Transparency of Outside Options in Bargaining*

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Abstract

This paper studies the effects of the transparency of an outside option in bilateral bargaining. A seller posts prices to screen a buyer over time, and the buyer may receive an outside option at a random time. We consider two information regimes, one in which the arrival of the outside option is public and one in which the arrival is private. The public arrival of the outside option works as a commitment device that forces the buyer to opt out immediately. The Coase conjecture holds in the unique equilibrium. In contrast, private information about the outside option leads to additional delay and multiplicity. The Coase conjecture fails in some equilibria. The buyer's preference about transparency is time-inconsistent: Ex ante, she prefers public arrivals, but ex post she prefers not to disclose her outside option if it is private.

Keywords: Bargaining, Arriving Outside Option, Dynamic Games, Coase Conjecture, Buyer's Commitment, Transparency of Outside Options, Disclosure

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1 Introduction

This paper studies the effects of the transparency of a trader's outside option on the dynamics of negotiation. More precisely, we consider a negotiation between a seller (he) and a buyer (she), and investigate how making the buyer's outside option public affects the dynamics and outcomes of a negotiation. We show that the buyer is better off when the arrival of her outside option is public than when it is private. Nevertheless, the buyer has no incentive to disclose her private outside option ex post, even if she is allowed to do so.

To understand the result, consider the following example. Suppose that a seller is selling a house to a buyer. The buyer informs the seller that she has a much better outside offer for a house in a different area, and she asks the seller to lower the price. What does the seller think? Naively, one may guess that the buyer can improve her bargaining power by disclosing the outside option; and thus the seller should lower the price to response to the competition. However, after more thought, the seller asks himself: "If the outside offer is so attractive, why is she still bargaining with me? She must like my house very much. If so, why should I lower the price?" So we see that when the seller is sufficiently sophisticated, making the buyer's outside option public may not necessarily enhance her bargaining power. The buyer's disclosure behavior effectively exposes her willingness to pay a high price and thus eliminates her information rent and undermines her bargaining power. In this paper, we construct a model to confirm this intuition.

Consider a bargaining game between a seller and a buyer over an indivisible good. The buyer privately knows her type which is her valuation of the good. For simplicity, we focus on the binary case where her type can only be high or low. In each period, the seller announces a price and the buyer decides whether to buy the good. If the buyer rejects the offer, an outside option arrives with positive probability. Once the outside option arrives, the buyer can exercise it in any period. We study two cases of the model, one in which the arrival of the outside option is public and one in which it is private information to the buyer.

In the model with the public outside option, there is a unique equilibrium which exhibits *Coasian dynamics*: On the equilibrium path, a rejection makes the seller more pessimistic about the buyer having a high valuation; and thus he lowers the price in the next period. In equilibrium, the buyer exercises her outside option immediately upon its arrival regardless of her type. The low-type buyer strictly prefers to opt out because the seller would not give her any share of the surplus in the future and waiting is costly. It follows that the high-type buyer also opts out, as otherwise she would be the lowest-type buyer remaining. Because both buyer types exit the game at the same rate, the model is essentially similar to the standard Coasian bargaining model except that the seller may lose the buyer each time the trade is delayed. The equilibrium satisfies a generalized Coase conjecture: When the seller can make offers arbitrarily frequently, the initial price becomes arbitrarily close to the low-type buyer's reservation price, and the trade occurs almost immediately.

In the model with the private outside option, the strategic interaction is richer. We first consider a "relaxed problem" in which the arrival of the outside option can not be disclosed. In this model, there may exist multiple equilibria, some of which do not exhibit the Coasian dynamics. The unraveling result no longer holds with a private outside option because the option to hide the outside option changes the buyer's incentives. The high-type buyer has a stronger incentive to trade, so she is less likely to exercise the outside option than the low-type buyer. After observing that the buyer has not left the negotiation, the seller believes that the buyer's type is more likely to be high. Put differently, the seller has an extra source of information to help assess the buyer's type: Beliefs are updated based not only on the buyer's previous rejections but also on the fact that the buyer is still negotiating. Coasian dynamics can be supported in an equilibrium only if the arrival probability of the outside option is low enough; when the arrival probability is high enough, there exists an equilibrium which does not exhibit Coasian dynamics. In this equilibrium, the aforementioned two sources of information exactly offset each other in every period so that the seller's belief does not change over time. Consequently, the seller offers the same randomized price in each period, leading to inefficient delay.

Next, we allow the buyer to disclose her outside option and we show that she has no incentive to do so. The key intuition behind this time-inconsistency is that the disclosure of the outside option may also work as the disclosure of the buyer's type. The low-type buyer exercises the outside option as soon as it arrives because she cannot obtain any information rent. Hence, if a buyer discloses her outside option, the seller thinks she is a high-valuation buyer. Since disclosure eliminates her information rent, the high-type buyer prefers to mimic a low-type buyer who has not yet received an outside option.

After analyzing the equilibria, we compare the buyer's ex ante welfare in the two cases to study the role of the transparency of the outside option. It is well known that a discrete-time sequential bargaining game is difficult to analyze, so we compare the buyer's ex ante welfare when the seller's commitment power becomes arbitrarily small. At the limit, we find that the buyer is weakly worse off with private outside options than with public outside options. In the public outside option case, the generalized Coase conjecture implies that the buyer takes all the bargaining surplus. With private outside options the model has multiple equilibria, and in the non-Coasian equilibrium the buyer's welfare is reduced due to a non-trivial delay of trade. As a result, making the buyer's outside option private creates strategic uncertainty, so it may prolong the screening process of the seller and thus lead to an inefficient real-time delay of the trade.

Interestingly, the buyer's preference regarding the transparency of the outside option is time-inconsistent: Although the buyer's surplus is maximized in the public case, she prefers not to disclose her privately known outside option if she is allowed to do so. One can interpret this time-inconsistency as commitment device. The public outside option case, the unraveling result implies that both types exercise the outside option upon its arrival. In the public outside option works as an endogenous commitment device, which ensures that the high-type buyer exercises the outside option. Consequently, the seller has an incentive to speed up the screening to avoid losing the customer; and thus the trade occurs at a low price after a short period of negotiation. On the contrary, when the arrival is confidential, the high-type buyer may have an incentive to hide the outside option, which slows down the seller's screening and induces inefficient delay. Because delay is costly, the buyer's ex ante payoff is undermined.

Literature Review

Our paper relates to a growing body of literature on the transparency of offers in dynamic trading, which starts with Swinkels (1999) and includes Hörner and Vieille (2009), Kim (2014), Fuchs *et al.* (2013), and Liu and Kaya (2014). This literature focuses on the effects of information about previous offers on future transactions and prices. In this paper, we study the effects of information regarding the arrival of the outside option, which can be interpreted as offers made currently or in the future, on the bargaining behavior before the option actually arrives.

In a recent paper, Board and Pycia (2014) consider a similar model where the buyer receives the outside option at the beginning of the game for certain. They show that there is a unique equilibrium in which the seller posts a constant price in every period and the Coase conjecture fails. Our model shows that when the outside option arrives stochastically, the Coase conjecture always holds in an equilibrium.

Fuchs and Skrzypacz (2010) consider sequential bargaining with random breakdown.[1] In their model, the game ends following the arrival of the public breakdown by assumption. In our model, the public arrival of the outside option plays a similar role to the breakdown in Fuchs and Skrzypacz (2010). Indeed, we show that the bargaining endogenously ends upon the arrival of a public outside option, which qualifies Fuchs and Skrzypacz (2010)'s exogenous ending assumption. In addition, we show that, if the outside option arrives privately, the game may not end upon its arrival.

In a complementary paper, Hwang (2013) considers a model with a private outside option and only analyzes in more detail an equilibrium that does not not exhibit Coasian dynamics. In this paper, we apply the results of Hwang (2013) in the private outside option model to identify the conditions under which there exists equilibria not exhibiting Coasian dynamics, and we also derive a condition under which there exists an equilibrium exhibiting Coasian dynamics. In addition, we allow the buyer to reveal her outside option while Hwang (2013) does not. Finally, by comparing the two environments, we are able to discuss the role of the transparency of the buyer's outside option.

Transparency of offers is also considered in the industrial organization literature. Krasteva and

[1] Also see Faingold *et al.* (2012), who consider the model when the arrival of the random event is uncertain. Inderst (2008) considers a sequential bargaining model with the arrival of new buyers who serves as the seller's outside option.

Yildirim (2012) consider a multilateral bargaining model where a buyer visits two sellers sequentially, and they study the role of the transparency of negotiation sequences and prices. They show that the buyer prefers private bargaining because it creates strategic uncertainty.[2] In our model, from the ex ante point of view, the buyer (weakly) prefers to avoid the cost of delay by having a public outside offer. Ex post, however, the buyer prefers to keep her outside option private once it arrives. In addition, in the private case of our model, the buyer's behavior (disclosing her outside option) can reveal her private reservation value, thus our result also relates to studies of consumer privacy by Taylor (2004) and Calzolari and Pavan (2006), which focus on firms sharing information about a buyer's purchase history.

We present the model in section 2 and study the public and private outside option cases in sections 3 and 4 respectively. In section 5, we consider a variation where the buyer is allowed to (truthfully) reveal her private outside option. In section 6, we study the limit properties of both models by allowing the seller to make offers frequently, and compare the buyer's welfare in the two cases. Section 7 concludes. All the proofs are contained in Appendix A. The limit properties as the probability of arrival converges to 1 are studied in Appendix B.

2 Model

We consider a bargaining game between a seller and a buyer. The seller owns an indivisible good that has zero value to him. Time is discrete and the length of each period is $\Delta > 0$, so periods are indexed by $t = 0, \Delta, 2\Delta, \ldots$.[3] At the beginning of period t, the seller makes a price offer $p(t)$. The buyer decides whether to accept or reject the seller's offer. If the buyer accepts the offer, the game ends. If the buyer declines the offer, then with probability $\lambda = 1 - e^{-\mu\Delta}$, she receives an outside option. The parameter $\mu > 0$ is the Poisson arrival rate for the outside option. A buyer who has an outside option decides whether to opt out or wait and hold the outside option, and whether to disclose the arrival. We assume that once the outside option arrives, the buyer can exercise it immediately or in every subsequent period. The result of the paper is robust to the case where the outside option is not permanent. If the buyer opts out, the game ends; otherwise, the game continues to the next period.[4]

The buyer is privately informed about her type θ, which is either high (H) or low (L). The buyer's type determines both her valuation of the good and the value of the outside option. Let $q(0) \in (0, 1)$ be the prior belief that the buyer is low type. If the buyer accepts a price offer p, then the buyer's payoff is

[2] Also see Krasteva and Yildirim (2014), who examine the optimal sequencing of trades with privately known values.

