Banking and Shadow Banking

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Abstract

Tightening financial regulation squeezes banking activities into the shadow banking sector, which may hurt financial stability and production (Plantin, 2014). Unlike Plantin's work, we investigate regulations that reduce banks' leverage and dampen financial amplification effects. Moreover, our paper studies the trade-off between economic growth and financial stability in light of shadow banking within a continuous-time, macro-finance framework. Shadow banking modeled as off-balance-sheet financing has an enforcement problem. We demonstrate that this problem leads to an endogenous leverage constraint for shadow banking and that the constraint tightens in economic downturns, which in turn forces shadow banks to conduct asset fire sales.

Keywords: shadow banking; off-balance-sheet financing; financial instability; financial regulation

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Introduction

The 2007-09 global financial crisis brought prominence to shadow banking—those bank-like activities that unregulated financial entities and off-balance-sheet vehicles undertake—in global discussions about the connection between financial instability and financial regulation. Over the course of these discussions, a new line of argument emerged, suggesting that since financial activities are capable of migrating to the unregulated sector, tightening financial regulation could endanger financial stability.[1] To the best of our knowledge, Plantin (2014) is the first theoretical work that examines the regulatory arbitrage channel through which financial regulation may incur adverse unintended consequences.

Given the importance of shadow banking and its influence on the channel through which financial regulation affects financial stability, this paper attempts to deepen our understanding of these topics with respect to three critical aspects that Plantin (2014) does not address. First, we focus on the class of financial regulations that restricts the use of bank leverage; in contrast, Plantin (2014) examines regulation that prohibits banks from issuing outside equity. Second, the concept of financial instability that we emphasize contrasts with that in Plantin (2014). Specifically, we recognize financial instability as the endogenous risk that the banking sector generates through the balance sheet mechanism (Kiyotaki and Moore, 1997; Bernanke, Gertler and Gilchrist, 1999). For Plantin (2014), meanwhile, the riskiness of outside equity reflects the instability that is counterproductive for the real sector. Finally, the dynamic general equilibrium setting in our paper allows us to characterize dynamic properties of shadow banking, so we may discuss the trade-off between economic growth and financial stability, as well as calibrate the model in order to offer specific policy suggestions. In characterizing these dynamic properties, the static setting in Plantin (2014) is not suitable.

In this paper, we argues that when financial regulation is sufficiently lenient, shadow banking activities are completely unsustainable due to their intrinsic frictions; in such circumstances, tight-

[1] For instance, Brunnermeier et al. (2009) mention the financial migration from the regulated to the unregulated sector and discuss the relevant regulatory perimeter issue. The IMF's October 2014 *Global Financial Stability Report* shows that countries that have a large shadow banking sector typically implement tight financial regulatory rules (IMF, 2014). Richard Berner, Director of the Office of Financial Research, mentioned the risk of financial migration towards the unregulated shadow banking sector in his speech for the Money Marketeers at New York University on October 15, 2014.

ening regulation lowers financial instability and improves social welfare. When financial regulation becomes stringent enough, the borrowing capacity of the shadow banking sector is sizeable, and further strengthening of regulation heightens financial instability and worsens social welfare. That said, in emphasizing the balance between economic growth and financial stability, we present a framework that can evaluate the welfare implications of financial regulation with respect to its impact on the shadow banking sector.

Further, our paper emphasizes that the borrowing capacity of shadow banking relies on market discipline and, more importantly, on the level of financial regulation that regular banks face. This emphasis differs from that of Plantin (2014), which assumes that a financial firm's internal optimal choice determines the extent of its shadow banking businesses. Moreover, our paper highlights that if the channel connecting the borrowing capacity of shadow banking to financial regulation is turned off, then tightening regulation can always lower financial instability because the contraction of the regular banking sector dominates the expansion of the shadow banking sector.

Similar to Thomas and Worrall (1988), Kehoe and Levine (1993), and Kocherlakota (1996), we find that a standard enforcement problem in our model gives rise to the maximum borrowing capacity of shadow banking, and this enforcement problem originates from shadow banking's institutional details. To be consistent with such details in our paper, we model regular banking as a regular bank's *on-balance-sheet financing* and shadow banking as the regular bank's *off-balance-sheet financing*. To regulatory authorities, a regular bank constructs its shadow bank (i.e., off-balance-sheet vehicle) as a legally separate entity to circumvent any regulation of financial activities that the shadow bank undertakes. To creditors, however, the regular bank portrays the shadow bank as part of its own business. Due to the absence of an authority, then, an enforcement problem arises, as the shadow bank's creditors cannot force its parent regular bank to protect them when the shadow bank is in trouble.

We next argue that tightening financial regulation raises the borrowing capacity of shadow banking (i.e., the increasing leverage that a regular bank can obtain via shadow banking). As in the limited enforcement literature, if a regular bank defaults on its shadow bank obligations, then the creditors of the shadow bank will refuse to lend in the future, which deprives the regular bank of its regulatory arbitrage opportunity. Therefore, the opportunity cost for the regular bank to default amounts to the present value of the future benefits that shadow banking offers. Since more stringent regulation leads to greater opportunities for regulatory arbitrage, the opportunity cost of default is larger in economies with tighter regulation, and the leverage of shadow banking is higher in such economies.

Because the leverage of shadow banking is endogenously determined, an interesting feedback loop between the opportunity cost of default and the leverage of shadow banking emerges. Specifically, if the cost of default declines due to loosening financial regulation, then the incentive to default will rise, and the shadow banking channel will contract. This shrinking shadow banking channel offers less benefits to regular banks, which consequently leads to an even lower opportunity cost of default. This feedback loop could amplify the effect of the initial drop in the cost of default so significantly that even a small drop can rule out shadow banking completely. Hence, shadow banking is unsustainable when regulation is sufficiently lenient.

We embed our modeling of banking and shadow banking in a standard, continuous-time, macro-finance framework (Brunnermeier and Sannikov, 2014). In this framework, because of a constraint on outside-equity financing, banks can only use leverage to finance their investments. We choose this macro-finance framework because we can model financial instability as an endogenous risk that the financial system itself generates. Financial regulation that limits the use of bank leverage can improve social welfare because the excessive use of bank leverage leads to high endogenous risk and causes a pecuniary externality for the entire economy (Lorenzoni, 2008; Stein, 2012). As a result, shadow banking emerges as regular banks' response to financial regulation.

The solution of a continuous-time, macro-finance model characterizes the full dynamics of an economy. With this advantage, our model captures two salient dynamic features of shadow banking observed during the 2007-09 financial crisis: the pro-cyclicality of shadow banking and reintermediation by shadow banks conducting fire sales of assets to regular banks. By analyzing both leverage dynamics and endogenous risk dynamics, we uncover a general equilibrium channel through which shadow banking adds to financial instability.

We first explain why shadow banking is pro-cyclical in our model. In economic booms, high asset prices and the corresponding low rates of return from holding assets lead to banking's low profitability. Hence, regular banks do not have strong incentives to leverage up via shadow banking with the intention to default. As a result, the enforcement problem is less severe in economic upturns, and the leverage of shadow banking also tends to be high in such upturns. In addition, the feedback loop between the cost of default and the leverage of shadow banking amplifies shadow banking's expansion in economic booms.

Shadow banking increases financial instability as a general equilibrium effect in our model. Since shadow banking is immune to financial regulation and its borrowing capacity is pro-cyclical, shadow banks accumulate substantial amounts of assets in upturns. Conversely, when a negative macroeconomic shock hits the economy, the shrinking shadow banking channel forces shadow banks to sell assets to regular banks at fire-sale prices (i.e., reintermediation). This asset fire-sale occurs because regular banks are reluctant to acquire assets due to financial regulations. Thus, the decline in asset prices has to be large enough such that regular banks are willing to purchase those assets. Naturally, the degree of financial instability increases in such situations.

We next elaborate on the U-shaped relationship between financial regulation and financial instability, as well as the hump-shaped relationship between financial regulation and social welfare. When regulation is loose enough, shadow banking is negligible. In this situation, tighter regulation of regular banking can lead to lower financial instability and higher economic welfare. When regulation is sufficiently tight, a considerable number of banking activities shift to the shadow banking sector, thanks to shadow banking's ample funding capacity. And, more stringent regulation, in this circumstance, gives rise to a larger shadow banking system and higher financial instability, which may diminish social welfare.

Related Literature. The literature on shadow banking is swiftly growing and diverse. Different papers model shadow banking in drastically different ways, and Adrian and Ashcraft (2012) provide a thorough survey of this growing literature. In our paper, we attempt to categorize models of shadow banking along two dimensions: the motive for shadow banking and the type of negative externalities that shadow banking causes.

The existence of shadow banking can be demand/preference driven. For example, in Gennaioli et al. (2013), infinitely risk-averse households only value securities' worst scenario payoffs, and shadow banking can increase such payoffs by pooling different assets together. Meanwhile, in Moreira and Savov (2014), the preference specification of households leads directly to a demand for the liquid securities that shadow banking generates.

The second motive for shadow banking is regulatory arbitrage, as we discuss in this paper. Luck and Schempp (2014), Ordonez (2013), and Plantin (2014) are papers that fall into this category.

Models of shadow banking differ with respect to the type of the externalities that shadow banking causes. The first category includes non-pecuniary externalities. In Plantin (2014), shadow banking exposes the real sector to counter-productive uncertainty. In both Luck and Schempp (2014) and Gennaioli et al. (2013), creditors of shadow banking suffer from unexpected default that bank runs or crises cause. Generally, investments financed by shadow banking in these models have worse or more volatile fundamentals than those investments financed by regular banking.

Unlike the first group of papers discussed above, we, as well as, Moreira and Savov (2014) assume that shadow banking does not involve any investments of inferior quality. Instead, we focus on the pecuniary externality; that is, the leverage choices of individual shadow banks cause excessive endogenous risk because they do not take into account the price impact of their actions in the competitive equilibrium.

This paper is also related to the literature on pecuniary externalities. One closely related paper is Bianchi (2011), whose quantitative examination of the pecuniary externality of excessive borrowing in a dynamic general equilibrium model highlights that raising borrowing costs can improve welfare.

For our methodology in this paper, we follow the emerging literature (e.g., Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012b, 2013)) that considers economies with financial frictions in a continuous-time setting. The methodology employed in this literature captures the exact characterization of full equilibrium dynamics particularly well. In our model, the tractability allows us to explicitly relate the leverage constraint to endogenous risk. Several papers (e.g., Adrian and Boyarchenko (2012), Danielsson et al. (2012), and Phelan (2012)) have similarly related the leverage constraint to endogenous risk by assuming various forms of value-at-risk constraints. The leverage constraint in our paper originates from the financial friction that bankers could strategically default on securities that shadow banking generates. The leverage of shadow banking is endogenously determined by the leverage constraint in equilibrium, which is in line with the market-based concept of shadow banking. More importantly, the leverage constraint draws the connection between shadow banking activities and financial regulation.

The paper is structured as follows. We first establish our baseline model in Section 1. In Section 2, we then characterize the non-sunspot equilibrium of this baseline model, in which the shadow banking system never collapses on the equilibrium path, and illustrate the main results with numerical examples. In Section 3, we next explore the welfare and policy implications of the baseline model, and we use Section 4 to highlight that the endogenous leverage constraint for shadow banking is essential for the U-shaped result and to demonstrate the robustness of the main results by varying agents' preferences.

1 The Baseline Model

To analyze how shadow banking changes conventional ideas about financial regulation and financial instability, this section models both shadow banking and regular banking and their interaction within a standard continuous-time macro-finance framework developed by Brunnermeier and Sannikov (2014). We will also specify the dynamic portfolio choice problem for each agent and list the equilibrium conditions that we will characterize in Section 2.

1.1 Model Setup

The general model setup is standard in the literature on financial frictions. As in Kiyotaki and Moore (1997), He and Krishnamurthy (2012b), and others, our model has heterogeneous agents: productive bankers and less-productive households. Bankers can raise funds from households through both regulated regular banking, modeled as on-balance-sheet financing, and unregulated shadow banking, modeled as off-balance-sheet financing.

1.1.1 Technology and Preferences

We consider a continuous-time infinite-horizon economy with two types of goods: durable physical capital goods and non-durable consumption goods.

At any time $t \in [0, \infty)$, a banker holding capital k_t produces consumption goods y_t according to

$$y_t = ak_t.$$

Households also have a linear production technology

$$y_t^h = a^h k_t^h,$$

although they are less productive; that is, $a^h < a$. Both bankers and households have the investment technology that an agent inverts $g(\iota_t)k_t$ units of consumption goods into $\iota_t k_t$ units of new capital, where

$$g(\iota_t) = \iota_t + 0.5\phi\left(\iota_t - \delta\right)^2$$

with δ denoting the depreciation rate.[2] Thus, the capital stock held by each agent grows at the rate $\iota_t - \delta$ in the absence of any shock.

The exogenous aggregate shock to the economy is driven by a Poisson process $\{N_t\}_{t=0}^{\infty}$ with intensity λ. Whenever the Poisson shock hits the economy, the capital stock held by each agent drops by a constant proportion, κ.[3] The law of motion of the aggregate capital K_t is

$$dK_t = K_{t-}\left(\iota_{t-} - \delta\right)dt - K_{t-}\kappa dN_t$$

conditional on all agents choosing the same investment rate ι_{t-}, where K_{t-} denotes $\lim_{s\uparrow t} K_s$; i.e., the left limit of the process $\{K_s, s \geq 0\}$ in period t. For purposes of exposition, we interpret time $t-$ as the period right before time t.

We assume that bankers have logarithmic utility, households are risk neutral, and both types

[2]We choose the functional form of capital adjustment cost to be consistent with Christiano et al. (2005), He and Krishnamurthy (2012a), and many other quantitative macroeconomic models.

[3]The capital in the model should be measured in efficiency units, and both the investment and the Poisson shock affect the quality of capital. The setup yields the tractability of the model, since the economy is scale-invariant with respect to the aggregate capital stock. Other recent papers that also use a macroeconomic capital quality shock include Gertler and Karadi (2011) and Gertler et al. (2012).

of agents have a time discount rate ρ. The expected discounted lifetime utility of a banker is

$$E_0\left[\int_0^\infty e^{-\rho t}u\left(c_t\right)dt\right], \tag{1}$$

where

$$u\left(c\right)=\begin{cases}\ln\left(c\right), & \text{if } c>0,\\ -\infty, & \text{otherwise.}\end{cases} \tag{2}$$

We assume that households can have negative consumption. In Section 4.2, we modify the baseline model such that households have Epstein-Zin preferences.

1.1.2 Physical Assets

The market for capital goods has no friction. The market price of capital goods is in units of consumption goods, denoted by q_t. The law of motion of q_t is

$$dq_t = q_{t-}\mu^q_{t-}dt - q_{t-}\kappa^q_t dN_t,$$

where μ^q_t and κ^q_t are endogenously determined in equilibrium. κ^q_t could be stochastic in each period t.

In period t, in the absence of a negative shock, the rate of return for a banker holding physical capital is

$$\frac{a-g(\iota_t)}{q_t}+\iota_t-\delta+\mu^q_t.$$

Other than the dividend yield $\left(a-g(\iota_t)\right)/q_t$, there are two sources of gain from holding capital: the growth in the banker's capital stock $\iota_t-\delta$ and the rise in the price of capital μ^q_t. Similarly, there are two types of risk for holding capital: exogenous and endogenous. Exogenous risk is the possible κ proportional decline in the banker's capital stock caused by the Poisson shock. Endogenous risk is the κ^q_t proportional change in the price of capital, which is the general equilibrium effect of the Poisson shock. Endogenous risk affects the banker's investment return through its impact on the $1-\kappa$ proportion of physical capital left to the banker. Formally, the rate of return for bankers from holding capital is

$$\underbrace{\left[\frac{a-g(\iota_{t-})}{q_{t-}}+\iota_{t-}-\delta+\mu^q_{t-}\right]}_{\equiv R_{t-}}dt-\kappa^Q_t dN_t,$$

where $\kappa^Q_t \equiv \kappa + (1-\kappa)\,\kappa^q_t$ is the overall investment risk. Similarly, the rate of return for households holding capital is

$$\underbrace{\left[\frac{a^h-g(\iota^h_{t-})}{q_{t-}}+\iota^h_{t-}-\delta+\mu^q_{t-}\right]}_{\equiv R^h_{t-}}dt-\kappa^Q_t dN_t.$$

1.1.3 The Financial Market and Regulatory Authority

The financial market is incomplete. The following four assumptions detail the incompleteness of the financial market.

Assumption 1 *Households do not hold equity issued by other agents.*

A banker can establish a regular bank. Via regular banking, bankers issue short-term debt and equity to finance their holdings of physical capital. The regulatory authority imposes the regulation in Assumption 2 on regular banks.

Assumption 2 *Regular banks' debt financing is taxed at rate τ_t in period t; total tax revenue is instantly redistributed back to regular banks as lump-sum subsidies and the amount of the subsidy is proportional to bankers' net worth.*

Under the regulation, regular banks have to pay tax τ_t for each dollar they raise. Even though there is a tax rebate, the tax rate τ_t affects the optimal leverage of a regular bank because the rebate is distributed as a lump-sum subsidy.[4]

To circumvent the above regulation, a banker can sponsor a shadow bank and earn its residual value as a management fee in each period. In practice, this activity is referred to as off-balance-sheet financing.[5]

Assumption 3 *The regulatory authority treats a shadow bank as a regular bank, if bankers hold the equity of the shadow bank.*

[4] The lump-sum tax rebate set-up cancels the wealth effect of tax τ_t.

