Assessing Asset Pricing Models
using Revealed Preference*

Jonathan B. Berk
Stanford University and NBER

Jules H. van Binsbergen
University of Pennsylvania and NBER

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Abstract

We propose a new method of testing asset pricing models that relies on using quantities rather than prices or returns. We use the capital flows into and out of mutual funds to infer which risk model investors use. We derive a simple test statistic that allows us to infer, from a set of candidate models, the model that is closest to the true risk model. Using this methodology, we find that of the models most commonly used in the literature, the Capital Asset Pricing Model is the closest. Given our current state of knowledge, we argue that the Capital Asset Pricing Model is the appropriate method to use to calculate the cost of capital of an investment opportunity. Despite the Capital Asset Pricing Model's success, we also document that a large fraction of mutual fund flows remain unexplained by existing asset pricing models.

The starting point of this paper is the insight that if an asset pricing model correctly prices risk, investors must be using it. Based on this idea we develop a novel test of asset pricing models.

All capital asset pricing models assume that investors compete fiercely with each other to find positive net present value investment opportunities, and in doing so, eliminate them. As a consequence of this competition, equilibrium prices are set so that the expected return of every asset is solely a function of its risk. When a positive net present value (NPV) investment opportunity presents itself in capital markets (that is, an asset is mispriced relative to the model investors are using) investors react by submitting buy or sell orders until the opportunity no longer exists (the mispricing is removed). These buy and sell orders reveal the preferences of investors and therefore they reveal which asset pricing model investors are using. By observing whether or not buy and sell orders occur in reaction to the existence of positive net present value investment opportunities as defined by a particular asset pricing model, one can infer whether investors price risk using that asset pricing model.

There are two criteria that are required to implement this methodology. First, one needs a mechanism that identifies positive net present value investment opportunities. Second, one needs to be able to observe investor reactions to these opportunities. We demonstrate that we can satisfy both criteria if we implement the methodology using mutual fund data. Under the assumption that a particular asset pricing model holds, we use the main insight from Berk and Green (2004) to show that positive (negative) abnormal return realizations in a mutual fund investment must be associated with positive net present value buying (selling) opportunities. We then measure investor reactions to these opportunities by observing the subsequent capital flow into (out of) mutual funds.

Using this methodology, we derive a simple test statistic that allows us to infer, from a set of candidate models, the model that is closest to the true asset pricing model. Our test can be implemented by running a simple univariate ordinary least squared regression using the t-statistic to assess statistical significance. We take as the set of candidate models, the Capital Asset Pricing Model (CAPM), originally derived by Sharpe (1964), Lintner (1965) and Mossin (1966), the reduced form factor models specified by Fama and French (1993) and Carhart (1997) and the dynamic equilibrium models derived by Merton (1973), Breeden (1979), Campbell and Cochrane (1999) and Bansal and Yaron (2004). We find that the CAPM is the closest model to the true model. Importantly, the CAPM better explains flows than no model at all, indicating that investors do price risk. Furthermore, it also outperforms a naive model in which investors ignore beta and simply chase any outperformance relative to the market portfolio. This result suggests

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that investors measure risk using the CAPM beta. However, much of the flows in and out of mutual funds remain unexplained. To that end the paper leaves as an unanswered question whether the unexplained part of flows result because investors use a superior, yet undiscovered, risk model, or whether investors use other, non-risk based, criteria to make investment decisions.

It is important to emphasize that implementing our test requires accurate measurement of the variables that determine the Stochastic Discount Factor (SDF). In the case of the CAPM, the SDF is measured using market prices which contain little or no measurement error, and more importantly, can be observed by investors as accurately as by empiricists. Testing the dynamic equilibrium models relies on observing variables such as consumption, which investors can measure precisely (they presumably know their own consumption) but empiricists cannot, particularly over short horizons. Consequently our tests cannot differentiate whether these models underperform because they rely on variables that are difficult to measure, or because the underlying assumptions of these models are flawed.

Ultimately, the reason financial economists are interested in discovering the true risk model is that it is required to calculate the cost of capital of an investment opportunity. It is tempting to assume that a model that better explains cross-sectional variation in average asset returns is a better model to use to calculate the cost of capital. The problem with this line of thinking is that it assumes that all cross-sectional variation in average returns results from risk differences. If not all variation in average returns results from risk differences, a model that better explains this cross sectional variation is not necessarily a better model to use to adjust for risk. Because the flow of funds reveals the risk preferences of investors, our methodology allows researchers to determine whether cross sectional variation not explained by a model represents an omitted risk factor. If this unexplained cross sectional variation does not represent an omitted risk factor, then even if a new model can explain this variation, it is inappropriate to use that model to calculate the cost of capital. So, for example, the fact the reduced-form factor models do not outperform the CAPM implies that these additional factors should not enter the cost of capital calculation. What our empirical work shows is that, given our current level of knowledge, the appropriate way to calculate the cost of capital of an investment opportunity is to use the CAPM.

Because we implement our methodology using mutual fund data, one might be tempted to conclude that our tests only reveal the risk preferences of mutual fund investors, rather than all investors. But this is not the case. When an asset pricing model correctly prices risk, it rules out positive net present value investment opportunities in all markets. Even

if no investor in the market with a positive net present value opportunity uses the asset pricing model under consideration, so long as there are investors in other markets that use the asset pricing model, those investors will recognize the positive net present value opportunity and will act to eliminate it. That is, if our test rejects a particular asset pricing model, we are not simply rejecting the hypothesis that mutual fund investors use the model, but rather, we are rejecting the hypothesis that any investor who could invest in mutual funds uses the model.