[3] Later we analyze the equilibrium behavior when $\Delta \to 0$, that is, when the seller's commitment power vanishes.

[4] The specification of the timing essentially requires the existence of a time gap during which the buyer can respond to the arrival of her outside option quicker than the seller. This results in a costly real-time delay between the buyer's decision time and the seller's.

$v_\theta - p$ and the seller's payoff is p. If the buyer opts out, then the buyer and the seller obtain payoffs of ω_θ and 0, respectively. Notice that ω_θ is not the face value of the outside option but a subjective object. We assume it is also the buyer's private information. The seller and the buyer share a common discount factor $\delta = e^{-r\Delta}$ where $r > 0$ is the discounting rate. To avoid a trivial case, we make the following assumption.

Assumption 1. $v_H - \omega_H > v_L - \omega_L > 0$.[5]

Assumption 1 is a "single-crossing condition" that ensures that the high type has a higher "residual value," $v_\theta - \omega_\theta$ on the current good conditional on the arrival of the outside option. Hence, for any given offer, the high type is more willing to accept the offer than the low type when they both have the option to exercise the outside option immediately.

We consider both the case in which the arrival of an outside option is public and the case in which it is private. Let $h^t \in \mathscr{H}$ be a public history and $\hat{h}^t \in \hat{\mathscr{H}}$ be a private history of the buyer. In the public outside option case, both h^t and $\hat{h}^t$ consist of a sequence of rejected price offers $\{p(\tau)\}_{\tau=0}^t$ and the history of the arrival of the outside option $\{o(\tau)\}_{\tau=0}^t$, where $o(t) = \mathbb{1}\{\text{an outside option is available at time } t\}$. In the private outside option case, the public history consists only of the price sequence $\{p(\tau)\}_{\tau=0}^t$ while the buyer's private history is $\hat{h}^t = \{p(\tau), o(\tau)\}_{\tau=0}^t$.

The seller's strategy is a pricing rule $P : \mathscr{H} \to \mathbb{R}_+$ such that $p(t) = P(h^{t-1})$, and the buyer's strategy is $\sigma : \hat{\mathscr{H}} \times \{L, H\} \times \mathbb{R}_+ \to [0,1]^3$, which specifies the probability of accepting the current offer, $\sigma_1(\hat{h}^{t-1}, \theta, p(t))$, the probability of exercising the outside option, $\sigma_2(\hat{h}^t, \theta)$, if $o(t) = 1$ given $\hat{h}^t$, and the probability of disclosing the outside option, $\sigma_3(\hat{h}^t, \theta)$, if $o(t) = 1$ given $\hat{h}^t$. In the public case, the disclosure choice is irrelevant; in the private case, $h^t = \hat{h}^t$ after the disclosure.

The solution concept is the *perfect Bayesian equilibrium* (PBE) which consists of a strategy profile (P, σ) and the beliefs $\{q(t)\}$ such that:

1. given the beliefs and $\sigma(\cdot)$, $P(\cdot)$ maximizes the seller's discounted payoff;

2. given the beliefs and $P(\cdot)$, $\sigma(\cdot)$ maximizes the buyer's discounted payoff; and

3. the beliefs $\{q(t)\}$ are derived from the equilibrium profile (P, σ) according to Bayes' rule whenever possible.

[5] Assumption 1 allows $\omega_H > \omega_L$ or $\omega_H \leq \omega_L$. A common outside option is an outside offer of a substitute good. By accepting the outside offer and consuming the good, the buyer obtains a payoff $\tilde{v}_\theta - \tilde{p}$ where $\tilde{p}$ is the price of the outside offer and $\tilde{v}_\theta$ is the buyer's utility obtained by consuming the substitute good. In general, $\tilde{v}_\theta$ and v_θ may be positively (which can be captured by $\omega_H > \omega_L$) or negatively correlated (which corresponds the case where $\omega_H < \omega_L$) or independent (which corresponds to $\omega_H = \omega_L$).

In general, multiple PBE may exist in our model. Some PBE satisfy *Coasian dynamics* (Fudenberg and Tirole, 1991). On the path of play of such an equilibrium,

- the price $p(t)$ strictly declines, and
- the belief $q(t)$ strictly rises over time.

If a PBE exhibits Coase dynamics, we refer to it as a *Coasian equilibrium*; otherwise, we refer to it as a *non-Coasian equilibrium*.

Note that the type-θ buyer's equilibrium payoff is bounded below by the payoff from rejecting the seller's offers and taking the outside option whenever it is available. Therefore in any PBE, the type-θ buyer's expected payoff given that she has an outside option is no less than w_θ. In any history where the buyer has not received an outside option (which includes the initial history), the buyer's discounted expected payoff is bounded below by

$$\lambda\omega_\theta + \delta\lambda(1-\lambda)\omega_\theta + \ldots = \frac{\lambda}{1-\delta(1-\lambda)}\omega_\theta \equiv W_\theta^*,$$

which is the *option* value of waiting for the arrival. Note that $W_\theta^* < w_\theta$ since waiting is costly.

3 Public Outside Option

In this section, we analyze the game when the outside option is public.

Lemma 1. *In any PBE of the model with a public outside option,*

1. *in any history where the buyer has an outside option, the seller never offers a price less than $v_L - \omega_L$.*
2. *in any history where the buyer does not have an outside option, the seller never offers a price less than $v_L - W_L^*$.*

Lemma 1 implies that the seller will never leave the low-type buyer any information rent after any history.

Lemma 2. *In any equilibrium, the buyer exercises the outside option upon its arrival regardless of her type, i.e., $\sigma_2(\hat{h}^t, \theta) = 1$ for $\theta = L, H$ as long as $o(t) = 1$.*

For intuition, note that Lemma 1 implies that once the outside option is available, the equilibrium price is bounded by $v_L - \omega_L$. Therefore, the low-type buyer receives no more than $\delta\omega_L$ by waiting and accepting the seller's offer. Thus opting out immediately upon the arrival of the outside option to obtain ω_L is the best response of the low-type buyer. This forces the high-type buyer to also opt out as soon as possible, since otherwise the seller will believe that the buyer is definitely a high type, and he will post a price no less than $v_H - \omega_H$, leaving the high type a payoff of no more than ω_H.

Lemma 2 implies that the seller will lose his customer with positive probability each time the trade is delayed. In order to obtain a positive profit, the seller has to set an appropriate price to serve the buyer before the arrival of the outside option. When the seller believes that the buyer is more likely to be a low type (that is, if $q(t)$ is large), he prefers to offer a low price and end the bargaining rather than to incur the cost of delay and the risk of losing the customer. So he offers a bargaining-ending price, $v_L - W_L^*$, and the buyer accepts it regardless of her type. On the other hand, if the seller believes that the buyer is more likely to be a high type ($q(t)$ is small), the seller screens the buyer by offering a high price. The high type accepts the price with positive probability but the low type declines it for sure. For this to work, the high type must be indifferent between accepting the current price and waiting every period. Her indifference condition is given by:

$$v_H - p(t) = \lambda \delta \omega_H + (1-\lambda)\delta(v_H - p(t+\Delta)), \tag{1}$$

where the left-hand side of (1) is her payoff from accepting the current offer, and the right-hand side is her payoff by waiting: If the outside option arrives in the current period, which occurs with probability λ, she opts out and obtains ω_H; otherwise, she obtains her discounted continuation value, which equals her payoff from accepting the next offer made by the seller in the next period. The seller updates his belief about the buyer's type if the buyer rejects his offer. Over time, the perceived belief that the buyer's type is low increases. Eventually, the seller becomes so pessimistic that he offers $v_L - W_L^*$ and ends the bargaining. The strategy profile described above is essentially the unique PBE.[6]

Proposition 1. *All PBE are Coasian equilibria. Generically, there exists a unique PBE.*

Naturally the players' bargaining behavior is affected by λ, the probability of the arrival of the outside option. The greater λ is, the higher the chance that the bargaining is going to end and the seller will receive zero payoff. When $\lambda = 1$, the outside option is always available, and our model becomes a two-type version of the model in Board and Pycia (2014). In this case, there is a unique equilibrium in which the seller charges a constant price in each period, and the buyer either accepts the offer or exercises the outside option in the first period. In Appendix B, we show that the equilibrium is continuous at $\lambda = 1$, implying that the equilibrium characterization of Board and Pycia (2014) is robust to a small perturbation of the arrival probability.

[6] In the non-generic case, the prior belief $q(0)$ takes value in a zero measure subset of $(0,1)$. In such a case, the seller has multiple optimal choices in the first period.

4 Private Outside Option

Next, we turn to the case with private outside option. We first study a "relaxed" private outside option model where the arrival cannot be disclosed, i.e., $\sigma_3 = 0$ in any history. We will drop this assumption in Section 5.

When the arrival of an outside option is private to the buyer, the unraveling result no longer holds because the buyer may have an incentive to hide her outside option. The buyer's behavior affects the seller's belief and equilibrium price offer. We show that there may exist multiple equilibria in the model with a private outside option. We show that a Coasian equilibrium still exists for small arrival probability, while there also exists an equilibrium that shows qualitatively different behavior when the outside option arrival probability is high enough.

In this section and the rest of the paper, we make a parametric assumption that is slightly stronger than Assumption 1:

Assumption 2. $v_H - \omega_H > v_L - W_L^*$.

Assumption 2 guarantees that an high-type buyer with an outside option may still find it optimal to wait for the seller's low price offer instead of exercising her outside option immediately. If Assumption 1 holds but Assumption 2 does not, both types exercise the outside option upon its arrival resulting in an equilibrium that is identical to the case with a public outside option. Under Assumption 2, we obtain a lower bound of the equilibrium price.

Lemma 3. *In any PBE of the model with a private outside option, in any history, the seller never offers a price less than $v_L - W_L^*$, and the low-type buyer exercises the outside option upon its arrival, i.e., $\sigma_2(\hat{h}^t, L) = 1$ if $o(t) = 1$.*

In the next two subsections, we construct two equilibria of the model — one that exhibits Coasian dynamics and another that does not — and analyze their characteristics.

