Interbank Counterparty Risk and Recovery Rates in Credit Default Swaps *

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Abstract

In this paper, I measure the interbank counterparty risk embedded in bank credit default swap (CDS) contracts. When a bank writes a CDS contract on the default of another bank, the buyer of that contract is faced with the risk of joint default by both banks. Using a unique feature of CDS data, I present a new approach for valuing CDS spreads that enables identification of the joint and conditional default probabilities, allowing for time variant recovery rates. I use the term structure of CDS spreads and option implied default probabilities to estimate time-variant joint default probabilities and time-variant recovery rates. In comparison to this approach, estimating interbank counterparty risk from CDS spreads assuming a fixed recovery rate underestimates joint default probability when the market is in distress. I apply the joint estimation method to measure interbank counterparty risk of CDS dealers from 2007 to 2010 and find that the fixed recovery rate model underestimates expected counterparty risk by approximately 21% when the market is in distress. In addition, I show that a bank's vulnerability to systemic risk, defined as the average conditional default probability of a bank conditional on default of its counterparties, is correlated with existing measure of systemic risk.

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1 Introduction

Contagion of systemic risk, through the interconnectedness of banks, has generated grave concerns since the financial crisis of 2007–2009. These concerns have triggered regulatory reform in which bank supervisors now monitor and address risks to financial stability, rather than focusing only on individual institutions. Given the importance of global financial stability, the interconnectedness of large banks and contagion of systemic risk are important issues. However, accurately quantifying interbank counterparty risk remains a challenge. To address this issue, I propose an approach to measure interbank counterparty risk, or the joint default probability of large banks, embedded in credit default swap (CDS) spreads.

When a bank writes a CDS contract on the default of another bank, the buyer of the CDS contract is faced with the risk of both banks defaulting. Let us assume that an investor buys a bond issued by bank i. This investor may buy a CDS contract from a CDS dealer, say bank j, to avoid loss on the bond if it were to default. The investor will then pay a CDS spread to bank j in exchange for the lump sum that bank j will pay when bank i goes bankrupt. Here, the question remains whether bank j can actually pay back the promised amount when bank i goes bankrupt. We know that multiple large banks, including Merrill Lynch and AIG, effectively went bankrupt within a few days of the Lehman Brothers default in September 2008. Therefore, counterparty risk exists in CDS contracts where the underlying firm is a bank.

While there has been some literature analyzing counterparty risk in CDS spreads, existing papers (e.g., Giglio (2013); Arora, Gandhi, and Longstaff (2012)) focus on a special setting. The present article targets three questions regarding interbank counterparty risk in CDS spreads. First, is it possible to measure the point estimate of joint default probability, and not the bounds of the probabilities without relying on proprietary CDS quote data? Second, does the fixed recovery rate assumption affect estimation of counterparty risk? Third, what implications does interbank counterparty risk have on systemic risk? I develop an approach toward addressing these questions.

The belief that counterparty risk is minimal in swap contracts was accepted market wisdom until the global financial crisis of 2007–2009. It was believed that the mark-to-market convention combined with collateral posting would mitigate counterparty risk, if any existed at all. Duffie and Huang (1996) empirically supported this view so it was reasonable for both practitioners and academics to ignore counterparty risk, not only in the interest rate swap market but also in the CDS market. Thus, it was reasonable that many subsequent papers not only in the interest rate swap literature such as Liu, Longstaff, and Mandell (2006), but also papers in other fields, such as Pan and Singleton (2008), chose to ignore counterparty credit risk. While such papers do not rule out the existence of counterparty risk, they were not able to detect an economically and statistically significant role of counterparty risk in swap markets at the time. Their findings were consistent with the argument that the standard mark-to-market procedures and the use of collateral were sufficient to mitigate counterparty risk in swap markets.

However, the global financial crisis of 2007–2009 shattered this conventional belief and motivated the measurement counterparty risk, to gauge the stability of the financial system. Giglio (2013) disentangles counterparty risk embedded in CDS spreads from bank specific credit risk using bank bond prices. This author uses linear programming to derive the upper and lower bounds of the joint default probabilities, while assuming a fixed recovery rate. While this is a useful approach, the bounds can sometimes be too wide and convey little information. Using a proprietary dataset of CDS quotes from different CDS dealers on the same underlying firm, Arora, Gandhi, and Longstaff (2012) empirically verify that counterparty risk is priced in CDS spreads. However, without access to a proprietary dataset, estimating counterparty risk precisely is a difficult task.

Using a unique feature of CDS data, I present a new approach to measure point estimates, and not the bounds, of the joint default probability without relying on proprietary data. The unique feature of CDS data is that a bank cannot price it's own CDS. Even the most secure bank cannot promise others that it will pay money back when it goes bankrupt. Thus, the CDS spread of a CDS dealer bank j, can be priced only by the counterparties of bank j, or by the CDS dealers excluding bank j. Since the readily available CDS spread data are the average of the quotes by different dealers, this implies that the interbank counterparty risk of bank j, or the average pairwise joint default probability of bank j and its counterparties, can be extracted from the CDS spread of bank j. Here we focus on measuring the average of pairwise joint default probabilities between bank j and its counterparties. Even when the number of banks, n, is a moderate number, estimating joint default probability separately for each bank i, j pair is inherently difficult. This is due to the fact that the number of bank pairs i, j grows exponentially while the number of available input data points grows linearly.

Second, I check the validity of the fixed recovery rate assumption and verify whether this assumption affects measurement of counterparty risk. While the fixed recovery rate assumption is widely used, I generate evidence supporting the view that recovery rates should vary over time. These findings are consistent with the literature analyzing time-variant recovery rates, such as Conrad, Dittmar, and Hameed (2013) and Doshi (2011). Next, I find that the fixed recovery rate assumption affects measurement of counterparty risk, and may yield biased estimates. A fixed recovery rate model assuming a constant fixed recovery rate parameter would underestimate the expected counterparty risk when the market is not performing well. I also check how the zero counterparty risk assumption would affect measurement of time-variant recovery rates. Conrad, Dittmar, and Hameed (2013) disentangles time-varying recovery rates and credit risk embedded in CDS spreads by using option-price-implied default probabilities and assuming zero counterparty risk. Although the zero counterparty risk method may be reasonable for measuring time-variant recovery rate of nonfinancial firms, counterparty risk should be critical at least for financial firms, as we learned from the recent crisis. I find that the zero counterparty risk model overestimates expected recovery rates regardless of market conditions. Thus, to correctly estimate counterparty risk and recovery rates for financial firms, we must measure counterparty risk while allowing the recovery rate to be time-variant.

Next, I present a framework for jointly estimating counterparty risk and time-variant recovery rates simultaneously. Our approach uses the term structure of CDS spreads and option-price-implied default probabilities to disentangle time-variant recovery rates with time-variant joint default probabilities. Since information from both CDS and equity option markets is used, I employ the Kalman filter to remove noise. This framework is applied to estimate counterparty risk and recovery rates of large financial institutions that dominated the CDS market in the period 2007 to 2010. I find that the counterparty risk estimated from fixed recovery rate model underestimates expected counterparty risk by approximately 21% when the market is in distress.

Finally, I examine the implications of interbank counterparty risk for systemic risk. First, I compare the conditional default probability with Marginal Expected Shortfall (MES) of Acharya, Pedersen, Philippon, and Richardson (2012), a popular measure of systemic risk. The average conditional default probability of a bank, conditional on default of its counterparties, is a proxy of a bank's vulnerability to systemic risk. Having calculated the joint default probabilities, measuring conditional default probability is a straightforward process. I show that conditional default probability is highly correlated with MES. Second, I compare idiosyncratic and systemic risk, as in Giglio (2013). I find that idiosyncratic risk, or average marginal default probability, increases gradually from July 2007 and first peaks during the Bear Stearns collapse. On the other hand, systemic risk, or average interbank counterparty risk, remains low until a sharp increase when Bear Stearns collapsed in March 2008. Both results are consistent with Giglio (2013).

The remainder of this paper proceeds as follows. Section 2 describes the CDS market, including existing methods used to infer counterparty risk or recovery rates from CDS spreads, along with their limitations. Section 3 discusses potential biases in existing models. Section 4 outlines the data and methodology. Section 5 sets forth the main results. Finally, Section 6 describes the study conclusions.

2 The CDS Market

2.1 The CDS Contract

An overview of a typical CDS contract is shown in the following figure.

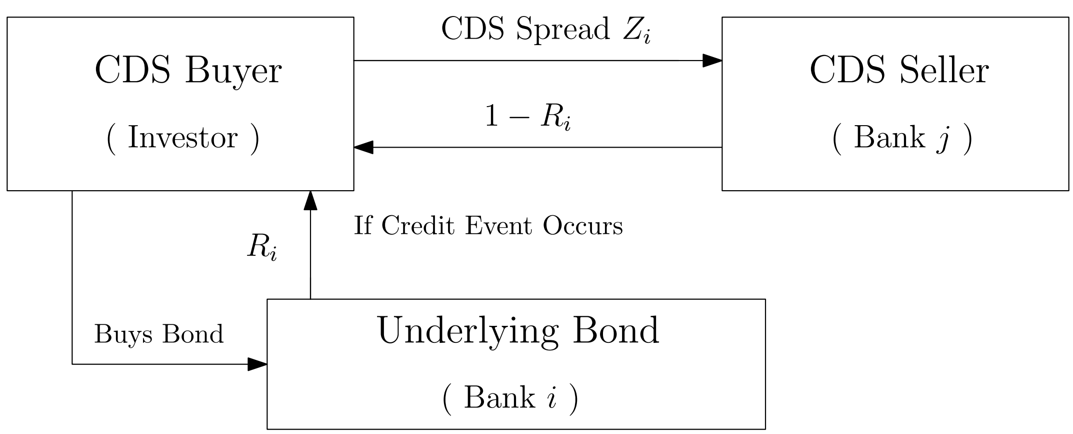


Figure 1: Overview of a CDS Contract

An investor who has bought a bond of an underlying bank i may choose to insure itself in the case of a credit event, such as the bankruptcy of bank i. When the underlying bank i experiences a credit event, the investor will only be able to recoup the recovery rate R_i of the face value. Therefore, the investor may want to buy a CDS contract written by bank j. The investor pays a premium Z_i to bank j, in exchange for the lump sum $1 - R_i$ that bank j will pay when the underlying bank i defaults.

If we ignore counterparty risk and assume that loss given default $1 - R_i$ will always be paid by bank j when bank i defaults, the market price of CDS spread Z_i depends primarily on the risk-neutral probability that the underlying bank i defaults $P(D_i)$, and the expected recovery rate $E[R_i]$. Typically, a fixed recovery rate parameter $\bar{R}$, within the range $[0.3, 0.5]$ will be used in place of the expected recovery rate. Thus, a simple risk-neutral valuation model for pricing a CDS contract yields $Z_i = P(D_i)(1 - \bar{R})$. [1]

2.2 Counterparty Risk in CDS Contracts

The global financial crisis of 2007–2009 has changed the conventional view that counterparty risk is negligible, especially among financial firms. Since CDS dealers try to act as market makers in the CDS market, ignoring counterparty risk may be a harmless abstraction when dealing with nonfinancial firms. Dealers selling CDS contracts may hedge themselves from the counterparty risk of the non-financials.

But this is not the case for financial firms. If the underlying bank and the CDS sellers are both in the same industry, they are exposed to joint default risk that cannot be ignored. Their day-to-day operations require collateral posting on a daily basis to settle net exposure. For example, Goldman Sachs argues that there were periods when AIG was not providing enough collateral, so Goldman Sachs had to hedge themselves in the CDS market.[2] Thus mark-to-market is an incomplete mechanism to mitigate counterparty risk among banks, and interbank counterparty risk should not be ignored.

Arora, Gandhi, and Longstaff (2012) discuss the collateral channel, as another reason why joint default risk should be non-negligible for financial firms. The collateral posted by the CDS seller to the underlying bank for their normal business operations (outside the CDS contract) may be rehypothecated by the underlying bank to a third party to obtain a loan. If the underlying bank were to default, the third party may seize and sell the rehypothecated collateral, leaving the CDS seller as a general unsecured creditor of the underlying bank subject to significant loss from the bankruptcy. Even if the collateral were not rehypothecated but not segregated from the underlying

[1] Refer to the Appendix for the derivation of this equation.

[2] Goldman Sachs (2009) and Harper, Westbrook, and Son (2010) document that Goldman Sachs bought CDS from other banks such as Citigroup and Lehman Brothers, to protect itself from the possible default of AIG.

bank's general assets, which was known to be typical practice prior to the Lehman default, the CDS seller would also be a general unsecured creditor of the underlying bank and would be subject to loss. Therefore, interbank counterparty risk is not negligible for financial firms.

Giglio (2013) suggests another reason that collateralization cannot fully eliminate counterparty risk. This author argues that the 5-year CDS contract on Lehman Brothers would have been in the money 15 cents on the dollar on the day before default, September 12, 2008. Thus, even under full collateralization, the counterparty would have posted only that amount as collateral. We know that the recovery rate on Lehman Brothers was realized at 8.625 cents per dollar, and the counterparty would have owed 91.375 cents per dollar after the default of Lehman Brothers on the 13th. Given the events that followed, without government support, it is highly likely that a joint default event would have occurred.[3] Thus, even on the day preceding these events, market participants were collateralized only a fraction of the damage that would have taken place.

Arora, Gandhi, and Longstaff (2012) test whether counterparty risk is actually priced in the CDS market. The authors find that CDS seller's credit quality negatively impacts CDS spread prices, and that counterparty risk in priced in CDS spreads.

2.3 Estimating Counterparty Risk in CDS Contracts

Giglio (2013) develops a model allowing for counterparty risk, or the joint default probability of the CDS selling bank j and the underlying bank i. Throughout the remainder of this paper, I use the terminology *counterparty risk* and *joint default probability* interchangeably. The CDS spread of bank i, Z_i, will be driven by the marginal default probability $P(D_i)$, expected recovery rate R_i of the underlying bank i, and joint default probability $P(D_i \cap D_j)$. Giglio (2013) estimates the counterparty risk embedded in CDS contracts using both CDS spreads and bond prices of banks. Assuming a fixed recovery rate $\bar{R}$, the CDS spread pricing equation is as follows:

$$Z_{i/j} = (1 - \bar{R}) \left[P(D_i) - (1 - S) P(D_i \cap D_j) \right] \tag{1}$$

where $Z_{i/j}$ is the CDS spread, where bank i is the underlying firm and bank j is the CDS seller,

[3] Shortly after the Lehman event, Merrill Lynch (September 14th) and AIG (September 16th) received government bailouts either directly or indirectly.

and $P(D_i \cap D_j)$ is the joint default probability of banks i and j. S represents the percentage collateralization, or the fraction of the claim that the buyer of the CDS contract recovers in case both banks i and j default. If the buyer is fully collateralized, $S = 1$ and the joint default probability or the counterparty risk can be ignored. If the buyer is not fully collateralized, $S < 1$, and the counterparty risk should not be ignored.

Therefore, one can use two methods based on the above equation 1 to extract the counterparty risk from CDS spreads. First, as in Giglio (2013), one can estimate the upper and lower bounds of the joint default probabilities using linear programming methods. Second, let us assume that a proprietary dataset of CDS quotes by different CDS dealers on the same underlying firm is given. Then, identifying the counterparty risk would be straightforward since we could easily identify banks i and j.