The first paper to use mutual fund flows to infer investor preferences is Guercio and Tkac (2002). Although the primary focus of their paper is on contrasting the inferred behavior of retail and institutional investors, that paper documents flows respond to outperformance relative to the CAPM. The paper does not consider other risk models. In work subsequent to ours, Barber, Huang, and Odean (2014) use our approach and confirm our result (implementing the approach using a different methodology) that the investors use the CAPM rather than the other reduced form factor models that have been proposed. They do not consider the dynamic equilibrium models, nor do they consider the possibility that investors use no model at all, and so do not show that risk-based models better explain flows than either the behavior model that investors just chase passed returns or a model of risk neutrality.[1]

1   A New Asset Pricing Test

The core idea that underlies every financial asset pricing model in economics is that prices are set by agents chasing positive net present value investment opportunities. When financial markets are perfectly competitive, these opportunities are competed away so that, in equilibrium, prices are set to ensure that no positive net present value opportunities exist. Under the neoclassical assumptions that underly these models, prices respond to the arrival of new information by instantaneously adjusting to eliminate any positive net present value opportunities that arise. It is important to appreciate that this price adjustment process is part of all asset pricing models, either explicitly (if the model is dynamic) or implicitly (if the model is static). The output of all these models – a prediction about expected returns – relies on the assumption that this price adjustment process occurs.

The importance of this price adjustment process has long been recognized by financial economists and forms the basis of the event study literature. In that literature, the

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[1]Readers interested in the exact chronology can consult "Note on the relation between the chronology of Barber, Huang and Odean and this paper" located at `http://www.gsb.stanford.edu/faculty-research/working-papers/assessing-asset-pricing-models-using-revealed-preference`.

asset pricing model is assumed to be correctly identified. In that case, because there are no positive net present value opportunities, the price change that results from new information (i.e., the part of the change not explained by the asset pricing model) measures the value of the new information.

Because prices always adjust to eliminate positive net present value investment opportunities, under the correct asset pricing model, expected returns are determined by risk alone. Modern tests of asset pricing theories test this powerful insight using return data. Rejection of an asset pricing theory occurs if positive net present value opportunities are detected, or, equivalently, if investment opportunities can be found that consistently yield returns in excess of the expected return predicted by the asset pricing model. The most important shortcoming in interpreting the results of these tests is that the empiricist is never sure that a positive net present value investment opportunity that is identified *ex post* was actually available *ex ante*.[2]

An alternative testing approach, that does not have this shortcoming, is to identify positive net present value investment opportunities *ex ante* and test for the existence of an investor response. That is, do investors react to the existence of positive net present value opportunities that result from the revelation of new information? Unfortunately, for most financial assets, investor responses to positive net present value opportunities are difficult to observe. As Milgrom and Stokey (1982) show, the price adjustment process can occur with no transaction volume whatsoever, that is, competition is so fierce that no investor benefits from the opportunity. Consequently, for most financial assets the only observable evidence of this competition is the price change itself. Thus testing for investor competition is equivalent to standard tests of asset pricing theory that use return data to look for the elimination of positive net present value investment opportunities.

The key to designing a test to directly detect investor responses to positive net present value opportunities is to find an asset for which the price is fixed. In this case the market equilibration must occur through volume (quantities). A mutual fund is just such an asset. The price of a mutual fund is always fixed at the price of its underlying assets, or the net asset value (NAV). In addition, fee changes are rare. Consequently, if, as a result of new information, an investment in a mutual fund represents a positive net present value investment opportunity, the only way for investors to eliminate the opportunity is by trading the asset. Because this trade is observable, it can be used to infer investments investors believe to be positive net present value opportunities. One can then compare those investments to the ones the asset pricing model under consideration identifies to be positive net present value and thereby infer whether investors are using the asset

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[2]For an extensive analysis of this issue, see Harvey, Liu, and Zhu (2014).

pricing model. That is, by observing investors' revealed preferences in their mutual fund investments, we are able to infer information about what (if any) asset pricing model they are using.

1.1   The Mutual Fund Industry

Mutual fund investment represents a large and important sector in U.S. financial markets. In the last 50 years there has been a secular trend away from direct investing. Individual investors used to make up more than 50% of the market, today they are responsible for barely 20% of the total capital investment in U.S. markets. During that time, there has been a concomitant rise in indirect investment, principally in mutual funds. Mutual funds used to make up less than 5% of the market, today they make up 1/3 of total investment.[3] Today, the number of mutual funds that trade in the U.S. outnumber the number of stocks that trade.

Berk and Green (2004) derive a model of how the market for mutual fund investment equilibrates that is consistent with the observed facts.[4] They start with the observation that the mutual fund industry is like any industry in the economy — at some point it displays decreasing returns to scale.[5] Given the assumption under which all asset pricing models are derived (perfectly competitive financial markets), this observation immediately implies that all mutual funds must have enough assets under management so that they face decreasing returns to scale. When new information arrives that convinces investors that a particular mutual fund represents a positive net present value investment, investors react by investing more capital in the mutual fund. This process continues until enough new capital is invested to eliminate the opportunity. As a consequence, the model is able to explain two robust empirical facts in the mutual fund literature: that mutual fund flows react to past performance while future performance is largely unpredictable.[6] Investors chase past performance because it is informative: mutual fund managers that do well (poorly) have too little (much) capital under management. By competing to take advantage of this information, investors eliminate the opportunity to predict future performance.

A key assumption of the Berk and Green (2004) model is that mutual fund managers

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[3]See French (2008).