[5] Examples of off-balance-sheet financing in the real world include securitization, Asset-Backed Commercial Paper, and Money Market Funds.

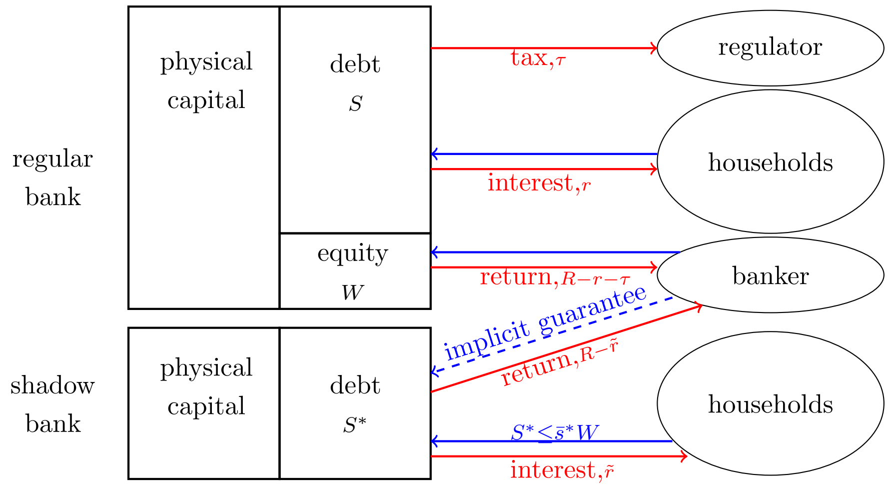

Figure 1: This figure details the financial side of the model. A regular bank's debt financing is taxed at rate τ. Households hold debt issued by regular banks and enjoy the rate of return r. Bankers earn regular banks' residual values at rate $R - r - \tau$ as their equity. Bankers also obtain shadow banks' residual value at rate $R - \tilde{r}$ as their guarantors, where $\tilde{r}$ is the rate of return that shadow banks promised to their household creditors. Bankers extend implicit guarantees to shadow banks. The maximum size of a shadow bank is the maximum leverage of shadow banking $\bar{s}$ times its banker's net worth W.

Shadow bank debt is also short term. Assumptions 1 and 3 imply that shadow banks are all debt financed. Any drop in the asset value of a shadow bank causes losses to its creditors unless the sponsor bails it out. Assumption 4 specifies the structure of the credit market for shadow banking and how households manage to secure the safety of their investments in shadow banks.

Assumption 4 *In any period t,*

i. each shadow bank offers a one-period debt contract $(\bar{s}^*_t, \tilde{r}_t)$;

ii. Given the contract, a shadow bank borrows up to $\bar{s}^*_t$ times the net worth of its sponsoring banker in period t and pays the principal and interest at rate $\tilde{r}_t$ in the following period.

iii. The market excludes bankers who default and allows their comebacks at rate ξ.

1.2 Problems for Bankers and Households

Suppose a banker's net worth is W_{t-} in period $t-$. S_{t-} denotes the value of debt that she raises via regular banking. The excess return from holding capital goods funded by regular banking is

$$S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)dt-S_{t-}\kappa_t^Q dN_t,$$

where r_t is the risk-free rate.

The banker also manages a shadow bank. The size of the shadow bank is S_{t-}^* in dollar terms. The banker earns the difference between the return from the capital investment $R_{t-}S_{t-}^*$ and the interest $\tilde{r}_{t-}S_{t-}^*$ promised to creditors. The size of the shadow bank is limited by the leverage constraint specified by its debt contract $\left(\bar{s}_{t-}^*,\tilde{r}_{t-}\right)$

$$S_{t-}^*\leq\bar{s}_{t-}^*W_{t-}. \tag{3}$$

In addition, the banker needs to decide whether she would default or not if a Poisson shock hits. This strategic choice is denoted by $\mathcal{D}_{t-}$. Given that the Poisson shock hits the economy, if the banker does not default ($\mathcal{D}_{t-}=0$), she will bear the loss $S_{t-}^*\kappa_t^Q$ for creditors of her shadow bank; otherwise, she will not do so. Thus, a banker's dynamic budget constraint is

$$\begin{aligned} dW_t &= \left(W_{t-}R_{t-}+S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+S_{t-}^*\left(R_{t-}-\tilde{r}_{t-}\right)+\pi_{t-}W_{t-}-c_{t-}\right)dt \\ &\quad -\left(W_{t-}\kappa_t^Q+S_{t-}\kappa_t^Q+\left(1-\mathcal{D}_{t-}\right)S_{t-}^*\kappa_t^Q\right)dN_t, \end{aligned} \tag{4}$$

where π_t is the ratio of subsidy to net worth and c_t is the banker's consumption.

Taking $\{q_t,r_t,\tau_t,\tilde{r}_t,\pi_t,\bar{s}_t^*\}_{t=0}^{\infty}$ as given, the banker chooses $\{c_t,S_t,S_t^*,\iota_t,\mathcal{D}_t\}_{t=0}^{\infty}$ to maximize her expected lifetime utility (1) subject to the leverage constraint (3) and the dynamic budget constraint (4).

Households can invest in both capital goods and debt issued by both regular and shadow banks. S_t^h denotes the value of capital that household h holds, and n_t the value of shadow bank debt that it holds. The wealth W_t^h of the household evolves according to

$$dW_t^h = \left(W_{t-}^h r_{t-} + S_{t-}^h \left(R_{t-}^h - r_{t-}\right) + n_{t-}\left(\tilde{r}_{t-} - r_{t-}\right) - c_{t-}^h\right) dt - \left(S_{t-}^h + \mathbf{d}_t n_{t-}\right) \kappa_t^Q dN_t, \quad (5)$$

where $\mathbf{d}_t$ denotes the fraction of shadow bank debt that defaults. Formally, a household chooses $\{c_t^h, n_t, S_t^h\}_{t=0}^{\infty}$ to maximize

$$U_0^h = E_0\left[\int_0^{\infty} e^{-\rho t} c_t^h dt\right].$$

1.3 Equilibrium

We make the following assumption to guarantee that the wealth share of bankers would not be large enough to undo all financial frictions.

Assumption 5 *Each banker retires independently at rate χ. If a banker retires, she can only save her wealth and earn risk-free rate r_t*

$\boldsymbol{I} = [0,1]$ and $\boldsymbol{J} = (1,2]$ denote sets of bankers and households, respectively. Individual bankers and households are indexed by $i \in \boldsymbol{I}$ and $j \in \boldsymbol{J}$.

Definition 1 *Given the initial endowments of capital goods $\left\{k_0^i, k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$ to bankers and households such that*

$$\int_0^1 k_0^i di + \int_1^2 k_0^j dj = K_0,$$

and a locally deterministic tax rate process $\{\tau_t\}_{t=0}^{\infty}$, an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{N_t\}_{t=0}^{\infty}$: the price of capital $\{q_t\}_{t=0}^{\infty}$, risk-free rate $\{r_t\}_{t=0}^{\infty}$, the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^{\infty}$, interest rate on notes $\{\tilde{r}_t\}_{t=0}^{\infty}$, the ratio of subsidy to net worth $\{\pi_t\}_{t=0}^{\infty}$, wealth $\left\{W_t^i, W_t^j\right\}_{t=0}^{\infty}$, capital holdings $\left\{k_t^i, k_t^j\right\}_{t=0}^{\infty}$, investment decisions $\{\iota_t^i\}_{t=0}^{\infty}$, default decisions $\{\mathcal{D}_t^i\}_{t=0}^{\infty}$, and consumption $\left\{c_t^i, c_t^j\right\}_{t=0}^{\infty}$ of banker $i \in \boldsymbol{I}$ and household $j \in \boldsymbol{J}$; such that

1. *$\left\{W_0^i, W_0^j\right\}$ satisfy $W_0^i = q_0 k_0^i$ and $W_0^j = q_0 k_0^j$, for $i \in \boldsymbol{I}$ and $j \in \boldsymbol{J}$;*

2. *bankers solve their problems given $\{q_t, r_t, \tau_t, \tilde{r}_t, \pi_t, \bar{s}_t^*\}_{t=0}^{\infty}$;*

3. *households solve their problems given $\left\{q_t, r_t, \tilde{r}_t, \mathcal{D}_t^i, i \in \boldsymbol{I}\right\}_{t=0}^{\infty}$;*

4. the budget of the regulatory authority is balanced;

5. markets for both consumption goods and capital goods clear

$$\int_0^1 c_i^i di + \int_1^2 c_t^j dj = \int_0^1 \left(a - g\left(\iota_t^i\right)\right) k_t^i di + \int_1^2 \left(a^h - g\left(\iota_t^j\right)\right) k_t^j dj, \tag{6}$$

$$\begin{aligned} \int_0^1 k_t^i di + \int_1^2 k_t^j dj &= K_t, \\ \text{where } dK &= \left(\int_0^2 \left(\iota_t^i - \delta\right) k_t^i di\right) dt - \kappa K_t dN_t; \end{aligned} \tag{7}$$

6. the credit market for shadow bank debt clears

Given the definition, the credit market for regular bank debt clears by Walras' Law.

1.4 Financial Frictions

It is worthwhile to summarize three types of financial frictions in the baseline model. First, we do not allow for the issuance of outside equity (Assumption 1). The restriction on equity financing gives rise to the standard balance sheet amplification mechanism in the financial friction literature (Krishnamurthy, 2010). Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012b) provide microfoundations for this restriction in continuous-time macro-finance models. Our model naturally inherits the amplification mechanism discussed in these two papers.

The second friction is the intervention of the regulatory authority (Assumption 2). Financial regulations are necessary since leverage chosen by bankers in a competitive market may not be socially optimal because bankers do not internalize the external impact of their private decisions, as discussed by Lorenzoni (2008) and Stein (2012). Section 3.1 shows that the tax on regular banking in Assumption 2 can improve bankers' welfare by adjusting the leverage choice of regular banks and diminishing the pecuniary externality. The tax rate can be interpreted as the shadow cost of financial regulation in the real world.[6]

[6]Kisin and Manela (2014) provide an estimate of the shadow cost of major banks' capital requirement constraint.

The third financial friction is the leverage constraint on shadow banking (Assumption 4). It is novel to the literature of modeling shadow banking closely based on the institutional details of off-balance-sheet financing. One implication of assumptions 1 and 3 is that shadow banks issue no equity. This is realistic because the equity portion of shadow banks in the real world, such as special purpose vehicles and money market mutual funds, is typically very thin.

2 Financial Regulation and Financial Instability

In this section, we use numerical examples to characterize non-sunspot equilibria of the baseline model. It is convenient to focus on non-sunspot equilibria because they do not involve a sudden collapse of the shadow banking sector. Thus, magnitudes of both endogenous risk κ_t^q and investment risk κ_t^Q are deterministic during each period in non-sunspot equilibria. With the model characterization, we will present our main result that the relationship between financial instability and financial regulation displays a U shape.

2.1 Equilibrium Characterization

2.1.1 Households' Optimal Choices

ι_t denotes a household's investment in period t. The expression of R_t^h implies that the optimal level of ι_t maximizes

$$\frac{-\iota_t - 0.5\phi\left(\iota_t - \delta\right)^2}{q_t} + \iota_t.$$

The first-order condition yields an expression that the optimal investment rate is a function of the price of capital q_t

$$\iota_t = \delta + \frac{q_t - 1}{\phi}. \tag{8}$$

Bankers have the same investment function $\iota(\cdot)$ as they have the same investment technology.

Since households are risk-neutral, the following conditions must hold in equilibrium to be consistent with the households' optimal consumption and portfolio choice.

Proposition 1 *A household's portfolio choices* $\{S_t^h, n_t, t \geq 0\}$ *satisfy*

$$r_t = \rho \tag{9}$$

$$R_{t-}^h - r_{t-} \leq \lambda \kappa_t^Q, \quad = \quad \text{if } S_{t-}^h > 0, \tag{10}$$

$$\tilde{r}_{t-} - r_{t-} \leq \lambda \mathbf{d}_t \kappa_t^Q, \quad = \quad \text{if } n_{t-} > 0, \tag{11}$$

for all $t \geq 0$.

2.1.2 Bankers' Optimal Choices

A banker's optimal overall leverage has an upper bound. This is because the second part of the utility specification (2) effectively imposes a nonnegative net worth constraint for bankers.

Proposition 2 *A banker's overall leverage has an upper bound in any period; in particular,*

$$(W_{t-} + S_{t-})\, \kappa_t^Q + S_{t-}^* \kappa_t^Q \mathbf{1}_{(d_t > 0)} < W_{t-}$$

always holds.

Proof. See Appendix A. ■

We next apply the stochastic control approach to solve for a banker's optimal consumption and portfolio choices given that she can access shadow banking. We first conjecture and later verify the functional form of the banker's continuation value

$$J_t \equiv E_t \left[\int_t^{\infty} e^{-\rho u} \ln(c_u) du \right] = \frac{ln(W_t)}{\rho} + h_t,$$

where W_t is the banker's net worth, and h_t is an additive term that depends on market conditions and evolves endogenously according to

$$dh_t = h_{t-} \mu_{t-}^h dt - h_{t\_} \kappa_t^h dN_t.$$

Second, we conjecture that if the banker defaults her continuation value is $\hat{J}_t = ln(W_t)/\rho + \hat{h}_t$, where $\hat{h}_t$ follows

$$d\hat{h}_t = \hat{h}_{t-}\mu_{t-}^{\hat{h}}dt - \hat{h}_{t-}\kappa_t^{\hat{h}}dN_t.$$

Now, we are ready to spell out the Hamilton-Jacobi-Bellman (HJB) equation for the banker's optimal control problem

$$0 = \max_{c_{t-},S_{t-},S_{t-}^*,\mathcal{D}_{t-}} \left\{(1-\mathcal{D}_{t-})\mathbf{HJB}_{\mathcal{N}} + \mathcal{D}_{t-}\mathbf{HJB}_{\mathcal{D}}\right\}, \text{ where} \tag{12}$$

$$\begin{aligned}
\mathbf{HJB}_{\mathcal{N}} &\equiv \max_{c_{t-},S_{t-},S_{t-}^*} \left\{ \begin{array}{c} \ln(c_{t-}) - \rho J_{t-} + \frac{\mu_W}{\rho} + h_{t-}\mu_{t-}^h + \chi\left(J_t^r(W_t) - J_{t-}\right) \\ +\lambda\left(\frac{1}{\rho}\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S_{t-}^*\right)\kappa_t^Q\right) + h_{t-}(1-\kappa_t^h) - J_{t-}\right) \end{array} \right\}, \\
\mathbf{HJB}_{\mathcal{D}} &\equiv \max_{c_{t-},S_{t-},S_{t-}^*} \left\{ \begin{array}{c} \ln(c_{t-}) - \rho J_{t-} + \frac{\mu_W}{\rho} + h_{t-}\mu_{t-}^h + \chi\left(J_t^r(W_t) - J_{t-}\right) \\ +\lambda\left(\frac{1}{\rho}\ln\left(W_{t-} - \left(W_{t-} + S_{t-}\right)\kappa_t^Q\right) + \hat{h}_{t-}(1-\kappa_t^{\hat{h}}) - J_{t-}\right) \end{array} \right\}, \\
\mu_W &\equiv \frac{1}{W_{t-}}\left(W_{t-}(R_{t-}+\pi_{t-}) + S_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + S_{t-}^*\left(R_{t-} - \tilde{r}_{t-}\right) - c_{t-}\right),
\end{aligned}$$

and $J_t^r(\cdot)$ is the banker's continuation value function if she retires in period t. While choosing her portfolio and consumption in period t, the banker also decides whether she would default to her shadow bank obligations ($\mathbf{HJB}_{\mathcal{D}}$) in the event of an adverse shock or not ($\mathbf{HJB}_{\mathcal{N}}$). Because of the time-consistency problem, a banker's portfolio choice (S_{t-}, S_{t-}^*) with respect to both $\mathbf{HJB}_{\mathcal{N}}$ and $\mathbf{HJB}_{\mathcal{D}}$ must satisfy their corresponding incentive-compatible constraints:

$$\frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S_{t-}^*\right)\kappa_t^Q\right)}{\rho} + h_{t-}(1-\kappa_t^h) \geq \frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-}\right)\kappa_t^Q\right)}{\rho} + \hat{h}_{t-}(1-\kappa_t^{\hat{h}})$$

for $\mathbf{HJB}_{\mathcal{N}}$ and

$$\frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S_{t-}^*\right)\kappa_t^Q\right)}{\rho} + h_{t-}(1-\kappa_t^h) \leq \frac{\ln\left(W_{t-} - \left(W_{t-} + S_{t-}\right)\kappa_t^Q\right)}{\rho} + \hat{h}_{t-}(1-\kappa_t^{\hat{h}})$$

for $\mathbf{HJB}_{\mathcal{D}}$. First-order conditions are straightforward to derive. We group them in the following proposition.