4.1 Coasian Equilibrium

In a Coasian equilibrium, the seller screens the buyer by gradually lowering the price over time. After finitely many periods, the price reaches the lower bound $v_L - W_L^*$, which both buyer types will accept. While the low type exercises her outside option upon its arrival, the high type may have an incentive to wait for a lower price. The high type's incentive to take the outside option depends crucially on the cost of delay, which is determined by the number of periods before a low price will be posted. As a result, there are two phases in the Coasian equilibrium:

- In Phase I, the price is high, the high type randomizes between accepting and waiting, and both types exercise the outside option upon arrival. Over time, the belief $q(t)$ rises and the price $p(t)$ declines. In each period, the high type's indifference condition (1) is satisfied.
- In Phase II, the price is low and the high type randomizes between taking the offer and rejecting it, but only the low type exercises the outside option. The high type is indifferent between accepting the current price and waiting, and her indifference condition is given as follows:

$$v_H - p(t) = \delta(v_H - p(t+\Delta)).$$

A Coasian equilibrium starts from Phase I: Initially, the price is relatively high, so the high type exercises the outside option when she can. Over time, as the price becomes sufficiently low, Phase II starts, and the high type prefers to wait for a low price.

Proposition 2. *For any discount factor $\delta < 1$, there exists a cutoff $\bar{\lambda}_\delta < 1$ such that there is a Coasian equilibrium if $\lambda \in [0, \bar{\lambda}_\delta)$. Moreover,* $\lim_{\delta\to 1} \bar{\lambda}_\delta \in (0,1)$.

In contrast to the standard sequential bargaining model, a Coasian equilibrium may fail to exist when λ is large enough. In Phase II, only the low type exercises her outside option upon arrival. As a result, the seller updates his belief based not only on the buyer's rejection choice but also on the fact that the buyer is still negotiating; and thus the equilibrium beliefs are updated in accordance:

$$\frac{q(t+\Delta)}{1-q(t+\Delta)} = \frac{q(t)}{1-q(t)} \frac{1-\lambda}{1-\sigma_1(\hat{h}^{t-1}, H, p(t))}, \tag{2}$$

because the low type opts out with probability λ and the high type accepts the offer $p(t)$ with probability $\sigma_1(\hat{h}^{t-1}, H, p(t))$.

To ensure the equilibrium exhibits the Coasian dynamics, that is, $q(t+\Delta) > q(t)$, we must ensure that $\sigma_1(\hat{h}^{t-1}, H, p(t)) > \lambda$ in each period, which requires that λ be small enough. [7]

4.2 Non-Coasian Equilibrium

Since the Coasian equilibrium exists only if the arrival probability is small, a natural question raises: What happens if the arrival probability is large?

Proposition 3. *For any discount factor $\delta < 1$, there exists a cutoff $\underline{\lambda}_\delta < 1$ such that there exists an equilibrium that does not satisfy Coasian dynamics for $\lambda \in (\underline{\lambda}_\delta, 1]$. Moreover,* $\lim_{\delta\to 1} \underline{\lambda}_\delta = 0$.

[7] In the standard sequential bargaining model (Fudenberg *et al.*, 1985) where $\lambda = 0$, the condition is automatically satisfied.

In this equilibrium, there exists a cutoff belief where the two belief-updating effects exactly offset one another. The seller begins by screening the buyer when the belief about her type being low is small, and his belief goes up over time. Once the crucial belief is achieved, the seller's belief stays the same; and thus the seller continues to make the same randomized price offer in every period.

More specifically, the equilibrium play at the threshold belief is as follows: The seller offers the bargaining-ending price $v_L - W_L^*$ with probability z and some high price x with the complementary probability; both types of buyer accept $v_L - W_L^*$; the high-type buyer accepts x with probability λ while the low-type buyer rejects it; only the low-type buyer opts out when the outside option arrives. Because $\sigma_1(\hat{h}^{t-1}, H, x) = \lambda$, we have $q(t) = q(t+\Delta)$ by Equation (2). Therefore, the seller's belief about the buyer's type remains the same, and the agents play the same strategies (prices and opt-out rules) in every period unless the bargaining ends. Clearly, this equilibrium does not exhibit Coasian dynamics.[8]

Note that by Propositions 2 and 3, if δ is large enough, then the Coasian equilibrium and the non-Coasian equilibrium coexist under the intermediate range of λ. The multiplicity can be understood as a self-fulfilling prophecy about the degree to which the buyer's outside option increases her bargaining power. Unlike the public case, Proposition 2 implies that, as λ converges to one, there exists no Coasian equilibrium. However, as we show in Appendix B, the non-Coasian equilibrium specified in Proposition 3 still exists and it converges to the monopoly pricing equilibrium of Board and Pycia (2014).

5 Buyer's Incentive to Disclose the Outside Option

In many markets, the buyer can credibly reveal her outside option to the current seller. In the house-selling example in the introduction, an outside option is an offer from another seller, and the buyer can truthfully disclose the offer. A natural question arises: Does the buyer have an incentive to disclose the outside option if she can? Naively, one may argue that disclosing the arrival may enhance the buyer's bargaining power and that the seller needs to lower the price to compete with the outside option. However, by disclosing the arrival of the outside option, the buyer may reveal her type and thereby lose her information rent. So she may rather hide the outside option.

Formally, we allow the buyer to reveal her outside option after it arrives, i.e., $\sigma_3(\hat{h}^t, \theta) \in [0,1]$, $\forall \hat{h}^t$ s.t. $o(t) = 1$. Once the arrival is disclosed, the continuation game becomes a public case like the one analyzed in Section 3. As $\sigma_3(\hat{h}^t, \theta) = 0, \forall \hat{h}^t$ s.t. $o(t) = 1; \theta = \{L, H\}$ is a feasible choice, one can view the private case in Section 4 as a "relaxed" problem of the current model.[9]

[8]The non-Coasian equilibrium described here shares many properties with the "deadlock equilibrium" in Hwang (2013).

[9]Clearly, the public case in Section 3 is also a "relaxed" problem of the current model because $\sigma_3(\hat{h}^t, \theta) = 1, \forall \hat{h}^t$ s.t. $o(t) = 1; \theta = \{L, H\}$ is a feasible choice.

Proposition 4. *In the private outside option case, suppose that the buyer is able to credibly reveal the arrival of her outside option. The equilibrium behavior of players specified by either Proposition 2 or Proposition 3 is still supported in an equilibrium. In each equilibrium, the buyer does not reveal the arrival of the outside option.*

When the arrival of the outside option is private, the low-type buyer always exercises it as soon as it arrives without disclosing its arrival, by the logic of Lemma 3. Hence, if a buyer discloses the arrival of the outside option, she will be treated as a high-type buyer, and the seller will extract all the information rent by offering her a price $v_H - \omega_H$ in the following periods. Consequently, after revealing the outside option, the high-type buyer will exercise the outside option as soon as it arrives to avoid the cost of delay as the low-type buyer does, and her payoff is ω_H. However, if the high-type buyer does not reveal the arrival of her outside option, she can mimic the low-type buyer who has not yet received an outside option so that she can obtain some information rent, so she has no incentive to reveal the outside option. As a result, one can restrict attention on the analysis of the "relaxed" private case without losing any generality.

6 Transparency of the Outside Option with Frequent Offers

In this section, we analyze the equilibrium behavior when the seller lacks commitment power. This limit exercise is appealing for two reasons. First, one can examine the validity of Coase conjecture in the presence of random arrival of the outside option. Second, one can compare the buyer's welfare in the model with the public outside option to her welfare in the model with the private outside option. It is well known that the discrete-time model is hard to analyze, but the limit of the model is relatively tractable. To make the comparison nontrivial, we focus on the case in which $q(0)$ is small enough so that the seller finds it optimal to screen the high type in both the Coasian and non-Coasian equilibrium. Specifically, we fix the discount rate r and the arrival rate of the outside option μ, and take the length of each period Δ to zero. At the limit where $\Delta \to 0$, the seller's commitment power vanishes as he can make offers arbitrarily frequently. Note that for any $r, \mu > 0$, the discount factor $\delta = e^{-r\Delta}$ and the arrival probability $\lambda = 1 - e^{-\mu\Delta}$ converge to one and zero, respectively, as Δ goes to zero.

First we examine the robustness of the Coase conjecture under the presence of the outside option. In the canonical sequential bargaining model without an outside option ($\lambda = 0$), the option value of waiting W_θ^* is zero for both types, and the Coase conjecture holds: The initial price is arbitrarily close to the lowest valuation of the buyer (v_L) as the seller can make offers arbitrarily frequently.

Proposition 5. *In both the public and private outside option cases, for any given $\mu > 0$, there exists a Coasian equilibrium in which, as $\Delta \to 0$,*

1. *the initial price converges to $v_L - \frac{\mu}{\mu+r}\omega_L$, and*

2. *the real-time delay of the bargaining goes to zero.*

Proposition 5 states that for any fixed arrival rate of the outside option, the initial price converges to the reservation price of the low-value buyer as the seller's commitment power vanishes. For reasons explained in section 3 and 4, the seller never offers a price less than $v_L - W_L^*$, the low-type's reservation price in equilibrium. As Δ goes to zero, the lower bound of the equilibrium price converges to $v_L - \frac{\mu}{\mu+r}\omega_L$. Therefore, the low-type buyer's equilibrium payoff converges to $\frac{\mu}{\mu+r}\omega_L$, and the high-type buyer's payoff converges to $v_H - v_L + \frac{\mu}{\mu+r}\omega_L$. Because the gains from trade are realized immediately and the seller's payoff is equal to the lowest price he will charge in any equilibrium, $v_L - \frac{\mu}{\mu+r}$, the buyer's ex ante payoff is maximized in a Coasian equilibrium.

In the private outside option case, there also exists a non-Coasian equilibrium as stated in Proposition 3. Its limit properties as $\Delta \to 0$ are very different from those in a Coasian equilibrium.

Proposition 6. *There exists $\mu^* > 0$ such that for any $\mu \geq \mu^*$, in the private outside option case, a non-Coasian equilibrium described in Proposition 3 exists as $\Delta \to 0$. Furthermore, there exists an associated $q^*(\mu) \in (0,1)$ such that, when $q(0) < q^*(\mu)$,*

1. *the real-time delay converges to a positive number bounded away from zero, and*

2. *the buyer's ex ante expected payoff is strictly less than her payoff in the Coasian equilibrium.*

Proposition 6 shows that the non-Coasian equilibrium in Proposition 3 exhibits many different behavior patterns when the seller's commitment power vanishes, i.e., when $\Delta \to 0$. First, when the arrival possibility of the outside option is large enough, bargaining delay (in terms of real time) does not disappear. This is because the seller is slower to update his belief in the non-Coasian equilibrium due to the buyer's type-dependent opt-out behavior, which in turn slows down the seller's screening process. Second, the buyer's ex ante expected payoff is strictly smaller than her payoff in the Coasian equilibrium. The inefficient bargaining delay reduces the bargaining surplus, so it undermines the payoff of both types. In addition, as the price does not decline quickly, the high-type buyer is willing to pay a high price instead of bearing a real-time delay that would further reduce her payoff.