[4]Stambaugh (2014) derives a general equilibrium version of this model based on the model in Pastor and Stambaugh (2012).

[5]Pastor, Stambaugh, and Taylor (2014) provide empirical evidence supporting this assumption.

[6]An extensive literature has documented that capital flows are responsive to past returns (see Chevalier and Ellison (1997) and Sirri and Tufano (1998)). Yet future investor returns are largely unpredictable (see Carhart (1997)).

are skilled and that this skill varies across managers. Berk and van Binsbergen (2013) verify this fact. They demonstrate that such skill exists and is highly persistent. More importantly, for our purposes, they demonstrate that mutual fund flows contain useful information. Not only do investors systematically direct flows to higher skilled managers, but managerial compensation, which is primarily determined by these flows, predicts future performance as far out as 10 years. Investors know who the skilled managers are and compensate them accordingly. It is this observation that provides the starting point of our analysis. Because the capital flows into mutual funds are informative, they reveal the asset pricing model investors are using.

1.2 Private Information

Most asset pricing models are derived under the assumption that all investors are symmetrically informed. Hence, if one investor faces a positive NPV investment opportunity, all investors face the same opportunity and so it is instantaneously removed by competition. The reality is somewhat different. The evidence in Berk and van Binsbergen (2013) of skill in mutual fund management implies that at least some investors have access to different information or have different abilities to process information. As a result, not all positive net present value investment opportunities are instantaneously competed away.

As Grossman (1976) argued, in a world where there are gains to collecting information and information gathering is costly, not everybody can be equally informed in equilibrium. If everybody chooses to collect information, competition between investors ensures that prices reveal the information and so information gathering is unprofitable. Similarly, if nobody collects information, prices are uninformative and so there are large profits to be made collecting information. Thus, in equilibrium, investors must be differentially informed (see, e.g., Grossman and Stiglitz (1980)). Investors with the lowest information gathering costs collect information so that, *on the margin*, what they spend on information gathering, they make back in trading profits. Presumably these investors are few in number so that the competition between them is limited, allowing for the existence of prices that do not fully reveal their information. As a result, information gathering is a positive net present value endeavor for a limited number of investors.

The existence of asymmetrically informed investors poses a challenge for empiricists wishing to test asset pricing models derived under the assumption of symmetrically informed investors. Clearly, the empiricist's information set matters. For example, asset pricing models fail under the information set of the most informed investor, because the key assumption that asset markets are competitive is false under that information set.

Consequently, the standard in the literature is to assume that the information set of the uninformed investors only contains publicly available information all of which is already impounded in all past and present prices (or returns), and to conduct the test under that information set. For now, we will adopt the same strategy but will revisit this assumption in Section 5.2, where we will explicitly consider the possibility that the majority of investors' information sets includes more information than just what is already impounded in past and present prices.

1.3 Methodology

To formally derive our testing methodology, let q_{it} denote assets under management (AUM) of fund i at time t and let θ_i denote a parameter that describes the skill of the manager of fund i.[7] At time t, investors use the time t information set I_t to update their beliefs on θ_i resulting in the distribution function $g_t(\theta_i)$ implying that the expectation of θ_i at time t is:

$$\bar{\theta}_{it} \equiv E\left[\theta_i \mid I_t\right] = \int \theta_i \, g_t\left(\theta_i\right) d\theta_i. \tag{1}$$

We assume throughout that $g_t(\cdot)$ is not a degenerate distribution function. Let R_{it}^n denote the excess return (that is, the net-return in excess of the risk free rate) earned by investors between time $t-1$ and t. Let R_{it}^B denote the risk adjustment prescribed by the asset pricing model over the same time interval. Note that q_{it}, R_{it}^n and R_{it}^B are elements of I_t. Let $\alpha_{it}(q)$ denote investors' subjective expectation of the risk adjusted return they make when investing in fund i that has size q between time t and $t+1$, also commonly referred to as the *net alpha*:

$$\alpha_{it}(q) = \bar{\theta}_{it} - h_i\left(q\right), \tag{2}$$

where $h_i\left(q\right)$ is a strictly increasing function of q, reflecting the fact that, under the assumptions underlying every asset pricing model, all mutual funds must face decreasing returns to scale in equilibrium. In equilibrium, the size of the fund q_{it} adjusts to ensure that there are no positive net present value investment opportunities so $\alpha_{it}(q_{it}) = 0$ and

$$\bar{\theta}_{it} = h_i\left(q_{it}\right). \tag{3}$$

At time $t+1$, the investor observes the manager's return outperformance,

$$\varepsilon_{it+1} \equiv R_{it+1}^n - R_{it+1}^B, \tag{4}$$

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[7]For expositional simplicity we do not allow θ_i to depend on q_{it}. This assumption is without loss of generality under the assumption that either the manager is allowed to borrow or can set his own fee, see Berk and Green (2004).

which is a signal that is informative about θ_i. The conditional distribution function of ε_{it+1} at time t, $f(\varepsilon_{it+1}|\alpha_{it}(q_{it}))$, satisfies the following condition in equilibrium:

$$E[\varepsilon_{it+1} \mid I_t] = \int \varepsilon_{it+1} f(\varepsilon_{it+1}|\alpha_{it}(q_{it})) \, d\varepsilon_{it+1} = \alpha_{it}(q_{it}) = 0. \tag{5}$$

Our testing methodology relies on the insight that good news, that is, $\varepsilon_{it} > 0$, implies good news about θ_i and bad news, $\varepsilon_{it} < 0$, implies bad news about θ_i. The following proposition shows that, in expectation, this condition holds generally. That is, on average, a positive (negative) realization of ε_{it} leads to a positive (negative) update on θ_i implying that before the capital response, the fund's alpha will be positive (negative).