Proposition 3 *A banker's optimal choice of consumption and portfolio weights* $\{c_t, s_t, s_t^*\}_{t=0}^{\infty}$ *sat-*

isfies

$$c_t = \rho W_t, \tag{13}$$

$$R_{t-} - r_{t-} - \tau_{t-} \leq \frac{\lambda\left(1-\mathcal{D}_{t-}\right)\kappa_t^Q}{1-\left(1+s_{t-}+s_{t-}^*\right)\kappa_t^Q} + \frac{\lambda \mathcal{D}_{t-}\kappa_t^Q}{1-\left(1+s_{t-}\right)\kappa_t^Q}, \quad = \ \textit{if } s_{t-} > 0, \tag{14}$$

$$R_{t-} - \tilde{r}_{t-} \geq \frac{\lambda\left(1-\mathcal{D}_{t-}\right)\kappa_t^Q}{1-\left(1+s_{t-}+s_{t-}^*\right)\kappa_{t-}^Q}, \quad = \ \textit{if } s_{t-}^* < \bar{s}_{t-}^*, \tag{15}$$

given $\{\mathcal{D}_t\}_{t=0}^{\infty}$ *for all* $t \geq 0$*, where* $s_t = S_t/W_t$ *and* $s_t^* = S_t^*/W_t$*.*

We can characterize a banker's default decision $\mathcal{D}_t$ given her optimal portfolio weights (s_{t-}, s_{t-}^*) under $\mathbf{HJB}_{\mathcal{N}}$ and $(\tilde{s}_{t-}, \tilde{s}_{t-}^*)$ under $\mathbf{HJB}_{\mathcal{D}}$. Intuitively, if

$$\begin{aligned} & \frac{s_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+s_{t-}^*\left(R_{t-}-r_{t-}\right)}{\rho} + \frac{\lambda}{\rho}\ln\Big(1-\left(1+s_{t-}+s_{t-}^*\right)\kappa_t^Q\Big) + \lambda h_{t-}(1-\kappa_t^h) \\ \geq \quad & \frac{\tilde{s}_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+\tilde{s}_{t-}^*\left(R_{t-}-r_{t-}\right)}{\rho} + \frac{\lambda}{\rho}\ln\Big(1-\left(1+\tilde{s}_{t-}\right)\kappa_t^Q\Big) + \lambda \hat{h}_{t-}(1-\kappa_t^{\hat{h}}), \end{aligned} \tag{16}$$

then the banker would choose to honor her shadow bank obligations in the event of an adverse shock.

So far, we are silent about a banker's continuation value $\hat{J}_t = ln(W_t)/\rho + \hat{h}_t$ in the case that she cannot access shadow banking due to default. The characterization of $\hat{J}_t$ is similar to that of J_t. The law of motion of the banker's net worth $\hat{W}_t$ is

$$d\hat{W}_t = \underbrace{\Big(\hat{W}_{t-}(R_{t-}+\pi_{t-}) + \hat{S}_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right) - \hat{c}_{t-}\Big)}_{\equiv \hat{W}_{t-}\mu_{t-}^{\hat{W}}} dt - \Big(\hat{W}_{t-}\kappa_t^Q + \hat{S}_{t-}\kappa_t^Q\Big)\, dN_t.$$

The HJB equation for the banker is

$$\begin{aligned} \rho \hat{J}_{t-} \quad = \quad & \hat{h}_{t-}\mu_{t-}^{\hat{h}} + \chi\big(J_t^r(\hat{W}_t) - \hat{J}_{t-}\big) + \xi(J_t - \hat{J}_{t-}) \\ + \quad & \max_{\hat{c}_{t-}, \hat{S}_{t-}} \left\{ \ln\left(\hat{c}_{t-}\right) + \frac{\mu_{t-}^{\hat{W}}}{\rho} + \lambda\bigg(\frac{1}{\rho}\ln\left(\hat{W}_{t-} - \left(\hat{W}_{t-}+\hat{S}_{t-}\right)\kappa_t^Q\right) + \hat{h}_{t-}\big(1-\kappa_t^{\hat{h}}\big) - \hat{J}_{t-}\bigg) \right\}. \end{aligned}$$

Similar to Proposition 3, we list the banker's first-order conditions

$$\begin{aligned} \hat{c}_t &= \rho \hat{W}_t \\ R_{t-} - r_{t-} - \tau_{t-} &= \frac{\lambda \kappa_t^Q}{1 - (1 + \hat{s}_{t-})\, \kappa_t^Q}. \end{aligned} \tag{17}$$

2.1.3 Shadow Bank Debt Market and Enforcement Problem

Shadow banks compete on two dimensions of their debt contract $(\bar{s}^*_{t-}, \tilde{r}_{t-})$ in the credit market. The no-outside-equity financing constraint for bankers (Assumption 1) simplifies our analysis because it implies that shadow bank debt issued in equilibrium must be risk-free and thus its interest rate $\tilde{r}_{t-}$ should equal the risk-free rate. No household would demand risky shadow bank debt because it is the combination of a risk-free debt component and an equity component. And, households refuse to hold equity issued by bankers (Assumption 1). The same idea implies that the maximum leverage of shadow banking $\bar{s}^*_{t-}$ should guarantee that banker would not default in the event of an adverse shock. Thus, the enforcement constraint (16) must hold in equilibrium.

To interpret the enforcement constraint more clearly, we focus on the case where $s_{t-} > 0$. In this case, we can verify that $s_{t-} + s^*_{t-} = \tilde{s}_t$ and $s^*_{t-} = \tilde{s}^*_{t-} = \bar{s}^*_{t-}$. Then, the enforcement constraint (16) reduces to

$$s^*_{t-} \le \frac{\rho\lambda\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big)}{R_{t-} - r_{t-} - \tau_{t-}}.$$

The above inequality has a clear message that both the increased opportunity cost of default $(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}}))$ and the decreased profitability of banking $(R_{t-} - r_{t-} - \tau_{t-})$ can alleviate the enforcement problem and raise the borrowing capacity of shadow banking.

We later will impose a simplification assumption to ensure the above simplification is general.[7] Thus, the maximum leverage of shadow banking in equilibrium satisfies

$$\bar{s}^*_{t-} = \frac{\rho\lambda\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big)}{R_{t-} - r_{t-} - \tau_{t-}} \tag{18}$$

To fully specify $\bar{s}^*_{t-}$, we need to know the difference between h_t and $\hat{h}_t$ denoted by H_t. The

[7] See Assumption 2′ and footnote 9.

following proposition characterizes H_t, whose interpretation is the opportunity cost for a banker to default on her shadow bank obligations in period t.

Proposition 4 *Given that $ln(\cdot)/\,\rho + h_t$ is the continuation value function of a banker who can access shadow banking and $ln(\cdot)/\,\rho + \hat{h}_t$ is the continuation value function of a banker who cannot use shadow banking due to prior default,*

$$H_t \equiv h_t - \hat{h}_t = E_t\left[\int_t^{\infty} \exp\left(-\left(\rho+\xi+\chi\right)\left(u-t\right)\right) f_u \; du\right], \tag{19}$$

where f_u equals

$$\frac{1}{\rho}\left(\begin{array}{c} \overbrace{\left(s_{u-}+s^*_{u-}-\hat{s}_{u-}\right)\left(R_{u-}-r_{u-}-\tau_{t-}\right)}^{\text{higher leverage benefit due to cheap credit}} + \overbrace{s^*_{u-}\tau_{u-}}^{\text{tax benefit}} \\ +\lambda\underbrace{\left(\ln\left(1-\left(1+s_{u-}+s^*_{u-}\right)\kappa^Q_u\right)-\ln\left(1-\left(1+\hat{s}_{u-}\right)\kappa^Q_u\right)\right)}_{\text{high risk due to high leverage}} \end{array}\right),$$

where s_t and s^*_t are the portfolio weights of the banker who can access shadow banking and $\hat{s}_t$ is the portfolio weight of the banker who cannot.

Proof. See Appendix A. ■

f_u is the tax benefit (i.e., regulatory arbitrage) that shadow banking offers, which is essentially the net worth growth rate difference between bankers who have the access to shadow banking and those who do not. The opportunity cost of default H_t is the present value of future tax benefits f_u that a banker will lose if she defaults in period t. The discount factor is the banker's time discount factor plus the "comeback" intensity ξ and the retirement rate χ. The interpretation is that once bankers return to the shadow banking sector or retire, the benefit of accessing shadow banking disappears.

We next highlight the feedback loop between the maximum leverage of shadow banking $\{\bar{s}^*_t\}_{t=0}^{\infty}$ and the cost of default $\{H_t\}_{t=0}^{\infty}$. First, the enforceability constraint (16) implies that the maximum leverage of shadow banking relies on a banker's cost of default to her shadow bank's obligations. Second, the probabilistic representation of the cost of default (19) indicates that the maximum leverage of shadow banking directly affects how costly default is for bankers.

This feedback loop gives rise to an equilibrium where shadow banking does not exist. Conjecture that $\{\bar{s}_t^* = 0\}_{t=0}^{\infty}$. The probabilistic representation (19) implies $\{H_t = 0\}_{t=0}^{\infty}$, and the enforceability constraint (16) justifies the conjecture. Therefore, the economy has an equilibrium in which shadow banking does not exist, as the following proposition summarizes.

Proposition 5 *There exists an equilibrium where shadow banking does not exist, that is,* $\{\bar{s}_t^* = 0, H_t = 0\}_{t=0}^{\infty}$.

We label this degenerate equilibrium the "bad" equilibrium since productive bankers are unable to leverage up via shadow banking. A non-degenerate equilibrium, where shadow banking emerges, may exist. We label it the "good" equilibrium. Equilibrium selection is beyond the scope of this paper. Given the importance of shadow banking in the real world, we assume that the "good" equilibrium prevails when both "good" and "bad" equilibria exist.

2.1.4 Miscellany

We group discussions of equilibrium conditions 4 and 5 in Definition 1 together, since all of them are straightforward. The budget of the regulatory authority is balanced if $\pi_t = s_t \tau_t$ for $t \geq 0$. Since households are risk-neutral, the market for consumption goods clears automatically. The market for physical capital clears if the fractions of physical capital held by bankers and households sum to 1. Let ψ_t denote the fraction of physical capital held by bankers, which equals $(1 + s_t + s_t^*)\omega_t$.

In our model, as in other continuous-time macro-finance papers, the wealth distribution matters for the dynamics of the economy. Later, we will capture the dynamics of an equilibrium with the bankers' wealth share $\omega_t \equiv \int_0^1 W_t^i di \Big/ q_t K_t$. Lemma 1 characterizes how ω_t evolves.

Lemma 1 *The law of motion of* ω_t *is*

$$
\begin{aligned}
d\omega_t &= \omega_{t-}\mu_{t-}^{\omega} dt - \omega_{t-}\kappa_t^{\omega} dN_t, && (20)\\
\textit{where } \mu_t^{\omega} &= R_t + s_t \left(R_t - r_t\right) + s_t^* \left(R_t - \tilde{r}_t\right) - \mu_t^q - \mu_t^K - \rho - \chi, && (21)\\
\textit{and } \kappa_t^{\omega} &= \frac{\left(s_{t-} + s_{t-}^*\right) \kappa_t^Q}{1 - \kappa_t^Q}. && (22)
\end{aligned}
$$

Proof. See Appendix A. ■

2.2 Markov Equilibrium

Our model has the property of scale-invariance with respect to K_t. This means that, for a given equilibrium in an economy with initial capital $\left\{k_0^i, k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$, there exists an equivalent equilibrium with the same laws of motion of ω_t, q_t, and H_t in any economy with initial capital $\left\{\varsigma k_0^i, \varsigma k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$, where $\varsigma \in (0, \infty)$.

Equations (3) – (19) can characterize an equilibrium specified by Definition 1. The scale-invariance property implies that we can characterize an equilibrium that is Markov in ω with a modification of Assumption 2.[8],[9]

Assumption 2′ *In period t, the tax rate τ_t equals $\min\{\tau, \tau s_t\}$, where τ is a positive constant and s_t denotes $\int_0^1 S_t^i \, di \Big/ \int_0^1 W_t^i \, di$; the tax rate is τ for bankers who cannot access shadow banking due to default.*

Note that the tax rate τ_t at any time t only depends on the aggregate variable. Thus, individual bankers take the tax rate as given.

In the Markov equilibrium, the dynamics of all endogenous aggregate variables can be fully described by the law of motion of the state variable and functions $q(\omega)$ and $H(\omega)$, which are defined over the domain $(0, \bar{\omega}]$. Thanks to Ito's Lemma, we derive the law of motion of $\{q_t, H_t\}_{t=0}^{\infty}$.

$$\mu_t^q = \frac{q'(\omega_t)}{q(\omega_t)} \omega_t \mu_t^\omega, \tag{23}$$

$$\kappa_t^q = \frac{q(\omega_{t-}) - q(\omega_{t-}(1 - \kappa_t^\omega))}{q(\omega_{t-})}, \tag{24}$$

$$\mu_t^H = \frac{H'(\omega_t)}{H(\omega_t)} \omega_t \mu_t^H, \tag{25}$$

$$\kappa_t^H = \frac{H(\omega_{t-}) - H(\omega_{t-}(1 - \kappa_t^\omega))}{H(\omega_{t-})}. \tag{26}$$

[8] A natural tax policy is that $\tau_t = \tau$ for $t \geq 0$. Given this policy, endogenous risk κ_t^q jumps as shadow banks become the marginal buyer of physical capital, which complicates the computation of an equilibrium. Our setup specified in Assumption 2′ makes the process that shadow banks become marginal buyers smooth and simplifies the computation.

[9] Given Assumption 2′, bankers would not have a conner solution ($s_{t-} = 0, s_{t-}^* > 0$). If this is true, then the tax rate τ would be zero due to Assumption 2′, which contracts the conjecture that ($s_{t-} = 0, s_{t-}^* > 0$) is the conner solution. Therefore, the simplification of the enforcement condition (16) considered in Section 2.1.3 is general under Assumption 2′.

The following proposition describes a system of delay differential equations and their boundary conditions, which define function $q(\omega)$ and $H(\omega)$.

Proposition 6 *$q(\omega)$ and $H(\omega)$ are defined over $(0, \bar{\omega}]$. Given $(\omega, q(\omega'), H(\omega'), 0 < \omega' \leq \omega)$, we compute $(q'(\omega), H'(\omega))$ using the following procedure:*

1. *Conjecture that $\psi < 1$, find $s + s^*$ such that*

$$\frac{a - a^h}{q} - \tau = \frac{\lambda \kappa^Q}{1 - (1 + s + s^*)\kappa^Q} - \lambda \kappa^Q,$$

   equations (22) and (24) hold. Derive $(\kappa^\omega, \kappa^q, \kappa^Q)$ according to Ito's Lemma and μ^q based on equation (14). Also compute ψ.

2. *If $\psi < 1$ does not hold, then $\psi = 1$ and $s + s^* = 1/\omega - 1$. Similarly, derive $(\kappa^\omega, \kappa^q, \kappa^Q, \kappa^H)$ according to Ito's Lemma and μ^q based on equation (14).*

3. *Given $s + s^* (= \tilde{s})$ and κ^Q, compute $\bar{s}^*$ such that equation (18) holds and then we derive s. Also, derive μ^ω according to equation (21).*

4. *Compute $q'(\omega)$ according to equation (23).*

5. *Finally, compute f based on (19) and then derive $H'(\omega)$ according to*

$$(\rho + \xi + \chi) H(\omega) = f + \omega \mu^\omega H'(\omega) + \lambda \left(H(\omega (1 - \kappa^\omega)) - H(\omega) \right). \tag{27}$$

Boundary conditions are

$$\mu^q(\bar{\omega}) = \mu^H(\bar{\omega}) = \mu^\omega(\bar{\omega}) = 0,$$

$$\lim_{\omega \to 0} q(\omega) = \underline{q} \text{ and } \lim_{\omega \to 0} H(\omega) = 0,$$

where $\underline{q}$ satisfies

$$a^h - \delta - \frac{\underline{q}^2 - 1}{2\phi} = \rho \underline{q}. \tag{28}$$

Proof. See Appendix A. ∎

2.2.1 Equilibrium Uniqueness

Within the class of Markov equilibria, we can identify the condition under which the "bad" equilibrium is unique. To prove this result, we define mapping Γ which takes the cost of default function $H(\omega)$ as the input,

$$\Gamma H(\omega) = E_t\left[\int_t^{\infty} \exp(-(\rho+\xi+\chi)(u-t)) f(\omega_u) du \middle| \omega_t = \omega\right]$$

where

$$f(\omega) = \frac{1}{\rho}\left(\begin{array}{c} (s+s^*)(R(\omega)-r-\tau(\omega)) + s^*\tau(\omega) - \hat{s}(R(\omega)-r-\tau) \\ +\lambda\left(\ln\left(1-(1+s(\omega)+s^*(\omega))\kappa^Q(\omega)\right) - \ln\left(1-(1+\hat{s}(\omega))\kappa^Q(\omega)\right)\right) \end{array}\right)$$

and

$$s^* \leq \frac{\rho\lambda H(\omega)}{R(\omega)-r-\tau(\omega)},$$

Where (s, s^*) are the portfolio weights of a banker who can access shadow banking given $\{q(\omega), \tau(\omega), \pi(\omega), \mu^{\omega}(\omega), \kappa^{\omega}(\omega)\}$ in an equilibrium and $\tilde{s}$ is the portfolio weight of a banker who cannot. Clearly, $H(\omega)$ in equilibrium is a fixed point of the mapping Γ. As we have noted, the mapping Γ allows for two possible fixed points: one leads to the "good" non-degenerate equilibrium, and the other yields the "bad" degenerate equilibrium. The following theorem provides a sufficient condition that justifies the uniqueness of the "bad" equilibrium.

