Targeted vs. Collective Information Sharing in Networks*

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Abstract

We introduce a simple two-stage game of endogenous network formation and information sharing for reasoning about the optimal design of social networks like Facebook or Google+. We distinguish between unilateral and bilateral connections and between targeted and collective information sharing. Agents value being connected to other agents and sharing and receiving information. We consider multiple utility specifications. We show that the game always has an equilibrium in pure strategies and then we study how the network design and the utility specifications affect welfare. Surprisingly, we find that in general, targeted information sharing is not necessarily better than collective information sharing. However, if all agents are either "babblers" or "friends", irrespective of whether the network is unilateral or bilateral, in equilibrium, targeted information sharing yields higher welfare than collective information sharing.

JEL Classificaiton: D47, D85, C72, C62

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1. Introduction

Online social networks are used by roughly one in four people worldwide and the number of users is estimated to reach almost 2 billions by the end of 2014 (eMarketer, 2013b). Excluding dating websites, there are more than 200 notable social networking websites (for short, networks) that are currently active. Among these networks, some have been more successful than others in attracting a very large number of users.[1] In particular, with more than 500 million users each, Facebook and Google+ are by far the largest networks.[2] However, even when restricting attention to only these two networks, little is know about how various network design options affect their success.

Facebook and Google+ esentially evolved over time, often following a "trial and error" approach, and their design was shaped by substantive lessons from both failures and successes. In some cases, less prominent options that one network made available to its users have been adopted by the other network, further developed, and then given prominent status. For example, Facebook had an option called "friends lists", which allowed a user to group a subset of his friends into a list and then share information only with the people in this list. However, this option was hidden deep in the user interface and Facebook did not take any steps to promote it or to encourage its usage. Indeed, before 2010, only 5% of Facebook's users have used "friends lists" (Eldon, 2010). In contrast, on it's launch in early 2011, Google+ showcased as one of its main features an option that allowed users to group their contacts into meaningful groups called "circles". Furthermore, it encouraged its users to share information selectively using these circles. Nowadays, although the friends list and the circles have roughly the same functionality, the emphasis placed on these options by each network remains very different. This is particularly interesting since these options are key in determining the information shared and received by users, and thus significantly influence the network's owner revenue from content specific advertising. Note that these revenues are impressive by any metric.[3]

[1]For example, networks that are estimated to have in excess of 100 million registered users include specialized networks such as Twitter (microblogging) or Linkedln (professional), or general networks such as Facebook, Google+, Bebo, Habbo, Netlog, Orkut, Qzone, Renren or Tagged.

[2]Other measures include the number of daily or monthly active users. According to the company's latest official report, in December 2013, Facebook had on average 757 million daily active users. Google+ reportedly attracted by mid 2013 around 359 million monthly active users representing 26% of all internet users worldwide and is now the second largest network in terms of both registered and active users (eMarketer, 2013a).

[3]Facebook obtained in 2013 a revenue of $7.87 billion (Facebook, 2014). Google Inc., the company owning and operating Google+, became on the 7th of February 2014 the second most valuable company in the world with a market capitalization of $395.42 billion. Google Inc. monetizes user activity across its products, which

We propose a new framework for reasoning about the optimal design of some of the options that networks like Facebook and Google+ make available to their users. In particular, we are interested in the design of the options that specify how agents may form connections and how they may share information via connections. Our model consists of a simple two-stage game.

In the first stage, we consider a process of endogenous network formation in which agents choose with what other agents to connect with. Given the type of connections that agents are allowed to form, we distinguish between unilateral and bilateral networks.

In the second stage, after the connections are formed, each agent receives some private information and has the option of sharing this information with his connections. Given the constraints upon sharing information, we distinguish between targeted and collective information sharing.

The network owner's revenue is derived from advertisement, which is proportional with the information shared and received in the network. Agents have quasilinear utilities from being connected to other agents and for sharing and receiving information. For each of our designs, we consider three utility specifications. In the most general specification, an agent who shares an information might value the information shared differently than the agent who receives it. We also consider two other special cases: a"babbler" network, a situation in which agents only value sharing information, and a "friends" network, a situation in which the information shared by one agent and received by another is valued identically by both of them.

We use subgame perfect Nash equilibrium as a solution concept for our two-stage game.

For all possible network designs and given the most general specification of the utility, we derive agents' equilibrium strategies and we show that for our two-stage game of endogenous network formation and information exchange, an equilibrium is always guaranteed to exist. We then turn to studying how the network design and the utility specifications affect the set of connections, the welfare of the agents, and the revenue of the network owner in equilibrium.

Surprisingly, in sharp contrast with the economic intuition that flexibility should be welfare enhancing, for the most general utility specification, for either unilateral or bilateral networks, we show that targeted information sharing is not necessarily better than collective information sharing for either the agents or the network owner. Thus, our result shows that in general there is no unambiguous recommendation for what design options an owner seeking to maximize

include but are not limited to Google+. It does not provide public reports concerning its revenues from Google+ alone but it these revenues are undoubtedly important. More generally, companies in the United States alone spent an estimated $6.1 billion in 2013 on ads in social media (BIA/Kelsey, 2014).

his utility should choose. However, for babbler or friends networks, we prove that targeted information sharing is always weakly better than collective information sharing for both the agents and the network owner.

Given any babbler or friends network with either targeted or collective information sharing, we also analyze the welfare implications of allowing for unilateral versus bilateral connections. We find that for babbler networks with targeted information sharing, networks with unilateral connections induce more connections and yield a weakly higher welfare than the networks with bilateral connections. However, for babbler networks with collective information sharing, the networks with unilateral connections yield a higher utility for every agent than the networks with bilateral connections, but the comparison with respect to the set of connections and the revenue of the owner of the network is ambiguous. Finally, we show that for friends networks with either targeted or collective information sharing, when comparing networks with unilateral connections versus networks with bilateral connections, the welfare effects are ambiguous.

2. Related Literature

Our modelling is in line with the game theoretical literature on multi-stage games with incomplete information (for an excellent textbook description, see Fudenberg and Tirole (1991)).