2.4 Estimating Average Counterparty Risk in CDS Contracts

However, there may be cases in which proprietary data is unavailable and simply knowing the range of counterparty risk is insufficient.[4] For example, when the range of counterparty risk becomes too wide, the estimate may become useless. Using a unique feature of CDS data that a bank cannot price it's own CDS contracts, I present a novel approach to estimate average interbank counterparty risk. I argue that the average pairwise joint default probability of a CDS dealer and its counterparties can be extracted from the CDS spread of that dealer.

The only CDS spread data that are publicly available are the mean CDS spread, or the average of the quotes submitted by CDS dealers. The mean CDS spread for the underlying bank i, $\bar{Z}_i$, is the average of the CDS quotes posted by other banks excluding bank i.

$$\bar{Z}_i = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} Z_{i/j}$$

where n is the total number of CDS dealer banks. Let us use the following notations for the average pairwise joint default probability,

[4]If proprietary data are available, estimating counterparty risk and recovery rate would be a much simpler process. Refer to the Appendix for details.

$$P(D_{-i} \cap D_i) = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} P(D_j \cap D_i) = P(D_{-i}|D_i)P(D_i)$$

where $P_q(D_{-i}) = \frac{1}{n-1} \sum_{j=1,j\neq i}^{n} P(D_j)$ is the average marginal default probability of the other $n-1$ banks, excluding bank i, and $P(D_{-i}|D_i)$ is the average pairwise conditional default probability such that $P_q(D_{-i}|D_i) = \frac{1}{n-1} \sum_{j=1,j\neq i}^{n} P(D_j|D_i)$. Thus, $P(D_{-i}|D_i)$ is the average pairwise default probability of other banks defaulting, conditional on bank i defaulting. As shown in later sections, along with the average joint default probability, $P(D_{-i} \cap D_i)$, conditional default probability is also a measure of systemic risk.

Based on the above definitions of joint default probability and conditional default probability, the CDS spread valuation equation is as follows:

$$\bar{Z}_i = (1 - \bar{R}) \left[P(D_i) - (1 - S) P(D_i \cap D_{-i}) \right]$$

Rather than trying to estimate joint default probabilities $P(D_i \cap D_j)$ for each pair of banks i and j, I abstract the interconnectedness into a single average pairwise joint default probability, or $P(D_i \cap D_{-i})$, for each bank i. Given n banks, trying to estimate the joint default probability for each bank pair would yield $\binom{n}{2}$ estimates. Since the number of inputs would be only $2n$, n default probabilities and n CDS spreads, trying to estimate $\binom{n}{2}$ variables would be very difficult even when n is a moderate number. Therefore, I suggest that estimating n average pairwise joint default probabilities from $2n$ inputs would be more straightforward.

3 Potential Biases in Existing Models

In the preceding section, I discuss existing methods used to estimate counterparty risk and suggest a new approach. However, the methods described are based on the assumption of using fixed recovery rates. I now first check the validity of the fixed recovery rate assumption, then discuss the existing method for estimating time-variant recovery rates.

Next, I examine how the fixed recovery rate assumption affects measurement of counterparty risk. I find that existing methods may produce biased estimates of counterparty risk or recovery rates. If we estimate counterparty risk using a mis-specified model assuming a fixed recovery rate, the estimates will be biased. More importantly, we will underestimate the expected counterparty risk when the market is in distress. On there other hand, if we estimate the recovery rate using a mis-specified model assuming zero counterparty risk, we will be overestimating the expected recovery rate regardless of market conditions.

3.1 Validity of the Fixed Recovery Rate Assumption

There has been a moderate body of literature on the time-varying nature of recovery rates. Altman, Brady, Resti, and Sironi (2005) suggest that the supply of defaulted bonds explains aggregate recovery rates. These authors find that default rates, or realized default probabilities, and recovery rates, show negative correlation. Bharath, Acharya, and Srinivasan (2007) shows similar results and finds evidence for the fire sale effect in Shleifer and Vishny (1992). The intuition for these results is simple. Let us assume that a market or industry-wide shock correlates with the default event. Then, the potential buyers of the defaulted assets, either distressed funds or firms in the same industry, may also be in trouble or reluctant to buy the assets, leading to a lower recovery value.

To further verify whether a fixed recovery rate assumption is valid, let us first check whether recovery rates vary over time. Imagine the ratio of CDS spread on senior bonds over CDS spread on junior bonds for the same underlying bank. Since these two bonds share the same risk characteristics, the ratio of these two spreads should depend only on the ratio of loss given default of senior bonds to loss given default of junior bonds. Let $Z^S_{i/j}, Z^J_{i/j}$ each represent the CDS spreads for the senior bond and junior bond, respectively. Let the fixed recovery rate for the senior and junior CDS be $\bar{R}_S, \bar{R}_J$. Then, since the CDS pricing equation would be $Z^S_{i/j} = (1-\bar{R}_S)\left[P(D_i) - (1-S)P(D_i \cap D_j)\right]$, $Z^J_{i/j} = (1-\bar{R}_J)\left[P(D_i) - (1-S)P(D_i \cap D_j)\right]$, the spread ratio would be

$$\frac{Z^S_{i/j}}{Z^J_{i/j}} = \frac{(1-\bar{R}_S)\left[P(D_i) - (1-S)P(D_i \cap D_j)\right]}{(1-\bar{R}_J)\left[P(D_i) - (1-S)P(D_i \cap D_j)\right]} = \frac{1-\bar{R}_S}{1-\bar{R}_J}$$

Thus, if CDS spreads were priced according to a fixed recovery rate, then the ratio should be constant over time. The following figure plots the average CDS senior spread to junior spread ratio of the 14 CDS dealer banks from 2007 to 2010. [5] The figure clearly suggests that recovery rates are time-varying.

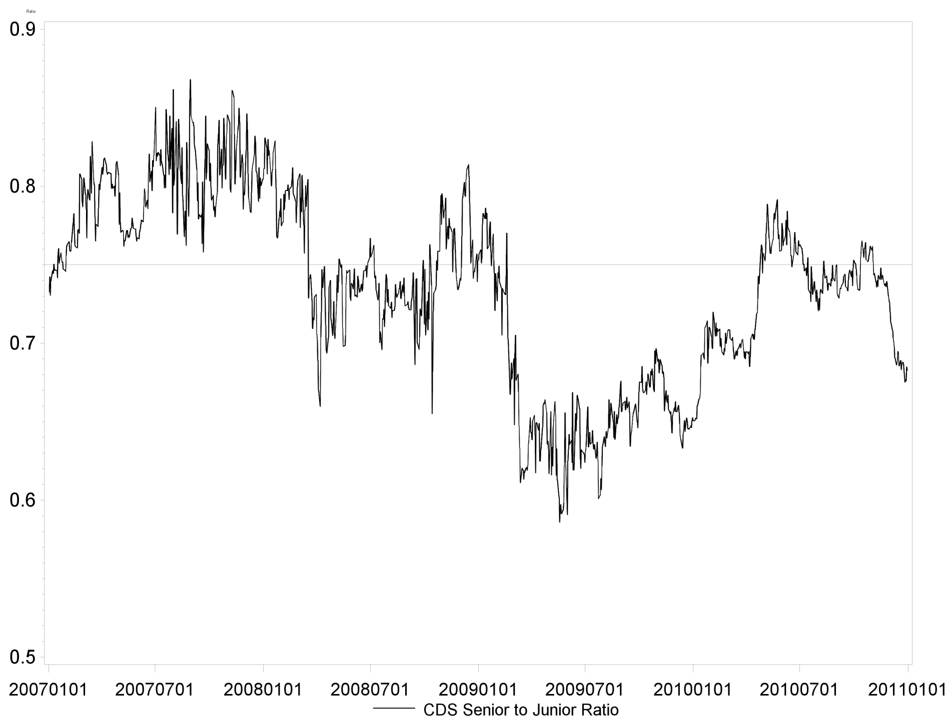


Figure 2: Average Senior to Junior CDS spread ratio

Next, let us check whether choosing a recovery rate as a fixed number between 0.3 to 0.5 is a good approximation. Here, I collect historical data from CDS auctions, or credit fixing events, from 2005 to 2014.[6] In figure 3, we plot the histogram of realized recovery rates, percentage of the par value on senior debt for corporate bonds.[7] The histogram shows that in only 8 out of 80 cases is the realized recovery rate within the range of 30% and 50%. That is, in 90% of actual default cases, the realized recovery rate is either smaller than 30% or larger than 50%. We acknowledge that these are ex-post valuations of recovery rates and that interpreting these result can be difficult. But given that the cross-sectional variation in ex-post recovery rates is so large, it would be reasonable

[5]The banks in the sample are Bank of America, Bear Stearns, Citigroup, Goldman Sachs, Lehman Brothers, JP Morgan, Merrill Lynch, Morgan Stanley, Wachovia, Barclays, Credit Suisse, Deutsche Bank, UBS, and Royal Bank of Scotland.

[6]http://www.creditfixings.com/CreditEventAuctions/disclaimerHist.jsp

[7]I remove LCDS and ELCDS credit events and focus exclusively on CDS events. Also, events where the underlying bond is either sovereign, government sponsored enterprise, or subordinate are removed. The full sample, including firm name and realized recovery rates, are shown in the Appendix.

to believe that ex-ante recovery rates should show some cross-sectional variance as well. Thus, it would not be an overstatement to argue that choosing a recovery rate parameter as a fixed number between 0.3 to 0.5 would probably lead to a biased estimate of actual recovery rates.[8]

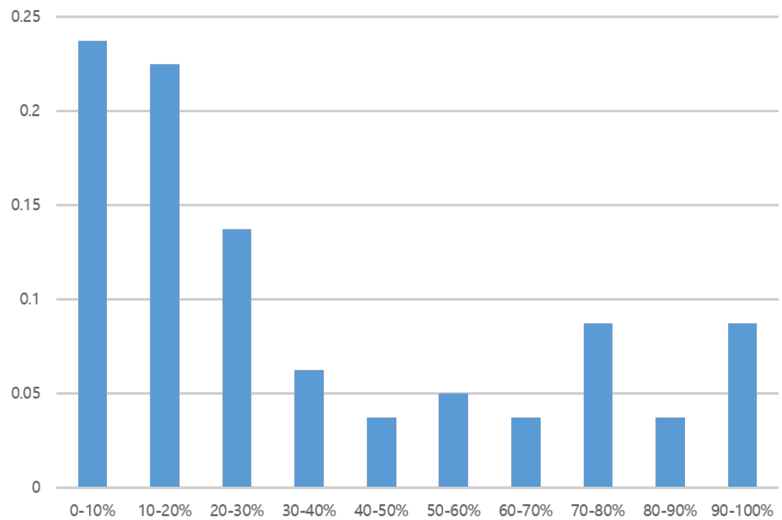


Figure 3: Historgram of actual CDS auction prices, by percentage of par value (% recovery rate)

3.2 Time-Variant Recovery Rate Ignoring Counterparty Risk

Here, I present the zero counterparty risk model, which allows for time-variant recovery rate. The CDS spread pricing equation used in Conrad, Dittmar, and Hameed (2013) is similar to the following:

$$Z_i = (1 - R_i) \times P_o(D_i)$$

where $P_o(D_i)$ is the option-price-implied default probability. Details on calculating option-price-implied default probability are presented in the Appendix. If one can calculate the $P_o(D_i)$ from option prices, the recovery rate R_i can be estimated from the observed CDS spread Z_i.

[8]In addition, the distribution of ex-post recovery rates changes over time. In the Appendix, I plot the histogram split by time periods and show that recovery rates are more dispersed during periods of distress.

3.3 An Example

Here, I illustrate a simple numerical example of how mis-specified models may produce estimates that are biased. For example, let us imagine that there are two states in the economy, the good state and the bad state. For simplicity, the probability of each state being realized is assumed to be 50%. In the good state, the observed CDS spread is 2% while the true default probability of the underlying firm is 5%. In the bad state, the observed CDS spread is 10% while the true default probability of the underlying is 20%. Also, let us assume that in the good state, true joint default probability is 1% and recovery rate is 50% so that the true model holds, $0.02 = (1 - 0.5) \times (0.05 - 0.01)$. In the bad state, true joint default probability is 7.5% and recovery rate is 20% so that $0.1 = (1 - 0.2) \times (0.2 - 0.075)$. Thus the true expected recovery rate would be 35% and the true expected joint default probability would be 4.25%.

Table 1: Estimates from the mis-specified models and the true model

	Fixed Recovery Rate Model	Zero Counterparty Risk Model	True Parameters from Joint Estimation
Good State	$\tilde{P}_G(D_i \cap D_j) = 1.6\dot{6}\%$	$\tilde{R}_G = 0.6$	$P_G(D_i \cap D_j) = 1\%$, $R_G = 0.5$
Bad State	$\tilde{P}_B(D_i \cap D_j) = 3.3\dot{3}\%$	$\tilde{R}_B = 0.5$	$P_B(D_i \cap D_j) = 7.5\%$, $R_B = 0.2$
Estimates	$E[\tilde{P}(D_i \cap D_j)] = 2.5\%$	$E[\tilde{R}_i] = 0.55$	$E[P(D_i \cap D_j)] = 4.25\%$, $E[R_i] = 0.35$

The fixed recovery rate model underestimates the expected counterparty risk, and the zero counterparty risk model overestimates the expected recovery rate

In the mis-specified fixed recovery rate model, let us use 40% as the fixed recovery rate parameter. Then, based on the observed CDS spreads and known true default probability, the joint default probabilities can be estimated. In the good state, $0.02 = (1 - 0.4) \times (0.05 - 0.016\dot{6})$, so joint default probability is estimated to be $1.6\dot{6}\%$. In the bad state, joint default probability is estimated as $3.3\dot{3}\%$, since $0.1 = (1 - 0.4) \times (0.2 - 0.03\dot{3}\%)$. So, the expected joint default probability from the fixed recovery rate model is 2.5%, and is smaller than the true expected joint default probability of 4.25%. The mis-specified model underestimates joint default probability, since the recovery rate parameter 40% is higher than the true expected recovery rate of 35%.

In the mis-specified zero counterparty risk model, the calculation is simpler. In the good state, the recovery rate is estimated to be 60%, since $0.02 = (1 - 0.6) \times 0.05$. In the bad state, the recovery rate is estimated to be 50%, since $0.1 = (1 - 0.5) \times 0.2$. Therefore, the expected recovery rate estimated from the zero counterparty risk model is 55%. Since joint default probability is ignored, the misspecified model overestimates the true expected recovery rate of 35%.

3.4 Fixed Recovery Rate Model

Now let us actually show how the fixed recovery rate model might be biased. Let us begin by assuming that the true CDS spread follows the following model. For simplicity, I assume zero collateralization.

$$Z_i = (1 - R_i)(P(D_i) - P(D_i \cap D_j))$$

Next, assume that one uses a mis-specified fixed recovery rate model, such as the following, to measure interbank counterparty risk:

$$Z_i = (1 - \bar{R})(P(D_i) - \tilde{P}(D_i \cap D_j))$$

where $\tilde{P}(D_i \cap D_j)$ is the estimated joint default probability from the mis-specified fixed recovery rate model.