Proposition 1 *On average, a positive (negative) realization of ε_{it} leads to a positive (negative) update on θ_i:*
$$E[\alpha_{it+1}(q_{it})\varepsilon_{it+1} \mid I_t] > 0.$$

PROOF:

$$
\begin{aligned}
E[\alpha_{it+1}(q_{it})\varepsilon_{it+1} \mid I_t] &= E[E[\alpha_{it+1}(q_{it})\varepsilon_{it+1} \mid \theta_i] \mid I_t] \\
&= E[(\theta_i - h_i(q_{it})) \, E[\varepsilon_{it+1} \mid \theta_i] \mid I_t] \\
&= E[(\theta_i - h_i(q_{it})) \, (\theta_i - h_i(q_{it})) \mid I_t] \\
&> 0.
\end{aligned}
$$

∎

Unfortunately this proposition is not directly testable because $\alpha_{it+1}(q_{it})$ is not observable. Instead what we do observe are the capital flows that result when investors update their beliefs. Our next objective is to restate the result in Proposition 1 in terms of capital flows.

What Proposition 1 combined with (3) tells us is that positive (negative) news must, on average, lead to an inflow (outflow). However, without further assumptions, we cannot quantify the magnitude of the capital response. Rather than lose generality by making further assumptions, we can sidestep this issue by focusing only on the direction of the capital response. With that in mind we begin by first defining the function that returns the sign of a real number, taking values 1 for a positive number, -1 for a negative number and zero for zero:

$$\phi(x) \equiv \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

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Next, let the flow of capital into mutual fund i at time t be denote by F_{it}, that is,

$$F_{it+1} \equiv q_{it+1} - q_{it}.$$

The following lemma proves that the sign of the capital inflow and the alpha inferred from the information in ε_{it+1} must be the same.

Lemma 1 *The sign of the capital inflow and the alpha inferred from the information in ε_{it+1} must be the same:*

$$\phi(F_{it+1}) = \phi(\alpha_{it+1}(q_{it})).$$

PROOF:

$$
\begin{aligned}
\phi(\alpha_{it+1}(q_{it})) &= \phi(\alpha_{it+1}(q_{it}) - \alpha_{it+1}(q_{it+1})) \\
&= \phi(h(q_{it+1}) - h(q_{it})) \\
&= \phi(q_{it+1} - q_{it}) \\
&= \phi(F_{it+1}).
\end{aligned}
$$

where the first line follows from (5) and the third line flows from the fact that $h(q)$ is a strictly increasing function. ∎

We are now ready to restate Proposition 1 as a testable prediction.

Proposition 2 *The regression coefficient of the sign of the capital inflows on the sign of the realized return outperformance is positive, that is,*

$$\beta_{F\varepsilon} \equiv \frac{cov(\phi(F_{it+1}), \phi(\varepsilon_{it+1}))}{var(\phi(\varepsilon_{it+1}))} > 0. \tag{6}$$

PROOF: See appendix. ∎

This proposition provides a testable prediction and thus a new method to reject an asset pricing model. Under our methodology, we define a model as working when investors' revealed preferences indicate that they are using that model to update their inferences of positive net present value investment opportunities. Because flows reveal investor preferences, a measure of whether investors are using a particular asset pricing model is the fraction of decisions for which outperformance (as defined by the model) implies capital inflows and underperformance implies capital outflows. The next Lemma shows that $\beta_{F\varepsilon}$ is a simple linear transformation of this measure.

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Lemma 2 *The regression coefficient of the sign of the capital inflows on the sign of the realized return outperformance can be expressed as follows:*

$$
\begin{aligned}
\beta_{F\varepsilon} &= \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] + \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}\right) = -1\right] - 1 \\
&= \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right].
\end{aligned}
$$

PROOF: See appendix. ∎

Practically, it is likely that not all asset pricing models under consideration will be rejected. In that case a natural question to ask is, of the models that cannot be rejected, which model is "best." By best we mean the model that comes closest to pricing risk correctly. To formalize this concept, consider a set of candidate risk models, indexed by $c \in C$, such that the risk adjustment of each model is given by R_{it}^c, so the risk-adjusted performance is given by:

$$
\varepsilon_{it}^c = R_{it}^n - R_{it}^c.
$$

Because at most only one element of the set of candidate risk models can be the true risk model, the rest of the models in C do not fully capture risk. We refer to these models as *false risk models*. We will maintain the assumption throughout this paper that conditional on knowing the true risk model, any false risk model cannot have additional explanatory power for flows:

$$
\Pr\left[\phi\left(F_{it}\right) \mid \phi\left(\varepsilon_{it}\right), \phi\left(\varepsilon_{it}^c\right)\right] = \Pr\left[\phi\left(F_{it}\right) \mid \phi\left(\varepsilon_{it}\right)\right]. \tag{7}
$$

Under the Null that the asset pricing model holds, there are no other reasons for aggregate flows to occur other than the existence of a positive NPV opportunity. For a false risk model $c \in C$, let β_{Fc} be the signed flow-performance regression coefficient of that model, that is,

$$
\beta_{Fc} \equiv \frac{cov\left(\phi\left(F_{it}\right), \phi\left(\varepsilon_{it}^c\right)\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)}.
$$

Notice, from Lemma 2, that $-1 \leq \beta_{Fc} \leq 1$. When outperformance relative to asset pricing model c is uninformative about flows, that is, $\Pr\left[\phi\left(F_{it}\right) \mid \phi\left(\varepsilon_{it}\right)\right] = \Pr\left[\phi\left(F_{it}\right)\right]$, then $\beta_{Fc} = 0$.