The first stage of our game connects with the literature on strategic network formation (Bala and Goyal, 2000; Galeotti et al., 2006, 2010; Jackson and Wolinsky, 1996). Jackson and Zenou (2013) provide an excellent overview of this literature. Closest to our work, Bala and Goyal (2000) propose a non-cooperative model of network formation and they characterize the architectures of the networks that arise in strong Nash equilibria. In contrast to their main results, we compare the set of connections, the utilities of the agents, and the revenue of the network, across the equilibria in different types of networks. We also assume that agents receive benefits not only from incoming connections, i.e., through receiving information, but also from outgoing connections, i.e., through sharing information. These type of benefits do not appear neither in Bala and Goyal (2000) nor in any other known to us paper.

The second stage of our game is constructed around the idea of studying information sharing, and idea that is inspired, although not directly relatable to, the studies on information sharing or disclosure (Austen-Smith, 1994; Okuno-Fujiwara et al., 1990; Ostrovsky and Schwarz, 2010; Pagano and Jappelli, 1993).

Finally, we note that social networks such as Facebook recently started to generate a lot of interest in economic literature (Batzilis et al., 2013; Chen et al., 2011; Hartline et al., 2008; Kleinberg and Ligett, 2013; Tarbush and Teytelboym, 2014, 2012). However, most of this literature is centred around empirical investigations. Closest to our work, Kleinberg and Ligett (2013) analyze information sharing and privacy in social networks. They study the behavior of rational agents in a network where an information shared by an agent may spillover from the agent who is the intended recipient to other agents, with potentially unpleasant consequences for the agent who shared the information. In their model, the utility of an agent takes one of two fixed values to reflect his relation with other agents, "friends" or "enemies", with whom he is in the same component. Thus, theirs is a simultaneous game concerned with the formation of Nash stable components. In contrast, in our setting agents are simultaneously concerned with the information shared and received. Our model is a multi-stage game with observed actions and incomplete information in which we study not just what connections are formed among agents but also what information gets shared. The network structure does not influence our results.

3. Model

There is a (network) owner, denoted by 0, and a finite set of agents $\mathcal{I} = (1, ..., I)$. Let $2^{\mathcal{I}}$ denote the power set of $\mathcal{I}$. The owner sets the "rules" of the network. These rules specify how agents may form connections among them and how they may share information via connections. Agents have preferences over forming connections and sharing private information. Each agent i has a constant value c^{ij} from being connected to agent j, values sharing his information $s^{ij}(x^i)$ with agent j, and values receiving information $r^{ij}(x^j)$ from agent j. We consider multiple design options for the rules that the owner may set. Once the rules are set, the agents play a two–stage game of endogenous network formation and information exchange. The two–stage game and the design options are as follows.

Stage 1 (*network formation*): Any two distinct agents i and j may form a connection ij. Each agent $i \in \mathcal{I}$ chooses a subset of other agents $N_i \in 2^{\mathcal{I}\setminus\{i\}}$ with whom he wants to connect with. We denote the choices of all agents except i as N_{-i}. The choices of all agents are $N = \{N_i\}_{i\in\mathcal{I}}$.

We consider two possible network designs. In networks with *unilateral connections*, for a connection ij to be formed, it is enough for agent i to choose another agent j. In contrast, in networks with *bilateral connections*, for a connection ij to be formed, it is not enough for agent i to choose another agent j; it also has to be the case that j chooses i. That is, we require agents i and j to choose each other.

Observe that under both possible network designs the choices of all agents N determine a finite network, where agents are the nodes and the connections formed are the edges. The choices of all agents are revealed at the end of Stage 1. For simplicity, with some abuse of the notation, we denote the resulting *network* by N. We denote by $\tilde{N}$ the set of all possible networks.

Stage 2 (*information sharing*): Each agent $i \in \mathcal{I}$ observes some *(private) information* x^i, with x^i independently distributed according to some function h^i with support X^i. We do not restrict X^i in any way. Each agent knows x^i but not x^{-i}. Let $\mathbf{x} \in X$ denote the profile of information, where $\mathbf{x} = \{x^i\}_{i \in \mathcal{I}}$ and $X = \prod_{i=1}^{I} X^i$. Any two agents who are connected may exchange information between them. We define a binary *action* of agent i regarding sharing his information x^i with agent j via connection ij at network N as $\mathbf{1}_{ij}(x^i, N) : X^i \times \tilde{N} \to \{0, 1\}$, where $\mathbf{1}_{ij}(x^i, N) = 1$ if ij is an edge in N and agent i shares his information, and $\mathbf{1}_{ij}(x^i, N) = 0$ otherwise.

We consider two possible information sharing designs. In networks with *targeted information sharing*, an agent can select individually with whom to share his information among his connections; formally, there are no restrictions on actions $\mathbf{1}_{ij}$. In contrast, in networks with *collective information sharing*, each agent can share his information only with all his connections at once; formally, $\mathbf{1}_{ij} = \mathbf{1}_{ij'}$ for all $j, j' \in N_i$ with ij, ij' being edges in network N.

We denote agent i's actions at each possible history as $\mathbf{1}_i = \{\mathbf{1}_{ij}(\cdot, \cdot)\}_{j \in \mathcal{I} \setminus \{i\}}$. The actions at each possible history of all agents are $\mathbf{1} = \{\mathbf{1}_i\}_{i \in \mathcal{I}}$.

Given the two stages described above a *pure strategy* of agent i is a pair $(N_i, \mathbf{1}_i)$.

In our model, the network formation stage precedes the information sharing stage because we consider that connection decisions are more long term than information sharing decisions. Thus, we consider the long term decisions as fixed when short term decisions are made. The flexibility of the design leads to four possible types of networks (Figure 1), allowing us to study two distinct externalities that agents impose on each other. Roughly speaking, in networks with bilateral connections, as each connection requires mutual consent, every two connected agents directly influence each others' payoffs as the information shared by one enters in the utility of

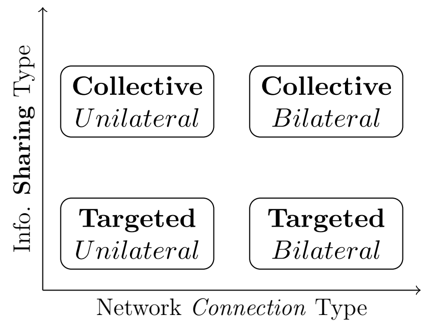


Figure 1. Network types.

the other. Thus, an agent i connected to another agent j, on top of the benefits from sharing his information with j, receives an externality via the benefits that he obtains from receiving information from j. In networks with collective information sharing, as each agent can only share his information with all his connections at once, each connection involving an agent i may influence his decision regarding potentially forming other connections. For example, an agent i connected to another agent m may choose to connect or not with n not solely based on her own intrinsic preferences, but also after taking into account the "compatibility" between m and n.