Lemma 1. $Cov(\bar{R} - R_i, P(D_i) - P(D_i \cap D_j)) > 0$

Proof. Refer to the Appendix □

Proposition 1 (A fixed recovery rate model underestimates counterparty risk when the market is in distress)**.** *The mis-specified model using fixed recovery rate underestimates the joint default probability when the true expected recovery rate $E[R_i]$ is not larger than the fixed recovery rate parameter $\bar{R}$ ($E[R_i] \leq \bar{R}$), or when the market is in distress.*

Proof.

$$Z_i = (1 - R_i)(P(D_i) - P(D_i \cap D_j)) = (1 - \bar{R})(P(D_i) - \tilde{P}(D_i \cap D_j))$$

With some rearranging,

$$(\bar{R}-R_i)(P(D_i)-P(D_i\cap D_j))=(1-\bar{R})(P(D_i\cap D_j)-\tilde{P}(D_i\cap D_j))$$

Taking expectation on both sides and using the definition $Cov(X,Y)=E[XY]-E[X]E[Y]$,

$$E[(\bar{R}-R_i)(P(D_i)-P(D_i\cap D_j))]=E[\bar{R}-R_i]E[P(D_i)-P(D_i\cap D_j)]+Cov(\bar{R}-R_i,P(D_i)-P(D_i\cap D_j))$$

$$=E[1-\bar{R}]E[P(D_i\cap D_j)-\tilde{P}(D_i\cap D_j)]=(1-\bar{R})E[P(D_i\cap D_j)-\tilde{P}(D_i\cap D_j)]$$

Since $Cov(\bar{R}-R_i,P(D_i)-P(D_i\cap D_j))>0$, $E[P(D_i)-P(D_i\cap D_j)]>0$, and $1-\bar{R}>0$, the condition $E[\bar{R}-R_i]\geq 0$ is sufficient condition for $E[P(D_i\cap D_j)-\tilde{P}(D_i\cap D_j)]>0$. Thus if $\bar{R}\geq E[R_i]$, then $E[P(D_i\cap D_j)]>E[\tilde{P}(D_i\cap D_j)]$. That is, if the fixed recovery rate parameter is not smaller than the true expected recovery rate, the misspecified model underestimates expected joint default probability.

When the market is in distress, it is highly likely that the fixed recovery rate parameter $\bar{R}\in[0.3,0.5]$, used regardless of market condition, overestimates the true expected recovery rate. Thus, if we assume that $E[R_i]\leq\bar{R}$ holds in times of distress, the fixed recovery rate model underestimates the expected joint default probability.

$\square$

An interesting implication of this proposition is that even when the fixed recovery rate parameter $\bar{R}$ is chosen such that it equals the true expected recovery rate, $\bar{R}=E[R_i]$, the misspecified model underestimates expected joint default probability. The covariance between the recovery rate and the default probability causes this bias.

3.5 Zero Counterparty Risk Model

On the other hand, if we use a mis-specified model that ignores counterparty risk to estimate recovery rates, the model yields recovery rate estimates that are overestimated. The mis-specified model is as follows:

$$Z_i = (1 - \tilde{R}_i)P(D_i)$$

where $\tilde{R}_i$ is the estimated recovery rate from the mis-specified model.

Proposition 2 (Zero counterparty risk model overestimates expected recovery rate)**.** *The mis-specified model assuming zero counterparty risk overestimates the expected recovery rate, such that* $E[\tilde{R}_i] > E[R_i]$.

Proof.

$$Z_i = (1 - R_i)(P(D_i) - P(D_i \cap D_j)) = (1 - \tilde{R}_i)(P(D_i))$$

With some rearranging,

$$\tilde{R}_i - R_i = \frac{(1 - R_i)P(D_i \cap D_j)}{P(D_i)}$$

Since $P(D_i \cap D_j) > 0$, $P(D_i)$, and $1 - R_i > 0$, it is straightforward that $\tilde{R}_i - R_i > 0$. Taking expectation yields $E[\tilde{R}_i] > E[R_i]$. That is, if joint default risk is positive then the estimated recovery rate from the mis-specified model ignoring counterparty risk will overestimate the true expected recovery rate.

□

4 Data and Methodology

Thus far, I have suggested that existing models may be biased when estimating counterparty risk or recovery rates. Here, I suggest an alternative method to jointly estimate counterparty risk and time-variant recovery rate. The term structure of CDS spreads is used to disentangle counterparty risk from recovery rate.

4.1 CDS Dealer Data

The sample includes the 14 dominant dealers in the CDS market. The sample begins with the top 15 dealers selected in Giglio (2013) by activity in July 2008. In addition, 2 additional banks from the 14-bank sample in Arora, Gandhi, and Longstaff (2012) are added, as 12 banks overlap. Due to lack of data, I remove ABN AMRO, BNP Paribas, and HSBC. The included banks are as follows: Bank of America, Bear Stearns, Citigroup, Goldman Sachs, Lehman Brothers, JP Morgan, Merrill Lynch, Morgan Stanley, Wachovia, Barclays, Credit Suisse, Deutsche Bank, UBS, and Royal Bank of Scotland. The CDS data are from Markit, from 2007 January to 2010 December. For each firm, the CDS spread on senior debt, with "SNRFOR" tier, are chosen with maturities of 6 months and 1 year. In addition, CDS contracts come with different restructuring clauses. Following Bai and Wei (2012), I use the CDS spreads from different clauses depending on the local law in terms of bankruptcy. For American firms before April 8, 2009, the "MR" clause is used, and after that date the "XR" clause is used. For European firms, the "MM" clause is used for the whole period.

4.2 Why Use Term Structure Data?

An important issue with the existing method, which uses CDS spreads to infer counterparty risk, is the choice of CDS term to use. There are only a handful of papers that use the term structure data of CDS spreads. Doshi (2011) explicitly focuses on the term structure of recovery rates, while Filipović and Trolle (2013) calculates the term structure of interbank risk. Except for rare papers such as these, most investigators using CDS spreads employ only a single term. The 5-year CDS spread has been the most popular choice, as it is known to be the most liquid contract.

Figure 4 shows the term structure of mean CDS spreads for the top 14 CDS dealers. The dates plotted in Figure 4 are as follows. First, BNP Paribas suspends redemption from three money market funds on August 9, 2007 and JP Morgan announces its acquisition of Bear Stearns on March 16, 2008. Next, Lehman Brothers files for bankruptcy on September 15, 2008 and the Federal Reserve begins to pay interest on required and excess reserve balances via the Emergency Economic Stabilization Act on October 9, 2008. Finally, the S&P500 Index hits its lowest point since 1997 (676.53) on March 9, 2009, and a downgrade of Greece's debt by S&P occurs on April 27, 2010.

It is clear that until late 2007, the term structure followed a normal upward sloping shape, incorporating future uncertainty. But toward the end of 2007 through right before the Lehman crisis, the term structure became flat. Immediately after September 2008 through mid 2009, we observe a reversed downward-sloping term structure. During this period, short-term uncertainty was higher than long-term uncertainty. The inverted term structure is easily visible in Figure 5, where the spread between the 5-year CDS spread and the 6-month CDS spread is plotted.

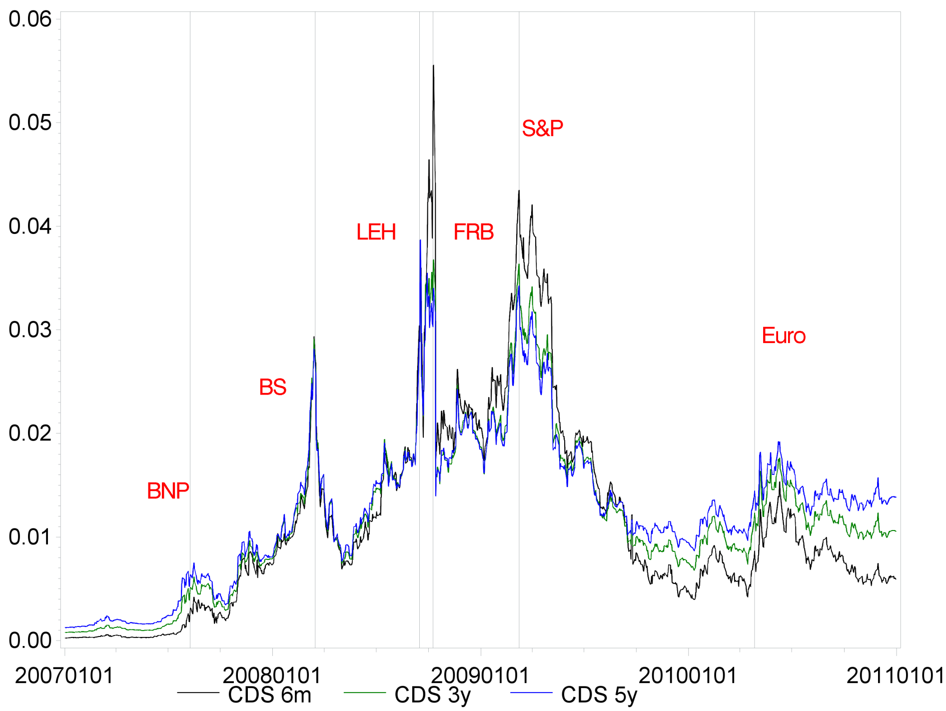


Figure 4: Term Structure of mean financial institution CDS spreads

Therefore, if we focus only on the 5-year term CDS spread, we miss the information embedded in short-term CDS spreads. The following graph of term structure for Lehman Brothers shows a more compelling story.

The actual CDS spreads for Lehman Brothers are presented in Table 1. Let us use the simple CDS pricing model $Z_i = (1 - \bar{R})P(D_i)$ to estimate the default probability of Lehman Brothers on August 1, 2008. The implied default probabilities differ depending on the choice of term. Using the 6-month rate of 6.28% (annualized), and $\bar{R} = 0.4$, the default probability is $10.4\dot{6}\%$. On the other hand, using the 5-year spread of 2.94% (annualized), the default probability is estimated to be 4.9%. The difference becomes even larger on September 10. Using the 6-month rate of 11.39%,

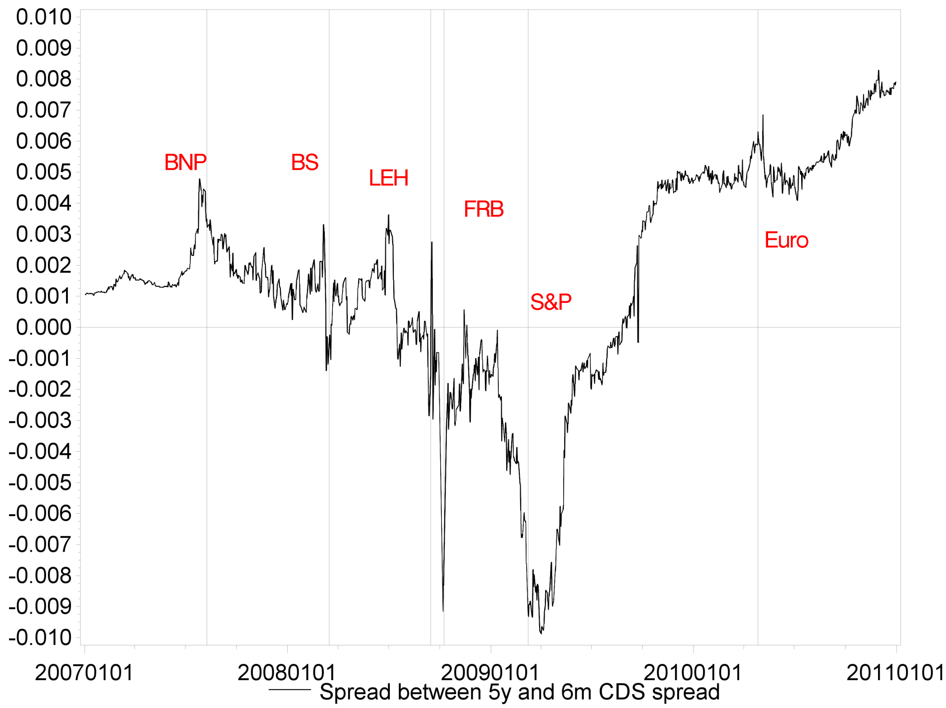


Figure 5: Spread between 5-year and 6-month mean financial institution CDS spreads

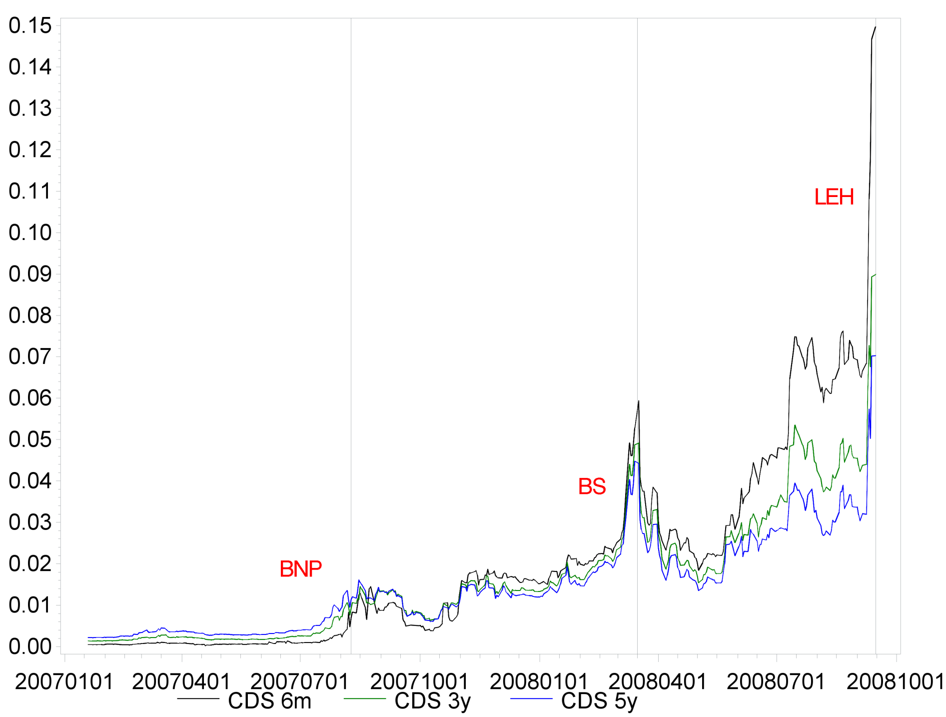


Figure 6: Term structure of Lehman Brothers CDS spreads

the implied default probability is 18.98%, while the default probability is 9.18% using the 5-year rate of 5.51%. Thus, we see a clear difference depending on the choice of CDS term. Choice of maturity is an important issue for practitioners as well. If one wanted to calculate the implied default probability of a bond that matured in, say, 2.5 years, one would have to use a model that incorporates term structure data for accurate calculation.