Theorem 1 *If $\tau < (\rho+\xi+\chi)\kappa$, the mapping Γ is a contraction mapping with the fixed point $H(\omega) = 0$ for all $\omega \in (0, \bar{\omega}]$.*

Proof. See Appendix A. ■

To show that Γ is a contraction mapping, we demonstrate that Γ satisfies Blackwell's sufficient conditions if $\tau < (\rho+\xi+\chi)\kappa$.

The feedback loop illustrated earlier explains why Γ could be a contraction mapping. Suppose the comeback rate ξ increases permanently. Then, the opportunity cost of default drops and the maximum leverage of shadow banking $\bar{s}^*$ declines accordingly (the enforceability constraint (18)). The decline in the leverage of shadow banking $\bar{s}^*$ reduces the opportunity cost of default H again (the probabilistic representation (19)). This cycle can make shadow banking unsustainable in equilibrium.

2.3 Numerical Example

In this section, we present the main dynamic properties of the baseline model via numerical examples. Thanks to the global solution provided by the continuous-time approach, we are able to show the endogenous variables as functions of the state variable (bankers' wealth share ω) as well as the dynamics of the economy at any state.

We restrict the choice of parameter values by calibrating our model. Parameter values that we choose are $\rho = 3\%$, $\chi = 16\%$, $a = 22.5\%$, $a^h = 10\%$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, $\tau = 3\%$, and $\xi = 7.5\%$. Appendix B contains the detail of our calibration.

2.3.1 Capital Misallocation, Endogenous Risk, and Pecuniary Externality

The constraint on issuing outside equity (Assumption 1) leads to capital misallocation and generates endogenous risk through the balance sheet amplification mechanism (He and Krishnamurthy, 2012b; Brunnermeier and Sannikov, 2014). As bankers' wealth share diminishes, they hold a declining fraction of capital goods and aggregate productivity deteriorates. The price of physical capital declines accordingly (Panel b in Figure 2). However, the excess rate of return for holding capital rises due to the low cost of purchasing capital (Panel d in Figure 2).

A negative pecuniary externality exists in our model because bankers do not internalize how their leverage choice will affect endogenous risk in the competitive economy. Therefore, the socially optimal leverage choice does not coincide with bankers' privately optimal choice. The tax on regular banking can twist bank leverage, lower endogenous risk, and improve social welfare. Welfare issues are discussed in detail in Section 3.1.

2.3.2 The Dynamics of the State Variable.

The evolution of the state variable is described by Equation (20). Figure 3 shows that for most of the time bankers hold about 38% of the wealth in the economy. An economy is rarely in a

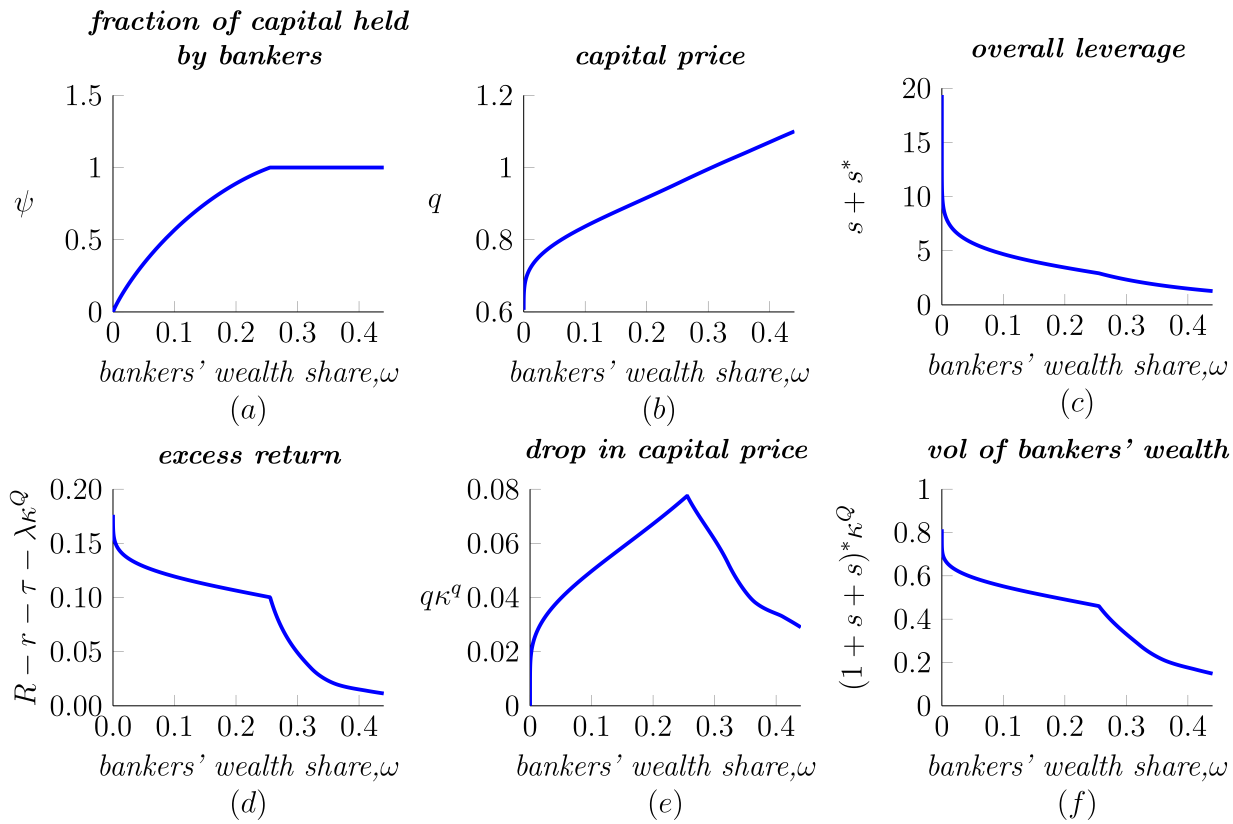

Figure 2: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For parameter values, see Section 2.3.

situation where bankers hold only a little wealth because the low price of physical capital and the high return from holding it help bankers to quickly build up their wealth and pull the economy out of recessions. Bankers' wealth share never exceeds the point $\bar{\omega}$, where $\mu^{\omega}(\bar{\omega}) = 0$ as bankers retire randomly.

2.4 The Feedback Loop in Shadow Banking

The feedback loop between the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^{\infty}$ and the cost of default $\{H_t\}_{t=0}^{\infty}$ is the driving force underpinning our main results: *i*) shadow banking is pro-cyclical; *ii*) shadow banking adds to financial instability through reintermediation; *iii*) financial regulation can raise or reduce financial instability under different circumstances. This section explains each of the three main results via dynamic and comparative statics analyses.

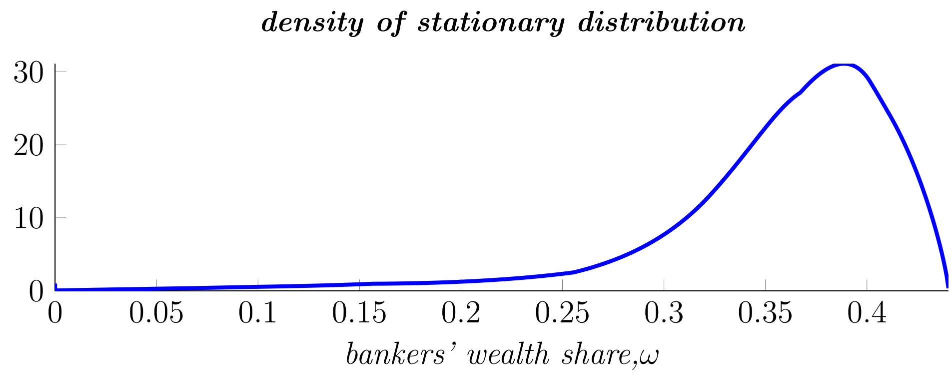

Figure 3: the density of stationary distribution as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For parameter values, see Section 2.3.

2.4.1 Dynamic Result: Pro-cyclical Leverage of Shadow Banking

We show that the leverage of shadow banking $\{s^*\}_{t=0}^{\infty}$ is pro-cyclical (Panel a in Figure 4). It is straightforward to see this from the expression for the maximum leverage of shadow banking (18). As the capital price increases in economic upturns (Panel b in Figure 4), the profitability of banking declines (Panel c in Figure 4) and the temptation to take high leverage declines. Therefore, the constraint that prevents bankers' opportunistic behaviours becomes less tight in economic upturns. Thus, the leverage of shadow banking rises in upturns. In addition, the rising leverage of shadow banking increases the opportunity cost of default for bankers, which alleviates the enforcement problem further. Hence, the feedback loop between $\{s_t^*\}_{t=0}^{\infty}$ and $\{H_t\}_{t=0}^{\infty}$ contributes to the substantial expansion of shadow banking.

2.4.2 Comparative Statics

We next compare different economies and see how the presence of shadow banking changes the conventional understanding of financial instability and its connection to financial regulation. First, we examine economies with and without shadow banking and show that shadow banking increases financial instability. Next, we vary parameter τ and explain the U-shaped relationship between financial instability and the regulation of the traditional banking sector.

Reintermediation. The combined effect of pro-cyclical shadow banking and reintermediation increases endogenous risk in the economy. In our model, the pro-cyclicality of shadow banking

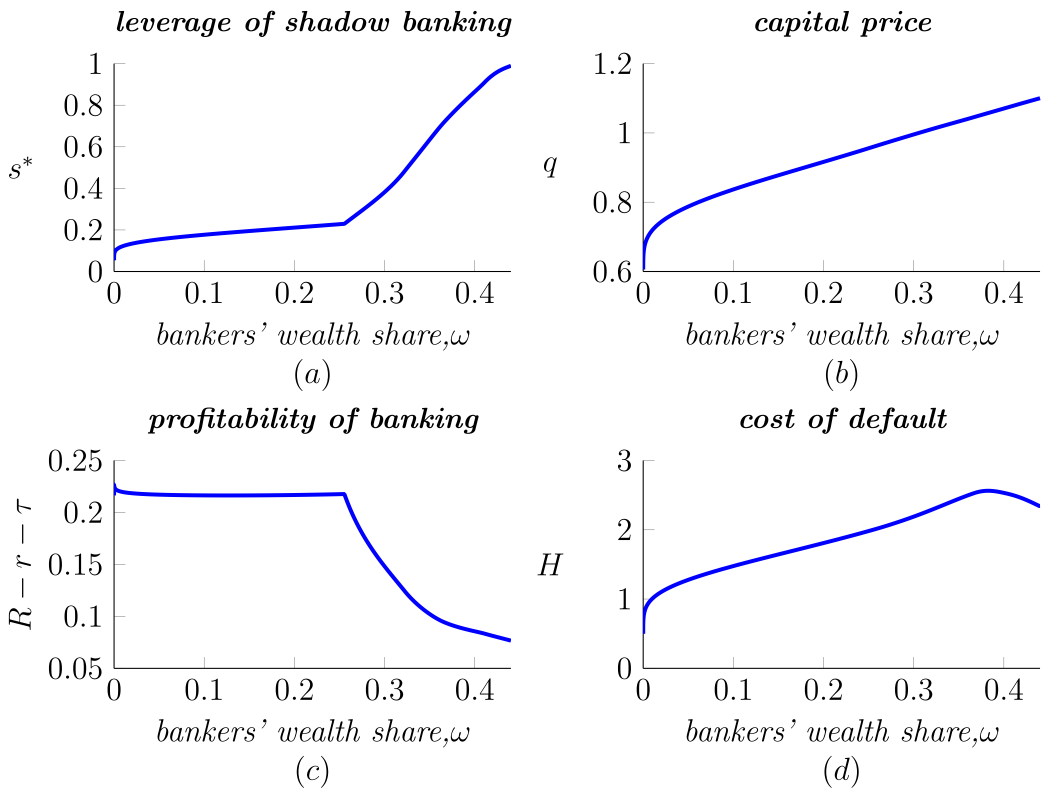

Figure 4: Pro-cyclical Shadow Banking
This figure presents the leverage of shadow banking s^*, capital price q, profitability of banking $R-r-\tau$, and the cost of default H as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For parameter values, see Section 2.3.

means that shadow banks purchase a large number of assets in economic upturns. The cost of funding the balance sheets of regular banks is expensive. Therefore, in economic booms, the scale of asset accumulation by shadow banks exceeds what regular banks would pursue in the absence of an accompanying shadow bank system. If an adverse shock hits the economy, shadow banks have to divest large amounts of assets as the leverage constraint tightens, and regular banks are reluctant to acquire these assets because it is so expensive to expand their balance sheets. As a result, the price of capital declines more than it would if there were no shadow banking (Panel b in Figure 5). Finally, the decline in the price of capital raises the profitability of banking as well as the incentive to take high leverage. Thus, the tightening enforcement constraint (18) leads to the further decline in the leverage of shadow banking. Overall, our calibrated model shows that shadow banking increases the price volatility of capital $q\kappa^q$ by 28% on average.

Shadow Banking: Innocent or Not? The answer is "Yes and No" in our model. The answer

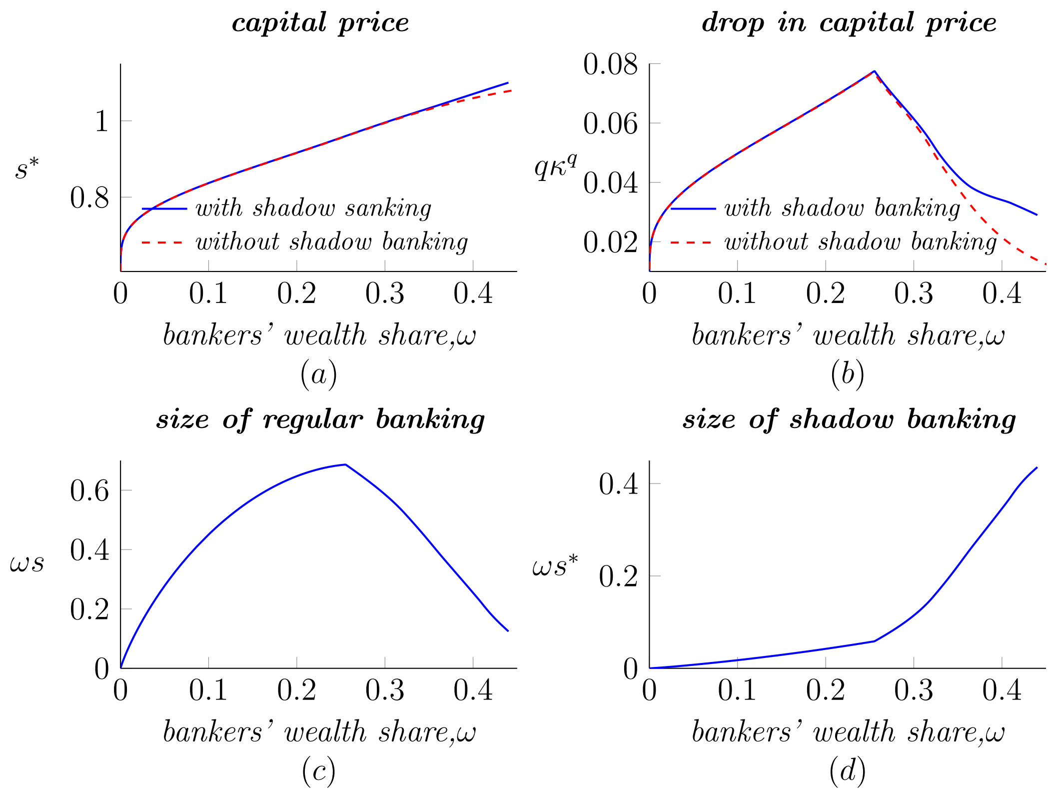

Figure 5: Reintermediation
This figure presents the price of capital (blue solid line in upper left panel), the jump in the price of capital (blue solid line in upper right panel), the size of the regular banking sector (lower left panel), and the size of the shadow banking sector (lower right panel) as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For comparison, this figure also shows the price of capital (red dashed line in upper left panel) and the jump in the price of capital (red dashed line in upper right panel) as functions of the state variable ω in the "bad" equilibrium. For parameter values, see Section 2.3

is "Yes" because all shadow banks hold the same type of physical capital as regular banks. The overall investment quality in the economy does not deteriorate because of shadow banking. Thus, the asset side of the shadow banking system is not responsible for increasing financial instability. Moreover, even if we move to the liability side, a single shadow bank that borrows up to the limit causes no harm, either. The answer is also "No" because when shadow banks expand in economic upturns, bankers fail to take into account negative pecuniary externalities of asset fire sales in economic downturns.