3.1. Preferences

We define the utilities of the agents after and before observing their private information.

The ex-post quasilinear utility of agent i who has private information x^i at network N, given that at Stage 2 he follows action $\mathbf{1}_i$ while the other agents follow actions $\mathbf{1}_{-i}$, is $U_i(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N)$, and we define $U_i(\cdot)$ contingent upon the Stage 1 network design.

In networks with unilateral connections, each agent i has a constant value c^{ij} and shares his information $s^{ij}(x^i)$ with each agent in $j \in N_i$. However, agent i can only receive information $r^{ij}(x^j)$ from agents j such that $i \in N_j$. Thus, recalling that we did not restrict X^i in any way, and denoting by E the mathematical expectation operator, agent i's utility is

$$U_i(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N) = \sum_{j \in N_i} \big(c^{ij} + s^{ij}(x^i)\mathbf{1}_{ij}(x^i, N)\big) + E \sum_{j: i \in N_j} r^{ij}(x^j)\mathbf{1}_{ji}(x^j, N).$$

In networks with bilateral connections, agents in a connection ij have each a constant value, and they can freely share and receive information between them. With some abuse of notation we denote agent i's utility as

$$U_i(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N) = \sum_{(i,j):j\in N_i, i\in N_j} \big(c^{ij} + s^{ij}(x^i)\mathbf{1}_{ij}(x^i, N) + E(r^{ij}(x^j)\mathbf{1}_{ji}(x^j, N))\big).$$

The ex-ante utility of agent i at network N is $U_i(N, \mathbf{1}) = E\, U_i(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N)$. This is agent i's utility before he observes his private information but after the network N is formed.

We assume that the utility of the network owner mainly comes as revenue from content specific advertising, which is proportional to the amount of relevant information exchanged among agents.[4] Consequently, we define the utility of the network owner as the sum of agents' ex-ante benefits corresponding to information exchange:

$$U_0(N, \mathbf{1}) = E \sum_{i\in\mathcal{I}} \sum_{j\in N_i} (s^{ij}(x^i)\mathbf{1}_{ij}(x^i, N) + r^{ij}(x^j)\mathbf{1}_{ji}(x^j, N)).$$

We assume that the network owner and the agents are all utility maximizers.

Next, we introduce two natural restrictions on the preferences of the agents.

First, consider an environment where agents value only sharing information, i.e., for each $i, j \in \mathcal{I}$ with $i \neq j$, we require $r^{ij} = 0$. We call such agents "*babblers*". A network in which all agents are babblers is called a *babbler network*.

Second, consider an environment where the preferences for sharing and receiving information coincide, i.e., for each $i, j \in \mathcal{I}$ with $i \neq j$, we require $r^{ij} = s^{ji}$. We call such agents "*friends*". A network in which all agents are friends is called a *friends network*.

3.2. Subgame Perfect Nash Equilibrium

To predict the outcome of our two–stage game, we use subgame perfect Nash equilibrium as a solution concept. A profile of strategies $(N, \mathbf{1})$ is a *subgame perfect Nash equilibrium* if

1. $U_i(N_i, N_{-i}, \mathbf{1}) \geq U_i(N_i', N_{-i}, \mathbf{1})$ for each $i \in \mathcal{I}$, each $N_i' \subseteq \mathcal{I}$, and

[4]The content specific advertising attempts to gain attention by providing content related to the user's experience. Common examples include Facebook Sponsored Stories and Twitter Promoted Tweets. The content specific advertising is usually called "native advertising" and currently accounts for 39% of the total US firms expenditure on social media advertising (see BIA/Kelsey, 2013).

2. $\mathbf{1}_i(\mathbf{x}^i, N) \in \text{argmax}_{S_i \in \Omega_i(N_i)} U_i(S_i, \mathbf{1}_{-i} | x^i, N)$ for each $x^i \in X^i$, each $N \in \tilde{N}$, where for targeted information sharing $\Omega_i(N_i) = \{0,1\}^{N_i}$ and for collective information sharing networks $\Omega_i(N_i) = \{0,1\}$.

For situations which result in identical payoffs, we assume that ties are broken consistently, e.g., by employing the following *tie-breaking* rule: (1) if an agent is indifferent between forming or not a connection with another agent, then he forms the connection, and (2) if an agent is indifferent between sharing or not his information with the other agent(s), then he shares his information.

For short, we refer to a subgame perfect equilbrium that satisfies the tie-breaking rule above simply as an *equilibrium*.

4. Equilibrium Strategies

We start by analyzing the strategic behavior of the agents at the information sharing stage given an existing network. At this stage all connections have been formed. Thus, the flow of fixed benefits is determined and agents cannot influence receiving information.[5] However, agents may choose to share or not information.

Without loss of generality, assume that agent i is connected to N_i agents.

In networks with targeted information sharing, by maximising his ex-post utility, an agent i shares information x^i with an agent $j \in N_i$ if and only if $s^{ij}(x^i) \geq 0$. Thus, for agent i, the expected ex-ante utility from sharing information is

$$E\Big(\sum_{j \in N_i} \max(s^{ij}(x^i), 0)\Big). \tag{1}$$

In networks with collective information sharing, an agent i shares information x^i with all agents $j \in N_i$ if and only if $\sum_{j \in N_i} s^{ij}(x^i) \geq 0$. Thus, for agent i, the expected ex-ante utility

[5]The assumption that agents cannot influence the benefits from receiving information can be relaxed. Alternatively, agents can decide to receive or block receiving information before the private information is realized based on the expectation regarding the connected agent's information distribution and the connected agent's information sharing strategy. In this case, we assume that the connected agent receives the same benefits from information sharing independently on whether he is blocked or not.

from sharing information is

$$E\big(\max(\sum_{j\in N_i} s^{ij}(x^i),0)\big). \tag{2}$$

Given that the maximum function is a convex function, the following remark follows.

