 0 when $\theta = 0$. ■

Because these two curves never cross by Claim 3 and the first one starts above the second one by Claim 4, it always remains above it. We conclude that $\theta^S < \theta^{SB}$: too many principals enter under the competitive search equilibrium. ■

Proof of Proposition 5: A welfare-maximizing planner faces a different problem

Problem 5

$$\max_{t(x),a} \frac{1 - p_0(\theta)}{\theta}\left(\pi(t, a) + U(t, a)\right) + \left[1 - \frac{1 - p_0(\theta)}{\theta}\right] u(h + y) - h$$

s.t. (4.1), (3.6) and (3.8).

with optimality conditions

$$\frac{1}{u'(t)} = 1+\gamma, \quad \gamma \geq 0 \tag{A.12}$$

$$0 = \pi_a + \frac{\theta}{1-p_0(\theta)} U_{aa}. \tag{A.13}$$

The third condition with respect to h depends on whether the agents' participation binds at u_0; we postpone it until necessary. Let $(t_1^W(x), a_1^W)$ denote a solution such that $U(t_1^W, a_1^W) > u_0$; when that is not true (i.e. $U(t_1^W, a_1^W) \leq u_0$) we need a constrained solution, denoted $(t_2^W(x), a_2^W)$.

Unconstrained planner: $U(t_1^W, a_1^W) > u_0$

In this case the third optimality condition is

$$u'(h+y)\left(1 - \frac{1-p_0(\theta)}{\theta}\right) = 1 \tag{A.14}$$

that is, the planner is unconstrained at u_0 and can freely engage in welfare-enhancing redistribution by equating the marginal benefit of the compensatory transfer to its cost. We contrast the planner's problem to the principals' in the search equilibrium; they maximize the Lagrangian

$$\begin{aligned}\mathcal{L} &= (1-p_0(\theta))\pi - \theta h + \nu\left[\frac{1-p_0(\theta)}{\theta}U(t,a) + \left(1 - \frac{1-p_0(\theta)}{\theta}\right)u(h+y)\right] \\ &+ \gamma[U(t,a) - u(h+y)] + \lambda[U(t,a) - u_0] + \mu U_a\end{aligned}$$

which rewrites

$$\begin{aligned}\frac{\mathcal{L}}{\theta} &= \frac{1-p_0(\theta)}{\theta}\left[\pi + \frac{\nu}{\theta}U(t,a)\right] + \frac{\nu}{\theta}\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u(h+y) \\ &+ \frac{\gamma}{\theta}[U(t,a) - u(h+y)] + \frac{\lambda}{\theta}[U(t,a) - u_0] + \frac{\mu}{\theta}U_a.\end{aligned} \tag{A.15}$$

It is immediate that the planner's and the equilibrium allocation are identical only if $\nu = \theta$ and $\mu^S/\theta = \mu^W$. That is, we must have $\theta = 1$; therefore $\nu = 1$ in equilibrium. In this case,

$$u'(h^S + y)\left(1 - \frac{1-p_0(\theta)}{\theta}\right) = 1, \ \theta = 1$$

also. Away from the knife-edge condition $\theta = 1$, we must distinguish between two cases – again.

Case 1: $\theta < 1$ **in equilibrium.** Then we want to show $a^S < a_1^W, h^S > h^W$ and that the transfer function $t^S(x)$ is not as steep as $t_1^W(x)$.

Claim 5 $\theta < 1 \Rightarrow \frac{\mu^S}{\theta} > \mu^W$.

Proof: Take $\frac{\mu^S}{\theta} > \mu^W$ and re-arrange as $\frac{\mu^S}{\mu^W} > \theta$

$$\frac{-\frac{\pi_a^S}{U_{aa}^S}(1-p_0(\theta))}{-\frac{\pi_a^S}{U_{aa}^S}\frac{1-p_0(\theta)}{\theta}} > \theta \Leftrightarrow \frac{-\frac{\pi_a^S}{U_{aa}^S}}{-\frac{\pi_a^S}{U_{aa}^S}} > 1$$

hence $\mu^S > \mu^W$ and therefore $\mu^S/\theta > \mu^W$ necessarily. ■

From Claim 5 it then follows that $a^S < a_1^W$. Second,

Claim 6 $\theta < 1 \Rightarrow \frac{\nu}{\theta} > 1$.