The next proposition proves that the regression coefficient of the true model must exceed the regression coefficient of a false model.

Proposition 3 *The regression coefficient of the sign of the capital inflows on the sign of the realized return outperformance is maximized under the true model, that is, for any*

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false model c,

$$\beta_{F\varepsilon} > \beta_{Fc}.$$

PROOF: See appendix. ∎

We are now ready to formally define what we mean by a model that comes closest to pricing risk. The following definition defines the best model as the model that maximizes the fraction of times outperformance by the candidate model implies outperformance by the true model and the fraction of times underperformance by the candidate model implies underperformance by the true model.

Definition 1 *Model c is a better approximation of the true asset pricing model than model d if and only if:*

$$\Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] + \Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right]$$
$$> \quad \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^d\right) = 1\right] + \Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^d\right) = -1\right]. \qquad (8)$$

With this definition in hand we now show that the models can be ranked by their regression coefficients.

Proposition 4 *Model c is a better approximation of the true asset pricing model than model d if and only if $\beta_{Fc} > \beta_{Fd}$.*

PROOF: See appendix. ∎

The next proposition provides an easy method for empirically distinguishing between candidate models.

Proposition 5 *Consider an OLS regression of $\phi\left(F_{it}\right)$ onto $\frac{\phi\left(\varepsilon_{it}^c\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} - \frac{\phi\left(\varepsilon_{it}^d\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)}$:*

$$\phi\left(F_{it}\right) = \gamma_0 + \gamma_1 \left(\frac{\phi\left(\varepsilon_{it}^c\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} - \frac{\phi\left(\varepsilon_{it}^d\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)} \right) + \xi_{it}$$

The coefficient of this regression is positive, that is, $\gamma_1 > 0$, if and only if, model c is a better approximation of the true asset pricing model than model d.

PROOF: See appendix ∎

2 Asset Pricing Models

All asset pricing models assume competitive capital markets and fully rational investors. Because we assume, under our Null Hypothesis, that the asset pricing model holds we make the same assumptions. Although these assumptions are clearly restrictive, it is important to emphasize that they are not part of our testing methodology, but instead are imposed on us by the models we test. Conceivably our methodology could be applied to behavioral models in which case these assumptions would not be required.

Our testing methodology can be applied to both reduced-form asset pricing models, such as the factor models proposed by Fama and French (1993) and Carhart (1997), as well as to dynamic equilibrium models, such as the consumption CAPM (Breeden (1979)), habit formation models (Campbell and Cochrane (1999)) and long run risk models (Bansal and Yaron (2004)). For the CAPM and factor models, R_{it}^B is specified by the beta relationship. We regress the excess returns to investors, R_{it}^n, on the risk factors over the life of the fund to get the model's betas. We then use the beta relation to calculate R_{it}^B at each point in time. For example, for the Fama-French-Carhart factor specification, the risk adjustment R_{it}^B is then given by:

$$R_{it}^B = \beta_i^{mkt}\text{MKT}_t + \beta_i^{sml}\text{SML}_t + \beta_i^{hml}\text{HML}_t + \beta_i^{umd}\text{UMD}_t,$$

where MKT_t, SML_t, HML_t and UMD_t are the realized excess returns on the four factor portfolios defined in Carhart (1997). Using this risk adjusted return, we calculate (4) over a T-period horizon ($T > 1$) as follows:

$$\varepsilon_{it} = \prod_{s=t-T+1}^{t} \left(1 + R_{is}^n - R_{it}^B\right) \quad - \quad 1. \tag{9}$$

In any dynamic equilibrium model returns must satisfy the following Euler equation in equilibrium:

$$E_t[M_{t+1}R_{it+1}^n] = 0, \tag{10}$$

where $M_t > 0$ is the stochastic discount factor (SDF). When this condition is violated a positive net present value investment opportunity exists. The dynamic equilibrium models are all derived under the assumption of a representative investor. Of course, this assumption does not presume that all investors are identical. When investors are not identical, it is possible that they do not share the same SDF. Even so, it is important to appreciate that, in equilibrium, all investors nevertheless agree on the existence of a

positive net present value investment opportunity. That is, if (10) is violated, it is violated for every investor's SDF.[8] Because our testing methodology only relies on the existence of this net present value investment opportunity, it is robust to the existence of investor heterogeneity.

The outperformance measure for fund i at time t is therefore

$$\alpha_{it} = E_t[M_{t+1}R^n_{it+1}]. \tag{11}$$

Notice that $\alpha_{it} > 0$ is a buying (selling) opportunity and so capital should flow into (out of) such opportunities. We calculate the outperformance relative to the equilibrium models over a T-period horizon as follows:

$$\varepsilon_{it} = \frac{1}{T} \sum_{s=t-T+1}^{t} M_s R^n_{is}. \tag{12}$$

Notice that in this case T must be greater than one because when $T = 1$, $\phi(\varepsilon_{it})$ is not a function of M_s. To compute these outperformance measures, we must compute the stochastic discount factor for each model at each point in time. For the consumption CAPM, the stochastic discount factor is:

$$M_t = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\gamma},$$

where β is the subjective discount rate and γ is the coefficient of relative risk aversion. The calibrated values we use are given in the top panel of Table 1. We use the standard data from the Bureau of Economic Analysis (NIPA) to compute consumption growth of non-durables and services.