Remark 1. *Given the same set of connections, each agent's benefit from information sharing in networks with targeted information sharing is weakly higher than in his benefit in networks with collective information sharing.*

Recall that our tie-breaking rule requires that if an agent is indifferent between sharing his information with his connection(s), then he shares his information. Thus, for each subgame at the information sharing stage, we have a Nash equilibrium. For networks with targeted information sharing, the Nash equilibrium is unique. For networks with collective information sharing, we may have a multiplicity of optimal sets, but the agents' utility in equilibrium is unique. For each information sharing design, we fix agents' continuation payoffs to be equal with their Nash equilibrium payoffs.

We now analyze the strategic behavior of the agents at the network formation stage.

In networks with unilateral connections, agent i cannot control what other agents connect to him. Thus, he cannot influence the information that he receives. However, agent i can choose the agents that he wants to connect with in order to send information.

In networks with unilateral connections and targeted information sharing, agent i decides individually regarding each of the connections that he makes in order to send information and chooses to connect with another agent j if and only if

$$c^{ij} + E(\max(s^{ij}(x^i),0)) \geq 0. \tag{3}$$

In networks with with unilateral connections and collective information sharing, agent i decides regarding the whole set of connections N_i that he makes in order to send information by maximizing the following expression:

$$\max_{N_i\subseteq \mathcal{I}}\big(\sum_{j\in N_i} c^{ij} + E\big(\max(\sum_{j\in N_i} s^{ij}(x^i),0)\big)\big). \tag{4}$$

Both optimization problems above are discrete maximization problems. Hence, both of them are well defined. This establishes the existence of a subgame perfect Nash equilibria for networks with unilateral connections.

In networks with bilateral connections the Nash equilibrium is a very weak equilibrium concept, and the existence of an equilibrium is straightforward. Each agent establishing no connection at the network formation stage and then following the actions that induce Nash equilibrium payoffs at the information sharing stage is a subgame perfect Nash equilibrium for both targeted and collective information sharing networks.

The above analysis proves the following theorem.

Theorem 1. *For all possible network designs, the two-stage game of endogenous network formation and information exchange has an equilibrium in pure strategies.*

5. Welfare Analysis

In this section we analyze the welfare implications of various network designs and the implications over the set of connections in equilibrium. We start by showing that for general agent's characteristics, when comparing networks with targeted information sharing against networks with collective information sharing, the welfare implications for the agents and for the owner are unclear.

Example 1. Targeted versus collective information sharing.

Let 0 denote the network owner and let $\mathcal{I} = \{1, 2, 3\}$. We assume that agents form the complete network with either unilateral or bilateral connections. For each agent, the constant value of being connected to another agent is null, i.e., $c^{ij} = 0$ for all $i \neq j$ where $i, j \in \mathcal{I}$. For convenience, for each agent $i \in \mathcal{I}$, we require X^i be a subset of $\mathbb{R}^2$ and we represent i's private information by a two-dimensional vector $\mathbf{x^i} = (x_1^i, x_2^i)$ that is uniformly distributed within the unit circle: $\{(x_1^i, x_2^i) : (x_1^i)^2 + (x_2^i)^2 \leq 1\}$. Agents' benefits from information sharing and receiving are linear: $s^{ij}(\mathbf{x}) = \mathbf{s}^{ij}\mathbf{x}$ and $r^{ij}(\mathbf{x}) = \mathbf{r}^{ij}\mathbf{x}$ with $\mathbf{s}^{ij}, \mathbf{r}^{ij} \in \mathbb{R}^2$ for all $i \neq j$. The information sharing weights, which can be thought of as reflecting the agents' preferences for

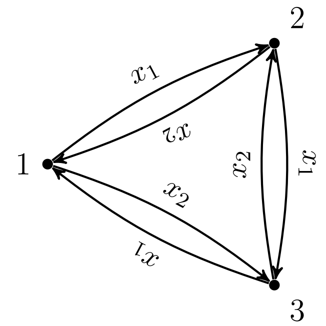


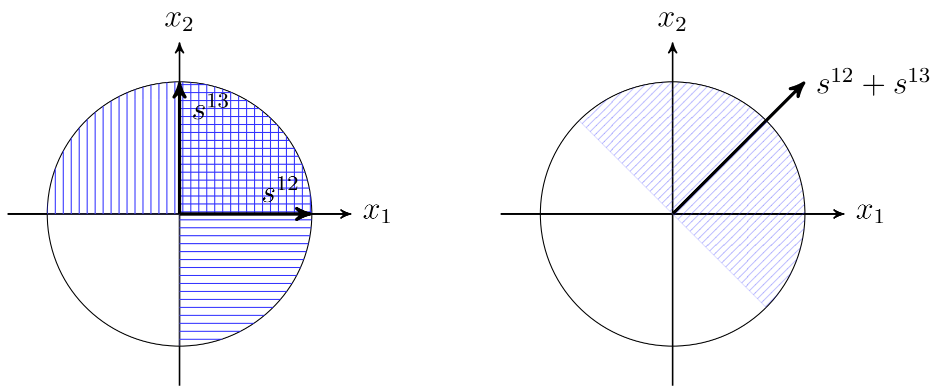
Figure 2. Nodes represent agents. An arc reflects that the "tail" agent wants so share the part of his private information that is specified on the label of the arc with the "head" agent.

sharing individual dimensions of their multidimensional private information, are: $\mathbf{s}^{12} = \mathbf{s}^{23} = \mathbf{s}^{31} = (1, 0)$ and $\mathbf{s}^{13} = \mathbf{s}^{32} = \mathbf{s}^{21} = (0, 1)$. We specify the information receiving weights $\mathbf{r}^{ij}$ later. Since agents are symmetric, we consider only agent 1, and for convenience, we omit the specific index of the agent. Figure 2 gives a visual representation of our example so far. Next, we analyze each agent's incentives at the information sharing stage.