Proof: When $\theta < 1, a^S < a_1^W$ and by FOSD of $F(\cdot|a)$, $F(x|a^S) > F(x|a_1^W)\ \forall x \in int(\mathcal{X})$. Therefore

$$\mathbb{E}\left[\frac{1}{u'(t^S)}\bigg|a^S\right] = \frac{\nu}{\theta} > \mathbb{E}\left[\frac{1}{u'(t^S)}\bigg|a_1^W\right] > \mathbb{E}\left[\frac{1}{u'(t_1^W)}\bigg|a_1^W\right] = 1,$$

where the second inequality follows from the proof of Claim 2 in the proof of Proposition 3, and $\mathbb{E}\left[\frac{1}{u'(t_1^W)}\bigg|a_1^W\right] = 1$ obtains by integrating Condition (A.12) in the planner's Problem 5. ■

From Claim 6 one immediately has $u'(h^S + y)\left(1 - \frac{1-p_0(\theta)}{\theta}\right) = \theta/\nu < 1$, whence $h^S > h^W$. Again from the proof of Claim 2 the proof of Proposition 3 we have that $t^S(x)$ is not as steep as $t_1^W(x)$. Last we need to compare the entry condition of the principals in the equilibrium given by (A.11) to the planner's condition

$$[1-p_0(\theta)][1-\eta(\theta)]\left[\pi_1^W + \frac{U(t_1^W, a_1^W) - u(h_1^W + y)}{\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u'(h_1^W + y)}\right] = k \tag{A.16}$$

where $\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u'(h_1^W + y) = 1$. Because $a^S < a_1^W, \pi^S < \pi_1^W$ and because $a^S < a_1^W, \mathbb{E}[1/u'(t^S)|a^S] > \mathbb{E}[1/u'(t_1^W)|a_1^W]$ and $h^S > h_1^W, U(t^S, a^S) - u(h^S + y) < U(t_1^W, a_1^W) - u(h_1^W + y)$ but again $\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u'(h^S + y) < 1$.

Denote $\psi^S(\theta) \equiv [1-p_0(\theta)][1-\eta(\theta)]\left[\pi^S + \frac{U(t^S,a^S)-u(h^S+y)}{\left(1-\frac{1-p_0(\theta)}{\theta}\right)u'(h^S+y)}\right]$ and $\psi^W(\theta) \equiv [1-p_0(\theta)][1-\eta(\theta)]\left[\pi_1^W + \frac{U(t_1^W,a_1^W)-u(h_1^W+y)}{\left(1-\frac{1-p_0(\theta)}{\theta}\right)u'(h_1^W+y)}\right]$. We know that Conditions (A.11)and (A.16) yield the same solution at for $k = k^*$. So at that point

$$\psi^S(\theta) = k = \psi^W(\theta).$$

Next, the two curves defined by these equations either cross or are just tangential at $\theta = 1$. For them to be tangential, their first derivatives have to agree at $\theta = 1$. We have

$$\begin{aligned}\frac{d\psi^S(\theta)}{d\theta} &= \frac{d\pi^S}{\theta} + \frac{(dU(t^S,a^S)/d\theta - u'dh^S/d\theta)\varphi(\theta^S) - \varphi'(\theta^S)[U(t^S,a^S) - u(h^S+y)]}{\varphi^2(\theta^S)} \\ \frac{d\psi^W(\theta)}{d\theta} &= \frac{d\pi^W}{\theta} + \frac{dU(t^W,a^W)}{d\theta} - u'\frac{dh^W}{d\theta}\end{aligned}$$

where $\varphi(\theta) \equiv \left(1 - \frac{1-p_0(\theta)}{\theta}\right) u'(h^S + y)$ and $\varphi(\theta = 1) = 1$. Taking all conditions together we must have

$$\begin{aligned}\frac{d\psi^S(\theta)}{d\theta} &= \frac{d\psi^W(\theta)}{d\theta}, \ \theta = 1 \\ \frac{d\psi^S(\theta)}{d\theta} &> \frac{d\psi^W(\theta)}{d\theta}, \ \theta < 1 \\ \frac{d\psi^S(\theta)}{d\theta} &< \frac{d\psi^W(\theta)}{d\theta}, \ \theta > 1.\end{aligned}$$