Table 2: CDS spreads and option-implied default probabilities by period

Panel A: CDS spread for Lehman Brothers in Aug and Sep 2008

Date	6-month CDS spreads	5-year CDS spreads
Aug 1, 2008	6.28%	2.94%
Sep 10, 2008	11.39%	5.51%

Panel B: CDS spreads by bank (1 year)

	Mean	Std	Min	Max	N
Bank of America	97.41	98.81	2.82	545.63	209
Barclays	69.95	60.97	1.77	272.08	209
Bear Stearns	143.61	194.49	5.96	1367.72	80
Citigroup	168.06	190.02	2.03	898.10	209
Credit Suisse	81.00	53.28	23.38	262.41	139
Deutsche Bank	55.77	43.14	2.26	184.36	208
Goldman Sachs	139.27	128.23	5.84	732.75	209
JP Morgan	57.84	48.33	3.19	251.12	209
Lehman Brothers	206.93	245.09	5.89	1422.78	89
Merrill Lynch	185.23	154.34	4.66	749.72	209
Morgan Stanley	234.21	335.03	6.22	3110.48	209
Royal Bank of Scotland	64.03	87.27	1.11	260.48	209
UBS	78.88	78.89	1.65	404.41	209
Wachovia Bank	75.66	91.44	2.38	467.50	186

4.3 CDS Valuation Equation

I use a generalized version of the CDS spread valuation equations used in Giglio (2013) and Conrad, Dittmar, and Hameed (2013).

$$Z_{i/j} = \frac{(1-R_i)(P_q(D_i) - (1-S)P_q(D_i \cap D_j))}{\frac{1}{4} + \frac{1}{8}P_q(D_i)}$$

where $Z_{i/j}$ is annual CDS spread on bank i sold by bank j, and $P_q(D_i)$ is the risk-neutral default probability of bank i on a quarterly basis. The details of this valuation equation are described in the Appendix. As in the preceding section, the mean CDS spread $\bar{Z}_i = \frac{1}{n-1}\sum_{j=1,j\neq i}^{n} Z_{i/j}$ is calculated as follows,

$$\bar{Z}_i = \frac{(1-R_i)(P_q(D_i)-(1-S)P_q(D_{-i}\cap D_i))}{\frac{1}{4}+\frac{1}{8}P_q(D_i)} = \frac{(1-R_i)P_q(D_i)(1-(1-S)P_q(D_{-i}|D_i))}{\frac{1}{4}+\frac{1}{8}P_q(D_i)}$$

4.4 Calculating Risk-neutral Default Probability from Option Prices

As in Eriksson, Ghysels, and Forsberg (2004), I assume that the risk-neutral probability density of the gross equity return follows a normal inverse gaussian (NIG) distribution. Following the methods outlined in Bakshi, Kapadia, and Madan (2003), the parameters of the distribution are estimated using out of the money (OTM) call and put option prices. Once the distribution is estimated, the risk-neutral default probability is calculated using the following equation,

$$P_o(D_i) = \int_{-\infty}^{0} f_{nig}(x, 1+R_{f,t}, Var_i, Skew_i, Kurt_i)\, dx$$

where f_{nig} is the probability density function of the NIG distribution, and the option-implied risk-neutral default probability $P_o(D_i)$ is the integrated probability density, where the gross return lies in the interval $(-\infty, 0]$. Details on the methodology are presented in the Appendix.

An alternative method to calculate the default probability would be to use a structural model, such as that of Merton (1974). Bharath and Shumway (2008) show that the Merton model and its variant may be useful in forecasting defaults. Unfortunately, this method requires a number of assumptions, including that the firm asset value follows a geometric Brownian motion; such assumptions are not required in our approach. Apart from the parametric assumptions, the Merton models have a disadvantage for estimating default probability in the interbank setting. To calculate implied default probability using the Merton model, one needs to estimate the value of outstanding debt and equity. At best, this is known at a quarterly frequency and would be a stale measure to calculate default probability in times of distress. For example, in the midst of the crisis in 2008, Goldman issued $5 billion worth of preferred shares to Berkshire Hathaway. It is highly unlikely that using quarterly data from Compustat would allow taking this event into account. Thus, a forward-looking approach that uses the market price of options is chosen to estimate risk-neutral default probability of banks.

4.5 Option Price Data

For options data, I use daily closing price data from OptionData, iVolatility, and OptionMetrics. The mean best bid and offer prices are used. Following standard practice, as in Conrad, Dittmar, and Hameed (2013), data with a mean price smaller than \$0.25 are removed. The option prices of banks traded on the American exchanges were collected from OptionData. I verify with existing data from OptionMetrics that the prices from the two databases match. The option prices of Barclays, Credit Suisse, Deutsche Bank, UBS, and Royal Bank of Scotland are collected from the European exchanges through iVolatility.[9] Although some of the equity options on the European banks are traded in US exchanges, only the shorter maturity options are traded on US exchanges.

The following process is executed for each bank in the sample. For each trading day, two maturity dates are chosen that are closest to the target maturity of 6 months so that one maturity is shorter and the other is longer than the target maturity. For each chosen maturity, all OTM put option and call option mean prices are collected, to calculate the parameters for the NIG distribution. The average parameters are then used to calculate the risk-neutral default probability. After calculating the two default probabilities for each maturity, the two default probabilities are interpolated to estimate the default probability of the target maturity. This entire process is repeated for the target maturity of 1 year. Therefore, the daily risk-neutral default probability of each bank is calculated with two terms, 6 months and 1 year. Either due to lack of data or when the restriction[10] imposed on the NIG parameters are not met, the risk-neutral default probability may not be calculated.

[9]Option prices of Barclays (L:BARC) and Royal Bank of Scotland (L:RBS) are from the UK exchange, those of Credit Suisse (S:CSGN) and UBS (S:UBSN) are from the Swiss exchange, and those of Deutsch Bank (DE:DBK) are from the German (DE) exchange.

[10]Refer to the Appendix for details on the restriction.

4.6 Identification Strategy

To estimate counterparty risk and recovery rates simultaneously, I rely on the term structure of CDS spreads. First, I define the counterparty risk coefficient $\beta^{\tau}_{i,t}$ as following:

$$\beta^{\tau}_{i} = \frac{P^{\tau}_{q}(D_i \cap D_{-i})}{P^{\tau}_{q}(D_i)P^{\tau}_{q}(D_{-i})} = \frac{P^{\tau}_{q}(D_i|D_{-i})}{P^{\tau}_{q}(D_i)}$$

where $P^{\tau}_{q}(\cdot)$ is the probability of an event in τ years such that $P^{0.5}_{q}(\cdot)$ will be the probability of an event in 6 months, while $P^{1}_{q}(\cdot)$ will be the probability of an event in 1 year. Thus, the time-varying bank specific counterparty risk coefficient β^{τ}_{i} measures how distant the joint default probability is compared to the independent case. If β^{τ}_{i} were to be equal to 1, this would mean that the joint default probability is the product of two marginal probabilities, as if these two events were independent. Given that we are looking only at banks, it is highly likely that $\beta^{\tau}_{i} > 1$.

Here, I introduce a single assumption to disentangle the joint default probability from the recovery rate of CDS spreads. First we define two different counterparty risk coefficients $\beta^{0.5}_{i}$ and β^{1}_{i} for the same bank i on different terms. [11]. I assume that the counterparty risk coefficient has a flat term structure, or that $\beta^{0.5}_{i} = \beta^{1}_{i}$. I acknowledge that while this may be a strong assumption made for modeling purposes, it is better than the zero counterparty risk model, which not only assumes that the counterparty risk coefficient has flat term structure, but it also assumes $\beta^{\tau}_{i} = 0$ for all firms in all periods. Also, this assumption is not stronger than the assumptions in existing fixed recovery rate models that examine only a single term.

4.7 Using the Kalman Filter for Estimation

For actual estimation, the unscented Kalman filter is used. The Kalman filter is a family of methods to estimate the time series of unobservable state variables from the observable time series that are a function of the state variables. The extended Kalman filter is the standard choice among different types of Kalman filters when the mapping function is nonlinear. The extended Kalman filter is used in Conrad, Dittmar, and Hameed (2013), and it is widely used in term structure modeling where

[11] $\beta^{0.5}_{i} = \frac{P^{0.5}_{q}(D_i \cap D_{-i})}{P^{0.5}_{q}(D_i)P^{0.5}_{q}(D_{-i})}$, $\beta^{1}_{i} = \frac{P^{1}_{q}(D_i \cap D_{-i})}{P^{1}_{q}(D_i)P^{1}_{q}(D_{-i})}$

the observable fixed income price is a nonlinear function of the state variables.

The unscented Kalman filter is a type of Kalman filter; introduced recently, it has gained popularity for highly nonlinear models. Julier and Uhlmann (2004) presents the unscented Kalman filter, creating a set of sample points and directly applying the original nonlinear function. These authors' intuition is that approximating a probability distribution is easier than approximating a highly nonlinear function. Christoffersen, Jacobs, Karoui, and Mimouni (2009) compare the extended and unscented Kalman filter methods in affine term structure models. These authors find that the unscented Kalman filter outperforms the standard extended Kalman filter in out of sample tests and propose the use of the unscented Kalman filter. Thus, the extended Kalman filter has gained popularity and is employed in such recent papers as Carr and Wu (2010), Doshi (2011), Filipović and Trolle (2013), and van Binsbergen and Koijen (2010). A deeper discussion of the Kalman filter, extended Kalman filter, and unscented Kalman filter is in the Appendix.

As with Pan and Singleton (2008) and Conrad, Dittmar, and Hameed (2013), the risk-neutral default probability is assumed to be log normally distributed over a discrete interval. The state variables are $q^1_{i,t}, q^2_{i,t}$, and $x_{i,t}$. First $q^1_{i,t} = ln(P^{0.5}_{q,t}(D_i))$ is the natural log of risk-neutral default probability maturing in 6 months and $q^2_{i,t} = ln(P^1_{q,t}(D_i))$ is the natural log of risk neutral default probability maturing in 1 year, both on a quarterly basis. The average joint default probabilities are governed by the state variable $x_{i,t}$ such that $j^1_{i,t} = \beta_{i,t} q^1_{i,t} Q^1_{-i,t}$ and $j^2_{i,t} = \beta_{i,t} q^2_{i,t} Q^2_{-i,t}$, where $\beta_{i,t} = A + \frac{K}{1+e^{-x_{i,t}}}$ is a generalized logistic function and $Q^1_{-i,t} = P^{0.5}_{q,t}(D_{-i}) = \frac{1}{n-1}\sum^n_{j=1, j\neq i} exp(q^{o1}_{j,t})$, $Q^2_{-i,t} = P^1_{q,t}(D_{-i}) = \frac{1}{n-1}\sum^n_{j=1,j\neq i} exp(q^{o2}_{j,t})$ is the average risk-neutral default probability of all other banks on a quarterly basis. Following Jaskowski and McAleer (2012), I assume that the recovery rate follows the following form, $R_{i,t} = b^i_2 + b^i_1 e^{b^i_0 \lambda^i_t}$, where λ^i_t is the default intensity calculated from the default probability. The state variables are assumed to follow VAR(1) dynamics,

$$\begin{pmatrix} q^1_{i,t} \\ q^2_{i,t} \\ x_{i,t} \end{pmatrix} = \phi^i_0 + \phi^i_1 \begin{pmatrix} q^1_{i,t-1} \\ q^2_{i,t-1} \\ x_{i,t-1} \end{pmatrix} + \eta_{i,t}$$

where $\eta_{i,t}$ is a vector of residuals, assumed to be normally distributed with mean 0 and covariance matrix Ω^i_1. Given the underlying state variable process as above, the observation model of the Kalman filter is as follows:

$$\bar{z}_i^1 = h_{i,t} + q_{i,t}^1 + ln(1-(1-S)Q_{-i,t}^1\beta_{i,t}^{0.5}) - ln(\frac{1}{4}+\frac{1}{8}exp(q_{i,t}^1)) + e_{s1,t}^i$$

$$\bar{z}_{i,t}^2 = h_{i,t} + q_{i,t}^2 + ln(1-(1-S)Q_{-i,t}^2\beta_{i,t}^1) - ln(\frac{1}{4}+\frac{1}{8}exp(q_{i,t}^1)) + e_{s2,t}^i$$

$$q_{i,t}^{o1} = q_{i,t}^1 + e_{o1,t}^i$$

$$q_{i,t}^{o2} = q_{i,t}^2 + e_{o2,t}^i$$

where $\bar{z}_i^1 = ln(\bar{Z}_{i,t}^1)$ and $\bar{z}_i^1 = ln(\bar{Z}_{i,t}^2)$ are the natural log of average CDS spread maturing in 6 months and 1 year, respectively, and $h_{i,t}$ is the natural log of loss given default $H_{i,t} = 1 - R_{i,t}$. Next, $q_{i,t}^{o1}$ and $q_{i,t}^{o2}$ are the natural log of option-implied risk-neutral probabilities for bank i with maturities of 6 months and 1 year, respectively, both on a quarterly basis. The errors in the observation model are assumed to be mean zero and i.i.d. such that the covariance matrix Ω_2^i is a diagonal matrix. The parameter $\Theta^i = [b^i, \phi^i, \Omega_1^i, \Omega_2^i]'$ is estimated through maximum likelihood estimation(MLE). Detailed procedures are documented in the Appendix.

5 Results

5.1 Descriptive Statistics

The CDS spreads and option-implied default probabilities by period are presented in Table 3. The same cutoff dates as in Conrad, Dittmar, and Hameed (2013) are used, to partition the full sample into three periods. The "Pre-Crisis" period is January 1, 2007 through August 1, 2007, the "Crisis" period is August 2, 2007 through July 1, 2009, and the "Post-Crisis" is July 2, 2009 through December 31, 2010.

In Panel A, we observe that average CDS spreads increase significantly during the crisis in comparison to the pre-crisis period, and fall during the post-crisis period, but they do not revert completely to the original level. These results are comparable with those of the systematically important financial institutions(SIFI) category of Table 3, Panel A in Conrad, Dittmar, and Hameed (2013). Our full sample mean is 106.16, while the SIFI full sample mean is 103.96 in Conrad, Dittmar, and Hameed (2013). Also, the subsample means of 7.52, 148.35, and 84.98 for each sub-

Table 3: CDS spreads and option-implied default probabilities by period

Panel A: CDS spreads (1 year)

	Full Sample	Pre-Crisis	Crisis	Post-Crisis
Mean	106.16	7.52	148.35	84.98
Median	67.35	6.62	98.72	77.35
Min	1.72	1.72	4.28	22.05
Max	2381.53	46.03	2381.53	271.32
N	1942	302	1018	622

Panel B: option-implied default probabilities

	Full Sample	Pre-Crisis	Crisis	Post-Crisis
Mean	3.857%	0.401%	5.449%	2.93%
Median	2.229%	0.371%	3.131%	2.593%
Min	0.0016%	0.0134%	0.0016%	0.0684%
Max	23.210%	3.17%	23.210%	15.211%
N	1942	302	1018	622

period are comparable to the subsample means of 19.88, 176.65, and 141.05 in Conrad, Dittmar, and Hameed (2013). The difference stems from the fact that the sample of banks and the sample period do not perfectly match that of Conrad, Dittmar, and Hameed (2013). The number of banks in our sample is 14, while only 8 banks are in the SIFI category in Conrad, Dittmar, and Hameed (2013). Also, the difference is largest for the pre-crisis period, which is from January 2006 to August 1, 2007 in Conrad, Dittmar, and Hameed (2013), and smallest for the crisis period, in which there is no difference in the sample periods.

The option-implied risk-neutral default probabilities are presented in Panel B. Like the results in Panel A, we observe a significant increase during the crisis period, followed by a decrease during the post-crisis period. The full sample mean default probability of 3.857% is also comparable to 5.44% in Conrad, Dittmar, and Hameed (2013). Further, the subsample means of 0.401%, 5.449%, and 2.93% are comparable to 2.46%, 8.05%, and 6.70% in Conrad, Dittmar, and Hameed (2013). As in Panel A, the difference is largest for the pre-crisis period and smallest for the crisis period.