Regulatory Paradox. The conventional wisdom that tough regulation always secures financial stability may not hold when we take shadow banking into account. In economies without shadow

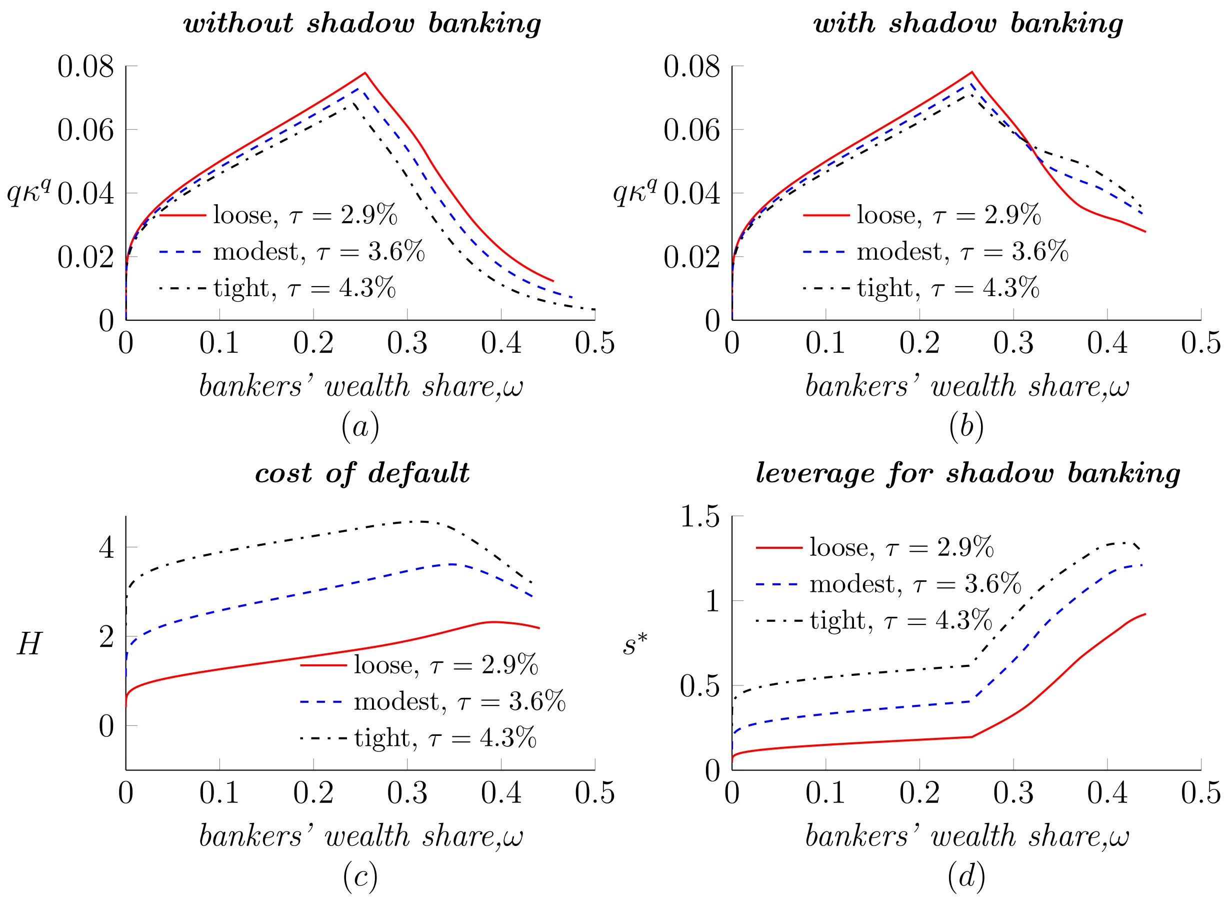

Figure 6: Regulatory Paradox
This figure shows the price of capital in the "bad" equilibrium (upper left panel), the price of capital in the "good" equilibrium (upper right panel), the cost of default (lower left panel), and the leverage of shadow banking (lower right panel). The red solid line is for the economy with loose financial regulation ($\tau = 2.9\%$); the blue dashed line for the economy with modest regulation ($\tau = 3.6\%$); the black dash-dot line for the economy with tight regulation ($\tau = 4.3\%$). For other parameter values, see Section 2.3

banking, if the regulatory authority tightens regulation by raising τ banks' leverage and the price volatility of capital decline accordingly (Panel a in Figure 6). However, in economies with shadow banking the economy with tighter regulation experiences higher market risk (Panel b in Figure 6). The intuition is that regular banks will face higher tax burdens if regulation becomes tighter. Thus, tighter regulation comes with the larger cost of default because regular banking becomes bankers' only option after default (Panel c in Figure 6). Furthermore, the larger cost of default leads to the higher leverage of shadow banking. Thus, the shadow banking sector is larger in economies with more stringent regulation. Since shadow banking adds to financial instability, tough regulation imposed on regular banks can jeopardize financial stability.

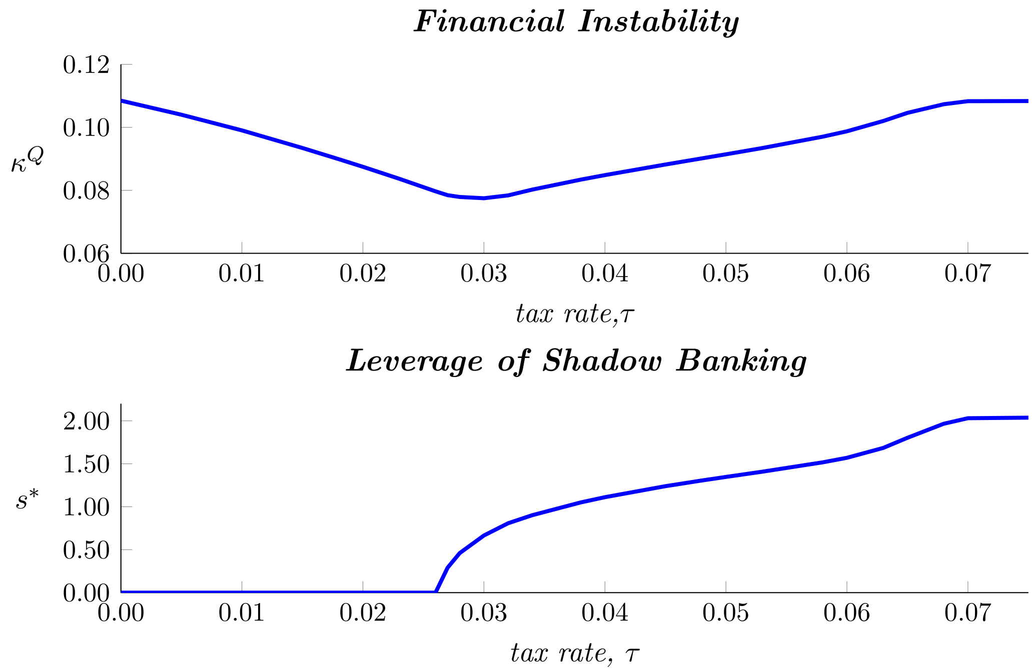

Figure 7: Regulatory Smile

This figure shows the investment risk κ^Q (upper panel) and the leverage of shadow banking (lower panel) at the stochastic steady state of economies with different tax rates τ. The stochastic steady state is the state where $\omega\mu^{\omega} - \lambda\omega\kappa^{\omega} = 0$. We assume that the "good" equilibrium prevails if it exists. For other parameter values, see Section 2.3

Regulatory Smile. The regulatory paradox does not mean that relaxing financial regulation always reduces financial instability. It depends on the relative size of the shadow banking system. Recall the feedback loop discussed earlier. If the regulatory authority lowers the tax rate τ, the benefit of shadow banking declines as well as the cost of default. This, in turn, lowers the maximum leverage of shadow banking and further reduces the cost of default. The feedback loop can be so significant that the shadow banking system becomes unsustainable. In our numerical example, when τ declines from 3% to 2.5% (lower panel in Figure 7), the shadow banking system disappears. In the regime where the level of financial regulation is lenient enough to eliminate the shadow banking system, the conventional wisdom that tightening regulation secures financial stability still holds.

Our model emphasizes the non-monotonic relationship between financial instability and financial regulation in the presence of shadow banking. The instability-regulation relationship is actually

U-shaped: when regulations are relaxed enough, the size of shadow banking is small and financial instability diminishes with decreased regulatory stringency; when regulation is so tight that a large shadow banking system emerges, the reverse is true.

3 Welfare and Policy Implications

In this section, we analyze the welfare and policy implications of our framework. First, we demonstrate the welfare improving property of financial regulation and highlight that a model without shadow banking can mislead welfare analysis. Second, we find that counter-cyclical regulation can generally improve financial stability because it alleviates the risk of the asset fire sales between shadow banks and regular banks. Third, we show that our regulatory smile result still holds when the price control analyzed in the baseline model is replaced by a quantity control (e.g., capital requirement constraint).

3.1 Welfare Implications of the Baseline Model

To avoid the problem of welfare aggregation, we reinterpret our model as one that consists of a representative banker and a representative household. Let K_0 denote the total capital stock in period 0. Note that the banker's wealth share ω_0 is exactly the fraction of capital goods that she owns. Thus, her net worth in period 0 is $\omega_0 K_0 q_0$, and the net worth of the representative household is $\big(1 - \omega_0\big)K_0 q_0$. Without loss of generality, we assume that $K_0 = 1$. Hence, the welfare pair of the representative household and banker is $\big((1 - \omega_0)q_0, \ln(\omega_0 q_0)/\,\rho + h_0\big)$. We will focus on the welfare of the representative banker due to our interest in the trade-off between economic growth and financial instability.[10] Our analysis will begin with the "bad" equilibrium and proceed to the "good" one.

Financial regulation can improve the banker's welfare in the "bad" equilibrium as the red dashed lines in Figure 15 show. The mechanism is that as the tax rate τ increases from zero the volatility of the banker's wealth declines (Panel b in Figure 9) and bankers' welfare improves. The

[10] Recall that households are risk-neutral.

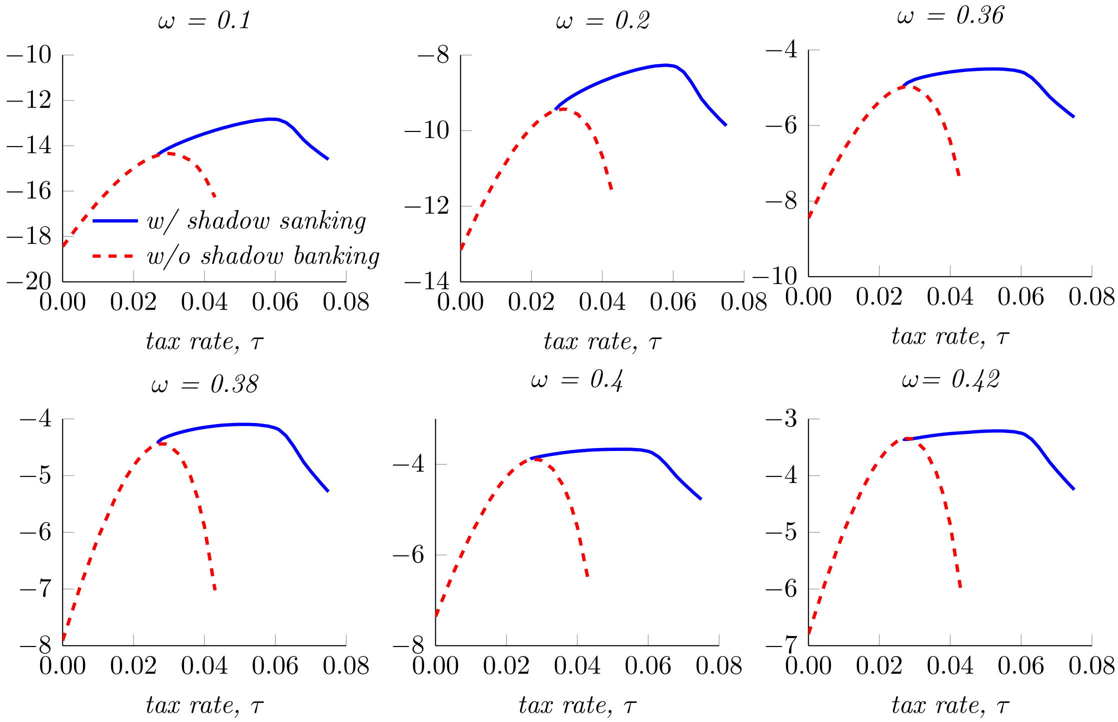

Figure 8: Welfare
This figure shows the level of the banker's continuation value $\ln(\omega q)/\rho + h$ in 6 different states ($\omega = 0.1$ in upper left panel, $\omega = 0.2$ in upper middle panel, $\omega = 0.36$ in the upper right panel, $\omega = 0.38$ in the lower left panel, $\omega = 0.4$ in the lower middle panel, and $\omega = 0.42$ in the lower right panel), given different tax rates. For parameter values other than τ, see Section 2.3.

unregulated competitive equilibrium ($\tau = 0$) is sub-optimal because the banker does not internalize the negative impact of her leverage choice on endogenous risk κ_t^q. This result essentially justifies the legitimacy of financial regulation in our framework.

We now move to the "good" equilibrium and discuss the optimal tax rate in light of shadow banking. In contrast to the "bad" equilibrium, the rise in tax rate τ increases both the growth and the volatility of the banker's wealth because *i*) the shadow banking sector expands as regulation tightens, and *ii*) the growth of shadow banking more than offsets the negative effect of regulation in terms of economic growth. The growth of shadow banking can benefit the banker's welfare because, as the shadow banking sector begins to grow, the growth benefit dominates the cost of increased risk. However, if regulation is too stringent the negative effect of tightening regulation will dominate the benefit of economic growth. Therefore, our baseline model indicates the optimal level of the tax rate with the impact of shadow banking taken into account.

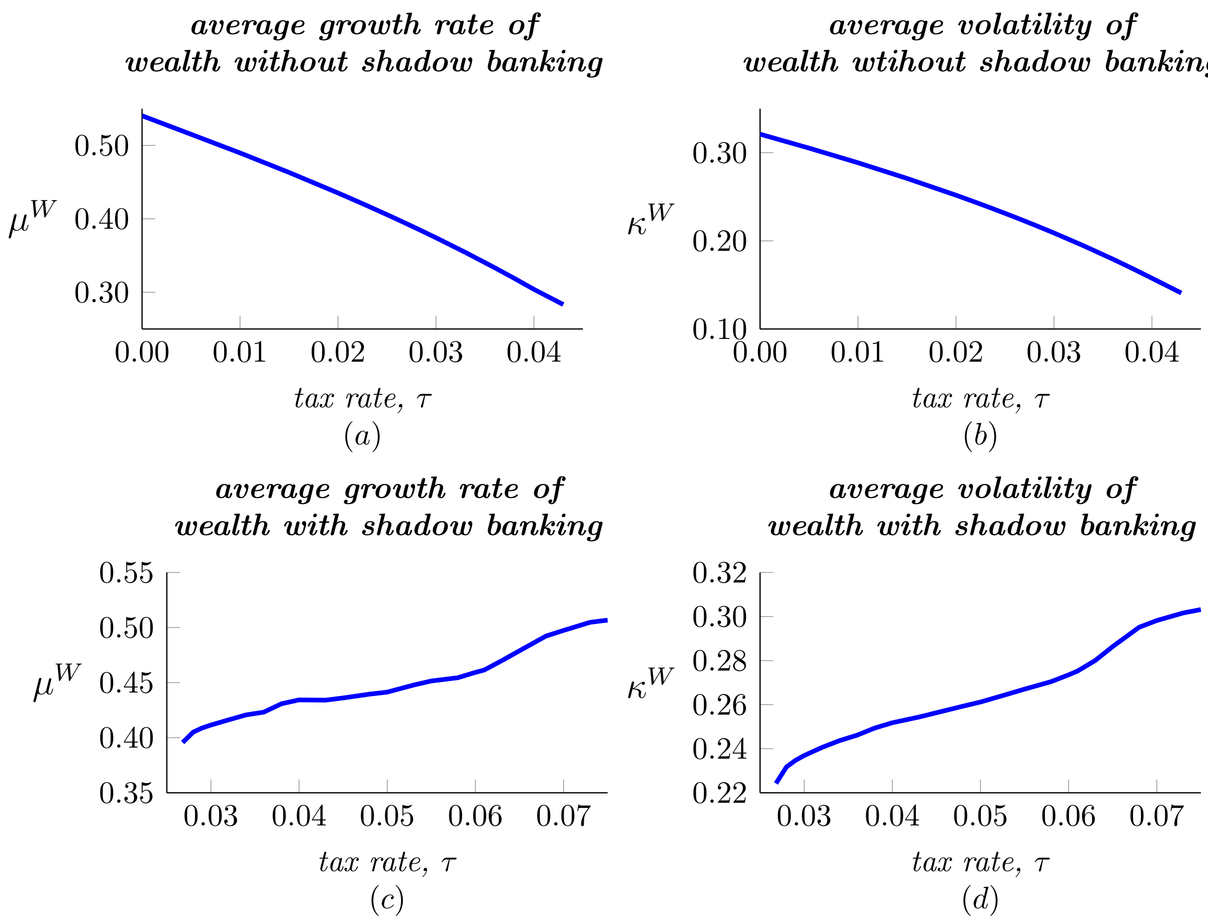

Figure 9: Welfare
This figure shows how the average growth rate (left panels) and the average volatility (right panels) of the representative banker's wealth change as the tax rate changes in economies without shadow banking (upper panels) and with shadow banking (lower panels). We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 2.3.