These conditions imply

$$\varphi'(\theta^S)[U(t^S,a^S) - u(h^S+y)] = 0, \quad \theta = 1$$

whereas we know that $\forall\theta, \ \varphi'(\theta^S)[U(t^S,a^S) - u(h^S+y)] < 0$. So we have a contradiction. Therefore $\psi^S(\theta)$ and $\psi^W(\theta)$ cannot be tangential at $\theta = 1$. But since $\psi^S(\theta = 1) = \psi^W(\theta = 1)$ they do cross. Hence

$$\begin{aligned}&\theta^S < \theta^W, && k < k^*; \\ &\theta^S = \theta^W = 1, && k = k^*; \\ &\theta^S > \theta^W, && k > k^*.\end{aligned}$$

Case 2: $\theta > 1$. This is the converse case; first take the converse of Claims 5 and 6. Then we have $u'(h^S + y)\left(1 - \frac{1-p_0(\theta)}{\theta}\right) = \theta/\nu > 1$ and again we compare the equilibrium entry condition (A.11) to the planner's condition (A.16) – evaluated at $h^S < h_1^W$. This case is easier: $\pi^S + U(t^S,a^S) - u(h^S+y) \leq \pi_1^W + U(t_1^W,a_1^W) - u(h_1^W+y)$ and now the RHS is divided by a number greater than 1. Hence (A.11) clears k for a higher θ than (A.16).

Constrained planner: $U(t_2^W, a_2^W) = u_0$.

Then the third optimality condition of the planner's Problem 5 is

$$u'(h+y)\left(1 - \frac{1-p_0(\theta)}{\theta} - \gamma\right) = 1 - \epsilon, \quad \epsilon \geq 0. \tag{A.17}$$

First we show that when the planner is constrained at u_0 he sets $h_2^W = 0$. Suppose not, the Lagrange multiplier $\epsilon = 0$ by complementary slackness. At a solution, $U(t_2^W, a_2^W) = u(h+y) > u_0$ so from (A.12), $\gamma = \mathbb{E}[1/u'(t_2^W)] - 1 > 0$. Substituting in (A.17),

$$u'(h+y)\left(1 - \frac{1-p_0(\theta)}{\theta}\mathbb{E}\left[\frac{1}{u'(t_2^W)}\right]\right) = 1, \text{ or } \frac{1}{u'(h+y)} + \frac{1-p_0(\theta)}{\theta}\mathbb{E}\left[\frac{1}{u'(t_2^W)}\right] = 1,$$

which implies that h and $t(x)$ are substitutes. But a binding constraint $U(t_2^W, a_2^W) = u(h_2^W + y)$ implies exactly the opposite. Hence we cannot have $h_2^W > 0$, and at a solution $U(t_2^W, a_2^W) = u_0$. From the equilibrium and the planner's conditions

$$\left(1 - \frac{1-p_0(\theta)}{\theta}\right)u'(h^S+y) = \frac{1}{\mathbb{E}\left[\frac{1}{u'(t^S)}\right]} \text{ and } u'(h_2^W+y)\left(1 - \frac{1-p_0(\theta)}{\theta}\mathbb{E}\left[\frac{1}{u'(t_2^W)}\right]\right) = 1-\epsilon < 1$$

one has

$$1 - \frac{1-p_0(\theta)}{\theta}\mathbb{E}\left[\frac{1}{u'(t_2^W)}\right] < \left(1 - \frac{1-p_0(\theta)}{\theta}\right)\mathbb{E}\left[\frac{1}{u'(t^S)}\right]$$

since $u'(h_2^W + y) > u'(h^S + y)$ and $\epsilon > 0$, and after some simple manipulations

$$1 < \frac{1}{1 - \frac{1-p_0}{\theta}} < \mathbb{E}\left[\frac{1}{u'(t^S)}\right] + \frac{1}{1 - \frac{\theta}{1-p_0}}\mathbb{E}\left[\frac{1}{u'(t_2^W)}\right]$$

where $\frac{1}{1-\frac{\theta}{1-p_0}} < -1$ and $\mathbb{E}\left[\frac{1}{u'(t_2^W)}\right] > 1$. Therefore

$$\mathbb{E}\left[\frac{1}{u'(t^S)}\right] > \mathbb{E}\left[\frac{1}{u'(t_2^W)}\right] > 1$$

and $\theta/\nu < 1$. With this apply Claims 5 and 6, whence $a^S < a_2^W$. Finally we compare the entry conditions (A.11) and (A.16), which now rewrites

$$[1 - p_0(\theta)][1 - \eta(\theta)]\pi_2^W = k. \tag{A.18}$$

To complete the proof, proceed as in the proof of Proposition 4; we conclude that $\theta^S > \theta_2^W$. ■