In networks with targeted information sharing, since $s^{12} = (1, 0)$ and $s^{13} = (0, 1)$, agent 1 prefers to share his information $X = (x_1, x_2)$ with agent 2 if $x_1 \geq 0$ and with agent 3 if $x_2 \geq 0$ (see the left panel on Figure 3). Thus, the ex-ante utility for sharing information for agent 1 is $E(max(x_1, 0)) + E(max(x_2, 0)) = \frac{2(2+\sqrt{2})}{3\pi}$.

In networks with collective information sharing, since $s^{12} = (1, 0)$ and $s^{13} = (0, 1)$, agent 1 prefers to share his information $X = (x_1, x_2)$ with agents 2 and 3 if $x_1 + x_2 \geq 0$ (see the right panel on Figure 3). Thus, the ex-ante utility for sharing information for agent 1 is $E(max(x_1 + x_2, 0)) = \frac{4\sqrt{2}}{3\pi}$.

When comparing the two types of information sharing networks from agent 1's perspective, since $\frac{2(2+\sqrt{2})}{3\pi} > \frac{4\sqrt{2}}{3\pi}$, his ex-ante utility from sharing information is higher under targeted information sharing than under collective information sharing. However, recall that the ex-ante utility of both the agents and the network owner includes not just the benefits from sharing information, but also the benefits from receiving information. Next, we show that the ex-ante utility from receiving information can be smaller in networks with targeted information sharing than in networks with collective information sharing, which can lead to situations in which the ex-ante utility for both the agents and the owner is smaller in networks with targeted information sharing than in networks with collective information sharing.



a) Targeted sharing **b) Collective sharing**

Figure 3. Targeted versus collective information sharing. *Agent* 1*'s information* $X = (x_1, x_2)$ *is uniformly distributed within the unit circle. The left panel illustrates targeted information sharing: agent* 1 *prefers to share with agent* 2 *his (private) information in the half circle filled with horizontal lines, and agent* 1 *prefers to share with agent* 3 *his information in the half circle filled with vertical lines. The right panel illustrates collective information sharing: agent* 1 *prefers to share with both agents* 1 *and* 2 *his information in the half circle filled with diagonal lines.*

Let the information receiving weights be $r^{ij} = (1, 1)$ for all $i \neq j$. Given that agents 2 and 3 use the same information sharing strategies as agent 1, agent 1's expected utility from receiving information in the networks with targeted information sharing is $E(x_1 + x_2 | x_1 \geq 0) + E(x_1 + x_2 | x_2 \geq 0) = \frac{4(1+\sqrt{2})}{3\pi}$, while his expected utility from receiving information in networks with collective information sharing is $2E(x_1 + x_2 | x_1 + x_2 \geq 0) = \frac{8\sqrt{2}}{3\pi}$. Our calculations are summarized in Table 1.

Agent's benefits / Owner's utility	*Targeted*	*Collective*
Sharing benefits:	$\frac{2(2+\sqrt{2})}{3\pi}$	$\frac{4\sqrt{2}}{3\pi}$
Receiving benefits:	$\frac{4(1+\sqrt{2})}{3\pi}$	$\frac{8\sqrt{2}}{3\pi}$
Owner's utility:	$\frac{8+6\sqrt{2}}{\pi}$	$\frac{12\sqrt{2}}{\pi}$

Table 1. The utility of the agents and of the owner for targeted and for collective information sharing.

Table 1 shows that given an existing network (recall that we assumed that agents form the complete network with either unilateral or bilateral connections), the utilities of the agents and the utility of the owner may be smaller in networks with targeted information sharing than in networks with collective information sharing. If we maintain the assumption that for each agent the value of being connected to another agent is null and agents 2 and 3 have no information to share, then the same example also applies to endogenously formed networks. Hence, in general, there is no clear relation between the utility of the agents in networks with targeted information sharing and their utility in networks with collective information sharing. A similar statement applies for the owner. ■

Example 1 shows that in general, for an owner who wishes to maximize his utility and who may choose the rules of his network to correspond to either targeted information sharing or to collective information sharing, it is not clear what type of network to choose. The unclarity persists even if the network owner is not maximizing his own utility but instead is an altruistic social planner who wishes to maximize the welfare of the agents. In the following subsections, we restrict the utility functions of the agents such that all agents are either babblers or friends (see the last paragraph of Section 3.1) and we compare [networks with targeted versus collective information sharing] and [networks with unilateral versus bilateral connections].

5.1. Targeted vs. Collective Information Sharing

While our previous example shows that in general it is difficult to obtain unambiguous results, the following theorem shows that in babblers or friends networks, flexibility is unambiguously better. Intuitively, in babbler networks, because the information received is made irrelevant, targeted information sharing can be thought of as a kind of perfect information sharing discrimination.[6] In friends networks, because the information shared by an agent and received by another is identically valued by both agents, the interests of these agents are perfectly aligned. Therefore, the individual optimum and the joint optimum coincide.

Theorem 2. *Consider a babblers network or a friends network with unilateral (bilateral) connections. In any equilibrium of the two-stage game with collective information sharing,* {*the set of connections, each agent's utility, and the utility of the owner*} *are weakly smaller than in any (some) equilibrium of the two-stage game with targeted information sharing.*

[6] Consider the following analogy. Imagine the information to be shared is a good to be sold and the utility from sharing information is the price received for the good. Then, in babblers networks, targeted information sharing corresponds to first degree price discrimination, whereas collective information sharing corresponds to the law of one price.

We prove Theorem 2 by proving a series of four lemmas that compare targeted versus collective information sharing in: babbler networks with unilateral connections (Lemma 1), babbler networks with bilateral connections (Lemma 2), friends networks with unilateral connections (Lemma 3), and in friends networks with bilateral connections (Lemma 4).