For the long-run risk model as proposed by Bansal and Yaron (2004), the stochastic discount factor is given by:

$$M_t = \delta^\theta \left(\frac{C_t}{C_{t-1}} \right)^{-\frac{\theta}{\psi}} (1 + R^a_t)^{-(1-\theta)}.$$

where R^a_t is the return on aggregate wealth and where θ is given by:

$$\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

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[8]In an incomplete market equilibrium investors may use different SDFs but the projection of each investor's SDF onto the asset space is the same.

The parameter ψ measures the intertemporal elasticity of substitution (IES). To construct the realizations of the stochastic discount factor, we use parameter values for risk aversion and the IES commonly used in the long-run risk literature, as summarized in the middle panel of Table 1. In addition to these parameter values, we need data on the returns to the aggregate wealth portfolio. There are two ways to construct these returns. The first way is to estimate (innovations to) the stochastic volatility of consumption growth as well as (innovations to) expected consumption growth, which combined with the parameters of the long-run risk model lead to proxies for the return on wealth. The second way is to take a stance on the composition of the wealth portfolio, by taking a weighted average of traded assets. In this paper, we take the latter approach and form a weighted average of stock returns (as represented by the CRSP value-weighted total market portfolio) and long-term bond returns (the returns on the Fama-Bliss long-term bond portfolio (60-120 months)) to compute the returns on the wealth portfolio. Given the calibration in Table 1, the implied value of θ is large making the SDF very sensitive to the volatility of the wealth portfolio. Because the volatility of the wealth portfolio is sensitive to the relative weighting of stocks and bonds, we calculate the SDF over a range of weights (denoted by w) to assess the robustness with respect to this assumption.[9]

For the Campbell and Cochrane (1999) habit formation model, the stochastic discount factor is given by:

$$M_t = \delta \left(\frac{C_t}{C_{t-1}} \frac{S_t}{S_{t-1}} \right)^{-\gamma},$$

where S_t is the consumption surplus ratio. The dynamics of the log consumption surplus ratio s_t are given by:

$$s_t = (1 - \phi)\bar{s} + \phi s_{t-1} + \lambda \left(s_{t-1} \right) \left(c_t - c_{t-1} - g \right),$$

where $\bar{s}$ is the steady state habit, ϕ is the persistence of the habit stock, c_t the natural logarithm of consumption at time t and g is the average consumption growth rate. We set all the parameters of the model to the values proposed in Campbell and Cochrane (1999), but we replace the average consumption growth rate g, as well as the consumption growth rate volatility σ with their sample estimates over the full available sample (1959-2011), as summarized in the bottom panel of Table 1. To construct the consumption surplus ratio data, we need a starting value. As our consumption data starts in 1959, which is long before the start of our mutual fund data in 1977, we have a sufficiently long period to initialize the consumption surplus ratio. That is, in 1959, we set the ratio to its steady

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[9]See Lustig, Van Nieuwerburgh, and Verdelhan (2013) for a discussion on the composition of the wealth portfolio and the importance of including bonds.

Consumption CAPM				
Subj. disc. factor	Risk aversion			
β	γ			
0.9989	10			

Epstein Zin preferences (LRR)				
Subj. disc. factor	Risk aversion	IES	Weight in bonds	
δ	γ	ψ	w	
0.9989	10	1.5	0%, 70%, 90%	

Habit formation preferences				
Subj. disc. factor	Risk aversion	Mean growth	Habit persistence	Consumption vol
δ	γ	g	ϕ	σ
0.9903	2	0.0020	0.9885	0.0076

Table 1: **Parameter Calibration** The table shows the calibrated parameters for the three structural models that we test: power utility over consumption (the consumption CAPM), external habit formation preferences (as in Campbell and Cochrane (1999)) and Epstein Zin preferences as in Bansal and Yaron (2004).

state value $\bar{s}$ and construct the ratio for the subsequent periods using the available data that we have. Because the annualized value of the persistence coefficient is 0.87, the weight of the 1959 starting value of the consumption surplus ratio in the 1977 realization of the stochastic discount factor is small and equal to 0.015.

3   Results

We use the mutual fund data set in Berk and van Binsbergen (2013). The data set spans the period from January 1977 to March 2011. We remove all funds with less than 5 years of data leaving 4394 funds.[10] Berk and van Binsbergen (2013) undertook an extensive data project to address several shortcomings in the CRSP database by combining it with Morningstar data, and we refer the reader to the data appendix of that paper for the details.

To implement the tests derived in Propositions 2 and 5 it is necessary to pick an observation horizon. For most of the sample, funds report their AUMs monthly, however in the early part of the sample many funds report their AUMs only quarterly. In order not to introduce a selection bias by dropping these funds, the shortest horizon we will

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[10]We chose to remove these funds to ensure that incubation flows do not influence our results. Changing the criterion to 2 years does not change our results. These results are available on request.

consider is three months. Furthermore, as pointed out above, we need a horizon length of more than a month to compute the outperformance measure for the dynamic equilibrium models. If investors react to new information immediately, then flows should immediately respond to performance and the appropriate horizon to measure the effect would be the shortest horizon possible. But in reality there is evidence that investors do not respond immediately. Mamaysky, Spiegel, and Zhang (2008) show that the net alpha of mutual funds is predictably non-zero for horizons shorter than a year, suggesting that capital does not move instantaneously. There is also evidence of investor heterogeneity because some investors appear to update faster than others.[11] For these reasons, we also consider longer horizons (up to four years). The downside of using longer horizons is that longer horizons tend to put less weight on investors who update immediately, and these investors are also the investors more likely to be marginal in setting prices.