Proof of Proposition 6: First consider an output-maximizing planner. Under the characterization of Proposition 2,

$$\begin{aligned} \mathbb{E}_{\mathcal{X}}\left[\frac{1}{u'(t^S)}\right] &= \frac{1}{u'(h+y)}\frac{\theta}{\theta - 1 + p_0} \\ h &= -p_0'(\theta)\pi^S - \frac{1 - p_0 + \theta p_0'}{\theta - 1 + p_0}\frac{U - u(h+y)}{u'(h+y)} \end{aligned}$$

while efficiency requires $h = 0,\ U = u_0$ and $\mathbb{E}_{\mathcal{X}}\left[\frac{1}{u'(t)}\right] = \lambda$. Consider any redistribution possible, and after redistribution, suppose there exists a transfer function $t(x)$ such that

$$\mathbb{E}_{\mathcal{X}}\left[\frac{1}{u'(t)}\right] = \lambda = \frac{1}{u'(h+y)}\frac{\theta}{\theta - 1 + p_0},$$

then $t^{SB} = t^S$ since $t(\cdot)$ is uniquely defined. Hence $a^{SB} = a^S$ and indeed $U = u_0$, and $h = 0$. But then $-p_0'(\theta) = 0$, which contradicts our assumption that search is directed (i.e. $-p_0'(\theta) > 0$). Indeed for any corrective transfer, that is, any argument entering $\pi(t,a), U(t,a)$ or $u(h+y),\ h = 0$ violates the optimality conditions (A.6) and (A.7). Similarly if considering the characterization of Lemma 3.

Second, a welfare-maximizing planner who is constrained by u_0 faces a similar problem: he must set $h_2^W = 0$ and $U(t_a^W, a_2^W) = u_0$. Hence our conclusion is the same.

Last, when the welfare-maximizing planner is *un*constrained by u_0, let τ^A, τ^P denote (vectors of) any fiscal instrument. We look for policies that simultaneously satisfy welfare optimality of the equilibrium allocation:

$$a^S = a_1^W;\ \mathbb{E}\left[\frac{1}{u'(t^S)}\right] = 1 \text{ and } \left(1 - \frac{1 - p_0(\theta)}{\theta}\right) u'(h^S + y) = 1;$$

budget balance

$$\sum_i \tau^i = 0,\ i = A, P$$

and optimality of free entry

$$\psi_1^W(\theta_1^W; \tau^A, \tau^P) = \psi^S(\theta^S; \tau^A, \tau^P) = k,\ \theta_1^W = \theta^S \tag{A.19}$$

where these functions are defined earlier and are now parametrized by the policy instruments τ^A, τ^P. Define post tax (or subsidy) transfers $h(\tau^A), t(x; \tau^A)$. Consider an equilibrium featuring excessive entry:

$$a^S < a_1^W;\ \mathbb{E}\left[\frac{1}{u'(t^S)}\right] > 1 \text{ and } \left(1 - \frac{1 - p_0(\theta)}{\theta}\right) u'(h^S + y) < 1$$

and

$$\psi_1^W(\theta_1^W) = \psi^S(\theta^S) = k,\ \theta_1^W < \theta^S.$$

We know that implementing the planner's solution requires

$$h(\tau^A) < h^S \text{ and } \mathbb{E}\left[\frac{1}{u'(t(x;\tau^A))}\right] < \mathbb{E}\left[\frac{1}{u'(t^S)}\right];$$

suppose there exist fiscal instruments τ^A achieving this, we have $\tau^A > 0$ (positive by convention) and $a^S(\tau^A) = a_1^W$. Recall now that $\pi^S \equiv \int [z - t(z)] dF(z|a)$, and observe that budget balancing requires directing tax receipts to principals. Therefore $\pi^S \equiv \pi^S(\tau^A, \tau^P)$, where $\tau^P < 0$. But this breaks Condition (A.19), unless π^S is invariant in τ^P. The reverse argument works when seeking to correct insufficient entry. ■