6.1 Public vs. Private Outside Option

We compare the buyer's surplus in the private case with that in the public case, and argue that making the buyer's outside option private may undermine her interest. Formally, we rank the bargaining game with public outside option and the bargaining game with private outside option in our model according to the buyer's surplus.

Definition 1. *Suppose that $E_A \subset \mathbb{R}$ is the set of the buyer's ex ante equilibrium payoff in game A, and $E_B \subset \mathbb{R}$ is that in game B. The buyer is* ***weakly worse off*** *in game A than game B if and only if (1) for any $e \in E_A, e' \in E_B$ and $e \leq e'$, and (2) there exists $e \in E_A$ and $e' \in E_B$ such that $e' > e$.*

Notice that the order is partial. In general, two bargaining games may not be comparable. However, we are able to compare the public case and the private case at the limit where $\Delta \to 0$. In the public outside option case, the Coasian equilibrium is a unique equilibrium and the buyer's ex ante payoff is maximized. On the other hand, there are multiple equilibria in the private outside option case. While the Coasian equilibrium exists, we show that there also exists a non-Coasian equilibrium in which the buyer's ex ante payoff is strictly smaller than the one in the Coasian equilibrium. The argument above is formalized as follows:

Corollary 1. *Suppose that $\mu \geq \mu^*$ and $q(0) < q^*(\mu)$ where $\mu^*, q^*(\mu)$ are specified in Proposition 6. As Δ goes to zero, the buyer is weakly worse off in the game where the outside option is private than in the game where the outside option is public.*

Corollary 1 implies that endowing the buyer with the privacy of her outside option may reduce her welfare. This argument seems counterintuitive: One may argue that an agent can enjoy information rent by having additional private information. However, in our setting, making the arrival of her outside option private may create additional strategic uncertainty, thereby slowing down the seller's screening process. Consequently, a delay occurs and both the buyer and the seller have to bear the cost of the real-time delay of trade.

Together, Corollary 1 and Proposition 4 imply that the buyer's preference regarding the transparency of her outside option is time-inconsistent: Even though she can maximize her ex ante welfare by committing to disclose the arrival of the outside option, she prefers not to do so when the outside option actually arrives. As a result, the buyer's commitment problem arises as well as the seller's in a sequential bargaining setting with an arriving outside option.

Remark 1. *While we show that the buyer is weakly worse off in the private outside option case, it remains to be seen whether the non-Coasian equilibrium described in Proposition 3 gives the buyer the worst equilibrium ex ante payoff in the private case.*

7 Concluding Remarks

In this section, we discuss some limitations and possible extensions of our analysis.

Binary-Type. Although we explain our economic intuition in a binary-type model, we believe it also applies in more general settings, such as in models with multiple types. In the public outside option model, the unraveling property still holds: Any type of buyer will exercise the outside option as soon as it arrives. Thus, the seller can only screen the buyer before the arrival of the buyer's outside option. By applying the technique used in the standard Coase conjecture models, we can fully characterize the unique equilibrium and show that the Coase conjecture holds. In the private outside option model, if the buyer types are finitely many, the skimming property still holds, so eventually the continuation games will become a binary-type model where our results hold. With a continuum of types, on the other hand, it is an open question whether a non-Coasian equilibrium exists. The key difficulty is that the buyer's incentive to exercise her outside option is type-dependent. As a result, the seller's belief about the buyer's type cannot be characterized by the truncated distribution of the original probability distribution.[10]

Timing Specification. In the public outside option model, a generalized Coase conjecture holds: There is no real-time-delay when the seller lacks commitment power. This result is directly driven by Lemma 2: Upon the arrival of the outside option, the buyer exercises it regardless of her type, and the seller's payoff is zero regardless of his belief. However, the validity of Lemma 2 relies on our specification of the timing, which requires that the buyer can respond to the arrival of her outside option earlier than the seller. In the current setting, it is reasonable to assume that the buyer knows her own outside option and can make her decision before the seller does. However, it is theoretically interesting to know what will happen under an alternative timing. For example, suppose that the timing is as follows in each period: (1) the outside option arrives first, (2) the seller makes an offer, and then (3) the buyer decides whether to exercise the outside option (if it is available) or to accept the offer. Notice that, upon the arrival of the outside option, the continuation game is a special case of Board and Pycia (2014): The seller charges either $v_H - \omega_H$ or $v_L - \omega_L$ depending on his belief; the buyer either accepts the offer or exercises the outside option; and the game ends immediately. Consequently, the seller's payoff conditional on the arrival of the outside option is decreasing in the belief that the buyer is of the low type, and thus the original game can be treated as a special case of Fuchs and Skrzypacz (2010) in which the Coase conjecture does not hold. On the other hand, in the private outside option model, the timing specification does not play a critical role because the seller cannot observe the arrival of an outside option. In general, we believe that in a sequential bargaining model with arrivals, the equilibrium prediction critically depends on the timing specification and observability of outside options.

Type-Dependent Arrival. We assume that the arrival rate of the outside option is type-independent. However, our model can be easily extended to incorporate type-dependent arrival rates. We conjecture that our main results are robust to type-dependent arrival rates as long as the corresponding parameters satisfy Assumption 2.

[10] See Hwang (2013) for additional discussion.

A Appendix: Omitted Proofs

We first prove Lemmas 1—3. Then we present a proof of Proposition 2 before the proof of Proposition 1 as the construction technique in the proof of Proposition 2 is used in the other proofs. After proving Proposition 3, we prove the limit result of the equilibria in both the public and private outside option cases (Proposition 5).

A.1 Proof of Lemma 1

1. We first show that the equilibrium price offer in any history in which the buyer holds an outside option is no less than $v_L - \omega_L$. Suppose, to the contrary, that there exists an equilibrium where the seller does offer such a price. Define $\underline{p} < v_L - \omega_L$ to be the infimum of the seller's offer after the arrival of the outside option. Then by Assumption 1, there exists $\varepsilon > 0$ sufficiently small that $v_\theta - (\underline{p} + \varepsilon) > \max\{\delta(v_\theta - \underline{p}), w_\theta\}$ for $\theta = H, L$. Let h^t be the history at which the seller offers a price $p < \underline{p} + \varepsilon/2$. Then both types of buyer must accept the offer as either's continuation payoff upon rejection is strictly lower than the value from acceptance. Then, however, the seller has a profitable deviation to offer $p' = p + \varepsilon/2$ as both types of buyers would still accept p' for sure, leading to a contradiction.

2. Recall that when the type-θ buyer does not have an outside option, she can achieve W_θ^* by rejecting the seller's offers and taking the outside option upon its arrival. Therefore she rejects any price higher than $v_\theta - W_\theta^*$. Note that Assumption 1 implies $v_H - W_H^* > v_L - W_L^* > 0$. Then the argument, similar to that in the above paragraph, shows that the seller never offers a price lower than $v_L - W_L^*$. ■

A.2 Proof of Lemma 2

By Lemma 1, the equilibrium price offer in any history in which the buyer holds an outside option is no less than $v_L - \omega_L$. Then the low-type buyer must opt out immediately after she receives an outside option, since the maximum payoff she could get after waiting is no more than $\delta\omega_L$ and she discounts the future payoffs ($\delta < 1$). In any equilibrium, the high-type buyer exercises the outside option immediately as well. Suppose that there exists an equilibrium where the high-type buyer does not opt out immediately. Then belief $q(t)$ becomes 1 after the arrival, and the argument (similar to that of the previous paragraph) implies that the equilibrium price offer in the continuation play is never below $v_H - \omega_H$, and because of discounting the high type strictly prefers to exercise the option immediately, which is a contradiction. Hence, $\sigma_1(\hat{h}^t, \theta) = 1$ for $\theta = L, H$ as long as $o(t) = 1$. ■

A.3 Proof of Lemma 3

Given Assumption 2, the proof of Lemma 3 is straightforward from the proof of Lemma 1. ■

A.4 Proof of Proposition 2

Equilibrium Construction. We will construct the Coasian equilibrium by using backward induction. There are two observations. First, recall from Lemma 3 that the seller's equilibrium offer after any history is no less than $p_0 \equiv v_L - W_L^*$. Second, we claim that in a Coasian equilibrium, p_0 will be charged when the belief $q(t)$ is close to 1. Suppose not, then the seller's payoff is bounded by $q(t)(v_H - W_H^*) + [1-q(t)]\delta p_0$ where $v_H - W_H^*$ is the highest price that can be accepted by the high type. On the contrary, if the seller posts p_0, then his payoff is p_0. As $q(t) \to 1$, the latter is higher than the former as long as $\delta < 1$.

We construct the equilibrium backwardly when $q(t)$ is large. The above observations imply that in any equilibrium, the price offer in the final period equals p_0 because the low-type buyer would never accept any offer above p_0. Moreover, it implies that the low-type buyer would always opt out once the outside option is available.

Now consider the equilibrium price in the penultimate period, which we denote as p_1. Since the high-type buyer must be indifferent between accepting p_1 and waiting for the final offer, the equilibrium price is determined uniquely. There are two cases. First, if $\omega_H \leq \delta(v_H - p_0)$, then the high type will not opt out even if the option is available. Then her indifference condition becomes

$$v_H - p_1 = \delta(v_H - p_0),$$

or $p_1 = (1-\delta)v_H + \delta p_0$. Second, if $\omega_H > \delta(v_H - p_0)$, then the high-type buyer will accept the outside option if it is available. In this case the high-type buyer's indifference condition is given by

$$v_H - p_1 = \lambda \omega_H + (1-\lambda)\delta(v_H - p_0),$$

or $p_1 = (1-\delta(1-\lambda))v_H + \delta(1-\lambda)p_0 - \lambda \omega_H$.