The difference between the results in Panel A and Panel B emphasizes the role of time-variant recovery rates and joint default probability. The increase in CDS spreads is almost 19.7 times (7.52bps to 148.35bps) during the crisis period, while the option-implied default probability increases by only 13.6 times (0.401% to 5.449%) in the same period. Using the simple model

$Z_i = (1 - R_i)P(D_i)$, it is trivial that an increase in $P(D_i)$ should be accompanied by a decrease in the recovery rate R_i, for Z_i to increase more than $P(D_i)$. Therefore, the descriptive statistics by themselves suggest a role for time-variant recovery rates. The fact that CDS spreads increase more than the default probability during the crisis period is consistent with the explanation that the recovery rate decreases in the crisis.

5.2 Filtered Results

5.2.1 Estimated Default Probabilities

Figure 7 presents the marginal default probability $P(D_i)$, the joint default probability $P(D_i \cap D_{-i})$, and the estimated recovery rate, all averaged across the CDS dealers. As with the raw results in Table 3, we observe that the marginal default probability prior to August 2007 was low but increased and stayed at an elevated level until around the second half of 2009. We observe a similar trend for the joint default probability.

The time series of average recovery rate in Figure 7 is also notable. In the beginning of the sample period the recovery rate maintained a high level around 65%, but started to drop in mid-July 2007 during the Bear Stearns event. Right after the acquisition of Bear Stearns, the recovery rate quickly increased, but during the fall of Lehman Brothers, the estimated recovery rate fell to its lowest point. The general trend of results is in line with that of Conrad, Dittmar, and Hameed (2013). However, as expected due to the inclusion of counterparty risk, the recovery rate estimated from the joint estimation model seems to be lower than that of Conrad, Dittmar, and Hameed (2013).

Next, the conditional default probability, the ratio of the joint default probability to the marginal default probability, is plotted in Figure 7. The marginal and joint default probability both trend in the opposite direction of the recovery rate. The correlation between the recovery rate and the marginal/joint/conditional probabilities are all negative and statistically significant, with a p-value of less than 0.01%. The negative correlation is natural given that the recovery rates are constructed to be negatively associated with default intensity.

Finally, I compare idiosyncratic and systemic risk, as in Giglio (2013), and obtain consistent results. In Giglio (2013), systemic risk is defined as the probability that many banks default, and

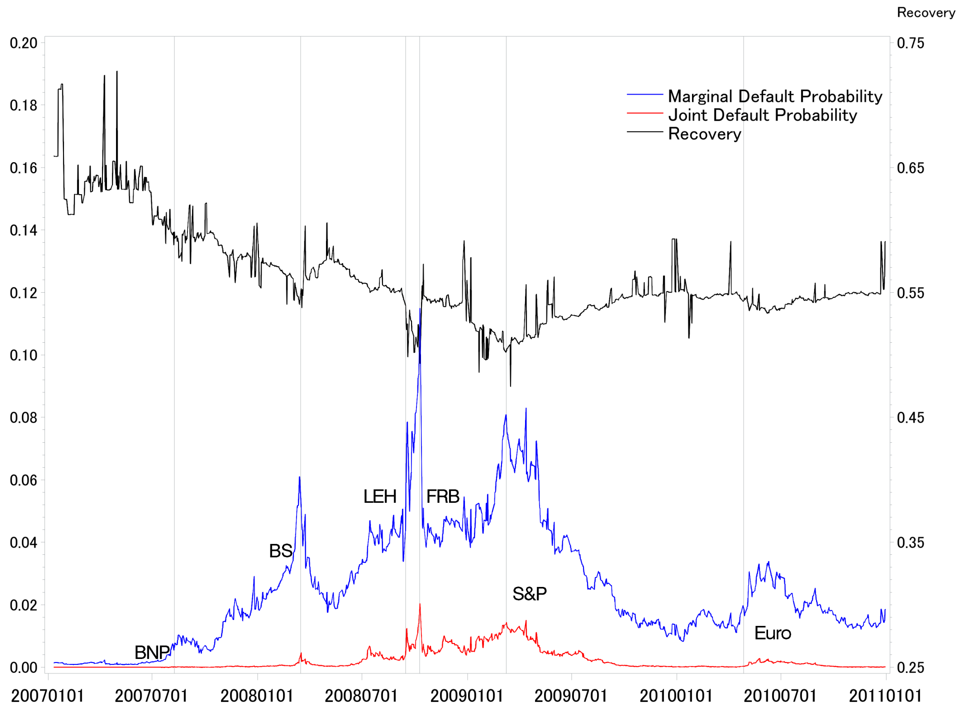


Figure 7: Joint model: recovery Rate, marginal, and joint default probabilities

idiosyncratic risk is defined as the probability that at least one bank defaults. Borrowing their definition, I proxy systemic risk with average joint default probability and idiosyncratic risk with average marginal default probability. I find that idiosyncratic risk has increased gradually since July 2007 and first peaks during the Bear Stearns collapse. On the other hand, systemic risk remained low until the sharp increase when Bear Stearns collapsed in March 2008. This finding is consistent with Giglio (2013) but is contrary to Segoviano and Goodhart (2009) and Huang, Zhou, and Zhu (2009), which suggests a sharp increase in systemic risk even at the beginning of 2008.

5.2.2 RMSE of the Filtered Results

Here, I show whether the Kalman filter estimates from the joint estimation model describe the data well. Figure 8 depicts the time series of the average root mean squared errors (RMSE) of CDS spreads. The median RMSE is 1.2 basis points and the mean RMSE is 3.8 basis points. Both are much smaller than the median CDS spread of 67 basis points. Thus, I argue that our model describes the data reasonably well. Our RMSE median of 1.2 basis points is much lower than the median RMSE of 15 basis points for non-financials and much smaller than the 59 basis points for financial firms as reported in Conrad, Dittmar, and Hameed (2013).[12] To explain why RMSE of financial firms are much larger than that of non-financials, Conrad, Dittmar, and Hameed (2013) suggests that financial firms may be subject to more complex dynamics, especially during the crisis. The fact that counterparty risk is taken into account may explain why the RMSE in the joint estimation model were significantly lower.

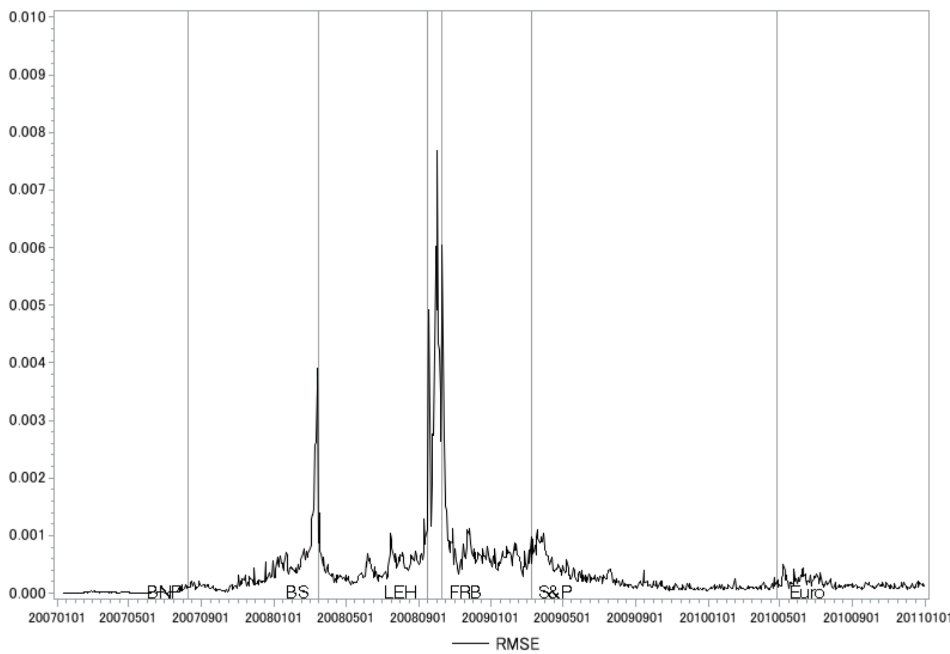


Figure 8: Joint estimation model: RMSE of CDS spreads

5.3 Comparison with the Fixed Recovery Model and the Zero Counterparty Model

5.3.1 Fixed Recovery Model

Estimates from the fixed recovery rate model are given in Figure 9, below. To test Proposition 1, the joint default probability estimates from the unrestricted joint estimation model are compared

[12]Although the RMSE in the joint estimation model is based on 6-month or 1-year CDS spreads, and Conrad, Dittmar, and Hameed (2013) uses 5-year CDS spreads, median CDS spreads do not differ by a large amount. The median CDS spread in our sample is 67 basis points, while the median CDS spread in Conrad, Dittmar, and Hameed (2013) is 72 basis points.

with the estimates from the restricted fixed recovery rate model. For the fixed recovery rate model estimation, parameter $\bar{R} = 0.4$ is used, as in Giglio (2013). Since the proposition specifies that the mis-specified fixed recovery rate model will underestimate joint default risk when the market is in distress, or $E[R_i] \leq \bar{R}$, the difference in the estimates of the recovery rates is tested. Using the same cutoff dates as in Conrad, Dittmar, and Hameed (2013), the results are presented in Table 4.

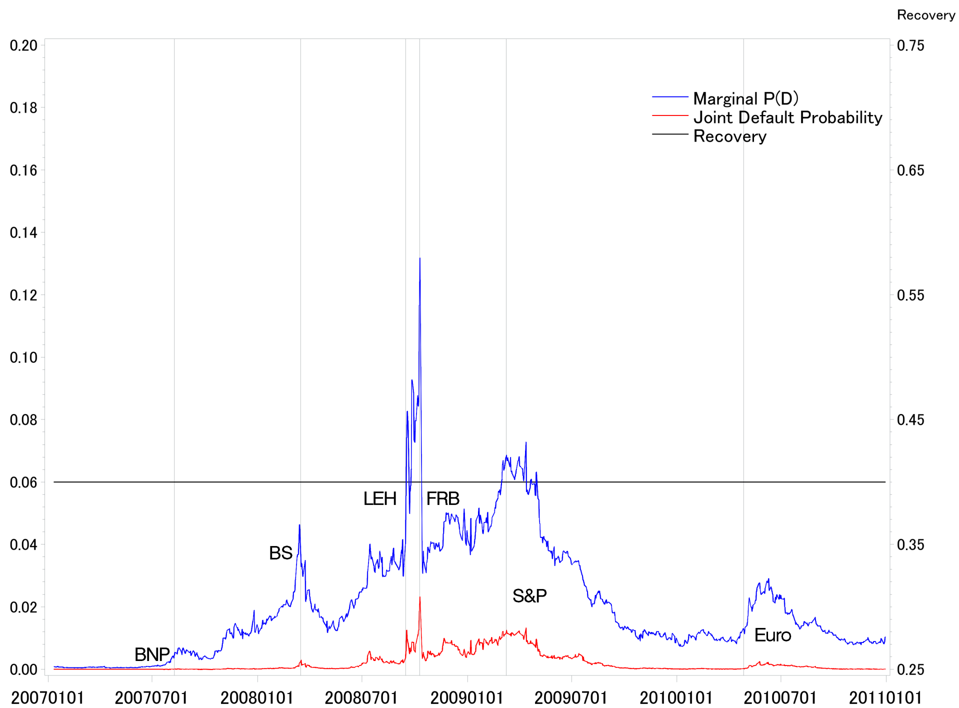


Figure 9: Fixed recovery model: recovery rate, marginal and joint default probabilities

In Panel A, the estimates of the 1-year joint default probabilities are presented. Our model estimates for each time period are 0.0008%, 0.2979%, and 0.0896%. As expected, the joint default probability soared during the crisis and subsequently decreased, but remained at an elevated level compared to before the crisis. We see a similar trend for the fixed recovery rate model. The fixed recovery rate model yields estimates of 0.0004%, 0.2305%, and 0.0628%. The main focus is in the "Crisis Period", as that is most likely the period when $E[R_i] \leq \bar{R}$ holds. The paired t-statistics are listed in the third column. The mean difference of 0.0674% is significantly larger than the standard error of 0.0023%, yielding a p-value statistically significant at the 0.1% level.

In Panel B, estimates of the 6-month joint default probabilities are provided. The estimates of the joint default probabilities for each time period are 0.0002%, 0.3374%, and 0.0888%. The fixed

Table 4: Estimated joint default probabilities, compared with fixed recovery model

Panel A: Comparison of 1-year joint default probabilities J_1 and $\tilde{J}_1$

	$E[J_1]$	$E[\tilde{J}_1]$	$E[J_1] - E[\tilde{J}_1]$	N
Pre-Crisis Period	0.0008 %	0.0004 %	$0.0004\%^{***}$ (0.00001%)	1293
Crisis Period	0.2979 %	0.2305 %	$0.0674\%^{***}$ (0.0023%)	5089
Post-Crisis Period	0.0896 %	0.0628 %	$0.0269\%^{***}$ (0.0008%)	3321

Panel B: Comparison of 6-month joint default probabilities $J_{0.5}$ and $\tilde{J}_{0.5}$

	$E[J_{0.5}]$	$E[\tilde{J_{0.5}}]$	$E[J_{0.5}] - E[\tilde{J_{0.5}}]$	N
Pre-Crisis Period	0.0002 %	0.0001 %	$0.0001\%^{***}$ (0.00006%)	1293
Crisis Period	0.3374 %	0.3065 %	$0.0310\%^{***}$ (0.0029%)	5089
Post-Crisis Period	0.0888 %	0.0725 %	$0.0163\%^{***}$ (0.0009%)	3321

Proposition 1 is tested in this table. $J_\tau = P_q^\tau(D_i \cap D_{-i})$ denotes the estimated joint default probability from our model, while $\tilde{J}_\tau = \tilde{P}_q^\tau(D_i \cap D_{-i})$ denotes the estimated joint default probability from the fixed recovery rate model. Firm-date matched paired t-test statistics for the difference in joint default probabilities between these models are provided. ***: $Pr < 0.001$, **: $Pr < 0.01$

recovery rate model yields estimates of 0.0001%, 0.3065%, and 0.0725%. Likewise, the focus is on the difference between the estimated joint default probabilities in the "Crisis Period". The mean difference of 0.0310% is significantly larger than the standard error of 0.0029%, yielding a p-value statistically significant at the 0.1% level.

Thus, the results in both Panels A and B support Proposition 1. It is interesting that the fixed recovery rate model seems to underestimate joint default probability both in the "Pre-Crisis" period and the "Post-Crisis" period. The same results are found for both the 6-month joint default probability and the 1-year joint default probability.

Finally, the figure below depicts the time series of the RMSE of CDS spreads in the fixed recovery rate model. The median RMSE is 5.9 basis points and the mean RMSE is 20.4 basis points. Both are much larger than the median and mean RMSE for the joint estimation model. Therefore, I argue that the joint estimation model describes the data better than the fixed recovery rate model.