Proof of Lemma 3: Contrast this to the case where $h \equiv 0$ exogenously. Then $u(y) = u_0$ and the constraint $U \geq u(h + y)$ becomes

$$U \geq u_0$$

with multiplier $\lambda \geq 0$. The FOC are

$$\begin{aligned} -[1 - p_0(\theta)]f + \lambda u' f + \mu u' f_a + \nu \frac{1 - p_0(\theta)}{\theta} u' f &= 0 \\ -p_0'(\theta) \int [x - t(x)] dF(x|a) - \nu \left[\frac{1 - p_0 + \theta p_0'}{\theta^2} \right] [U - u_0] &= 0 \end{aligned}$$

with complementary slackness conditions

$$\begin{aligned} \nu[V - \widetilde{V}] &= 0 \\ \lambda[U - u_0] &= 0. \end{aligned}$$

Immediately we must have $\nu > 0$ and $\lambda = 0$ by direct inspection of the second condition. So again

$$\frac{1}{u'(t)} = \frac{\nu}{\theta} + \hat{\mu} \frac{f_a}{f}$$

where however

$$\nu(h \equiv 0) = -\frac{\theta^2}{1 - p_0 + \theta p_0'} \frac{p_0' \pi}{U - u_0}$$

and the equilibrium transfer function clearly entails a distortion when compensatory transfers cannot be used. ■

Proof of Proposition 7: By Constraint (3.7) – the MUP,

$$U(h > 0) < U(h = 0)$$

necessarily (since $\widetilde{V}$ if fixed for any h) – the contracting agent receives a lower rent in equilibrium when using compensatory transfers. We'd like to know what that means in terms of equilibrium action. The comparison to the Lagrange multiplier $\nu(h^S)$ is not quite direct since

$$\nu(h^S) = -\frac{\theta^2}{1 - p_0 + \theta p_0'} \left[\frac{p_0' \pi}{U - u(h + y)} + \frac{h}{U - u(h + y)} \right]$$

from (A.7), with h positive but $U - u(h+y) < U - u_0$. In addition, the equilibrium transfer functions $t(x)$ are not the same in both problems. To tackle this problem we must understand which contract is cheaper. Once again we turn to the representation of the cost of effort given by Jewitt (1997) and reproduced in JKS.

$$C(a) = \max_{\hat{\nu},\hat{\mu}} \left[\hat{\nu}(h)[u(h+y) + c(a)] + \hat{\mu}c'(a) - \int \rho\left(\hat{\nu}(h) + \hat{\mu}\frac{f_a}{f}\right) dF(z|a) \right]$$

where $\hat{\nu} \equiv \nu/\theta = \nu/\Theta$. Consider

$$\left.\frac{\partial \nu(h)}{\partial h}\right|_{h=0} = -\frac{\theta^2}{(1 - p_0 + \theta p_0')[U - u(h+y)]} \left[1 - \frac{(p_0'\pi + h)u'(h+y)}{U - u(h+y)}\right] < 0$$

since $(p_0'\pi + h) < 0$ as $\nu(h) > 0 \;\; \forall h$. So the principal always wants to offer at least some h, hence

$$\nu(h^S) < \nu(h \equiv 0).$$

Therefore the cost of effort is always lower when using compensatory transfers. ■

Proof of Proposition 8: Add the constraint $\theta h \leq H$ to Problem 2 with new multiplier $\phi \geq 0$. The FOC w.r.t. $t(x), h, \theta$ are

$$-[1 - p_0(\theta)]f + \gamma u' f + \mu u' f_a + \nu \frac{1 - p_0(\theta)}{\theta} u' f \quad = \quad 0 \qquad \text{(A.20)}$$

$$-\theta - \gamma u'(h+y) + \epsilon - \phi\theta + \nu \left[1 - \frac{1 - p_0(\theta)}{\theta}\right] u'(h+y) \quad = \quad 0 \qquad \text{(A.21)}$$

$$-p_0'(\theta) \int [x - t(x)] dF(x|a) - h - \nu \left[\frac{1 - p_0 + \theta p_0'}{\theta^2}\right] [U - u(h+y)] - h \quad = \quad 0. \qquad \text{(A.22)}$$

The only new terms are $-\phi\theta$ in (A.21) and $-h$ in (A.22). As before the complementary slackness conditions are

$$\begin{aligned} \nu[V - \widetilde{V}] &= 0 \\ \epsilon h &= 0 \\ \gamma[U - u(h+y)] &= 0. \end{aligned}$$