Let p_k be the equilibrium price offer k periods before the final period. Using a similar argument, we construct a sequence $\{p_k\}$ recursively as follows:

- if $p_{k-1} \leq v_H - \omega_H/\delta$, then
$$p_k = (1-\delta)v_H + \delta p_{k-1}. \tag{3}$$
- if $p_{k-1} > v_H - \omega_H/\delta$, then
$$p_k = (1-\delta(1-\lambda))v_H + \delta(1-\lambda)p_{k-1} - \lambda \omega_H. \tag{4}$$

Notice that the $\{p_k\}$ constructed above is increasing and converges to $v_H - W_H^*$. Define $k' = \min\{k : p_{k-1} > v_H - \omega_H/\delta\}$. Then the high-type buyer takes the outside option if and only if there are $k \geq k'$ periods left until the final period. Also note that by Assumption 2, if δ is large enough, we have $\omega_H < \delta(v_H - p_0) = \delta(v_H - v_L + W_L^*)$, so $k' > 1$ and Phase II exists.

Given the sequence of price offers, we construct a decreasing sequence of beliefs $\{q_k\}$ with $q_0 = 1$ such that in the Coasian equilibrium, the seller offers p_k if $q(t) \in (q_{k+1}, q_k]$. We construct $\{q_k\}$ only with the necessary property that if the seller's belief is equal to q_k, then he must be indifferent between offering p_k (which will move the seller's belief to q_{k-1}) and offering p_{k-1} (which will move his belief to q_{k-2}). As is common in dynamic bargaining models, the single crossing property completes the rest of the proof: A seller strictly prefers offering p_k to p_{k-1} if his belief is below q_k, while he strictly prefers p_{k-1} to p_k if his belief is above q_k.

In order to describe the following analysis in a concise way, define $\beta(q,q')$ to be the acceptance probability of the high-type buyer by which the seller's belief changes from q to q', given that both types of buyers take the outside option. That is,

$$\frac{q'}{1-q'} = \frac{q}{1-q}\frac{1}{1-\beta(q,q')} \Leftrightarrow \beta(q,q') = 1 - \frac{q}{1-q}\frac{1-q'}{q'}.$$

Similarly, define $\beta'(q,q')$ as the acceptance probability of the high type by which the seller's belief changes from q to q', given that only the low-type buyer takes the outside option. That is,

$$\frac{q'}{1-q'} = \frac{q}{1-q}\frac{1-\lambda}{1-\beta'(q,q')} \Leftrightarrow \beta'(q,q') = 1 - \frac{q}{1-q}\frac{1-q'}{q'}(1-\lambda).$$

First look at the case where $k = 1$. At $q = q_1$, if the seller offers p_1, then the high-type buyer accepts it for sure, and the low-type buyer opts out if the option arrives; otherwise she accepts p_0 in the next period. On the other hand, if the seller offers p_0, then both types accept the offer and the game ends immediately.

$$(1-q_1)p_1 + q_1\delta(1-\lambda)p_0 = p_0,$$

so q_1 is given by

$$q_1 = \frac{p_1 - p_0}{p_1 - \delta(1-\lambda)p_0}.$$

Note that the high-type buyer accepts p_1 for certain, so we do not need to consider the high-type buyer's behavior regarding the outside option.

For $k = 2, \ldots$, however, the seller's indifference condition depends on whether the high-type buyer would opt out if the outside option is available, that is, whether $k \geq k'$.

1. If $k < k'$ or $p_{k-1} \leq v_H - \omega_H/\delta$ (Phase II): Recall that at $q = q_k$, the seller offers p_k in equilibrium and he is indifferent between offering p_k and p_{k-1}. If the seller offers p_k, then the high-type buyer

accepts with probability $\beta'(q_k, q_{k-1})$, and only the low-type buyer takes the outside option. So his payoff is given by

$$V(q_k) = (1-q_k)\beta'(q_k,q_{k-1})p_k + (1-(1-q_k)\beta'(q_k,q_{k-1}) - q_k\lambda)\delta V(q_{k-1})$$

where $V(q)$ is the seller's payoff in the Coasian equilibrium when the belief is q. On the other hand, if the seller offers p_{k-1}, the high-type buyer accepts with a higher probability $\beta'(q_k, q_{k-2})$ and only the low-type buyer takes the outside option. In this case, we write the seller's payoff as the sum of (1) the payoff from selling the good at price p_{k-1} to the high-type buyer with probability $\beta(q_k, q_{k-1})$, and (2) the equilibrium payoff when $q = q_{k-1}$:

$$(1-q_k)\beta(q_k,q_{k-1})p_{k-1} + (1-(1-q_k)\beta(q_k,q_{k-1}))V(q_{k-1}).^{11} \tag{5}$$

Since $(1-q)\beta(q,q') = 1 - \frac{q}{q'}$ and $(1-q)\beta'(q,q') = 1 - \frac{q}{q'}(1-\lambda(1-q'))$, the seller's indifference condition is given by

$$\begin{aligned} V(q_k) &= (1-\gamma_k(1-\lambda+\lambda q_{k-1}))p_k + \gamma_k(1-\lambda)\delta V(q_{k-1}) \qquad (6) \\ &= (1-\gamma_k)p_{k-1} + \gamma_k V(q_{k-1}), \qquad (7) \end{aligned}$$

where $\gamma_k = \frac{q_k}{q_{k-1}}$. From (6) and (7), we have

$$(1-\delta(1-\lambda))\gamma_k V(q_{k-1}) = (1-\gamma_k(1-\lambda+\lambda q_{k-1}))(1-\delta)v_H + (\delta\lambda\gamma_k(1-q_{k-1}) - (1-\delta)(1-\gamma_k))p_{k-1}. \tag{8}$$

Putting (8) into (7), we have

$$(1-\delta(1-\lambda))V(q_k) = (1-\gamma_k(1-\lambda+\lambda q_{k-1}))(1-\delta)v_H + (1-q_k)\lambda\delta p_{k-1}. \tag{9}$$

Putting (9) again into (8) and simplifying, we have

$$\gamma_k = \frac{v_H - p_{k-1}}{(v_H - p_{k-1}) + (1-\lambda)v_H(1-\gamma_{k-1}) - \lambda\delta(v_H - p_{k-2})(1-q_{k-1})}. \tag{10}$$

2. If $k \geq k'$ or $p_{k-1} > v_H - \omega_H/\delta$ (Phase I): The seller offers p_k, which the high-type buyer accepts with probability $\beta(q_k, q_{k-1})$ and all types of buyers take the outside option. On the other hand, if he offers p_{k-1}, which the high-type buyer accepts with probability $\beta(q_k, q_{k-2})$. Similar to equation (5), we write the seller's payoff as the sum of the payoff from selling the good at price p_{k-1} to the high-type buyer with probability $\beta(q_k, q_{k-1})$ and the equilibrium payoff when $q = q_{k-1}$.

[11] It is equal to $(1-q_k)\beta'(q_k,q_{k-2})p_{k-1} + (1-(1-q_k)\beta'(q_k,q_{k-2}) - q_k\lambda)\delta V(q_{k-2})$ by simple algebra.

Therefore, the seller's indifference condition is given by

$$\begin{aligned}V(q_k) &= (1-q_k)\beta(q_k,q_{k-1})p_k+(1-(1-q_k)\beta(q_k,q_{k-1})-\lambda)\delta V(q_{k-1}) \\ &= (1-q_k)\beta(q_k,q_{k-1})p_{k-1}+(1-(1-q_k)\beta(q_k,q_{k-1}))V(q_{k-1}).\end{aligned}$$

Simplifying, we have

$$V(q_k) = (1-\gamma_k)p_k+\gamma_k(1-\lambda)\delta V(q_{k-1}) \tag{11}$$

$$= (1-\gamma_k)p_{k-1}+\gamma_k V(q_{k-1}). \tag{12}$$

From (11) and (12), we have

$$\gamma_k V(q_{k-1}) = (1-\gamma_k)(v_H-p_{k-1}-W_H^*) \tag{13}$$

Putting (13) into (12), we have

$$V(q_k) = (1-\gamma_k)(v_H-W_H^*). \tag{14}$$

Putting (14) again into (13) and simplifying, we have

$$\gamma_k = \frac{(v_H-W_H^*)-p_{k-1}}{(2-\gamma_{k-1})(v_H-W_H^*)-p_{k-1}}. \tag{15}$$

Because $\gamma_{k-1}<1$, $\gamma_k<1$.

Equilibrium Profile. We describe the Coasian equilibrium profile by using $\{p_k\}$ and $\{q_k\}$ as constructed above. Let K be an integer such that a decreasing sequence of $\{q_k\}$ goes below the prior $q(0)$ for the first time. That is,

$$K = \min\{k : q_k \le q(0) \text{ and } \gamma_j < 1 \text{ for all } j \le k\},$$

where $\gamma_k = \frac{q_k}{q_{k-1}}$. Note that if $\gamma_{k'} \ge 1$ for some k', then there is no such K for any prior $q(0) < q_{k'}$. Consider a generic case where $q_K < q(0)$. Then the equilibrium behavior of the Coasian equilibrium is as follows:

- On the equilibrium path, the seller offers a price

$$p(t) = \begin{cases} p_{K-1} & \text{if } q(t) \in [q(0), q_{K-1}], \\ p_j & \text{if } q(t) \in (q_{j+1}, q_j] \text{ for } j=1,\dots,K-2, \\ p_0 & \text{if } q(t) \in (q_1, 1]. \end{cases}$$

- Both on and off the path, the high type accepts $p(t)$ with probability

$$\sigma_1(\hat{h}^{t-1}, H, p(t)) = \begin{cases} 0 & \text{if } p(t) > p_{K-1}, \\ \max\{\beta(q(t), q_j), 0\} & \text{if } p(t) \in (p_j, p_{j+1}] \text{ for } j = 1, \ldots, K-2; p(t) > v_H - \omega_H, \\ \max\{\beta'(q(t), q_j), 0\} & \text{if } p(t) \in (p_j, p_{j+1}] \text{ for } j = 1, \ldots, K-2; p(t) \leq v_H - \omega_H, \\ 1 & \text{if } p(t) \leq p_1. \end{cases}$$

  where $q(t)$ is consistent with $\hat{h}^{t-1}$.

- The high type opts out at period t if and only if $p(t) > v_H - \omega_H$.

To see the behavior off the equilibrium path, observe that the buyer's deviation does not change the continuation play because it is unobservable. Once a seller deviates by charging a "wrong" price at time t, players are off the path of play. As long as the price is higher than $v_L - W_L^*$, the low type will decline it, and the high type will mix between accepting it and rejecting it, and the probability of accepting the offer ensures that the updated belief $q(t+\Delta) = q_k$ for some $k = 1, 2, \ldots$. To sustain the high-type buyer's indifference condition, the seller must find it optimal to randomize between different (equilibrium) prices at time $t + \Delta$ and the probability distribution of the seller's randomization depends on the deviation price. The seller's incentive to randomize over multiple prices is ensured as the belief that the buyer's type being low is $q(t+\Delta) = q_k$ for some k so that the seller is indifferent between p_k and p_{k-1}.