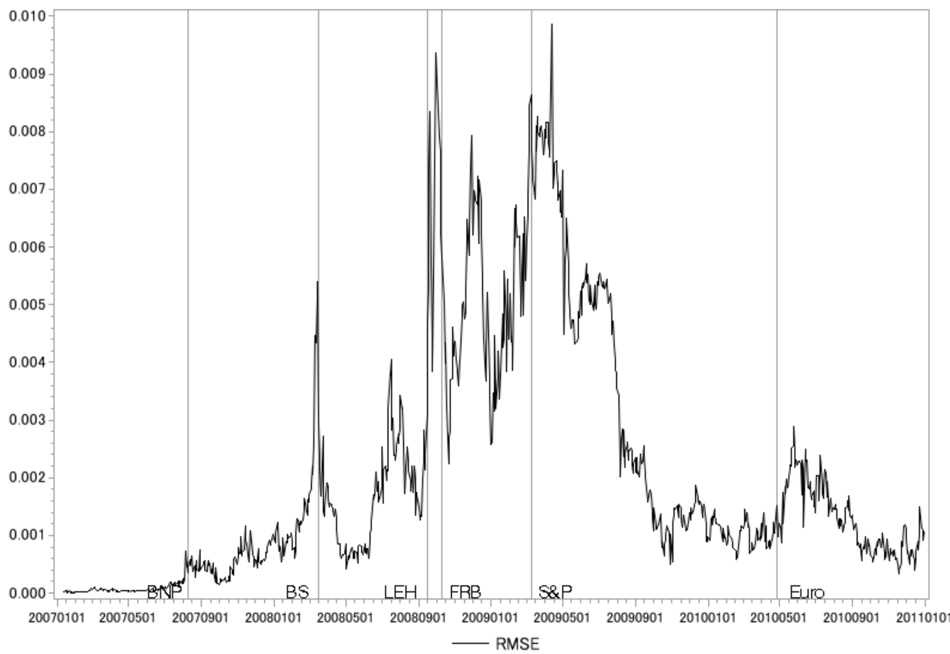


Figure 10: Fixed recovery rate model: RMSE of CDS spreads

5.3.2 Zero Counterparty Risk Model

Estimates of the zero counterparty risk model are shown in Figure 11 below. Rarely falling below 50%, it is clear that the estimated recovery rates in the zero counterparty risk model are higher than in the unrestricted model, as shown in Figure 8.

Although Figure 11, in comparison with Figure 8, clearly suggests that the zero counterparty risk model overestimates the recovery rate, we need a formal test to validate this claim. If counterparty risk is positive, Proposition 2 states that the zero counterparty risk model overestimates the recovery rate. To test Proposition 2, the recovery rate estimated from the unrestricted joint estimation model is compared with the recovery rate estimated from the zero counterparty risk model.

The results are tabulated in Table 5. The full sample mean recovery rate of our model yields 56.19%, while the zero counterparty risk model estimates the recovery rate as 61.92%. This 5.72

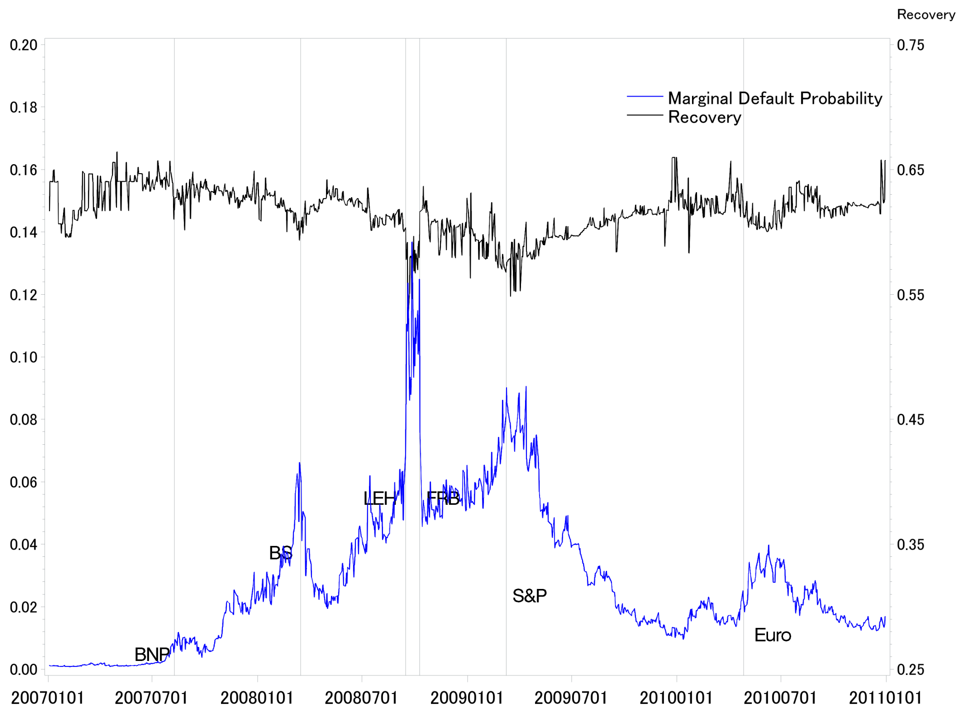


Figure 11: Zero counterparty risk model: marginal default probability and recovery rate

percentage point difference is much larger than the 0.086% standard error, yielding a p-value less than 0.01%. The same results hold for all sub-periods. For all periods, our model suggests a significantly lower recovery rate compared to the zero counterparty risk model, all with p-value less than 0.01%. Thus, the results are consistent with Proposition 2.

Since Conrad, Dittmar, and Hameed (2013) do not tabulate the mean of the recovery rate or loss given default, I cannot directly compare my results with theirs. However the authors plot the filtered loss given default of SIFI banks in Figure 5, Panel C in Conrad, Dittmar, and Hameed (2013). From 2007 to 2010, their loss given default is mostly within the range of [0.1, 0.3] and only very infrequently does it go beyond that boundary. Therefore, their mean recovery rate is estimated to be between 70% and 90%. Given our full sample mean of 56.19%, it is highly likely that this indirect comparison with Conrad, Dittmar, and Hameed (2013) also strengthens the prediction given in Proposition 2.

Table 5: Estimated recovery rate, compared with zero counterparty risk model

	$E[R]$	$E[\tilde{R}]$	$E[R] - E[\tilde{R}]$	N
Full Sample	56.19 %	61.92 %	-5.72^{****} (0.0860)	8208
Pre-Crisis Period	62.72 %	65.12 %	-2.40^{****} (0.237)	997
Crisis Period	55.64 %	61.50 %	-5.85^{****} (0.118)	4354
Post-Crisis Period	54.76 %	61.45 %	-6.69^{****} (0.142)	2857

Proposition 2 is tested in this table. Firm-date matched paired t-test statistics of $H_0 : E[R] - E[\tilde{R}] = 0$ against $H_a : E[R] - E[\tilde{R}] < 0$ are provided. ****: $Pr < 0.0001$, ***: $Pr < 0.001$, **: $Pr < 0.01$

5.3.3 Likelihood Ratio Tests

Finally, the log likelihood values from the MLE estimation are presented in Table 6, for each bank. The results strongly suggest that the likelihood ratio test rejects the null that the unrestricted model and the fixed recovery model are similar. For all banks, p-values are less than 1%. The results are less clear but still strong when comparing the unrestricted model with zero counterparty risk. The p-value is less than 0.1% for all but one bank. Therefore, I suggest that the unrestricted model provides a better fit for the data.

5.4 Conditional Default Probability as a Measure of Systemic Risk

In this section, I discuss using the conditional default probability as a measure of bank vulnerability to systemic risk. I proxy bank i's vulnerability to a systemic crisis with $P(D_i|D_{-i})$, the default probability of bank i conditional on its counterparty going into default. Having measured the joint default probability $P(D_i \cap D_{-i})$, estimating the conditional default probability is a straightforward process, since $P(D_i|D_{-i}) = \frac{P(D_i \cap D_{-i})}{P(D_{-i})}$. Next, I compare the conditional default probability with existing measures of vulnerability, and find that it is highly correlated with Marginal Expected Shortfall (MES), a popular measure of systemic risk suggested in Acharya, Pedersen, Philippon, and Richardson (2012). MES_i is defined as the expected loss of bank i's equity, conditional on a systemic crisis such that,

Table 6: Log likelihood values and the likelihood ratio for each bank

Ticker	Unrestricted	Zero Counterparty Risk	LR	Fixed Recovery	LR
GS	2899.4	2491.1	816.4***	1205.1	3388.4***
LEH	1325.3	1148.9	352.8***	-209.1	3068.9***
BAC	1938.9	1609.4	659.1***	-5984.6	15847.1***
MER	1298.6	1242.5	112.3***	102.5	2392.3***
JPM	2173.4	2058.6	229.5***	-9573.9	23494.5***
MS	3060.5	3003.9	113.2***	1774.1	2572.8***
WB	332.8	466.6	-267.5	-2576.8	5819.2***
BARC	991.8	437.9	1107.9***	613.0	757.7***
BSC	897.9	736.2	323.4***	-168.2	2132.2***
UBSN	2433.5	2206.2	454.7***	-1260.7	7388.4***
DBK	741.3	617.8	247.1***	-4772.9	11028.5***
CSGN	2248.1	2004.1	488.1***	-1418.8	7334.0***
RBS	1255.9	585.2	1341.5***	-613.9	3739.6***

Here, the log likelihood values for the unrestricted model, zero counterparty risk model, and the fixed recovery rate model are presented. The p-values for the chi-squared log likelihood ratios are as presented $***: Pr < 0.001$, $**: Pr < 0.01$, $*: Pr < 0.05$

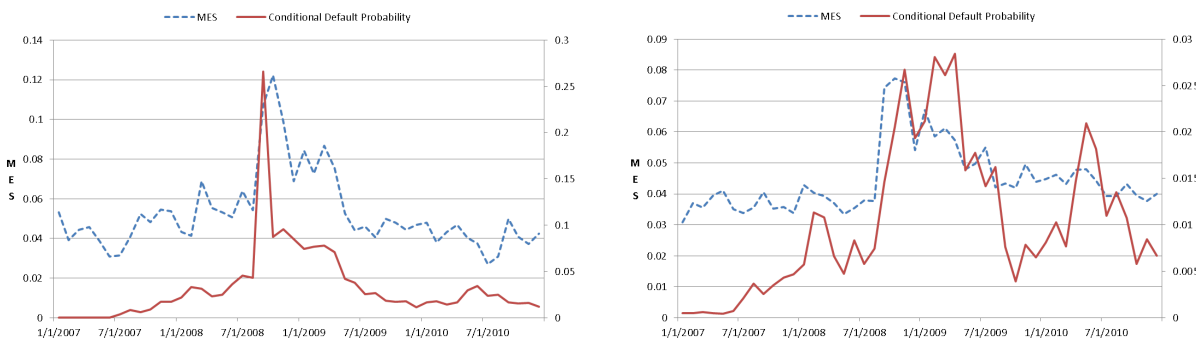
$$MES_i = E[Bank\ i's\ Equity\ loss|Systemic\ Crisis],$$

where the systemic crisis event is proxied by an extreme negative market return, say the market average return reaching negative 40% in a 6-month period. The bank's vulnerability is modeled as expected equity loss, given an extreme left-tail event in the market return. Therefore, a bank's vulnerability to a systemic risk and the interconnectedness of banks is proxied through the joint tail movement between bank equity and the market.

Here, I suggest using bank vulnerability V_i, the default probability of a bank i conditional on a systemic crisis, to measure bank i's vulnerability to systemic risk. V_i is given as follows,

$$V_i = E[Bank\ i's\ Default\ Probability|Systemic\ Crisis] = P(D_i|D_{-i}) = \frac{P(D_i \cap D_{-i})}{P(D_{-i})}$$

where the systemic crisis event is proxied by a default of a CDS dealer bank. Due to the nature of the CDS market, only the largest, most secure banks can act as CDS dealers. Therefore, it is not a surprise that CDS dealer banks are a subset of SIFIs. Therefore, it seems reasonable to match the systemic crisis event with the bankruptcy of a CDS dealer bank. In addition, this measure proxies interconnectedness among banks more directly, compared to MES, which proxies interconnectedness through the joint tail movement of bank equity and the market. Both MES and the bank vulnerability measure V_i rely only on publicly available market data.



(a) Morgan Stanley (b) Deutsche Bank

Figure 12: MES and conditional default probability of Morgan Stanley and Deutsche Bank

Figure 12, below, plots MES_i and bank vulnerability V_i, or the conditional default probabilities for Morgan Stanley and Deutsche Bank. The monthly MES measures for these banks were collected from Volatility Lab(V-Lab) at New York University Stern School of Business.[13] As is evident in the graphs, the bank vulnerability measure V_i is highly correlated with the MES measure. The correlation coefficients are 0.683 for the 1-year conditional default probability and 0.623 for the 6-month conditional default probability. The peaks overlap and there is no clear lead-lag relationship between these variables.

A distinction between MES and the conditional default probability V_i emerges when we compare the levels of each variable in "Pre-Crisis" and "Post-Crisis" periods. The average level of MES before July 2007 does not appear to be significantly different from the level of MES after July 2009. On the other hand, the conditional probabilities are almost negligible in the "Pre-Crisis" period but did not revert to the original level even after crisis. The fact that CDS spread level permanently rises seems to affect the difference between the findings. Another interesting difference between

[13]V-Lab: http://vlab.stern.nyu.edu

these two variables is that MES seems to be weaker in capturing the Eurozone crisis of 2010 around April 2010, especially for Deutsche Bank. I observe a similar trend in the plots of other CDS dealer banks, available in the Appendix.

6 Conclusion

This article presents a new approach toward measuring interbank counterparty risk embedded in bank CDS contracts. This approach utilizes the unique nature of CDS data, that the CDS spreads of a CDS dealer bank can be priced only by bank counterparties in the CDS dealer network. Therefore, it becomes possible to extract the average pairwise joint default probability between a bank and its counterparties. Next, I highlight the importance of the fixed recovery rate assumption in measuring counterparty risk in CDS spreads. I show a fixed recovery rate model underestimates the counterparty risk when the market is in distress. In addition, I find that the zero counterparty risk model overestimates the expected recovery rate.

To verify the claim empirically, I present a method to estimate time-variant joint default probabilities and recovery rates from the term structure of CDS spreads and option-implied default probabilities. From the sample of large financial institutions that dominated the CDS market from 2007 to 2010, I document the following main findings. First, I verify that the fixed recovery rate model underestimates expected counterparty risk when the market is in distress, and that the zero counterparty risk model overestimates expected recovery rates. Second, a bank's vulnerability to systemic risk, the average conditional default probability of a bank conditional on the default of its counterparties, is highly correlated with the MES of Acharya, Pedersen, Philippon, and Richardson (2012). Finally, I find that systemic risk or average interbank counterparty risk remain low until the sharp increase when Bear Stearns collapsed in March 2008.

A Appendix

A.1 CDS Pricing

A.1.1 A Simple Model of CDS

If we ignore counterparty risk, the market price of CDS spread Z_i depends primarily on $P(D_i)$, the risk-neutral probability that the underlying bank i defaults, and $E[R_i]$, which is the expected recovery rate. Assuming a fixed recovery rate parameter $\bar{R}$, a simple risk-neutral valuation model for pricing a CDS contract with 1$ notional value is as follows,

$$\frac{Z_i}{4}\sum_{k=1}^{T}\delta(k)(1-P_q(D_i))^k = (1-\bar{R}) \times \sum_{k=1}^{T}\delta(k)P_q(D_i)(1-P_q(D_i))^{k-1}$$

where T is the maturity of the contract in units of quarters, $P_q(D_i)$ is the risk-neutral probability that bank i defaults in 1 year on a quarterly basis so that $1-P(D_i) = (1-P_q(D_i))^4$, and $\delta(k)$ is the discount rate from current period to k. The left-hand side is the expected cost of the CDS spreads paid through maturity, while the right-hand side is the expected value of the CDS payment. Thus we end up with the naive model,

$$Z_i \approx (1-\bar{R})P(D_i)$$

since $P(D_i) \approx 4P_q(D_i)$ and $\frac{\sum_{k=1}^{T}\delta(k)(1-P_q(D_i))^{k-1}}{\sum_{k=1}^{T}\delta(k)(1-P_q(D_i))^{k}} \approx 1$ for small $P_q(D_i)$. Thus, $P(D_i) = \frac{Z_i}{1-\bar{R}}$ is the estimated risk-neutral default probability under the naive model. Jarrow (2011) provides a continuous time version of this formula.