Lemma 1. *Consider a babbler network with unilateral connections. In any equilibrium of the two-stage game with collective information sharing, {the set of connections, each agent's utility, and the utility of the owner} are weakly smaller than in any equilibrium of the two-stage game with targeted information sharing.*

To prove this lemma, let us consider a babblers network with unilateral connections. Let us consider some equilibrium of the two-stage game with collective information sharing. Then, the optimal set of agent i's unilateral connections N_i^* solves the maximization problem in (4). Assume $|N_i^*| \geq 2$ and let $k \in N_i^*$. We can then write the following set of inequalities

$$\begin{aligned}\sum_{j \in N_i^* \setminus k} c^{ij} + E(\max(\sum_{j \in N_i^* \setminus k} s^{ij}(x^i), 0)) + c^{ik} + E(\max(s^{ik}(x^i), 0)) \geq \\ \sum_{j \in N_i^*} c^{ij} + E(\max(\sum_{j \in N_i^*} s^{ij}(x^i), 0)) \geq \\ \sum_{j \in N_i^* \setminus k} c^{ij} + E(\max(\sum_{j \in N_i^* \setminus k} s^{ij}(x^i), 0)).\end{aligned} \tag{5}$$

The first inequality follows from the convexity of the maximum function and the second from the optimality of the set N_i^*. Comparing the first and the third expressions we deduce that

$$c^{ik} + E(\max(s^{ik}(x^i), 0)) \geq 0, \tag{6}$$

which by inequality (3) implies that agent i is connected to agent k in any equilibrium of the two-stage game with targeted information sharing.[7] Hence, the set of connections in any equilibrium of the two-stage game with collective information sharing is a subset of the set of connections in any equilibrium of the two-stage game with targeted information sharing.

Let us now compare agent i's utility from being connected to the set of agents N_i^* in the

[7] Note that for $|N_i^*| < 2$ there is no distinction between collective and targeted information sharing.

networks with collective and targeted information sharing. Since agent i can better tailor his information sharing decisions when information sharing is targeted, his utility is greater for the latter network, i.e.,

$$\sum_{j\in N_i^*} c^{ij} + E\big(\max(\sum_{j\in N_i^*} s^{ij}(x^i),0)\big) \leq \sum_{j\in N_i^*} c^{ij} + E\big(\sum_{j\in N_i^*} \max(s^{ij}(x^i),0)\big). \tag{7}$$

When information sharing is targeted, agent i might also be connected to some agents $k \in \mathcal{I}\backslash N_i^*$. These connections, however, necessarily satisfy (6) and, hence, agent i's utility in any equilibrium of the two-stage game with collective information sharing is smaller than in any equilibrium of the two-stage game with targeted information sharing. Summing across all agents, we also obtain that the revenue of the network owner utility in any equilibrium of the two-stage game with collective information sharing is smaller than in any equilibrium of the two-stage game with targeted information sharing. ■

Lemma 2. *Consider a babbler network with bilateral connections. In any equilibrium of the two-stage game with collective information sharing, {the set of connections, each agent's utility, and the utility of the owner} are weakly smaller than in any equilibrium of the two-stage game with targeted information sharing.*

To prove this lemma, let us consider a babblers network with bilateral connections. Let us consider some equilibrium of the two-stage game with collective information sharing. Then, the optimal set of agent i's unilateral connections N_i^* solves the maximization problem

$$\max_{N_i\subseteq N_{-i}^*} \big(\sum_{j\in N_i} c^{ij} + E(\max(\sum_{j\in N_i} s^{ij}(x^i),0))\big), \tag{8}$$

which is analogue to the maximisation problem (4) in Lemma 1. The proof then follows by the same arguments as in Lemma 1. ■

Lemma 3. *Consider a friends network with unilateral connections. In any equilibrium of the two-stage game with collective information sharing, {the set of connections, each agent's utility, and the utility of the owner} are weakly smaller than in any equilibrium of the two-stage game* *with targeted information sharing.*

To prove this lemma, let us consider a friends network with unilateral connections. Let us consider some equilibrium of the two-stage game with collective information sharing. Let N^* denote the set of connections in this equilibrium. As agents cannot influence the benefits from receiving information, the strategic decision of each agent is influenced only by the constant value of a connection and by the benefits from sharing information. But then, when considering *only* the benefits from information sharing, the same arguments as in Lemma 1 apply and the same conclusions obtain, i.e., if agent $ik \in N^*$, then agent i is connected to agent k in any equilibrium of the two-stage game with targeted information sharing, the set of connections in any equilibrium in the former game is smaller, and agents derive smaller benefits from sharing information in the two-stage game with collective information sharing.

Let us now consider the benefits from receiving information. Consider some agent k who is among i's connections, $k \in N_i^*$. But then, as we established above, in any equilibrium of the two-stage game with collective information sharing agent k derives smaller benefits from sharing information with i than in any equilibrium of the two-stage game with targeted information sharing

$$E\big(s^{ki}(x^k)\mathbf{1}^*_{ki}(x^k, N^*)\big) \leq E\big(max(s^{ki}(x^k), 0)\big), \tag{9}$$

where $\mathbf{1}^*_{ki}$ is an optimal information sharing strategy, i.e., $\mathbf{1}^*_{ki}(x^k, N^*) = 1$ if $\sum_{l \in N_k^*} s^{kl}(x^k) \geq 0$ and 0 otherwise. Since agents i and k are friends, this implies that i also enjoys smaller benefits from receiving information when information is shared collectively. Overall, in any equilibrium of the two-stage game with collective information sharing, agent i obtains a smaller utility than in any equilibrium of the two-stage game with targeted information sharing. Summing across all agents, the same comparison for the utility of the network owner follows straightforwardly.■

Lemma 4. *Consider a friends network with bilateral connections. In any equilibrium of the two-stage game with collective information sharing,* {*the set of connections, each agent's utility, and the utility of the owner*} *are weakly smaller than in any equilibrium of the two-stage game with targeted information sharing.*