Note that the equilibrium profile does not need to specify the behavior when the posterior belief is less than the prior. This is because, given the equilibrium behavior, the seller's belief is never less than the prior after any history.

Existence Condition. It remains to show that there exists an upper bound $\bar{\lambda}_\delta \in (0,1)$ such that the Coasian equilibrium exists for any $\lambda < \bar{\lambda}_\delta$. It suffices to show that for any $\lambda < \bar{\lambda}_\delta$, the sequence of cutoff beliefs $\{q_k\}$ is decreasing in k and converges to zero. Since Equation (11) implies that when $p_{k-1} \leq v_H - \omega_H/\delta$ (that is, when the equilibrium is in Phase I), $\{q_k\}$ is decreasing in k and converges to zero. Then it suffices to show that for any k in Phase II, $\gamma_k < 1$.

Now suppose $\gamma_{k-1} < 1 - \varepsilon$. From (10), we have

$$\begin{aligned} \gamma_k &= \frac{v_H - p_{k-1}}{v_H - p_{k-1} + (1 - \gamma_{k-1}) v_H - \lambda \left[(1 - \gamma_{k-1}) v_H + (1 - q_{k-1}) \delta (v_H - p_{k-2}) \right]} \\ &< \frac{\frac{v_H - p_{k-1}}{v_H}}{\frac{v_H - p_{k-1}}{v_H} + \varepsilon - \lambda(\varepsilon + \delta)}. \end{aligned}$$

Set $\bar{\lambda}_\delta = \varepsilon \frac{1 - \varepsilon - \frac{v_H - (v_L - W_L^*)}{v_H}}{1 + \varepsilon}$, then $\gamma_k < 1 - \varepsilon$. Note that $\lim_{\delta \to 1} \bar{\lambda}_\delta$ is bounded away from zero. ■

A.5 Proof of Proposition 1

Recall from Lemma 2 that the game ends as soon as the outside option arrives, and from Lemma 1 that the equilibrium price offer is no less than $v_L - W_L^*$. Because both types of buyers exit the game at the same rate, the model is essentially similar to the standard sequential bargaining model of Fudenberg *et al.* (1985) except that the seller may lose the buyer each time the trade is delayed. Therefore, the equilibrium construction and the uniqueness argument are straightforward applications of Fudenberg *et al.* (1985) in which the players need to take the arrival of the outside option into account, and on the equilibrium path, the lowest price is $v_L - W_L^*$ rather than v_L. On the equilibrium path,

1. the equilibrium price sequence $p(t)$ declines over time;
2. the high-type buyer accepts the price with probability $\beta(p) \in (0,1]$ when the equilibrium price is p;
3. since $\beta(p(t)) > 0$ in each period, the belief $q(t)$ increases over time on the path of play;
4. when the belief $q(t)$ is high enough, the seller posts a price $v_L - W_L^*$ and the buyer accepts the offer regardless of her type.

Since the proof of Proposition 1 is a special case of the proof of Proposition 2, we only provide the outline of the equilibrium construction here. (It only includes Phase I.)

Constuction of Sequences of Prices and Cutoff Beliefs. The equilibrium screening process is characterized by a sequence of cutoff beliefs $\{q_k\}$ and a sequence of prices $\{p_k\}$ where $k \in \mathbb{N}$. When the seller's belief is $q(t) = q_1$, he is indifferent between charging p_0 and p_1, in which

$$\begin{aligned} p_0 &= v_L - W_L^*, \\ p_1 &= v_H - \lambda\omega_H - (1-\lambda)\delta(v_H - v_L + W_L^*), \textit{and} \\ q_1 &= \frac{p_1 - p_0}{p_1 - \delta(1-\lambda)p_0.} \end{aligned}$$

If $q(t) > q_1$, the seller charges p_0 and the game ends. If $q(t) \leq q_1$, the seller screens the high type by charging a higher price.

For $k \geq 2$, the equilibrium dynamics is similar to one of Phase I of the Coasian equilibrium in the private outside option case (Proposition 2): If $q(t) = q_k$, the seller charges p_k; only the high type accepts price p_k with probability β_k; both types immediately exercise the outside option; the updated belief $q(t+\Delta) = q_{k-1}$. Therefore, the price sequence $\{p_k\}_{k\geq 2}$ is pinned down by (4), and the belief sequence

$\{q_k\}_{k\geq 2}$ is pinned down by (15). Finally, the high-type buyer's acceptance probability β_k is derived by the Bayes' rule:

$$\frac{q_{k-1}}{1-q_{k-1}} = \frac{q_k}{1-q_k}\frac{1}{1-\beta_k}.$$

Owing to the arrival of the outside option, the cutoff belief q_1 and the equilibrium price sequence are different from those in the standard sequential bargaining model. Since the sequence $\{q_k\}$ is unique, for any initial belief $q(0) \in (0,1)$ there is a unique finite K such that $q(0) \in (q_K, q_{K-1}]$.

Equilibrium Profile. The equilibrium behavior of the Coasian equilibrium is as follows:

- On the equilibrium path, the seller offers a price

$$p(t) = \begin{cases} p_{K-1} & \text{if } q(t) \in [q(0), q_{K-1}], \\ p_j & \text{if } q(t) \in (q_{j+1}, q_j] \text{ for } j = 1, \ldots, K-2, \\ p_0 & \text{if } q(t) \in (q_1, 1]. \end{cases}$$

- The high type accepts $p(t)$ with probability

$$\sigma_1(\hat{h}^{t-1}, H, p(t)) = \begin{cases} 0 & \text{if } p(t) > p_{K-1}, \\ \max\{\beta(q(t), q_j), 0\} & \text{if } p(t) \in (p_j, p_{j+1}] \text{ for } j = 1, \ldots, K-2; p(t) > v_H - \omega_H, \\ \max\{\beta'(q(t), q_j), 0\} & \text{if } p(t) \in (p_j, p_{j+1}] \text{ for } j = 1, \ldots, K-2; p(t) \leq v_H - \omega_H, \\ 1 & \text{if } p(t) \leq p_1. \end{cases}$$

  where $\beta(q,q') = 1 - \frac{q}{1-q}\frac{1-q'}{q'}$.

- The low type accepts $p(t)$ only if $p(t) \leq v_L - \omega_L$ and $o(t) = 0$.

- Both types exercise the outside option upon its arrival: $\sigma_2(\hat{h}^t, L) = \sigma_2(\hat{h}^t, H) = 1$ if $o(t) = 1$.

The players' behavior off-the-path of play are specified as in Proposition 2. One can verify that no player has an incentive to deviate from the equilibrium strategy profile.

Uniqueness. By applying the argument in Lemma 3 of Fudenberg *et al.* (1985), we claim that, in any equilibrium, the price reaches $v_L - W_L^*$ in finitely many periods; otherwise, the low type does not accept the offer in finitely many periods, and there must exist an interval of beliefs $[\underline{q}, \bar{q}]$ where $\bar{q} - \underline{q}$ can be arbitrarily small such that the equilibrium belief $q(t)$ does not "jump across the interval" in finitely many periods. Because $\sigma_1(h^{t-1}, L, p) = 0$, the belief $q(t)$ does not go down. Hence, there must exist a history in which the seller's equilibrium payoff arbitrarily close to zero. As a result, the seller will be better off by charging $p_0 \equiv v_L - W_L^*$ to end the game immediately. Hence, the Coasian equilibrium we constructed is generically the unique PBE.[12] ■

A.6 Proof of Proposition 3

We construct a non-Coasian equilibrium to prove the proposition. The equilibrium construction is an application of the *deadlock equilibrium* in Hwang (2013), in which the author considers a dynamic lemon market situation. In the equilibrium, there exist $q^* \in (0,1)$, $x > v_L - W_L^*$ and $z \in (0,1)$, such that the equilibrium behavior is as follows:

- If $q(0) > q^*$, then the seller offers $v_L - W_L^*$ and both types of buyer accept the offer. Bargaining ends in the first period with probability one.
- If $q(0) < q^*$, then the game may last more than one period. The equilibrium dynamics exhibit the following two phases:
  1. Phase A lasts as long as the seller's belief $q(t)$ is smaller than q^*. The seller offers a price $p(t)$ higher than x. Responding to that, the high-type buyer accepts the offer with positive probability and the low-type buyer rejects it. Both buyer types opt out immediately upon receiving an outside option. In Phase A, the seller's belief $q(t)$ increases over time and his offer $p(t)$ decreases over time.
  2. Phase B begins when the seller's belief $q(t)$ reaches q^*. The seller randomizes between offering $v_L - W_L^*$ (with probability z) and offering x (with probability $1-z$). If $p(t) = v_L - W_L^*$, the buyer accepts the offer and the game ends with probability one. If $p(t) = x$, the high type accepts the offer with probability λ and the low type rejects it. Only the low type exercises her outside option when it arrives. In Phase B, the seller's belief $q(t) = q^*$ stays the same since both types of buyer exit the game with the same probability (λ). The players use the same strategy in every period as long as the bargaining continues.

In the following analysis, we pin down the values of the parameters (q^*, z, x) by the players' incentive conditions in Phase B. Then we continue to analyze the behavior in Phase A.

Phase B. First, we show that x must be equal to $v_H - \omega_H$. It is easy to verify that $x \leq v_H - \omega_H$, otherwise the high-type buyer takes the outside option. Now suppose that $x < v_H - \omega_H$. Offering $x + \varepsilon < v_H - \omega_H$

[12] In the generic case, $q(0) \in (q_{K-1}, q_K)$, so the optimal equilibrium price is unique. In the non-generic case, $q(0) = q_K$. The seller is indifferent between charging p_K and p_{K-1} in the first period, which leads to multiple equilibrium paths.

is then a profitable deviation for the seller, since the high-type buyer's unique consistent response is to accept $x+\varepsilon$ with probability λ and not to take the outside option.