Figure 15 shows that the optimal tax rate in an economy with shadow banking differs from that in an economy without shadow banking. Although extremely tight regulation hurts the banker's welfare in both cases, the underlying channels are different. In the economy without shadow banking, tightening regulation slows down economic growth (Panel a in Figure 9). However, in the economy with shadow banking raising tax rate τ increases financial instability (Panel d in Figure 9). Therefore, a framework that ignores shadow banking will completely misguide policy-making if the actual economy has a fast-growing shadow banking sector.

The policy suggestion of our framework is straightforward. Within the class of financial regulation that our baseline model considers, the regulatory authority could improve the welfare of

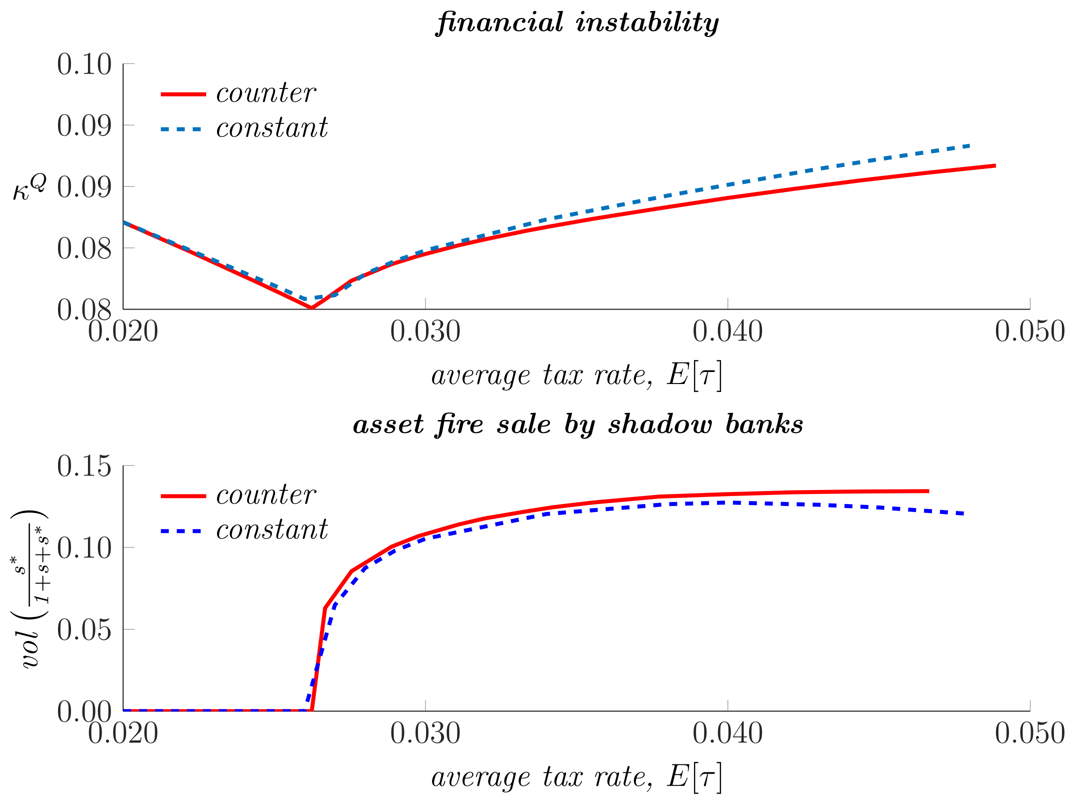

Figure 10: Financial Instability and Counter-Cyclical Regulation
This figure shows how the average volatility (red solid line, upper panel) and the volatility of the securitization ratio (red solid line, lower panel) change with the average tax rate in the "good" equilibrium of an economy with a counter-cyclical tax rate policy. All moments are based on the stationary distribution. For comparison, upper and lower panels show how the average volatility and the volatility of the securitization ratio changes, respectively, as the tax rate moves in the same economy with a constant tax rate. For parameter values, see both Section 2.3.

the banker by increasing the tax rate, i.e., the shadow cost of financial regulation, from 3%.[11] The optimal level of the shadow cost depends on the ratio of the banking sector's capitalization to the total wealth in the economy.

3.2 Counter-Cyclical Regulation

In this section, we substitute the constant tax rate regulation (Assumption 2′) with a counter-cyclical regulation specified by the following assumption.

Assumption 2″ *In period t, the tax rate τ_t equals* $\min\{\tau(\omega_t), \tau(\omega_t)s_t\}$. $\tau(\omega)$ *is defined by* $\left(\underline{\tau}\left(\tilde{\omega}-\omega\right)+\bar{\tau}\omega\right)/\tilde{\omega}$,

[11]The policy advice will be the same for this calibrated case if we take into account the household's welfare and give them an equal weight. See Figure 15 in Appendix B.

where $\underline{\tau}$, $\bar{\tau}$, and $\tilde{\omega}$ are constants, $\underline{\tau} < \bar{\tau}$, and $\tilde{\omega}$ is larger than $\bar{\omega}$.

The interpretation of Assumption $2''$ is that the regulatory authority alleviates the tax burden on the regular banking sector in recessions and discourage the bankers' use of leverage in economic booms. We use the same algorithm to solve for the equilibrium of the modified model.

The "regulatory paradox" result still holds in the model with the counter-cyclical policy, although the financial market is more stable. The upper panel of Figure 10 shows that counter-cyclical regulation can enhance financial stability when the average tax is high. This is true despite of the fact that the volume of reintermediation is larger in the economy with counter-cyclical regulation (bottom panel in Figure 10). This is because regular banks face declining borrowing costs when they need to raise funds to acquire assets dumped by shadow banks in economic downturns. Therefore, counter-cyclical regulation can mitigate the magnitude of asset fire-sales and lower the financial instability of an economy.

3.3 Quantity Control

In this section, we investigate a modified model, in which the regular banking sector is subject to a quantity control instead of the price control in the baseline model. In particular, we consider the capital-requirement constraint that commercial banks in the real world often face, and this constraint imposes an upper bound $\bar{s}$ for a regular bank's liability-to-equity ratio.

With the price control replaced by the quantity control, a banker's dynamic budget constraint becomes

$$dW_t = \left(W_{t-}R_{t-} + \left(S_{t-} + S^*_{t-}\right)\left(R_{t-} - r_{t-}\right) - c_{t-}\right)dt - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa_t^Q dN_t.$$

In addition to the leverage constraint for shadow banking (3), the banker in the modified model faces the capital-requirement constraint $S_t \leq \bar{s}W_t$. Similar changes apply to bankers who cannot access shadow banking due to default. We use the same numerical procedure to solve for the Markov equilibria of the modified model.

We first focus on the dynamics of endogenous variables and then move to the regulatory smile result of the quantity-control model. A number of endogenous variables have dynamics similar to

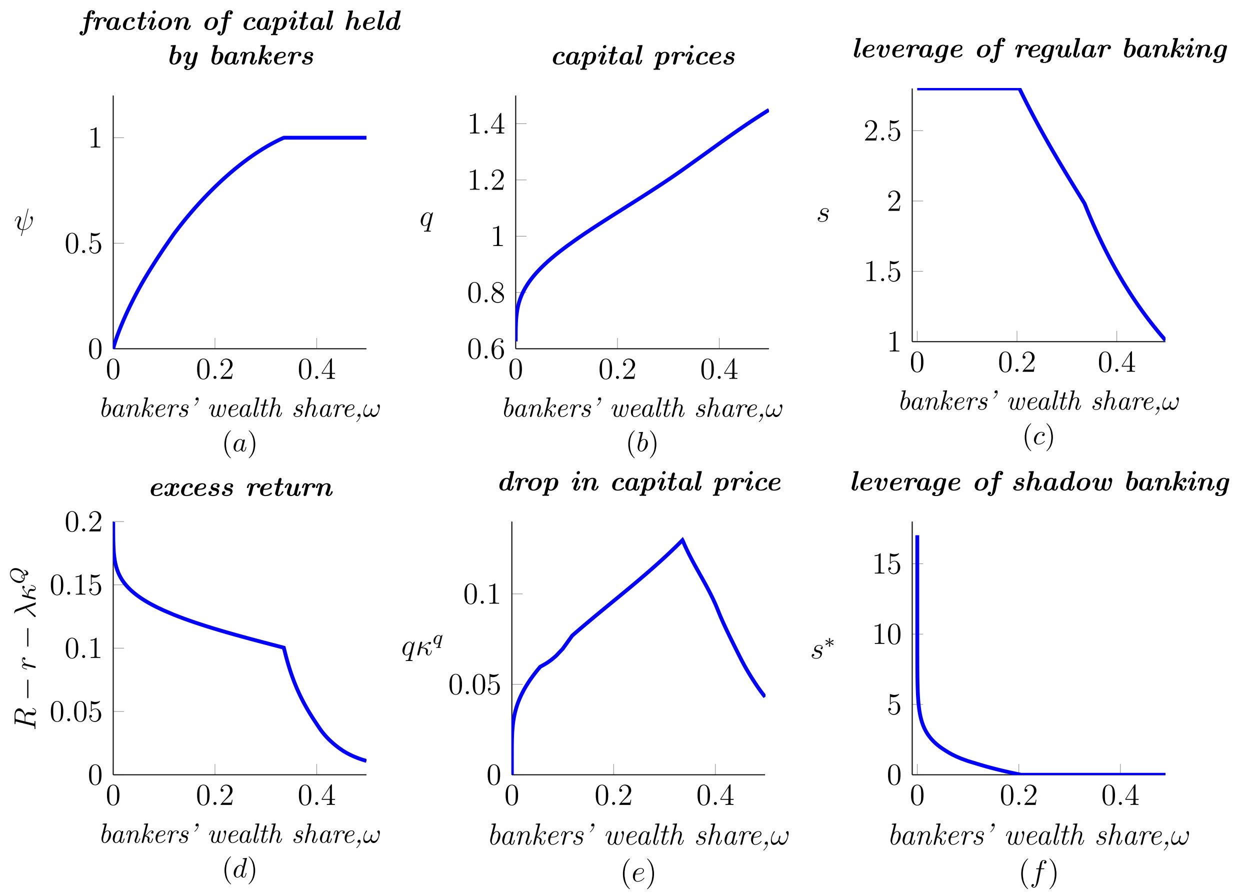

Figure 11: $\psi, q, (s+s^*), R-r-\tau, q\kappa^q$, and $(1+s+s^*)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in "good" equilibrium of the modified model with the capital requirement constraint. The choice of parameter values follows: $\rho = 3\%$, $\chi = 1\%$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, and $\bar{s} = 2.8$.

the baseline model (Panels a-e in Figure 11). However, the leverage dynamics of shadow banking change drastically. This is the consequence of the fact that when bankers' share of wealth is small the excess return is high and the incentives to build up leverage are strong. In these states, it is extremely costly to default to shadow bank debt because regular banking only allows for considerably low leverage. Therefore, when bankers' share of wealth is small, the enforcement problem is not severe and the leverage of shadow banking is high. This property is absent in the baseline model because bankers do not face a binding leverage constraint for regular banking.

Figure 12 shows that the regulatory smile result continues to hold in the modified model with quantity control. Very lenient financial regulation comes with the low leverage of shadow banking and high financial instability. As financial regulation tightens (i.e., the maximum leverage of

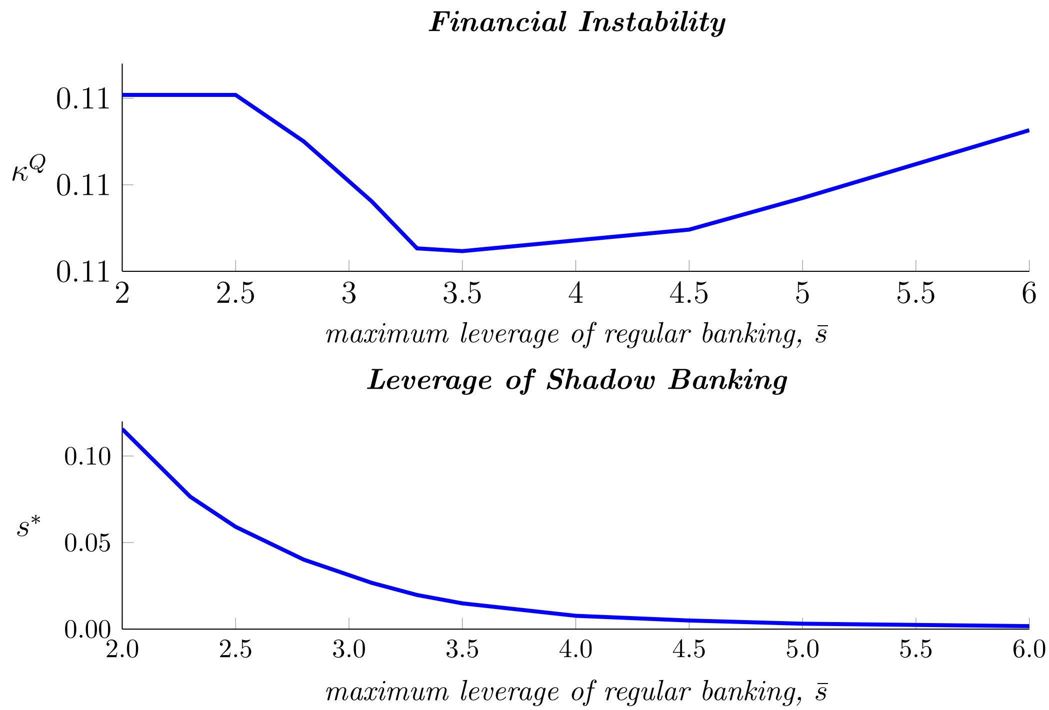

Figure 12: This figure shows the investment risk κ^Q (upper panel) and the leverage for shadow banking (lower panel) at the stochastic steady state of different economies with different capital-requirement constraints in the "good" equilibrium of the modified model. The stochastic steady state is the state where $\omega\mu^\omega - \lambda\omega\kappa^\omega = 0$. The maximum leverage of regular banking $\bar{s}$ is 2.8. For the choice of other parameter values, see Section 3.3.

regular banking $\bar{s}$ declines), financial instability initially diminishes. However, if the regulation is so tight that the shadow banking sector becomes sizeable, tighter regulation causes higher financial instability.

4 Robustness

To identify what specification is critical for our main results, we vary the setup of the baseline model in two dimensions: the opportunity cost of default and the preference of households. We find that it is crucial to have the cost of default depend on financial regulation in the baseline model and that households' preferences are not essential for our main results.

4.1 Exogenous Leverage Constraint for Shadow Banking

For "regulatory paradox" result to obtain, the maximum leverage of shadow banking must depend on financial regulation. To demonstrate this, we characterize a variant of the baseline model where the endogenous cost of default $\{H_t\}_{t=0}^{\infty}$ is replaced by a constant $\bar{H}$.

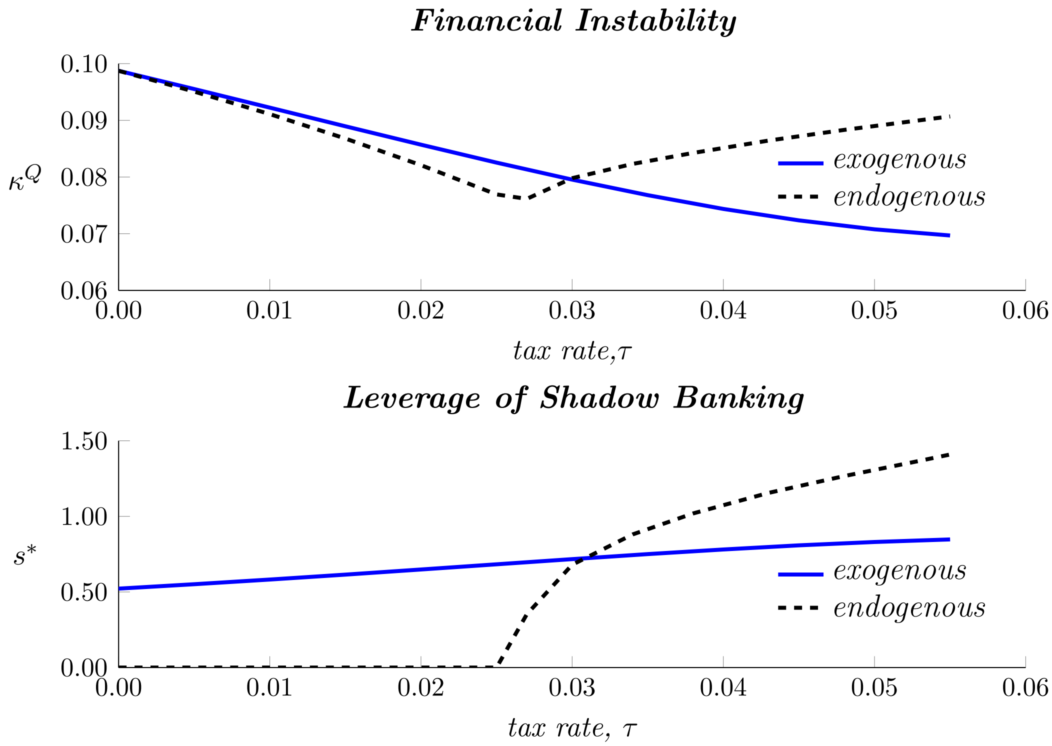

Figure 13: Financial Instability and Financial Regulation.
This figure shows how the average volatility (the blue solid line in the upper panel) and the average leverage for shadow banking (the blue solid line in the lower panel) changes with the tax rate in economies where bankers face exogenous leverage constraint for shadow banking. For comparison, two panels also display their counterparts in the baseline model (the black dashed line). The exogenous cost of default H equals 2.3973, which is the average cost of default in the calibrated model in Section 2.3. For other parameter values, see Section 2.3.