A.1.2 CDS Valuation Equation

The CDS pricing equation used in this paper is based on the model in Houweling and Vorst (2005), as in Conrad, Dittmar, and Hameed (2013), except that we incorporate the counterparty risk as in Giglio (2013). Let Z_t^i be the annual percentage spread for a CDS contract sold by bank j, with bank i as the underlying firm. As in other papers, we assume that when default occurs, the accrued premium is paid to the issuer. Then, the risk-neutral valuation of the payments is as follows:

$$\frac{Z_t^i}{4}\sum_{k=1}^{T} D_t(k)(1-P_{q,t}(D_i))^k + \frac{Z_t^i}{8}P_{q,t}(D_i)\sum_{k=1}^{T} D_t(k)(1-P_{q,t}(D_i))^k$$

where the first term refers to the present value of payments while the underlying bank does not default, and the second term is the present value of accrued payments if bank i defaults. Since the timing of default is random within the quarter, $\frac{Z_t^i}{8}$ is the amount of accrued payment.

Next, we present the expected value of payments by the contract seller,

$$(1-R_i)\times\sum_{k=1}^{T} D_t(k)(P_{q,t}(D_i)-(1-S)P_{q,t}(D_i\cap D_j))(1-P_{q,t}(D_i))^k$$

where the joint default probability $P_{q,t}(D_i \cap D_j)$ is introduced to incorporate the possibility that the contract seller will not be able to fully pay out the loss given default $1-R_i$ when the underlying bank defaults. Since risk-neutral valuation implies that both values must be equal, we have the following:

$$\left(\frac{Z_t^i}{4}+\frac{Z_t^i}{8}P_{q,t}(D_i)\right)\sum_{k=1}^{T} D_t(k)(1-P_{q,t}(D_i))^k$$
$$= (1-R_i)(P_{q,t}(D_i)-(1-S)P_{q,t}(D_i\cap D_j))\times\sum_{k=1}^{T} D_t(k)(1-P_{q,t}(D_i))^k$$

Thus,

$$\frac{Z_t^i}{4}+\frac{Z_t^i}{8}P_{q,t}(D_i) = (1-R_i)(P_{q,t}(D_i)-(1-S)P_{q,t}(D_i\cap D_j))$$

Finally,

$$Z_t^i = \frac{(1-R_i)(P_{q,t}(D_i)-(1-S)P_{q,t}(D_i\cap D_j))}{\frac{1}{4}+\frac{1}{8}P_{q,t}(D_i)}$$

Note that the CDS pricing equation $S_t = (1 - R_t)\frac{Q_t}{\frac{1}{4}+\frac{1}{8}Q_t}$ used in Conrad, Dittmar, and Hameed (2013) is a special case of our equation where $S = 1$.

A.2 Option-implied Default Probability

To calculate the option-implied risk-neutral default probability, we follow the method outlined in Conrad, Dittmar, and Hameed (2013). First, we assume that the risk-neutral probability density of gross equity return follows a NIG distribution, as in Eriksson, Ghysels, and Forsberg (2004). To calculate the parameters of the risk-neutral distribution, we follow the methods in Bakshi, Kapadia, and Madan (2003). Bakshi, Kapadia, and Madan (2003) uses OTM call options and put options to calculated the risk-neutral moments of the risk-neutral gross return. Once the risk-neutral moments are given, the parameters of the NIG distribution can be calculated. The estimated NIG distribution is used to calculate the risk-eutral default probability by integrating the probability density where the gross stock return lies in the range $(-\infty, 0]$. That is, we interpret that the default occurs when the equity value reaches zero, that is, gross equity return equals zero.

First Bakshi, Kapadia, and Madan (2003) defines a quadratic, cubic, and quartic contract so that each contract has a payoff of log equity return to the power of 2, 3, and 4, respectively. Next Bakshi, Kapadia, and Madan (2003) relies on ideas from earlier papers such as Bakshi and Madan (2000) and Carr and Madan (2010). The idea is that any payoff function $H(S)$ that is twice differentiable can be spanned by a continuum of OTM European call options and put options. Their equation is given as:

$$H(S) = H(\bar{S}) + (S - \bar{S})H_S(\bar{S}) + \int_{\bar{S}}^{\infty} H_{SS}(K)(S-K)^+ \, dK + \int_0^{\bar{S}} H_{SS}(K)(K-S)^+ \, dK$$

where $H_S[\cdot]$ and $H_{SS}[\cdot]$ represents the first order and second-order derivative of the payoff function with respect to S.

Thus, one can apply the payoff function of the quadratic, cubic, and quartic contracts with the above equation to estimate the payoff of each contract with the OTM call and put option prices.[14] The following equations result from this application:

$$V_t(\tau) = \int_{S_t}^{\infty} \frac{2 - 2ln(K/S_t)}{K^2} C_t(\tau; K)\, dK + \int_0^{S_t} \frac{2 - 2ln(K/S_t)}{K^2} P_t(\tau; K)\, dK$$

$$W_t(\tau) = \int_{S_t}^{\infty} \frac{6ln(K/S_t) - 3ln(K/S_t)^2}{K^2} C_t(\tau; K)\, dK + \int_0^{S_t} \frac{6ln(K/S_t) - 3ln(K/S_t)^2}{K^2} P_t(\tau; K)\, dK$$

$$X_t(\tau) = \int_{S_t}^{\infty} \frac{12ln(K/S_t)^2 - 4ln(K/S_t)^3}{K^2} C_t(\tau; K)\, dK + \int_0^{S_t} \frac{12ln(K/S_t)^2 - 4ln(K/S_t)^3}{K^2} P_t(\tau; K)\, dK$$

where S_t is the stock price at time t, K is the strike price, $C_t(\tau; K)$ is the price of the call option, which matures in τ, and $P_t(\tau; K)$ is the price of the put option.

Once the price of the contracts has been calculated, the risk-neutral moments are calculated from the prices of those contracts, as follows:

$$Var_t(\tau) = e^{r\tau} V_t(\tau) - \mu_t(\tau)^2$$

$$Skew_t(\tau) = \frac{e^{r\tau} W_t(\tau) - 3\mu_t(\tau) e^{r\tau} V_t(\tau) + 2\mu_t(\tau)^3}{(e^{r\tau} V_t(\tau) - \mu_t(\tau)^2)^{1.5}}$$

$$Kur_t(\tau) = \frac{e^{r\tau} X_t(\tau) - 4e^{r\tau} \mu_t(\tau) W_t(\tau) + 6e^{r\tau} \mu_t(\tau)^2 V_t(\tau) - 3\mu_t(\tau)^4}{(e^{r\tau} V_t(\tau) - \mu_t(\tau)^2)^2}$$

where $\mu_t(\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2} V_t(\tau) - \frac{e^{r\tau}}{6} W_t(\tau) - \frac{e^{r\tau}}{24} X_t(\tau)$.

Once the variance, skewness, and kurtosis are calculated for the risk-neutral distribution, we calculate the option-implied default probability according to the following integration of the NIG density. Since we are calculating the risk-neutral probability, the mean of the NIG distribution should be the gross risk-free return.

$$P_t^O(D; \tau) = \int_{-\infty}^{0} f_{nig}(x, 1 + R_{f,t,\tau}, Var_t(\tau), Skew_t(\tau), Kurt_t(\tau))\, dx$$

[14] For example in the quadratic case, we have $H(x) = ln(\frac{x}{S})^2$. Then $H'(x) = \frac{2}{x} ln(\frac{x}{S})$, and $H''(x) = \frac{2 - 2ln(\frac{x}{S})}{x^2}$.

where f_{nig} is given as,

$$f_{nig}(x; M, V, S, K) = f(x; \alpha, \beta, \mu, \delta)$$

where $\alpha, \beta, \mu, \delta$ are given as,

$$\alpha = \frac{3(\frac{4}{\rho} + 1)}{K\sqrt{1 - \frac{1}{\rho}}}$$

$$\beta = sign(S)\alpha\frac{1}{\sqrt{\rho}}$$

$$\mu = M - sign(S)\sqrt{3(\frac{4}{\rho} + 1)\frac{V}{K\rho}}$$

$$\delta = \sqrt{3(\frac{4}{\rho} + 1)(1 - \frac{1}{\rho})\frac{V}{K}}$$

where $\rho = \frac{3K}{S^2} - 4$, $sign(\cdot)$ is the sign function and f is given as

$$f(x; \alpha, \beta, \mu, \delta) = \frac{\alpha}{\pi\delta}exp\left(\sqrt{\alpha^2 - \beta^2} + \frac{\beta(x - \mu)}{\delta}\right) \times \frac{K_1\left(\alpha\sqrt{1 + (\frac{x-\mu}{\delta})^2}\right)}{\sqrt{1 + (\frac{x-\mu}{\delta})^2}}$$

where $K_1(\cdot)$ is the modified Bessel function of the third kind with index 1. It is important to note that one condition, $\rho > 1$, must be satisfied for the estimation to work. If this restriction is not satisfied, the risk-neutral default probability cannot be calculated.

To calculate integrals from discrete option prices, we use trapezoidal method as in Dennis and Mayhew (2002). [15]

[15] I thank Patrick Dennis and Stewart Mayhew for providing us with the sample code for the trapezoidal integration.

A.3 Kalman Filter Estimation

Here we state the Kalman filter, the extended Kalman filter, and the unscented Kalman filter.[16] The Kalman filter is a family of methods to estimate the time series of unobservable state variables from the observable time series, which is a function of the state variables. Let us assume that we wish to estimate an unobservable time series of state variable x_t from an observable time series of data, y_t. If the state variables are constant over time we could use the recursive Bayes filter. If the state variables are observable and if we can evaluate the likelihood function, we would use the maximum likelihood method. When the underlying states are unobservable and time-varying, the Kalman filter is a suitable choice for estimation.

A.3.1 Kalman Filter

Let x_t be a $n \times 1$ vector of state variables at time t. Then, the following describes a linear state-space system:

$$x_{t+1} = \phi_0 + \phi_1 x_t + e^s_{t+1}$$

where ϕ_0 is a $n \times 1$ vector, ϕ_1 is a $n \times n$ transition matrix, and $e^s_{t+1} \sim N(0, v(x_t))$, where $v(x_t)$ is the conditional covariance matrix of state vector x_t.

And the following is a description of the measurement equation, where y_t is the $N \times 1$ vector of observable variables, which is a function of the underlying state variables. Here, we assume that the observation function g is a linear function. Then,

$$y_t = g(x_t) + e^o_{t+1} = c + dx_t + e^o_{t+1}, \qquad t = 1 \ldots T,$$

where c is a $N \times 1$ vector, and d is a $N \times n$ matrix.

Given this model, the Kalman filter generates the minimum MSE defined as follows,

[16] Christoffersen, Jacobs, Karoui, and Mimouni (2009) provides more detailed discussion on these filters.

$$MSE = \frac{1}{T}\sum_t (y_t - y_{t|t-1})'(y_t - y_{t|t-1})$$

The Kalman filter is as follows. At each time period,

$$x_{t|t} = \hat{x}_{t|t-1} + K_t(y_t - y_{t|t-1})$$

$$P_{xx(t|t)} = P_{xx(t|t-1)} - K_t P_{yy(t|t-1)} K_t'$$

where

$$\hat{x}_{t|t-1} = \phi_0 + \phi_1 x_{t-1|t-1}$$

$$P_{xx(t|t-1)} = \phi_1 P_{xx(t-1|t-1)} \phi_1' + v(x_{t-1})$$

$$K_t = P_{xy(t|t-1)} P_{yy(t|t-1)}^{-1}$$

$$\hat{y}_{t|t-1} = E_{t-1}[g(x_t)]$$

Starting with the initial estimate of state vector x_0, the Kalman filter provides the optimal estimate of $\hat{x}_{1|0} = \phi_0 + \phi_1 x_0$, the estimate of state at time $t = 1$ given information about the transformation (ϕ_0, ϕ_1), and the estimate of current state at $t = 0$. Also, the filter provides the optimal estimate of $\hat{y}_{1|0} = g(x_{1|0})$, the expected observation given the optimal estimate of $\hat{x}_{1|0}$. When y is observed at $t = 1$, the difference $y_1 - \hat{y}_{1|0}$ gives information on how much the original estimate $x_0, \hat{x}_{1|0}$ differed from the true value. Thus we are able to get a better estimate of state variable x_1. The whole process is iterated for all periods. K_t, the multiplier to $y_t - \hat{y}_{t|t-1}$ in finding the updated x_t is called the Kalman gain.

A.3.2 Extended Kalman Filter

The extended Kalman filter is the standard choice among different types of Kalman filters when the mapping function g is nonlinear. The extended Kalman filter addresses nonlinearity by applying the first-order Taylor approximation:

$$y_t = g(x_{t|t-1}) + J_t(x_t - x_{t|t-1}) + u_t$$

where $J_t = \frac{\partial g}{\partial x_t}\Big|_{x_t = x_{t|t-1}}$ is the Jacobian matrix of g evaluated at $x_{t|t-1}$.

While the state updates for $x_{t|t}$ and covariance updates for $P_{xx(t|t)}$ are identical to the original Kalman filter,

the covariance matrices $P_{xy(t|t-1)}, P_{yy(t|t-1)}$ and the Kalman gain K_t are calculated differently, since we are applying Taylor approximation to the mapping function. Thus,

$$P_{xy(t|t)} = P_{xx(t|t-1)} J_t$$

$$P_{yy(t|t)} = J_t P_{xx(t|t-1)} J_t' + R$$

$$K_t = P_{xx(t|t-1)} J_t P_{yy(t|t-1)}^{-1}$$

A.3.3 Unscented Kalman Filter

The unscented Kalman filter is another type of Kalman filter; introduced recently, it has gained popularity in highly nonlinear models. While the extended filter is applicable when the first-order Taylor approximates the nonlinear mapping function well, it may not be suitable in cases where the function is highly nonlinear. Instead of approximating the nonlinear function, the unscented Kalman filter creates a set of sample points in a deterministic fashion that preserves current information about the current sample state, and applies true nonlinear function directly to the set of sample points. Julier and Uhlmann (2004) shows that the unscented Kalman filter estimates of the mean and covariance are correct up to the second order for any nonlinear function.