Then z is determined by the high-type buyer's indifference condition

$$v_H - x = \delta[(1-z)(v_H - x) + z(v_H - (v_L - W_L^*)],$$

or

$$z = \frac{1-\delta}{\delta} \frac{\omega_H}{(v_H - \omega_H) - (v_L - W_L^*)}. \tag{16}$$

Finally, q^* is determined by the seller's indifference condition

$$v_L - W_L^* = (1-q^*)\lambda x + \delta(1-\lambda)(v_L - W_L^*),$$

or

$$q^* = 1 - \frac{(1-\delta(1-\lambda))(v_L - W_L^*)}{\lambda(v_H - \omega_H)}. \tag{17}$$

Phase A. The equilibrium dynamics of Phase A are similar to those in the Coasian equilibrium. Specifically, the equilibrium dynamics are characterized by an increasing sequence of prices $\{\hat{p}_k\}$ and a decreasing sequence of beliefs $\{\hat{q}_k\}$, where $\hat{p}_0 = v_L - W_L^*$, $\hat{p}_1 = x = v_H - \omega_H$, $\hat{q}_0 = 1$, and $\hat{q}_1 = q^*$. Similar to the proof of Proposition 2, we use the recursive method to construct $\{\hat{p}_k\}$ and $\{\hat{q}_k\}$ for $k \geq 2$.

Since $p_k > v_H - \omega_H$ for any $k \geq 2$, both high-type and low-type buyer exercise the outside option when it arrives. Therefore, the recursive formula is identical to Phase I of the Coasian equilibrium (Proposition 2), where the price sequence $\{\hat{p}_k\}_{k\geq 2}$ is pinned down by (4) and the belief sequence $\{\hat{q}_k\}_{k\geq 2}$ is pinned down by (15). We restate the recursive equations:

$$\hat{p}_k = (1-\delta(1-\lambda))v_H + \delta(1-\lambda)\hat{p}_{k-1} - \lambda\omega_H,$$

and

$$\hat{\gamma}_k = \frac{(v_H - W_H^*) - \hat{p}_{k-1}}{(2-\hat{\gamma}_{k-1})(v_H - W_H^*)) - \hat{p}_{k-1}},$$

where $\hat{\gamma}_k = \frac{\hat{q}_k}{\hat{q}_{k-1}}$.

Equilibrium Behavior. The equilibrium behavior of the non-Coasian equilibrium is as follows:

- on the equilibrium path, the seller offers a price

$$p(t) = \begin{cases} \hat{p}_{K-1} & \text{if } q(t) \in [q(0), \hat{q}_{K-1}], \\ \hat{p}_j & \text{if } q(t) \in (\hat{q}_{j+1}, \hat{q}_j) \text{ for } j = 1, \ldots, K-2, \\ \hat{p}_1 & \text{if } q(t) = \hat{q}_1 \text{ and } p(t-\Delta) = \hat{p}_2, \\ \gamma \circ \hat{p}_1 + (1-\gamma) \circ \hat{p}_0 & \text{if } q(t) = \hat{q}_1 \text{ and } p(t-\Delta) = \hat{p}_1, \\ \hat{p}_0 & \text{if } q(t) \in (\hat{q}_1, \hat{q}_0]. \end{cases}$$

  where K is the integer such that $q(0) \in (q_K, q_{K-1}]$.

- both on and off the path, the high type accepts $p(t)$ with probability

$$\sigma_1(\hat{h}^{t-1}, H, p(t)) = \begin{cases} 0 & \text{if } p(t) > \hat{p}_{K-1}, \\ \max\{\beta(q(t), \hat{q}_j), 0\} & \text{if } p(t) \in (\hat{p}_j, \hat{p}_{j+1}] \text{ for } j = 1, \ldots, K-2\,, \\ \max\{\beta'(q(t), \hat{q}_1), 0\} & \text{if } p(t) \in ((1-\delta)v_H + \delta\hat{p}_0, \hat{p}_1], \\ 1 & \text{if } p(t) \leq (1-\delta)v_H + \delta\hat{p}_0. \end{cases} \tag{18}$$

- the high type opts out at period t if and only if she receives an outside option and $p(t) > \hat{p}_1 = v_H - \omega_H$.

- the low type accepts the offer if and only if $p(t) \leq \hat{p}_0$ and opts out immediately when she receives an outside option.

As in the construction of the Coasian equilibrium, once the seller charges a "wrong" price at time t, the players are off the path. When $q(t) < q^*$, as long as the price is higher than $v_L - W_L^*$, the low type declines it and the high type accepts it with positive probability, which ensures that the updated belief $q(t+\Delta) = \hat{q}_k$ for some $k = 1, 2, \ldots$. In order to make the high type indifferent, at time $t+\Delta$, the seller must find it optimal to randomize between two equilibrium prices and the distribution of the randomization depends on the deviation price at time t. The seller's incentive to randomize is ensured as $q(t+\Delta) = \hat{q}_k$ for some k so that the seller is indifferent between charging $\hat{p}_k$ and $\hat{p}_{k-1}$.

When $q(t) = q^*$, after a deviation price $p(t)$, to ensure the high type's best response is consistent with (18), the seller's continuation strategy needs to depend on his deviation price. For example, if $\sigma(t) \in (0,1)$, at time $t+\Delta$, the seller charges $v_H - \omega_H$ with probability z' and charges $v_L - W_L^*$ with the complementary probability so that

$$v_H - p(t) = \delta z' \omega_H + (1-z')(v_H - v_L + W_L^*).$$

Optimality and the Existence Condition. Next we show that there exists a lower bound $\underline{\lambda}_\delta < 1$ such that the above equilibrium exists if and only if $\lambda > \underline{\lambda}_\delta$. Similarly to the public outside option case, the optimality of the profile for $q \neq q^*$ is satisfied by the construction of the sequence $\{\hat{q}_t\}$.

Thus it remains to verify the seller's optimality at $q = q^*$. Recall that the seller's equilibrium strategy at $q = q^*$ is a randomized price offer between x and $\hat{p}_0$. From (18), the high-type buyer's response to the seller's offer $p(t)$ when $q(t) = q^*$ is given by

$$\sigma_1(\hat{h}^{t-1}, H, p(t)) = \begin{cases} 0 & \text{if } p(t) > \hat{p}_1, \\ \lambda & \text{if } p(t) \in ((1-\delta)v_H + \delta\hat{p}_0, \hat{p}_1], \\ 1 & \text{if } p(t) \leq (1-\delta)v_H + \delta\hat{p}_0. \end{cases}$$

We check the seller's optimality for each of the following five ranges of non-equilibrium prices:

1. $p > x = v_H - w_H$: The seller is better off by charging x since any price greater than x is rejected for sure.

2. $p \in ((1-\delta)v_H + \delta\hat{p}_0, x)$: The seller is better off by charging x.

3. $p = (1-\delta)v_H + \delta\hat{p}_0$: See the paragraph below.

4. $p \in (\hat{p}_0, (1-\delta)v_H + \delta\hat{p}_0)$: The seller is better off by charging $(1-\delta)v_H + \delta\hat{p}_0$.

5. $p < \hat{p}_0$: The seller is better off by charging $\hat{p}_0$.

Finally, it remains to show that offering $p = (1-\delta)v_H + \delta\hat{p}_0$ is not profitable. In this case, the high-type buyer will accept the offer with probability one, and only the low-type buyer (who does not receive an outside option) will remain in the second period. In that case, the seller's payoff is given by

$$(1-q^*)((1-\delta)v_H + \delta(v_L - W_L^*)) + q^*\delta(1-\lambda)(v_L - W_L^*). \tag{19}$$

Therefore, the seller does not have an incentive to charge $(1-\delta)v_H + \delta(v_L - W_L^*)$ if and only if (19) is no more than $v_L - W_L^*$. Plugging in q^* from (17) and arranging, we have

$$(1-\delta)v_H \leq \lambda\left((v_H - w_H) - \delta(v_L - \frac{\lambda}{1-\delta(1-\lambda)}w_L)\right). \tag{20}$$

For any fixed $\delta < 1$, the right-hand side of (20) is continuous and increasing in λ while the left-hand side is constant. Note that the right-hand side converges to zero as λ goes to zero, and converges to $(v_H - w_H) - \delta(v_L - w_L)$ as λ goes to one. Thus if δ is large such that it satisfies $\omega_H - \delta\omega_L < \delta(v_H - v_L)$, there exists $\underline{\lambda}_\delta < 1$ such that (20) is satisfied if and only if $\lambda > \underline{\lambda}_\delta$. Finally, it is easy to check that as δ becomes arbitrarily close to one, (20) is satisfied with $\lambda = 0$; that is, $\lim_{\delta\to 1}\underline{\lambda}_\delta = 0$.

A.7 Proof of Proposition 4

First, if the arrival is revealed, the continuation game becomes a public outside option case. By the proof of Lemma 1, the low type's payoff is bounded by ω_L. To avoid the cost of delay, the low type opts out upon the arrival of the outside option rather than revealing it in any equilibrium. As a result, if there is an equilibrium in which the buyer reveals the arrival on the path of play, she must be treated as a high-type buyer for sure. However, the seller will leave no rent for her, so her continuation payoff is ω_H. By the proof of Proposition 3, the high type's equilibrium payoff is $v_H - x = v_H - (v_H - \omega_H) = \omega_H$, which is greater than the continuation payoff by revealing the arrival, $\delta\omega_H$ as $\delta < 1$. Hence, both types do not deviate from the deadlock equilibrium; and therefore, the seller has no incentive to deviate either. By applying the same argument, one can also show that both types do not reveal the outside option in the Coasian equilibrium. ■

A.8 Proof of Proposition 5

Fix any r and μ. It suffices to show that there exists $\bar{\Delta} > 0$ such that for any $\Delta < \bar{\Delta}$, there exists $\sigma_2 > 0$ such that $\gamma_k < 1 - \sigma_2$ for all k. The rest of the proof is identical to the proof in the standard Coase conjecture literature, so we provide only a heuristic argument here to illustrate the idea.