Figure 16 in Appendix B shows that most basic results of our baseline model are preserved in the modified model. However, the "regulatory paradox" result does not hold (upper panel of Figure 13). The primary reason is that the size of the shadow banking sector does not change much as the tightness of financial regulation varies (lower panel of Figure 13). When the regulatory authority raises the tax on regular banking, not many banking activities migrate to the shadow banking sector. As a result, the magnitude of the reintermediation does not change as significantly as it does in the baseline model. Therefore, financial instability does not change much as the tax rate varies.

4.2 Households with Epstein-Zin Preference

In this section, we demonstrate that our main results are robust to the preference specification of households. In particular, we modify the baseline model so that households have Epstein-Zin preferences with the time discount rate ρ, the relative risk-aversion coefficient γ, and the elasticity of intertemporal substitution b^{-1}. To simplify the characterization of the model, we modify Assumption 5 such that bankers become households at rate χ.

In the modified model, a household chooses $\{c_t^h, S_t^h, n_t, t \geq 0\}$ to maximize

$$U_0^h = E_0 \left[\int_0^\infty f\left(c_s^h, U_s^h\right) ds \right],$$

where

$$f\left(c^h, U^h\right) = \frac{1}{1-b} \left\{ \frac{\rho \left(c^h\right)^{1-b}}{\left(\left(1-\gamma\right) U^h\right)^{\frac{\gamma-b}{1-\gamma}}} - \rho \left(1-\gamma\right) U^h \right\}$$

and

$$U_t^h = E_t \left[\int_t^\infty f\left(c_s^h, U_s^h\right) ds \right], \text{ for } t > 0,$$

subject to the dynamic budget constraint

$$dW_t^h = \left(W_{t-}^h r_{t-} + S_{t-}^h \left(R_{t-}^h - r_{t-} \right) + n_{t-} \left(\tilde{r}_{t-} - r_{t-} \right) - c_{t-}^h \right) dt - S_{t-}^h \kappa_t^Q dN_t.$$

In the interest of space, we skip the standard analysis of households' optimal choices and directly emphasize two market-clearing conditions that differ from their counterparts in the baseline model. First, the market for consumption goods does not clear automatically in the modified model since households are risk-averse. Second, the risk-free rate is jointly determined by the portfolio choices of both bankers and households and the dynamics of households' continuation value.

As in our previous analyses, we focus on the Markov equilibrium of the modified model where shadow banking exists. Results found in the baseline model survive in the modified model. Figure

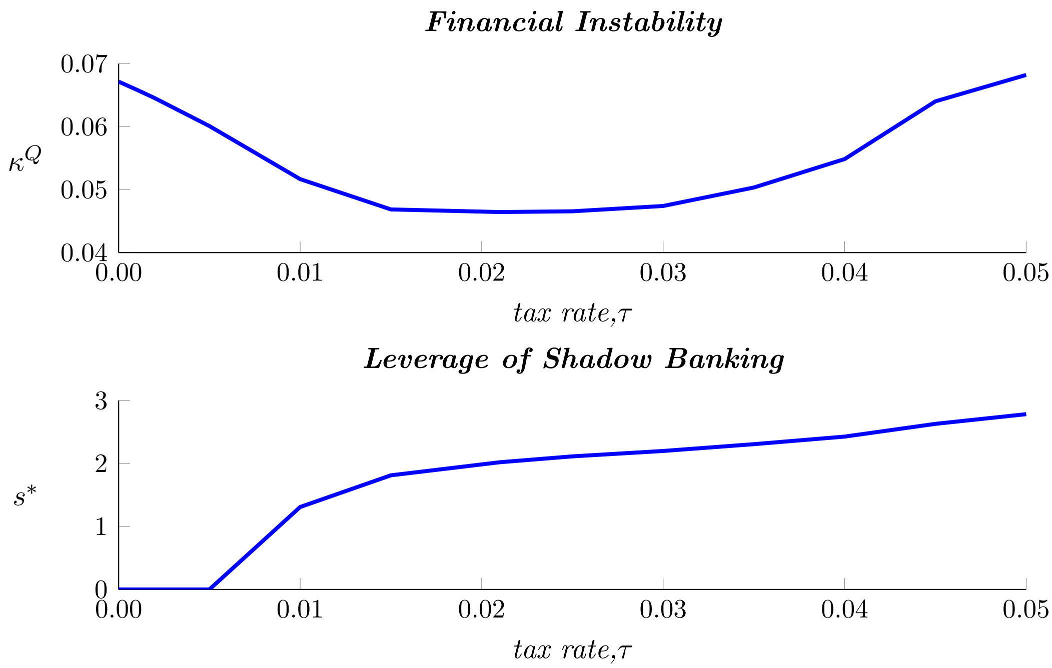

Figure 14: This figure shows the investment risk κ^Q (upper panel) and the leverage for shadow banking (lower panel) at the stochastic steady state of different economies with different tax rates τ in the modified model with households of Epstein-Zin preference. The stochastic steady state is the state where $\omega\mu^{\omega} - \lambda\omega\kappa^{\omega} = 0$. The choice of parameter values is that $\rho = 4\%$, $\gamma = 2$, $b = 0.5$, $\chi = 0.1$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, $\tau = 3\%$, and $\xi = 5\%$.

17 in Appendix B shows that the main endogenous variables have dynamic properties similar to those in the baseline model. The regulatory smile result holds in the modified model, as Figure 14 displays.

5 Final Remarks

This paper provides a framework for evaluating financial regulatory rules in the modern financial environment where the unregulated shadow banking sector plays a critical role. Our framework explicitly takes into account the unintended and indirect influence of bank regulation on the shadow banking sector. Thus, our paper provides a more comprehensive framework for policy evaluation.

The framework proposed in this paper could be extended in the following three directions. The first and most straightforward follow-up work is to characterize the social planner's constrained efficient regulatory rule by, say, a process of tax rate $\{\tau_t, t \geq 0\}$. However, this exercise requires a completely new methodology that can characterize the set of *all* competitive equilibria under *all* sorts of regulations. Recall that our paper focuses only on Markov equilibria with a single state variable.

Second, one could investigate the collapse of the shadow banking system by exploring the stability property of the "good" equilibrium and endogenizing the regime switch between the "good" equilibrium and the "bad" one. By combining certain quantitative work, one can develop a framework that provides early warning signs of a financial crisis.

Third, how creditors respond to the default of a shadow bank could also be endogenized as an equilibrium outcome. One can calibrate this extension by feeding it with the deep parameters found in credit markets.

Appendix

A Proofs

Proof of Proposition 2. Now suppose in period t, the banker's net worth W_t is negative. The law of motion for the banker's net worth is

$$dW_t = \left(W_{t-}R_{t-} + S_{t-}\left(R_{t-} - r_{t-}\right) + S^*_{t-}\left(R_{t-} - r_{t-}\right) - c_{t-}\right)dt - \left(W_{t-}\kappa^Q_t + S_{t-}\kappa^Q_t + S^*_{t-}\kappa^Q_t\right)dN_t$$

Given a fixed time T, we can construct a new measure under which

$$R_{s-} - r_{s-} = \tilde{\lambda}_{t-}\kappa^Q_s$$

for each time s between t and T. Under this new measure,

$$\tilde{E}_t\left[W_T \exp\left(-\int_t^T r_u du\right) + \int_t^T \left(c_s \exp\left(-\int_t^s r_u du\right)\right) ds\right] \leq W_t < 0$$

Suppose the banker retires at the stopping time S with positive net worth W_S. After the banker retires, her net worth evolves as

$$dW_{S+u} = W_{S+u} \left(r_{S+u} - \rho \right) du.$$

Her net worth in period $S+s$ is $W_S \exp\left(\int_S^{S+s} r_u du - \rho s \right)$. It is easy to see that

$$\lim_{s \to \infty} E_S \left[\exp\left(- \int_S^{S+s} r_u du \right) W_{S+s} \right] = 0.$$

Thus, if T is large enough, $E_t \left[W_T \exp\left(- \int_t^T r_u du \right) \right]$ could be arbitrarily small. Since $W_t < 0$, the consumption of the banker must be negative at some point between t and T with a strictly positive probability. Since the banker has logarithm preference, the banker's expected lifetime discounted utility in period t must be negative infinity. Therefore, we show that it is never optimal for the banker to have negative net worth and that a banker's overall leverage must have an upper bound.

■

Proof of Proposition 4. Without loss of generality, we focus a banker with net worth W_t in period t and explicitly express her continuation value in different cases.

We start with the case that the banker is retired. Since logarithmic agents only consumer ρ fraction of their net worth, the growth rate of her net worth is $r_{t+v} - \rho$ in period $t+v$. Hence, the banker's net worth in period $t+u$ will be $W_t \exp\left(\int_0^u r_{t+v} - \rho \, dv \right)$. The banker's continuation value in period t is

$$\begin{aligned}
& \int_0^\infty \exp\left(-\rho u\right) \left(\ln\left(\rho W_t\right) + \int_0^u r_{t+v} - \rho \; dv \right) du \\
= \; & \frac{\ln\left(W_t\right)}{\rho} + \frac{\ln\left(\rho\right)}{\rho} + \int_0^\infty \exp\left(-\rho u\right) \int_0^u r_{t+v} - \rho \; dv \; du \\
= \; & \frac{\ln\left(W_t\right)}{\rho} + \frac{\ln\left(\rho\right)}{\rho} + \frac{1}{\rho} \int_0^\infty \exp(-\rho v) \left(r_{t+v} - \rho \right) \; dv,
\end{aligned}$$

which is denoted by $\ln\left(W_t\right) / \rho + h_t^r$.

We use the same idea to express the continuation value of a banker who can access shadow banking. Given the banker's optimal portfolio choices $\left(s_{t+u}, s_{t+u}^* \right)$, if she does not retire in period

$t+u$, her net worth is

$$W_t \exp\left(\begin{array}{c} \int_0^u \left(R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho\right) dv \\ + \int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v} \end{array}\right)$$

Let $t+T$ denote the stopping time that the banker retires. Her continuation value in period t is

$$\begin{aligned} & E_t\left[\begin{array}{c} \int_0^T \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho\ dv\right) du \\ + \int_0^T \exp\left(-\rho u\right)\int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v}\ du + \exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho} + h^r_{t+T}\right). \end{array}\right] \\ = & \frac{\ln\left(W_t\right)}{\rho} + \frac{\rho\ln\left(\rho\right)}{\rho+\chi} \\ & + E_t\left[\int_0^\infty \exp\left(-\left(\rho+\chi\right)v\right)\left(\begin{array}{c} R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho \\ +\lambda\ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) \end{array}\right) dv\right] \\ & + E_t\left[\int_0^\infty \chi\exp\left(-\left(\rho+\chi\right)v\right) h^r_{t+v}\ dv\right], \end{aligned}$$

which we denote as $\ln\left(W_t\right)/\rho + h_t$.

Finally, we consider the case that the banker who cannot use shadow banking but obtain such opportunity at intensity ξ. Let $\hat{s}_{t+u}$ denote her optimal portfolio choices and T_ξ the stopping when the banker obtain the access to shadow banking. Her continuation value in period t is

$$E_t\left[\begin{array}{c} \int_0^{\min\left(T,T_\xi\right)} \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u R_{t+v} + \hat{s}_{t+v}\left(R_{t+v} - r_{t+v} - \tau_{t+v}\right) + s_{t+v}\tau_{t+v} - \rho\ dv\right) du \\ + \int_0^{\min\left(T,T_\xi\right)} \exp\left(-\rho u\right)\int_0^u \ln\left(1 - \left(1 + \hat{s}_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v}\ du \\ + \exp\left(-T_\xi\rho\right)\left(\frac{\ln\left(W_{t+T_\xi}\right)}{\rho} + h_t\right)\mathbf{1}_{\left(\min\left(T_\xi,T\right)=T_\xi\right)} + \exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho} + h^r_{t+T}\right)\mathbf{1}_{\left(\min\left(T_\xi,T\right)=T\right)} \end{array}\right]$$

$$\begin{aligned} = & E_t\left[\begin{array}{c} \int_0^T \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho\ dv\right) du \\ + \int_0^T \exp\left(-\rho u\right)\int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v}\ du \exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho} + h^r_{t+T}\right) \end{array}\right] - \\ & E_t\left[\int_0^{\min\left(T,T_\xi\right)} \exp(-\rho u)\int_0^u \left(\begin{array}{c} \left(\left(s_{t+v} + s^*_{t+v} - \hat{s}_{t+v}\right)\left(R_{t+v} - r_{t+v} - \tau_{t+v}\right) + s^*_{t+v}\tau_{t+v}\right) dv \\ + \left(\ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) - \ln\left(1 - \left(1 + \hat{s}_{t+v}\right)\kappa^Q_{t+v}\right)\right) dN_{t+v} \end{array}\right) du\right] \\ = & \ln\left(W_t\right)/\rho + h_t - H_t, \end{aligned}$$

where

$$H_t = E_t \left(\int_0^{\infty} \exp\left(-\left(\rho+\chi+\xi\right)v\right) \left(\begin{array}{c} \left(s_{t+v}+s^*_{t+v}-\hat{s}_{t+v}\right)\left(R_{t+v}-r_{t+v}-\tau_{t+v}\right)+s^*_{t+v}\tau_{t+v} \\ +\lambda\left(\ln\left(1-\left(1+s_{t+v}+s^*_{t+v}\right)\kappa^Q_{t+v}\right)-\ln\left(1-\left(1+\hat{s}_{t+v}\right)\kappa^Q_{t+v}\right)\right) \end{array} \right) dv \right]$$

■

Proof of Lemma 1. W_t denotes $\int_0^1 W_t^i di$. In a Markov equilibrium, bankers' dynamic budget constraint, the optimal choice of bankers, and the market-clearing for notes, and the balanced budget of the regulatory authority imply that

$$\begin{aligned} dW_t &= W_{t-}\left(\left(R_{t-}+s_{t-}\left(R_{t-}-r_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)-\rho-\chi\right)dt-\left(s_{t-}+s^*_{t-}\right)\kappa^Q_t dN_t\right) \\ &= W_{t-}\left(\left(R_{t-}+s_{t-}\left(R_{t-}-r_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)-\rho-\chi\right)dt-\left(s_{t-}+s^*_{t-}\right)\kappa^Q_t dN_t\right). \end{aligned}$$

Note bankers retire at the intensity χ. Next, consider the scaling factor $1/\left(q_t K_t\right)$.

$$d\left(q_t K_t\right) = q_{t-}K_{t-}\left(\left(\mu^q_{t-}+\mu^K_{t-}\right)dt-\kappa^Q_t dN_t\right),$$

and

$$d\left(\frac{1}{q_t K_t}\right) = \left(\frac{1}{q_{t-}K_{t-}}\right)\left(-\left(\mu^q_{t-}+\mu^K_{t-}\right)dt+\frac{\kappa^Q_t}{1-\kappa^Q_t}dN_t\right).$$

Then,

$$\begin{aligned} d\omega_t &= \omega_{t-}\left(\mu^\omega_{t-}dt-\kappa^\omega_t dN_t\right) \\ \text{where } \mu^\omega_t &= R_{t-}+s_{t-}\left(R_{t-}-r_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)-\mu^q_{t-}-\mu^K_{t-}-\rho-\chi \\ \text{and } \kappa^\omega_t &= \frac{\left(s_{t-}+s^*_{t-}\right)\kappa^Q_t}{1-\kappa^Q_t}. \end{aligned}$$

■

Proof of Proposition 6. To justify the algorithm proposed by Proposition 6, we need to show that if the leverage constraint for shadow banking is satisfied bankers' HJB equation can reduce to

$$0 = \mathbf{HJB}_{\mathcal{N}}$$

without considering the incentive-compatible constraint. First, we know that the optimal choice of $\mathbf{HJB}_{\mathcal{D}}$ is dominated by that of $\mathbf{HJB}_{\mathcal{N}}$ by the definition of the maximum leverage of shadow banking. Thus, what we need to show next is that if the portfolio choice (s_{t-}, s^*_{t-}) satisfies the leverage constraint for shadow banking it automatically meets the incentive-compatible constraint. Now, suppose (s_{t-}, s^*_{t-}) is such that

$$s^*_{t-} \leq \frac{\rho\lambda\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big)}{R_{t-} - r_{t-} - \tau_{t-}}.$$

Thus,

$$s^*_{t-} \leq \rho\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big)\frac{1-(1+s_{t-}+s^*_{t-})\kappa_t^Q}{\kappa_t^Q},$$

which comes from the first-order condition with respect to (s_{t-}, s^*_{t-}). Since $x > \ln(1+x)$ for $x > 0$,

$$\ln\left(1 + \frac{s^*_{t-}\kappa_t^Q}{1-(1+s_{t-}+s^*_{t-})\kappa_t^Q}\right) < \frac{s^*_{t-}\kappa_t^Q}{1-(1+s_{t-}+s^*_{t-})\kappa_t^Q} \leq \rho\big(h_{t-}(1-\kappa_t^h) - \hat{h}_{t-}(1-\kappa_t^{\hat{h}})\big).$$

Hence, we show that the incentive-compatible constraint is satisfied. ■

Proof of Theorem 1. We will apply the contraction mapping theorem to show the uniqueness of the solution $H(\omega) = 0$. First, we define a complete metric space. Since the state variable ω is between 0 and $\bar{\omega}$, we focus on the space $B((0,\bar{\omega}])$ of bounded continuous functions $h : (0,\bar{\omega}] \to R$ under sup norm. Theorem 3.1 in Stokey et al. (1989) implies that $B((0,\bar{\omega}])$ is a complete metric space.