First, the sample points are generated so that the mean and covariance are preserved. Given the mean μ_x and covariance P_{xx}, we first create $2n + 1$ number of weighted sample points $\bar{x}$.

$$\bar{x}_0 = \mu_x$$

$$\bar{x}_i = \mu_x + \left(\sqrt{(n+\xi)P_{xx}}\right)_i, \qquad \text{for } i \in [1,n]$$

$$\bar{x}_i = \mu_x - \left(\sqrt{(n+\xi)P_{xx}}\right)_i, \qquad \text{for } i \in [n,2n]$$

with weights

$$W_0^m = \frac{\xi}{n+\xi}, \qquad W_0^c = \frac{\xi}{n+\xi} + (1-\rho^2+\theta)$$

$$W_i^m = W_i^c = \frac{1}{2(n+\xi)}, \qquad \text{for } i \in [1,2n]$$

where $\xi = \rho^2(n+\chi)$, $\left(\sqrt{(n+\xi)P_{xx}}\right)_i$ is the ith column of the matrix square root of $(n+\xi)P_{xx}$, or the covariance matrix multiplied by the number of states plus ξ. Next, ρ is a positive scaling parameter, and χ is a positive scaling parameter to guarantee the positivity of the covariance matrix. Once the sample points are calculated, the vector of sample points, $\mathcal{X}$, is formed as follows:

$$\mathcal{X}_{t-1|t-1} = \begin{bmatrix} x_{t-1|t-1} & x_{t-1|t-1} \pm \sqrt{(n+\xi)P_{xx(t-1|t-1)}} \end{bmatrix}$$

Then comes the prediction step,

$$\hat{\mathcal{X}}_{t|t-1} = \phi_0 + \phi_1 \mathcal{X}_{t-1|t-1}$$

$$\hat{x}_{t|t-1} = \sum_{i=0}^{2n} W_i^m \hat{\mathcal{X}}_{i,t|t-1}$$

$$P_{xx(t|t-1)} = \sum_{i=0}^{2n} W_i^c [\hat{\mathcal{X}}_{i,t|t-1} - \hat{x}_{t|t-1}][\hat{\mathcal{X}}_{i,t|t-1} - \hat{x}_{t|t-1}]'$$

$$\hat{\mathcal{Y}}_{t|(t-1)} = g(\hat{\mathcal{X}}_{t|t-1})$$

$$\hat{y}_{t|t-1} = \sum_{i=0}^{2n} W_i^m \hat{\mathcal{Y}}_{i,t|t-1}$$

Finally, the measurement update

$$P_{xy(t|t-1)} = \sum_{i=0}^{2n} W_i^c [\hat{\mathcal{X}}_{i,t|t-1} - \hat{x}_{t|t-1}][\hat{\mathcal{Y}}_{i,t|t-1} - \hat{y}_{t|t-1}]'$$

$$P_{yy(t|t-1)} = \sum_{i=0}^{2n} W_i^c [\hat{\mathcal{Y}}_{i,t|t-1} - \hat{y}_{t|t-1}][\hat{\mathcal{Y}}_{i,t|t-1} - \hat{y}_{t|t-1}]'$$

The state updates for $x_{t|t}$ and covariance updates for $P_{xx(t|t)}$ are identical to the original Kalman filter.

A.3.4 Estimation

The log likelihood function is given as,

$$\mathcal{L}(\Theta) = -\frac{1}{2} \sum_{t=1}^{T} \Big(n\, log(2\pi) + log|P_{yy(t|t-1)}| + (y_t - \hat{y}_{t|t-1})' P_{yy(t|t-1)}^{-1} (y_t - \hat{y}_{t|t-1}) \Big)$$

We use two algorithms for constrained maximization, the limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm and the truncated Newton method, within reasonable bounds for the coefficients.

A.4 Proof of Lemma

Lemma. *Assume $\rho_i > \rho_{ij}$ where $\rho_i = Corr(-R_i, P(D_i))$ and $\rho_{ij} = Corr(-R_i, P(D_i \cap D_j))$, and that $Var(P(D_i)) > Var(P(D_i \cap D_j))$. Then, $Cov(-R_i, P(D_i \cap D_j)) < Cov(-R_i, P(D_i))$, or $Cov(\bar{R} - R_i, P(D_i) - P(D_i \cap D_j)) > 0$.*

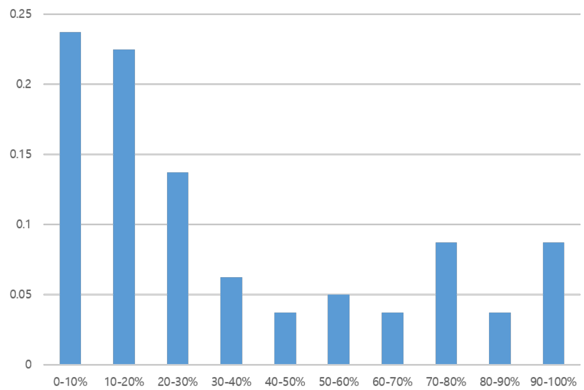
Proof. The assumption, $\rho_i > \rho_{ij}$, should be trivial since the correlation between firm i's negative recovery rate $-R_i$ and $P(D_i)$, the default probability of firm i should be higher than the correlation between firm i's negative recovery rate $-R_i$ and $P(D_i \cap D_j)$, the joint default probability of firm i and j. That is, the joint default probability is not only affected by firm i's fundamentals, but also by firm j's fundamentals. Next, we assume that $Var(P(D_i)) > Var(P(D_i \cap D_j))$. Given that $P(D_i) > P(D_i \cap D_j)$ holds and $P(D_i) >> P(D_i \cap D_j)$ in most cases, we believe that this is a reasonable assumption.

$$Cov(-R_i, P(D_i \cap D_j)) = \rho_{ij}\sigma(-R_i)\sigma(P(D_i \cap D_j))$$

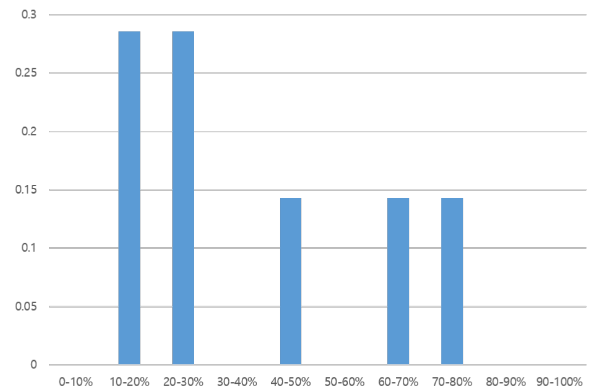
$$Cov(-R_i, P(D_i)) = \rho_i\sigma(-R_i)\sigma(P(D_i))$$

Thus $Cov(-R_i, P(D_i \cap D_j)) < Cov(-R_i, P(D_i))$. $\square$

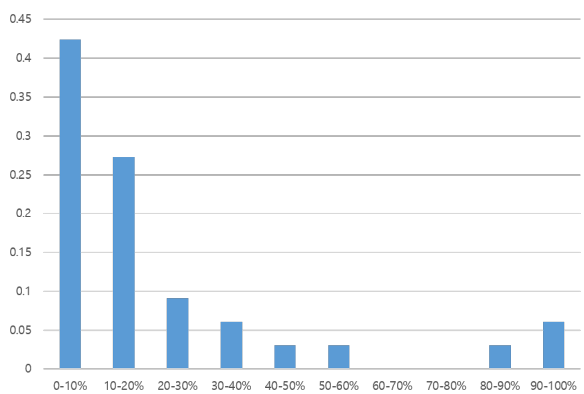
A.5 Additional Tables and Figures



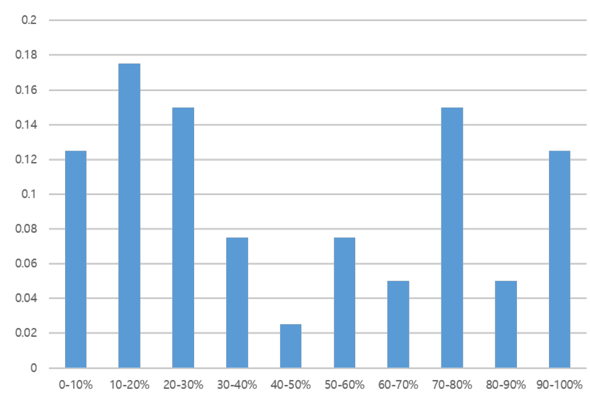
(a) Full Sample, 2005-2014



(b) 2005-2007.6

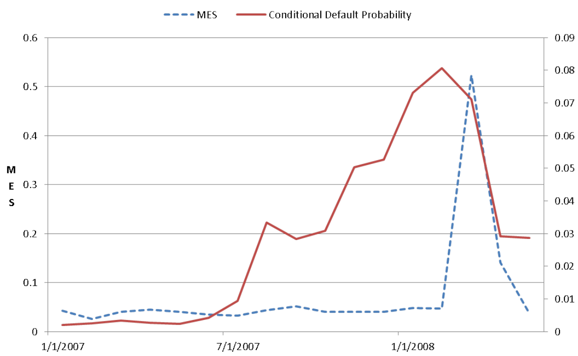


(c) 2007.7-2009.6

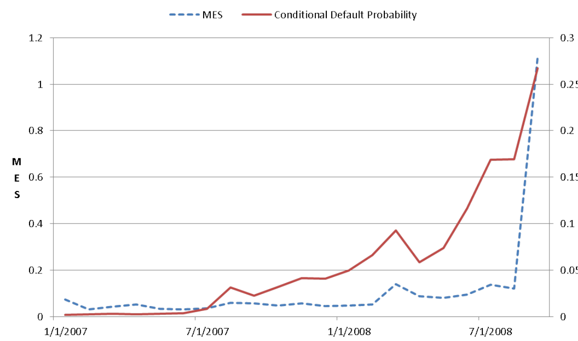


(d) 2009.7-2014

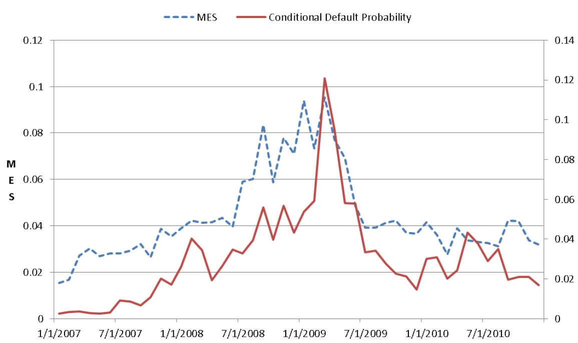
Figure 13: Historgram of realized CDS auction prices, by percentage of par value (% recovery rate)



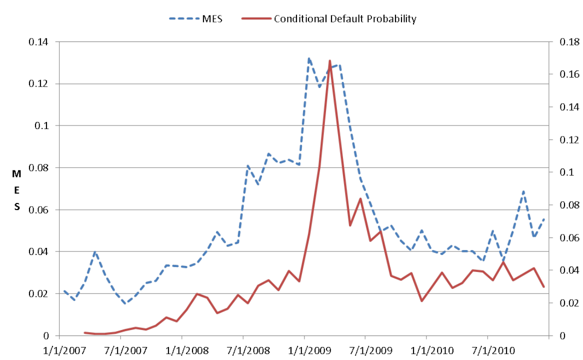
(a) Bear Stearns



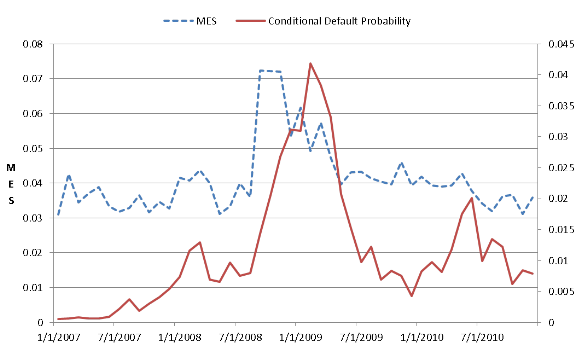
(b) Lehman Brothers



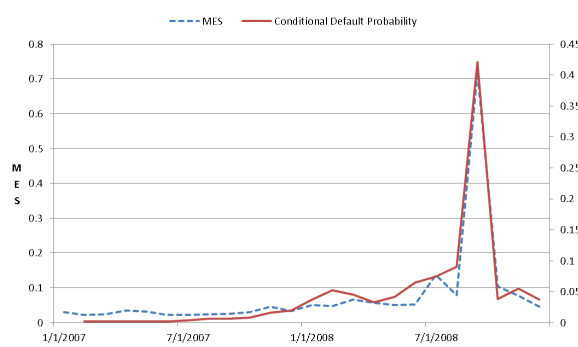
(c) JP Morgan



(d) Bank of America



(e) Credit Suisse



(f) Wachovia Bank

Figure 14: MES and Conditional Default Probability of CDS dealers

Table 7: List of CDS event auctions with realized recovery rates

Panel A: 2005.6.14-2009.5.12

Date	Name	% of par value
2005.6.14	Collins & Aikman	43.625
2005.10.11	Northwest Airlines	28
2005.10.11	Delta Air Lines	18
2005.11.4	Delphi Corporation	63.375
2006.1.17	Calpine Corporation	19.125
2006.3.31	Dana Corporation	75
2006.11.28	Dura	24.125
2007.10.23	Movie Gallery	91.5
2008.2.19	Quebecor World	41.25
2008.10.2	Tembec Inc	83
2008.10.10	Lehman Brothers	8.625
2008.10.23	Washington Mutual	57
2008.11.4	Landsbanki	1.25
2008.11.5	Glitnir	3
2008.11.6	Kaupthing	6.625
2009.1.6	Tribune	1.5
2009.1.14	Republic of Ecuador	31.375
2009.2.3	Millennium America Inc	7.125
2009.2.3	Lyondell	15.5
2009.2.3	EquiStar	27.5
2009.2.10	Nortel Ltd.	6.5
2009.2.10	Nortel Corporation	12
2009.2.19	Smurfit-Stone	8.875
2009.2.26	Ferretti	10.875
2009.3.9	Aleris	8
2009.3.31	Station Casinos	32
2009.4.14	Chemtura	15
2009.4.14	Great Lakes	18.25
2009.4.15	Rouse	29.25
2009.4.16	LyondellBasell	2
2009.4.17	Abitibi	3.25
2009.4.21	Charter Communications	2.375
2009.4.22	Capmark	23.375
2009.4.23	Idearc	1.75
2009.5.12	Bowater	15

Panel B: 2009.5.13-2012.6.6

Date	Name	% of par value
2009.5.13	General Growth Properties	44.25
2009.5.27	Syncora	15
2009.5.28	Edshcha	3.75
2009.6.11	R.H. Donnelley Corp.	4.875
2009.6.12	General Motors	12.5
2009.6.18	JSC Alliance Bank	16.75
2009.6.23	Visteon	3
2009.7.9	Six Flags	14
2009.7.21	Lear	38.5
2009.11.20	CIT Group Inc.	68.125
2009.12.9	Thomson	77.75
2009.12.15	Hellas II	1.375
2009.12.16	NJSC Naftogaz of Ukraine	83.5
2010.1.7	Financial Guarantee Insurance Company	26
2010.2.18	CEMEX	97
2010.3.25	Aiful	33.875
2010.4.15	McCarthy and Stone	70.375
2010.4.22	Japan Airlines Corp.	20
2010.6.4	Ambac Assurance Corp.	20
2010.7.15	Truvo Subsidiary Corp.	3
2010.9.9	Truvo (formerly World Directories)	41.125
2010.9.21	Boston Generating LLC	40.75
2010.10.28	Takefuji Corp.	14.75
2010.12.9	Anglo Irish Bank	18.25
2010.12.10	Ambac Financial Group	9.5
2011.11.29	Dynegy Holdings, LLC	71.25
2011.12.9	Seat Pagine Gialle	10
2011.12.13	PMI Group	16.5
2011.12.15	AMR Corp.	23.5
2012.2.22	Eastman Kodak Co	22.875
2012.3.22	Elpida Memory	20.125
2012.3.29	ERC Ireland Fin Ltd	0
2012.5.9	Sino Forest Corp	29
2012.5.30	Houghton Mifflin Harcourt Publishing Co	55.5
2012.6.6	Residential Cap LLC	17.625

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