The flow of funds is important in our empirical specification because it affects the alpha generating technology as specified by $h(\cdot)$. Consequently, we need to be careful to ensure that we only use the part of capital flows that affects this technology. For example, it does not make sense to include as an inflow of funds, increases in fund sizes that result from inflation because such increases are unlikely to affect the alpha generating process. Similarly, the fund's alpha generating process is unlikely to be affected by changes in size that result from changes in the price level of the market as a whole. Consequently, we will measure the flow of funds over a horizon of length T as

$$q_{it} - q_{it-T}(1 + R_{it}^V),$$

where R_{it}^V is the cumulative return to investors of the appropriate Vanguard benchmark fund as defined in Berk and van Binsbergen (2013) over the horizon from $t - T$ to t. This benchmark fund is constructed by projecting fund i's return onto the space spanned by the set of available Vanguard index funds which can be interpreted as the investors alternative investment opportunity. Thus, in our empirical specification, we only consider capital flows into and out of funds net of what would have happened had investors not invested or withdrawn capital and had the fund manager adopted a purely passive strategy.

We begin by examining the correlation structure of performance between mutual funds. One would not expect mutual fund strategies to be highly correlated because otherwise the informational rents would be competed away. It is nevertheless important that we check that this is indeed the case, because otherwise our assumption that $h(\cdot)$ is a function of the size of the fund (rather than the size of the industry) would be subject to ques-
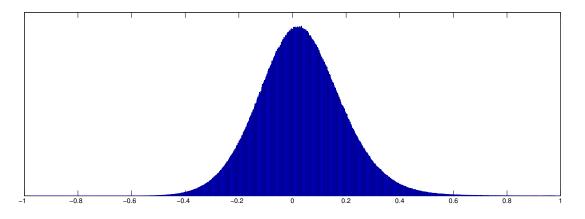
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[11]See Berk and Tonks (2007).

Figure 1: **Correlation Between Funds**
The histogram displays the distribution of the pairwise correlation coefficients between funds of outperformance relative to the Vanguard benchmark.

tion. To examine this correlation, we calculate outperformance relative to the Vanguard benchmark defined in Berk and van Binsbergen (2013), that is, for each fund we calculate ε_{it} using the Vanguard benchmark. We then compute the correlation coefficients of outperformance between every fund in our sample for which the two funds have at least 4 years of overlapping data. Figure 1 is a histogram of the results. It is clear from the figure that managers are not using the same strategies — the average correlation between the funds in our sample is 0.03. Furthermore, 43% of funds are negatively correlated and the fraction of funds that have large positive correlation coefficients is tiny (only 0.55% of funds have a correlation coefficient over 50%).

We implement our tests as follows. For each model, c, in each fund, i, we compute monthly outperformance, ε_{it}^c, as we explained in Section 2. That is, for the factor models we generate the outperformance measure for the horizon by using (9) and for the dynamic equilibrium models, we use (12). At the end of this process we have a fund flow and outperformance observation for each fund over each measurement horizon. We then implement the test in Proposition 2 by estimating $\beta_{F\varepsilon}$ for each model by running a single linear regression. Table 2 reports our results. For ease of interpretation, the table reports

$$\frac{\beta_{F\varepsilon} + 1}{2} = \frac{\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] + \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]}{2},$$

that is, the average conditional probability that the sign of outperformance matches the sign of the fund inflow. If flows and outperformance are unrelated, we would expect this measure to equal 50%, that is, $\beta_{F\varepsilon} = 0$. The main takeaway from Table 2 is that none of our candidate models can be rejected based on Proposition 2, that is, $\beta_{F\varepsilon}$ is significantly

Model	Horizon					
	3 month	6 month	1 year	2 year	3 year	4 year
Market Models (CAPM)						
CRSP Value Weighted	**63.21**	**63.08**	**63.07**	**62.98**	62.36	**62.34**
S&P 500	62.04	61.47	61.25	61.05	59.96	59.65
No Model						
Return	58.52	58.72	58.87	59.80	60.64	60.69
Excess Return	58.17	58.49	58.80	60.37	60.98	60.58
Return in Excess of the Market	61.75	61.37	61.18	61.07	60.71	60.43
Multifactor Models						
FF	62.94	62.52	62.63	62.96	**62.56**	61.85
FFC	63.02	62.63	62.81	62.72	62.31	61.98
Dynamic Equilibrium Models						
C-CAPM	58.18	58.35	58.68	60.07	60.59	60.54
Habit	58.14	58.23	58.64	60.00	60.67	60.43
Long Run Risk – 0% Bonds	57.30	58.32	59.31	62.07	61.43	58.63
Long Run Risk – 70% Bonds	57.07	57.53	58.56	58.06	58.20	59.33
Long Run Risk – 90% Bonds	57.14	57.70	58.81	59.05	59.59	60.04

Table 2: **Flow of Funds Outperformance Relationship (1977-2011):** The table reports estimates of (6) for different asset pricing models. For ease of interpretation, the table reports $(\beta_{F\varepsilon} + 1)/2$ in percent, which by Lemma 2 is equivalent to the average conditional probability that the sign of outperformance matches the sign of the fund inflow: $(\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] + \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}\right) = -1\right])/2$. Each row corresponds to a different risk model. The first two rows report the results for the market model (CAPM) using the CRSP value weighted index and the S&P 500 index as the market portfolio. The next three lines report the results of using as the benchmark return, three rules of thumb: (1) the fund's actual return, (2) the fund's return in excess of the risk free rate, and (3) the fund's return in excess of the return on the market as measured by the CRSP value weighted index. The next two lines are the Fama-French (FF) and Fama-French-Carhart (FFC) factor models. The final four lines report the results for the dynamic equilibrium models: the Consumption CAPM (C-CAPM), the habit model derived by Campbell and Cochrane (1999), and the long run risk model derived by Bansal and Yaron (2004). For the long run risk model we consider three different versions, depending on the portfolio weight of bonds in the aggregate wealth portfolio. The maximum number in each column (the best performing model) is shown in bold face.