To prove this lemma, let us consider a friends network with bilateral connections. Let us consider some equilibrium of the two-stage game with collective information sharing. Let N^* denote the set of connections in this equilibrium. Since now agent i needs the approval of other agents to form connections, the optimal set of i's connections N_i^* solves

$$\max_{N_i \subseteq N^*_{-i}} \big(\sum_{j \in N_i} c^{ij} + E(\max(\sum_{j \in N_i} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i} s^{ji}(x^j) \mathbf{1}^*_{ji}(x^j, N^*))\big), \tag{10}$$

where $\mathbf{1}^*_{ji}(x^j, N^*) = 1$ if $\sum_{l \in N_j^*} s^{jl}(x^j) \geq 0$ and 0 otherwise. Consider an agent k who is among agent i's connections $k \in N_i^*$. We then have the following set of inequalities

$$\begin{aligned}
\textstyle\sum_{j \in N_i^* \setminus \{k\}} c^{ij} &+ E(\max(\sum_{j \in N_i^* \setminus \{k\}} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i^* \setminus \{k\}} s^{ji}(x^j) \mathbf{1}^*_{ji}(x^j, N^*))\big) + \\
c^{ik} &+ E(\max(s^{ik}(x^i), 0)) + E(\max(s^{ki}(x^k), 0)) \geq \\
\textstyle\sum_{j \in N_i^*} c^{ij} &+ E(\max(\sum_{j \in N_i^*} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i^*} s^{ji}(x^j) \mathbf{1}^*_{ji}(x^j, N^*))\big) \geq \qquad (11) \\
\textstyle\sum_{j \in N_i^* \setminus \{k\}} c^{ij} &+ E(\max(\sum_{j \in N_i^* \setminus \{k\}} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i^* \setminus \{k\}} s^{ji}(x^j) \mathbf{1}^*_{ji}(x^j, N^*))\big),
\end{aligned}$$

where we have $\mathbf{1}^*_{ji}(x^j) = 1$ if $\sum_{l \in N_j^*} s^{jl}(x^j) \geq 0$, and 0 otherwise. The first inequality follows from convexity of the maximum function. Here we also use that agents are friends: the information agent i prefers to receive from k coincides with the information that k prefers to share with i. The second inequality follows from the optimality of set N_i^*: agent i does not want to drop any of his connections. Comparing the first inequality with the third, we obtain

$$c^{ik} + E(\max(s^{ik}(x^i), 0)) + E(\max(s^{ki}(x^k), 0)) \geq 0. \tag{12}$$

Hence, each agent i proposing the set of connections N_i^* and following his optimal strategies is an equilibrium of the two-stage game with targeted information sharing and bilateral connections. In this equilibrium, agents also receive at least as large benefits from sharing and receiving information. The same comparison holds for the revenue of the network owner. Thus, for any equilibrium of the two-stage game with collective information sharing, there exists some equilibrium of the two-stage game with targeted information sharing that results in at least the same set of connections, delivers larger utility to each agent, and yields a higher utility for the owner of the network. ■

5.2. Unilateral vs. Bilateral Connections

In this subsection, we compare unilateral versus bilateral connections when agents are babblers or friends. Unlike Theorem 2 in Subsection 5.1, however, agents being babblers or friends leads to different implications. We first consider babblers networks, and then friends newtorks.

Theorem 3. *Consider a babbler network.*

i. When information sharing is targeted, in any equilibrium of the two-stage game with unilateral connections, {the set of connections, each agent's utility, and the utility of the owner} are weakly larger than in any equilibrium of the two-stage game with bilateral connections.

ii. When information sharing is collective, in any equilibrium of the two-stage game with unilateral connections, each agent's utility is larger than in any equilibrium of the two-stage game with bilateral connections. The comparison with respect to the set of connections and the utility of the network owner is ambiguous.

To prove *i.*), let us consider a babblers network with targeted information sharing. Let us consider some equilibrium of the two-stage game with bilateral connections. In this equilibrium, if agents i and j form a bilateral connection ij, then they choose each other. Hence, the following inequalities are satisfied:

$$c^{ij} + E(\max(s^{ij}(x^i), 0)) \geq 0,$$

$$c^{ji} + E(\max(s^{ji}(x^i), 0)) \geq 0.$$

Thus, in any equilibrium of the two-stage game with unilateral connections, agent i establishes a unilateral connection with j, and j establishes a unilateral connection with i. Hence, the set of connections in any equilibrium of the two-stage game with bilateral connections is a subset of the set of connections in the two-stage game with unilateral connections, and the implications regarding each agent's utility and the utility of the owner immediately follow.

To prove *ii.*), let us consider a babblers network with collective information sharing. For each agent, the set of connections formed in any equilibrium of the two-stage game with bilateral connections is also available in the two-stage game with unilateral connections. Thus,

a) Bilateral connections **b) Unilateral connections**

Figure 4. Bilateral versus unilateral connections in babblers networks with collective information sharing. Nodes represent agents and arcs represent connections. The number of connections and the utility of the network owner can be smaller in networks with unilateral connections than in networks with bilateral connections.

in any equilibrium of the two-stage game with unilateral connections, each agent has a weakly larger utility than in any equilibrium of the two-stage game with bilateral connections. The implications for the set of connections and the revenue of the network owner, however, can be ambiguous, as the following example illustrates. ■

Example 2. Bilateral versus unilateral connections in babblers networks with collective information sharing.

Let us consider a babblers network with collective information. Let $\mathcal{I}_{\prime} = \{a, 1, 2, 3\} \cup \{0\}$. Agent a has a constant value from connecting to 1, but not to the other agents, i.e., $c^{a1} = 1$, $c^{a2} = c^{a3} = 0$; agent a prefers to share information with 1 that is opposite to the information he prefers to share 2 and 3, i.e. $s^{a1} = (-1, -1), s^{a2} = (1, 1), s^{a3} = (1/2, 1/2)$; agent a's information is distributed such that if he optimally shares with 1 his utility is $E(\max(-x_1 - x_2, 0) = 1$, and if he optimally shares with 2 and 3 his total utility is $3/2E(\max(x_1+x_2, 0) = 3/2$.

Agent 1 prefers to remain unconnected and to not share anything, i.e., $c^{1a} = c^{12} = c^{13} = -\infty$, $s^{1a} = s^{12} = s^{13} = (0, 0)$. Agents 2 and 3 prefer to share any information and they prefer to be connected only with a, i.e., $c^{2a} = c^{3a} = 1$, and the constant value from connecting to any other agent are infinitely negative.

In the two-stage game with bilateral connections, agent a cannot connect with 1 because 1 blocks the connection. Thus, in the equilibrium with the largest number of connections, agent a connects 2 and 3 (see the left panel on Figure 4), and the utility of the network owner is $3/2$.