We will use Blackwell's sufficient conditions to show Γ is a contraction mapping. Suppose both h, $\tilde{h} \in B((0,\bar{\omega}])$ and $h(\omega) \geq \tilde{h}(\omega)$, for all $\omega \in (0,\bar{\omega}]$, since

$$\bar{s}^* = \frac{\rho\lambda H}{R - r - \tau}$$

all portfolio choices permitted under $\tilde{h}(\omega)$ are feasible under $h(\omega)$. Hence, $\Gamma h \geq \Gamma\tilde{h}$, for all $\omega \in (0,\bar{\omega}]$. Next, we need to show that there exists a positive constant $\beta < 1$ such that $\Gamma(h+v) \leq$

$\Gamma h+\beta v$, for all $h\in B\left((0,\bar{\omega}]\right)$, $v\geq 0$, $\omega\in(0,\bar{\omega}]$. Consider

$$\Gamma\left(h+v\right)\left[\omega\right]=E_0\left[\int_0^{\infty}\exp\left(-\left(\rho+\xi+\chi\right)u\right)f\left(\omega_u\right)du\middle|\omega_0=\omega\right],$$

where

$$f\left(\omega\right)=\frac{1}{\rho}\left(\begin{array}{c}\left(s+s^*\right)\left(R\left(\omega\right)-r-\tau(\omega)\right)+s^*\tau\left(\omega\right)-\hat{s}\left(R\left(\omega\right)-r-\tau\right)\\+\lambda\left(\ln\left(1-\left(1+s+s^*\right)\kappa^Q\left(\omega\right)\right)-\ln\left(1-\left(1+\hat{s}\right)\kappa^Q\left(\omega\right)\right)\right)\end{array}\right)$$

and

$$s^*\leq\frac{\rho\lambda H}{R(\omega)-r-\tau(\omega)},$$

where (s,s^*) is the optimal portfolio choice of a banker who has the access to shadow banking given $\{q(\omega),\tau(\omega),\pi(\omega),\mu^{\omega}(\omega),\kappa^{\omega}(\omega)\}$ and $\tilde{s}$ is the portfolio choice of a banker who does not have. Since the lower bound of h is zero, then

$$\begin{aligned}\bar{s}^* &= \frac{\rho\lambda(h+v)}{R-r-\tau}=\frac{\rho\lambda h}{R-r-\tau}+\frac{\rho\lambda v}{R-r-\tau}\\ &= \frac{\rho\lambda h}{R-r-\tau}+\frac{\rho v(1-(1+\tilde{s})\kappa^Q)}{\kappa^Q}\\ &= \frac{\rho\lambda h}{R-r-\tau}+\rho v\left(\frac{1}{\kappa^Q}-(1+\tilde{s})\right)\\ &\leq \frac{\rho\lambda h}{R-r-\tau}+\frac{\rho v}{\kappa}\end{aligned}$$

With the assistance of above inequality, we derive that

$$\begin{aligned}\Gamma\left(h+v\right)\left[\omega\right] &\leq \Gamma h+E_0\left[\int_0^{\infty}\exp\left(-\left(\rho+\xi+\chi\right)u\right)\frac{v}{\kappa}\tau du\middle|\omega_0=\omega\right]\\ &\leq \Gamma h+\frac{\tau}{\left(\rho+\xi+\chi\right)\kappa}v.\end{aligned}$$

If $\tau<\left(\rho+\xi+\chi\right)\kappa$, Γ is a contraction mapping. ∎

B Calibration, Data, Tables, and Figures

We set the time discount factor ρ to 3% to match the real interest rate estimated by Campbell and Cochrane (1999). Bankers' retirement rate χ is set at 16% to target the average Sharpe ratio. We set bankers' productivity at 22.5% so that the average investment-to-capital ratio is close to 11% (He and Krishnamurthy, 2012a). The productivity of less-productive households is chosen at 10% to match the fact that the Sharpe ratio during the 2007-09 financial crisis was approximately 15 times the average level (He and Krishnamurthy, 2012a). Choices of the depreciation rate and the capital adjustment cost ϕ are standard in the macroeconomic literature (Christiano, Eichenbaum and Evans, 2005). We set the Poisson shock parameters to target the conditional volatility of the growth rate of bankers' wealth in distressed periods and in non-distressed periods. The distressed periods are defined as periods with the lowest 33% Sharpe ratios. The regulation parameter, tax rate τ, is set at 3% to target the average leverage of the entire banking sector. We set the intensity with which bankers can re-access shadow banking after default at 6% to target the ratio of securitization by non-agency issuers in the third quarter of 2006.

Table 1: Moments[1]

Moment	Model	Target	Source
average Sharpe ratio	33.7%	40%	Wachter (2013)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	15.3	15	He and Krishnamurthy (2012a)
average $\frac{\text{investment}}{\text{capital}}$ ratio	11.2%	11%	He and Krishnamurthy (2012a)
average ratio of securitization	25.5%	25.1%	ratio of securitization in third quarter of 2006
bankers' overall leverage	2.9	3	He and Krishnamurthy (2012a)
volatility of bankers' wealth growth rate			
in distress periods[2]	35.1%	31.5%	He and Krishnamurthy (2012a)
in non-distress periods	18.9%	17.5%	He and Krishnamurthy (2012a)

[1] We use the density of the stationary distribution to calculate all moments.
[2] The distress periods are those with highest 33% Sharpe ratio.

Securitization. We follow Loutskina (2011) to compute the ratio of securitization. The difference is that we focus on securitization done by non-agency security issuers. All data are drawn from the "Flow of Funds Accounts of the United States". There are 5 loan categories. The details of items for each category are listed in Table 2.

Table 2: Details of Securitization Data

	Outstanding	Securitized
Home Mortgages	FL383165105	FL673065105
Multifamily Residential Mortgages	FL143165405	FL673065405
Commercial Mortgages	FL383165505	FL673065505
Commercial and Industrial Loans[1]	FL253169255 FL263168005 FL263169255	FL673069505
Consumer Credit	FL153166000	FL673066000

[1] Becuase item FL253169255 is not available now, we use the ratio of securitization calculated by Loutskina (2011) to estimate the outstanding commercial and industrial loans.

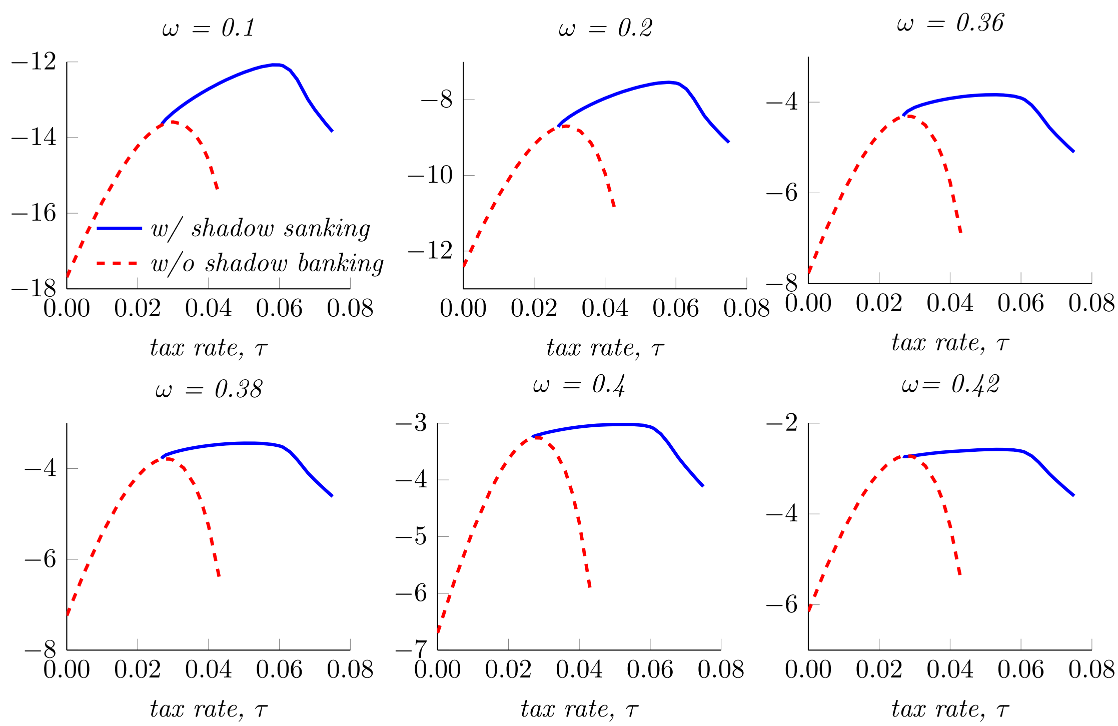

Figure 15: Welfare
This figure shows the equal-weighted social welfare in 6 different states ($\omega = 0.1$ in upper left panel, $\omega = 0.2$ in upper middle panel, $\omega = 0.36$ in the upper right panel, $\omega = 0.38$ in the lower left panel, $\omega = 0.4$ in the lower middle panel, and $\omega = 0.42$ in the lower right panel), given different tax rates. For parameter values other than τ, see Section 2.3.

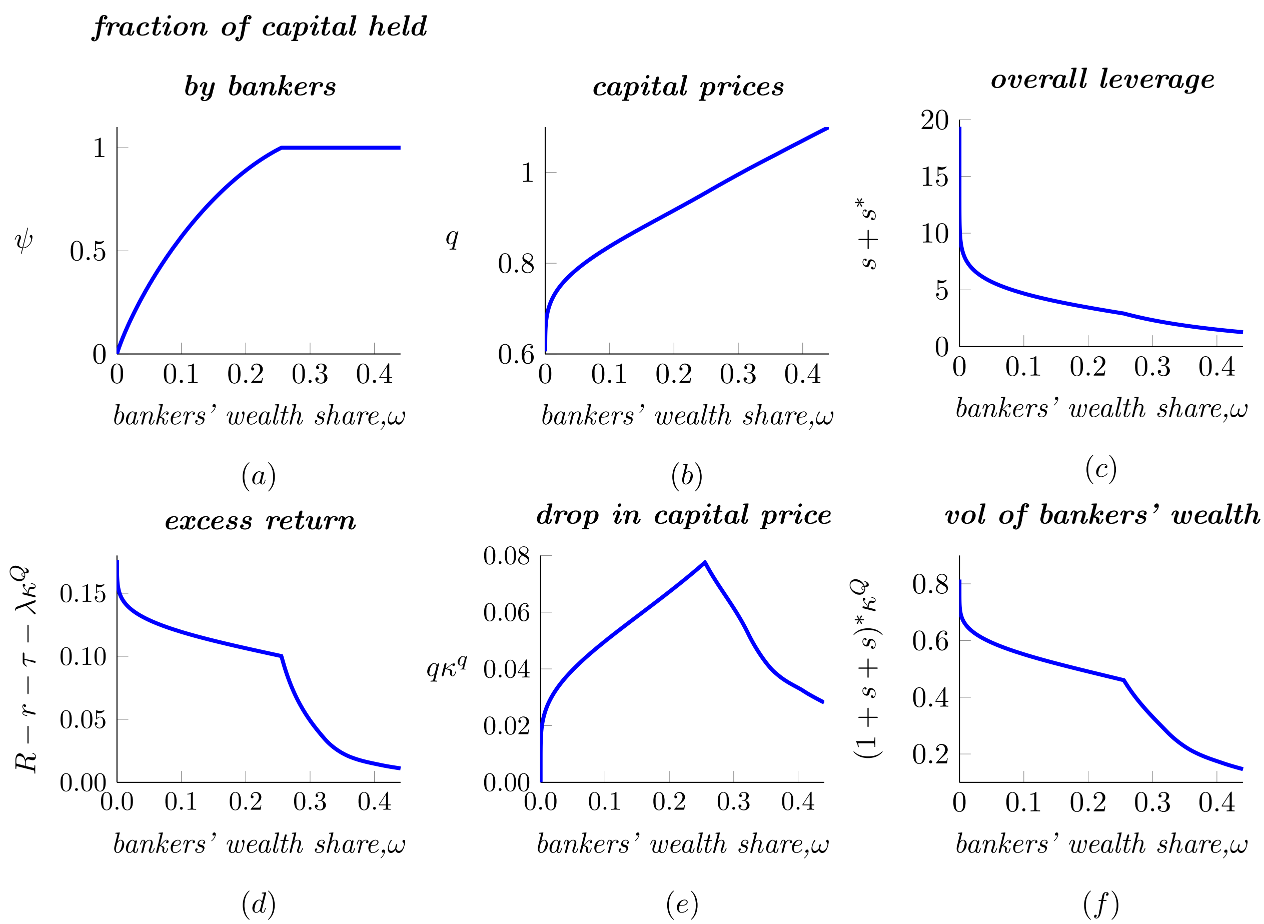

Figure 16: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω in the modified model with a constant cost of default $\bar{H}=2.3973$, which equals the average cost of default in the calibrated model in Section 2.3. For other parameter values see the same section.

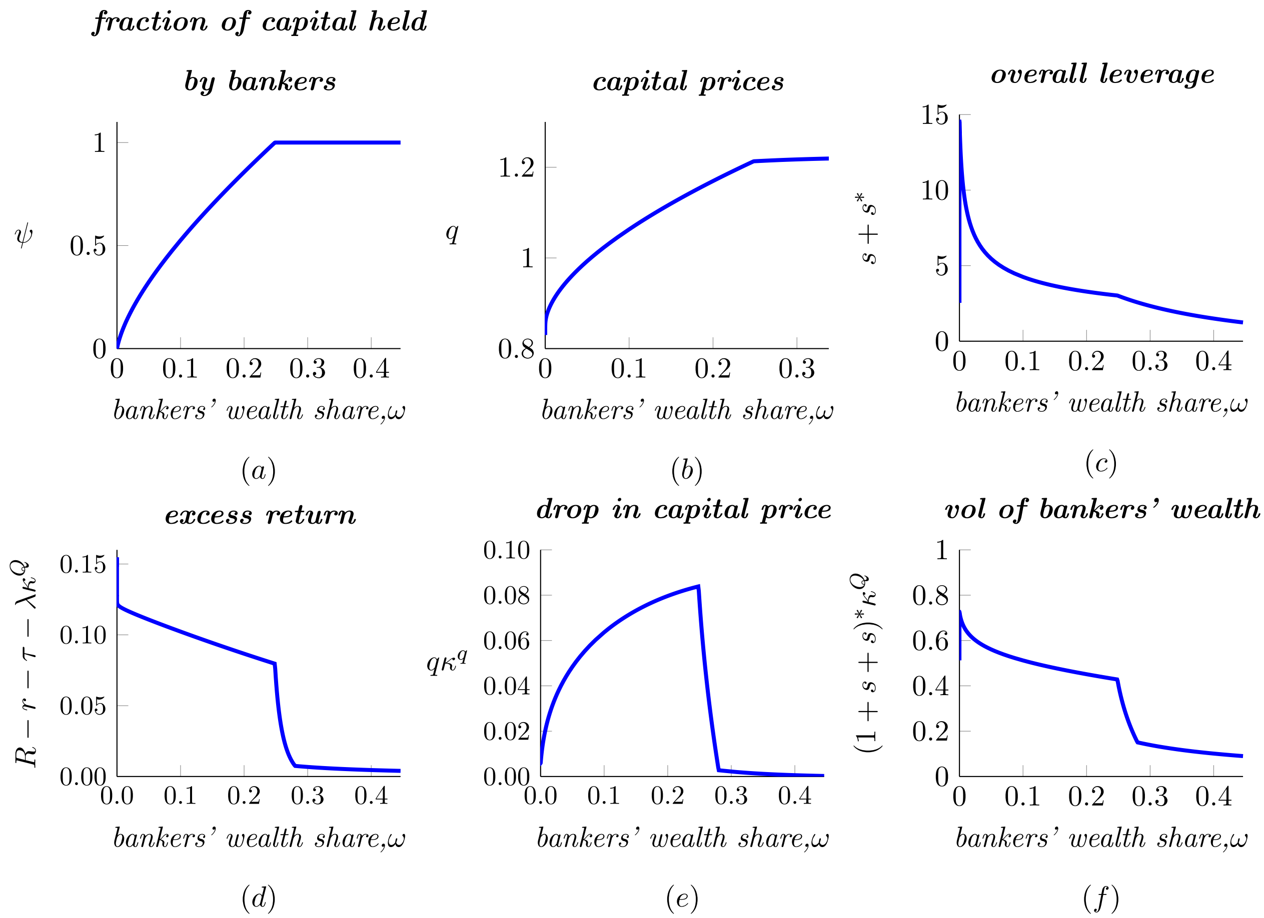

Figure 17: $\psi, q, (s+s^*)\,, R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in the "good" equilibrium of the modified model in which households have Epstein-Zin preferences. For the choice of parameter values, see Section 4.2.

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