By arguments that are now standard, $\nu > 0, \gamma = 0$ so

$$\frac{1}{u'(t)} = \frac{\nu}{\theta} + \hat{\mu}\frac{f_a}{f}$$

The case of interest is when $\phi > 0$, then $h = \frac{H}{\theta}$ and the last condition gives

$$\nu(H/\theta) = -\left[p_0'\pi + \frac{2H}{\theta}\right] \frac{\theta^2(1 - p_0 + \theta p_0')}{U - u(H/\theta + y)}$$

and since $\frac{2H}{\theta} > 0$ we have $\nu(H/\theta) < \nu(h \equiv 0)$. Meanwhile Condition (A.22) yields

$$\nu(H/\theta) = \frac{\theta(1+\phi)}{u'(H/\Theta + y)} \frac{\theta^2}{\theta - 1 + p_0}.$$

Substituting in $\frac{1}{u'(t)} = \frac{\nu}{\theta} + \hat{\mu}\frac{f_a}{f}$ gives (5.3), and since $\gamma > 0$, $\nu(H/\theta) > \nu(h^S)$ as claimed. ■

B Small market: validation

This section validates the use of a large market in the main paper. For conciseness on the case of $h \equiv 0$ – which is also the norm in the literature.

First we note that in a finite economy limiting the scope to symmetric equilibria amounts to imposing that agents use mixed strategies on and off the equilibrium path in the continuation game. This is not restrictive: Bland and Loertscher (2012) provide a refinement that selects mixed strategies in the buyers' continuation game. They show that all equilibria other than directed search equilibria violate a monotonicity property of agent' strategies because they require at least one agent to visit a principal with higher probability after this principal increases his price. Their result provides a rationale for focusing on directed search equilibrium with agents mixing off the equilibrium path. Furthermore, Galenianos and Kircher (2012) show in a canonical directed search model that the symmetric equilibrium is also unique. In other words, there are no asymmetric equilibria if one focuses on agents playing mixed strategies off the equilibrium path.

With this preliminary we can proceed. There is a finite number N of agents and a finite number M of principals. The rest of the environment is as described in the main text. To solve for an equilibrium in finite market, we postulate a candidate (symmetric) equilibrium contract $\mathcal{C}^* = \{t_n^*(x), a_n^*\}_{n=1}^N$, posted by all Principals $k \in \mathcal{M}\backslash j$, and consider the benefit of Principal j deviating to contract $\mathcal{C}^j = \left\{t_n^j(x), a_n^j\right\}_{n=1}^N$.

To simplify notation let $\sigma^j = \sigma$. Furthermore, let $B_n(N,\sigma) = \binom{N}{n}\sigma^n(1-\sigma)^{N-n}$ the probability of n agents selecting the deviating Principal. Similarly, let $B_n(N-1,\sigma) = \binom{N-1}{n}\sigma^n(1-\sigma)^{N-n-1}$ be the probability of n other agents selecting the deviating Principal from the perspective of an agent considering selecting that Principal. Both these functions are clearly continuous and differentiable in σ. Noting that $B_n(N-1,\sigma) = \frac{B_n(N,\sigma)}{\sigma N}$, the deviating Principal solves:

$$\max_{\{t_n, a_n\}_{n=1}^N, \sigma \in (0,1)} \sum_{n=1}^N B_n(N,\sigma)\pi(t_n(x), a_n)$$

$$\text{s.t. } U(t_n, a_n) \geq u_0 \tag{B.1}$$

$$a_n \in \arg\max U(t_n, a_n) \tag{B.2}$$

$$V_P(\sigma) \geq V_P(\sigma^*) \tag{B.3}$$

$$\sigma + (M-1)\sigma^* = 1 \tag{B.4}$$

where

$$V_P(\sigma) \equiv \sum_{n=1}^{N} \frac{B_n(N,\sigma)}{\sigma N}(U(t_{n,}a_n) - u_0) + u_0,$$

and

$$V_P(\sigma^*) \equiv \sum_{n=1}^{N} \frac{B_n(N,\sigma^*)}{\sigma^* N}(U^*(t^*_{n,}a^*_n) - u_0) + u_0.$$

σ^* represent agents selection strategy for any non deviating Principals. Note that it is responsive to σ so the Market Utility Property may not be applied. The Lagrangian for the problem is:

$$\max_{\{t_n,a_n\}_{n=1}^N,\sigma\in(0,1)} \sum_{n=1}^{N} B_n(N,\sigma)\pi(t_n(x),a_n) + \mu U_{a_n} + \lambda(U-u_0) + \nu[V_P(\sigma) - V_P(\sigma^*)].$$