In the two-stage game with unilateral connections, in the unique equilibrium, agent a connects to 1 and obtains a large fixed benefit, i.e., the constant value, from this connection. Agent a does not want to connect to 2 and 3 because they impose a negative externality: agent a's utility from connecting to all three agents is 1.5 compared to his utility of 2 from connection to agent 1 only (see the right panel on Figure 4). Also, note that the utility of the owner of the network is null as there is no information shared among the agents.

Thus, overall, the network with bilateral connections may have an equilibrium with more connections and may generate a higher utility for the owner. ■

Next, we compare networks with unilateral and bilateral connections when agents are friends.

Theorem 4. *Consider a friends network with targeted (collective) information sharing. The comparison in terms of {the set of connections, each agent's utility, and the utility of the network owner} between an equilibrium of the two-stage game with unilateral connections and some equilibrium of the two-stage game with bilateral connections is ambiguous.*

Let us consider a friends network wither either targeted or collective information sharing. The result that the equilibrium of a two-stage game with unilateral connections can yield more connections, a higher agent's utility, and more utility for the owner, than an equilibrium of a two-stage game with bilateral connections is straightforward.

Below, we show that there may exist an equilibrium of the two-stage game with bilateral connections which may have more connections and yield a higher utility for the agents and for the owner than the the unique equilibrium of a two-stage game with unilateral connections. Intuitively, we provide an example of a setting with only two agents (hence, targeted and collective information sharing coincide) in which the agents fail to agree on a connection in the unique equilibrium of the two-stage game with unilateral connections, whereas there exists and an equilibrium of the two-stage game with bilateral connections in which a connection is formed.

Let us consider two symmetric agents. Each agent has negative fixed benefits from establishing the connection equal to -1.5. The agents are friends and enjoy sharing and receiving the same type of information. We assume that when agents share information optimally, their benefits from sharing and receiving information are equal to 1. Agent utilities from various

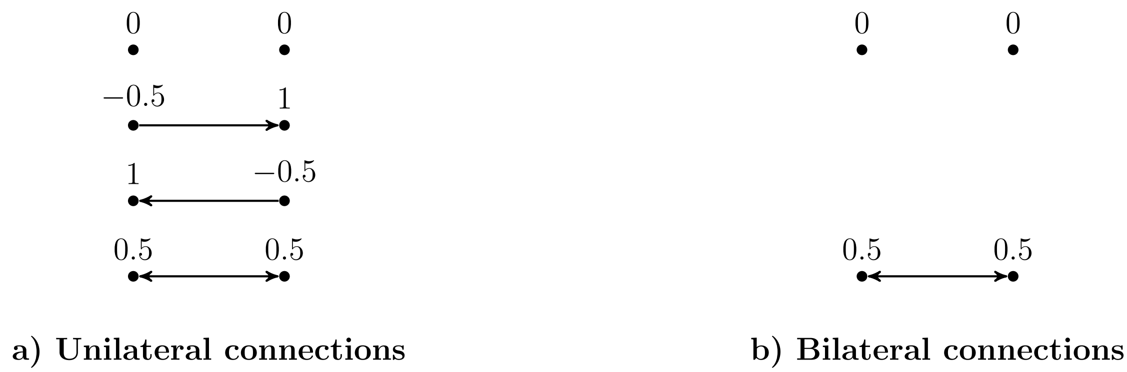


Figure 5. Unilateral versus bilateral connections in friends networks. Nodes represent agents, arcs represent possible connections, and the numbers represent agents' utilities for the respective possible connections. In the two-stage game with unilateral connections, no connection is the unique equilibrium. In the two-stage game with bilateral connections, the possible connection depicted may be an equilibrium.

network configurations for both unilateral and bilateral networks are depicted on Figure 5.

Consider the two-stage game with unilateral connections. Forming the connection is costly for each agent: fixed benefits are -1.5 and benefits from information sharing equal only 1. Hence, each agent prefers to drop his connection independently of the action of the other agent. Thus, in equilibrium, no connection is formed.

Consider the two-stage game with unilateral connections. Now each agent realizes that if he drops his connection he does not obtain benefits from receiving information. Thus, both agents connecting to each other is an equilibrium. ■

6. Conclusion

We introduced a *new* model for reasoning about the design of some of the most important options that networks make available to their users. Our model is deliberately parsimonious. We employ what we believe is the minimal formalism needed for a first–order approximation of the most important options related to network formation and information sharing that are made available by online social networks such as Facebook or Google+ to their users. Apart from the design options, our model is also the first to allow users' preferences to account for not just the information shared but also for the one received.

Surprisingly, we found that *in general*, enhancing and promoting the tools for targeted information sharing is *not* necessarily in the benefit of the users or in the benefit of the owner of the network. Finding out what combination of network design options is best suited for the owners of networks such as Facebook or Google+ depends on the preferences of the typical user of the network. Pinning down the preferences of the typical user is a delicate empirical question which we can't answer with certainty in absence of direct access to data but we conjecture that a friends network with bilateral connections is best suited to proxy today's real life online social networks. For this environment, we found that in equilibrium targeted information sharing is at least as good as collective information sharing, which may explain why Google+ chose to refine and emphasize its "circles" option.

While we aimed to keep our model as general as possible, we did make some simplifying assumptions.

First, we required each agent's utility be separable in terms of benefits from information sharing and receiving. Our proofs, however, use only the implication stated in Remark 1: given the same set of connections, each agent derives higher benefits from targeted information sharing compared with collective information sharing. Hence, any non-separable utility specification that satisfies the above implication should lead to the same results.

Second, we used a very permissive notion of Nash equilibrium to analyze the network formation. We conjecture that our main results extend to other solution concepts such as pair-wise stability Jackson and Wolinsky (1996), bilateral rationality Kim and Wong (2007), set-wise stability Echenique and Oviedo (2006) or coalition-proof Nash equilibrium Dutta and Mutuswami (2005).

Finally, we considered targeted and collective information sharing, which are two extreme types of information sharing. The possibility of sharing information with each of the connections relates to the assumption there is no costs associated to writing personal messages. In the presence of such costs, each agent might prefer to form several groups of agents to share similar information with, i.e. a "family" group, a "work" group, etc. We believe that our analysis extends to the environment with several, but limited number of groups. We leave this question for future research.

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