- For $k = 1$, in both the public and the private outside option models, we have $\gamma_1 = q_1 = \frac{p_1 - p_0}{p_1 - \delta(1-\lambda)p_0} = \frac{v_H - p_0}{v_H - p_0 + \frac{1-\delta(1-\lambda)}{1-\delta}p_0}$. As Δ goes to zero, $\gamma_1 \to \frac{v_H - p_0^*}{v_H + \frac{\mu}{r}p_0^*} < 1$ where $p_0^* = \lim_{\Delta\to 0} p_0 = v_L - \frac{\mu}{r+\mu}\omega_L$.
- For any $k \in \mathbb{N}$, $p_k \to p_0^*$ as $\Delta \to 0$ in both the public and the private outside option models.
- When the outside option is private, for $k > 1$, in Phase II,

  $$\begin{aligned}\gamma_k &= \frac{v_H - p_{k-1}}{v_H - p_{k-1} + (1-\gamma_{k-1})v_H - \lambda\left[(1-\gamma_{k-1})v_H + (1-q_{k-1})\delta(v_H - p_{k-2})\right]} \\ &\to \frac{v_H - p_0^*}{(2-\gamma_{k-1})v_H - p_0^*},\end{aligned}$$

  as Δ goes to zero. Since the function $f(x) = \frac{v_H - p_0^*}{(2-x)v_H - p_0^*}$ is convex and has fixed points of 1 and $1 - \frac{p_0^*}{v_H}$, if $\gamma_k \in (1 - \frac{p_0^*}{v_H}, 1)$, then $\gamma_{k+1} < \gamma_k$.
- Similarly, when the outside option is private, for $k > 1$, in Phase I,

  $$\gamma_k \to \frac{v_H - W_H^{**} - p_0^*}{(2-\gamma_{k-1})(v_H - W_H^{**}) - p_0^*}, \tag{21}$$

  as Δ goes to zero, where $W_H^{**} = \lim_{\Delta\to} W_H^* = \frac{\mu}{\mu+r}\omega_H$.

- When the outside option is public, for $k > 1$, the updating of beliefs is identical to that in Phase I of the private outside option model.

Hence, for any $q(0)$, as $\Delta \to 0$, there exists a finite K such that $q(0) > q_k$ when $k > K$. So the proof is complete. ■

A.9 Proof of Proposition 6

As $\Delta \to 0$, equation (20) converges to

$$rv_H \leq \mu\left((v_H - w_H) - (v_L - \frac{\mu}{r+\mu}w_L)\right), \tag{22}$$

which holds for sufficiently large μ. Denote the cutoff arrival rate as μ^*. For each $\mu \geq \mu^*$, a deadlock equilibrium exists as Δ goes to zero. Furthermore, by equation (17), $q^*(\mu) = \lim_{\Delta \to 0} q^*$ is strictly positive and less than 1. What remains to complete the proof is to show that when $q(0) < q^*(\mu)$, the equilibrium has inefficient delay.

As shown in Proposition 3, the equilibrium behavior when the prior $q(0)$ is less than q^* is as follows. In Phase A, the seller offers a price higher than $x = v_H - w_H$, and the high-type buyer accepts the offer with positive probability while the low-type buyer rejects it. As a result, the seller's belief increases in each period. When the belief reaches the cutoff belief q^*, the "deadlock" phase begins: The seller randomizes between the bargaining-ending offer $v_L - W_L^*$ and the high price offer x; the high-type buyer accepts x with probability λ while the low-type buyer rejects it; only the low-type buyer opts out when the outside option arrives; and the seller's belief stays same at q^*.

As in the Coasian equilibrium, when $\Delta \to 0$, the real-time delay of Phase A goes to zero, the seller's price offer in Phase A converges to x, and the number of periods of the initial phase remains bounded. So the trade occurs almost immediately at prices close to x until the seller's belief reaches q^*.

However, the bargaining delay (in terms of real time) of Phase B does not vanish. Recall from Proposition 3 that when $q(t) = q^*$, the bargaining ends with probability $z + (1-z)\lambda$ in each period: Bargaining is ended by 1) the seller's low-price offer (with probability z), 2) the seller's high-price offer and the high-type buyer's acceptance (with probability $(1-z)(1-q^*)\lambda$); or 3) the low-type buyer's decision to opt out (with probability $(1-z)q^*\lambda$). Hence the expected number of periods of the negotiation is $\frac{1}{z+(1-z)\lambda}$. Therefore, as Δ goes to zero, the expected real-time duration of the negotiation becomes

$$\begin{aligned}
\lim_{\Delta \to 0} \frac{\Delta}{z + (1-z)\lambda} &= \lim_{\Delta \to 0} \frac{1}{\frac{1-\delta}{\delta\Delta}\frac{\omega_H}{(v_H-\omega_H)-(v_L-W_L^*)} + \frac{(1-z)\lambda}{\Delta}} \\
&= \frac{1}{\frac{r\omega_H}{(v_H-\omega_H)-(v_L-W_L^*)} + \mu} > 0.
\end{aligned}$$

It is straightforward from the above argument that as Δ goes to zero, the low-type buyer's surplus becomes $\frac{\mu}{\mu+r}\omega_L$ and the high-type buyer's surplus becomes ω_H. ■

A.10 Proof of Corollary 1

First, $v_L - W_L^*$ is the lower bound of the price in any equilibrium. The low type's payoff is bounded by W_L^* in any equilibrium. Second, the high type's payoff is bounded by $\max\{W_H^*, v_H - (v_L - W_L^*)\}$, which equals $v_H - (v_L - W_L^*)$ by Assumption 1. As $\Delta \to 0$, $W_L^* \to \omega_L$ and $v_H - (v_L - W_L^*) \to v_H - (v_L - \omega_L)$, which are the low type and the high type's payoff in the limit of the Coasian equilibrium. Hence, in the private outside option model, $q(0)\omega_L + [1 - q(0)][v_H - (v_L - \omega_L)]$ is the upper bound of the buyer's ex ante surplus. The rest of the proof immediately follows Propositions 5 and 6. ■

B Appendix: Large Arrival Probability

Naturally, the players' bargaining behavior is affected by the probability of the arrival, λ. The greater λ is, the higher the chance that the bargaining will end and the seller will receive zero payoff. In this section, we derive the limit result in both the public and private outside option models as the arrival probability λ goes to 1 by fixing the discount factor. A natural implementation is to fix the discount rate r and the duration of each period Δ, but take the arrival rate μ to infinity. We will show that in both the public and private cases, there exists an equilibrium which converges to the equilibrium in Board and Pycia (2014).

B.1 Public Outside Option

In the following proposition, we characterize the limit properties of the equilibrium in which the outside option arrives almost surely in each period.

Proposition 7. *In the (Coasian) equilibrium, for any $\delta \in (0,1)$, as λ goes to 1,*

1. *If $q(0) \leq \frac{(v_H - \omega_H) - (v_L - \omega_L)}{v_H - \omega_H}$, the initial price $p(0)$ converges to $v_H - \omega_H$; the high-type buyer accepts the initial offer with probability one; the low-type buyer exercises the outside option as soon as it arrives; the game ends in the first period almost surely.*

2. *If $q(0) > \frac{(v_H - \omega_H) - (v_L - \omega_L)}{v_H - \omega_H}$, the initial price converges to $v_L - \omega_L$; both types of buyer accept the offer in the first period; and the game ends in the first period with probability one.*

Proof. On the equilibrium path, when $q(t) > q_1$, $p(0) = v_L - W_L^*$. When $q(t) \in (q_{k+1}, q_k]$ for $k = 1, 2, ...$, the seller charges $p(t) = p_k$. As $\lambda \to 1$, $W_L^* \to \omega_L$, so $p_0 \to v_L - \omega_L$ and $p_1 \to v_H - \omega_H$. Similarly,

$q_1 \to \frac{v_H-\omega_H-v_L+\omega_L}{v_H-\omega_H}$. By equation (4), $p_k \to v_H - \omega_H$ for $k \geq 1$. Hence, if $q(0) > \frac{v_H-\omega_H-v_L+\omega_L}{v_H-\omega_H}$, the seller believes the buyer is low-type with very high probability, so he will discontinue the screening and the initial price $p(0) \to p_0$ as $\lambda \to 1$; As $p(0) = p_1$, the high-type buyer accepts the offer for sure. The low-type buyer exercises the outside option if it is available. At the limit, the outside option arrives with probability one in the first period. If $q(0) \leq \frac{v_H-\omega_H-v_L+\omega_L}{v_H-\omega_H}$, the seller would screen the buyer, but the initial price goes to $v_H - \omega_H$ as λ goes to 1. Moreover, by Equation (11), $\gamma_2 = \frac{q_2}{q_1} \to 0$ as $\lambda \to 1$, thus $q_2 \to 0$, which implies that the game ends in at most three periods. □

The intuition behind Proposition 7 is as follows. When λ is large enough, the players believe that the outside option almost surely arrives in the first period, and therefore, as λ goes to 1, the buyer's willingness to pay converges to $v_\theta - \omega_\theta$ since $\lim_{\lambda\to 1} W_\theta^* = \omega_\theta$. As the buyer will exercise the outside option once it arrives, the seller's problem essentially becomes a static price posting problem as λ converges to one:

- When $q(0)$ is large, the seller charges a low price that is accepted by both types, so the game ends immediately, and
- when $q(0)$ is small, the seller initially charges a high price and targets only the high-type buyer.

When $\lambda = 1$, our model is a two-type analogy of Board and Pycia (2014). They consider a bargaining model where the buyer has the outside option in the beginning of the negotiation. They show that there is a unique equilibrium in which the seller posts a constant take-it-or-leave-it "generalized static monopoly" price in each period, and the buyer either takes the offer or opts out in the first period. Proposition 7 says that given a fixed δ, when the arrival of the outside option is public, the equilibrium correspondence is continuous at $\lambda = 1$.

B.2 Private Outside Option

In the private outside option model, when λ is close enough to 1, there exists no Coasian equilibrium, but there exists a non-Coasian equilibrium specified by Proposition 3.

Corollary 2. *Fix $\delta \in (0,1)$. As λ goes to zero, the limit properties of the equilibrium specified by Proposition 3 are given as follows:*

1. *If $q(0) \leq \frac{(v_H-\omega_H)-(v_L-\omega_L)}{v_H-\omega_H}$, the initial price $p(0)$ converges to $v_H - \omega_H$; the high-type buyer accepts the initial offer with probability one; the low-type buyer exercises the outside option as soon as the outside option arrives; and the game ends in the first period almost surely.*
2. *If $q(0) > \frac{(v_H-\omega_H)-(v_L-\omega_L)}{v_H-\omega_H}$, the initial price converges to $v_L - \omega_L$; both types of buyer accept the offer in the first period; and the game ends in the first period with probability one.*

Proof. By Equation (11) again, the real-time delay of Phase A goes to zero as $\lambda \to 1$. In Phase B, the limits of q^* and x as $\lambda \to 1$ are $\frac{(v_H - \omega_H) - (v_L - \omega_L)}{v_H - \omega_H}$ and $v_H - \omega_H$ respectively. As $\lambda \to 1$, the probability that the low type receives and exercises the outside option in the first period goes to zero. In addition, the probability that a high type accepts price x is $\beta = \lambda$, which goes to one. $\square$

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