We optimize over σ as in Proposition 1 (envelope condition). The necessary conditions are:

$$\begin{aligned}
t_n \quad &: \quad \pi_{t_n} + \mu U_{a_n t_n} + \lambda U_{t_n} + \nu\frac{U_{t_n}}{\sigma N} = 0, \ \forall n \leq N \\
a_n \quad &: \quad \pi_{a_n} + \mu U_{a_n a_n} + \lambda U_{a_n} + \nu\frac{U_{a_n}}{\sigma N} = 0, \ \forall n \leq N \\
\sigma \quad &: \quad \sum_{n=1}^{N} \frac{\partial B_n(N,\sigma)}{\partial\sigma}[\pi + \mu U_{a_n} + \lambda(U-u_0)] \\
&+ \quad \nu\left[\sum_{n=1}^{N} \frac{\partial\frac{B_n(N,\sigma)}{\sigma N}}{\partial\sigma}(U-u_0) - \sum_{n=1}^{N} \frac{\partial\frac{B_n(N,\sigma^*)}{\sigma^* N}}{\partial\sigma}\frac{\partial\sigma^*}{\partial\sigma}(U^*-u_0)\right] = 0
\end{aligned} \tag{B.5}$$

where $\frac{\partial\sigma^*}{\partial\sigma} = -\frac{1}{M-1}$. The complementary slackness conditions associated with (B.3) and (B.1) read $\nu(V - V^*) = 0$ and $\lambda(U - u_0) = 0$. Since we focus on symmetric equilibrium where $\sigma = \sigma^* = \frac{1}{M}$, $U = U^*$, $t_n = t^*_n$ and $a_n = a^*_n$ for all n, (B.5) becomes:

$$\sum_{n=1}^{N} \frac{\partial B_n(N,\sigma)}{\partial\sigma}[\pi + \mu U_{a_n} + \lambda(U-u_0)] + \nu\left[\sum_{n=1}^{N} \frac{\partial\frac{B_n(N,\sigma)}{\sigma N}}{\partial\sigma}\frac{M}{M-1}(U-u_0)\right] = 0$$

Next, suppose $\lambda > 0$ then $U = u_0$ by complementary slackness; combining with $U_{a_n} = 0$ generates a contradiction by the third condition. So $\lambda = 0$ necessarily. To show $\nu > 0$, rewrite (B.5) again as

$$\nu = \frac{\sum_{n=1}^{N} \frac{\partial B_n(N,\sigma)}{\partial\sigma}\pi(t_n,a_n)}{\left[-\sum_{n=1}^{N} \frac{\partial\frac{B_n(N,\sigma)}{\sigma N}}{\partial\sigma}\frac{M}{M-1}(U(t_n,a_n) - u_0)\right]}.$$

It is easy to show that $\frac{\partial B_n(N,\sigma)}{\partial \sigma} > 0$ and $\frac{\partial \frac{B_n(N,\sigma)}{\sigma N}}{\partial \sigma} < 0$ for all n. Therefore $\nu > 0$ and is unique and *independent* of n,. Rewriting the first two necessary conditions again using $\sigma = \frac{1}{M}$ and $\Theta = \frac{N}{M}$,

$$\frac{1}{u_{t_n}} = \frac{\nu}{\Theta} + \mu \frac{f_{a_n}}{f}, \ \forall n \leq N \tag{B.6}$$

$$\pi_{a_n} + \mu U_{a_n a_n} = 0, \ \forall n \leq N.$$

Here too we can make use of Proposition 1, which is not specific to a large market. Then things simplify further:

$$\nu = \frac{(1 - \frac{1}{M})^{N-1} \pi}{\left[\frac{1 - (1 - \frac{1}{M})^N - \Theta (1 - \frac{1}{M})^{N-1}}{(\Theta)^2} \frac{M}{M-1} (U - u_0) \right]} > 0.$$

Substituting in (B.6) yields

$$\frac{1}{u_t} = \mu \frac{f_a}{f} + \frac{\Theta (1 - \frac{1}{M})^{N-1} \pi}{\left[(1 - (1 - \frac{1}{M})^N - \Theta (1 - \frac{1}{M})^{N-1}) \frac{M}{M-1} (U - u_0) \right]},$$

and taking the limit as $N, M \to \infty$ (but maintaining Θ finite)

$$\frac{1}{u'} = \lambda + \mu \frac{f_a}{f} + \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta} - \Theta e^{-\Theta}} \frac{\pi}{U - u_0}$$

just as in the large market.

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