Horizon (months)					
3	6	12	24	36	48
CAPM	CAPM	CAPM	CAPM	FF	CAPM
FFC	FFC	FFC	FF	CAPM	FFC
FF	FF	FF	FFC	FFC	FF
CAPM SP500	CAPM SP500	CAPM SP500	LRR 0	LRR 0	Return
Excess Market	Excess Market	Excess Market	Excess Market	Excess Return	Excess Return
Return	Return	LRR 0	CAPM SP500	Excess Market	C-CAPM
C-CAPM	Excess Return	Return	Excess Return	Habit	Habit
Excess Return	C-CAPM	LRR 90	C-CAPM	Return	Excess Market
Habit	LRR 0	Excess Return	Habit	C-CAPM	LRR 90
LRR 0	Habit	C-CAPM	Return	CAPM SP500	CAPM SP500
LRR 90	LRR 90	Habit	LRR 90	LRR 90	LRR 70
LRR 70	LRR 70	LRR 70	LRR 70	LRR 70	LRR 0

Table 3: **Model Ranking:** The table shows the ranking of all the models at each time horizon. Factor models are shown in red, dynamic equilibrium models in blue, and black entries are models that have not been formally derived. The CAPM is coded in both red and blue since it can be interpreted as both a factor model and an equilibrium model. The number following the long run risk models denotes the percentage of the wealth portfolio invested in bonds.

greater than zero in all cases,[12] implying that regardless of the risk adjustment, a flow-performance relation exists. On the other hand, none of the models performs better than 64%. It appears that a large fraction of flows remain unexplained. Investors appear to be using other criteria to make a non-trivial fraction of their investment decisions.

Which model best approximates the true asset pricing model? Table 3 ranks each model by its β_{Fc}. The best performing model, at almost all horizons, is the CAPM with the CRSP value weighted index as the market proxy. To assess whether this ranking reflects statistically significant outperformance, we implement the pairwise linear regression specified in Proposition 5 and report the double clustered t-statistics of these regressions in Table 4.

We begin by first focusing on the behavioral model that investors just react to past returns, the column marked "Ret" in the table. By looking down that column one can see that the factor models all statistically significantly outperform this model at horizons of two years or less. For example, the t-statistic that $\beta_{F,CAPM} > \beta_{F,Ret}$ at the 3-month

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[12]Table 4 reports the double clustered t-statistics.

Panel A: 3 Month Horizon

Model	$\beta_{F\varepsilon}$	Univ t-stat	CAPM	FFC	FF	CAPM SP500	Ex. Mkt	Ret	C-CAPM	Ex. Ret	Habit	LRR 0	LRR 90	LRR 70
CAPM	0.264	35.82	0.00	0.81	1.22	6.36	9.08	7.01	8.16	8.25	8.20	9.87	10.02	10.59
FFC	0.260	37.38	-0.81	0.00	0.76	3.14	4.63	6.44	7.47	7.56	7.52	9.13	9.15	9.70
FF	0.259	38.02	-1.22	-0.76	0.00	2.89	4.48	6.28	7.30	7.38	7.34	8.99	9.02	9.55
CAPM SP500	0.241	28.41	-6.36	-3.14	-2.89	0.00	1.06	5.45	6.41	6.49	6.44	7.68	8.34	8.47
Excess Market	0.235	31.17	-9.08	-4.63	-4.48	-1.06	0.00	4.54	5.35	5.41	5.39	6.91	6.93	7.35
Return	0.170	14.92	-7.01	-6.44	-6.28	-5.45	-4.54	0.00	2.22	2.26	2.41	1.86	4.82	3.83
C-CAPM	0.163	13.94	-8.16	-7.47	-7.30	-6.41	-5.35	-2.22	0.00	0.16	1.32	1.43	4.12	3.31
Excess Return	0.163	14.00	-8.25	-7.56	-7.38	-6.49	-5.41	-2.26	-0.16	0.00	0.34	1.45	4.50	3.55
Habit	0.163	13.80	-8.20	-7.52	-7.34	-6.44	-5.39	-2.41	-1.32	-0.34	0.00	1.38	3.91	3.19
LRR 0	0.146	11.14	-9.87	-9.13	-8.99	-7.68	-6.91	-1.86	-1.43	-1.45	-1.38	0.00	0.31	0.54
LRR 90	0.143	11.91	-10.02	-9.15	-9.02	-8.34	-6.93	-4.82	-4.12	-4.50	-3.91	-0.31	0.00	0.29
LRR 70	0.141	11.46	-10.59	-9.70	-9.55	-8.47	-7.35	-3.83	-3.31	-3.55	-3.19	-0.54	-0.29	0.00

Panel B: 6 Month Horizon

Model	$\beta_{F\varepsilon}$	Univ t-stat	CAPM	FFC	FF	CAPM SP500	Ex Mkt	Ret	Ex Ret	C-CAPM	LRR 0	Habit	LRR 90	LRR 70
CAPM	0.262	35.28	0.00	1.71	2.12	8.60	9.10	5.83	6.96	7.06	6.56	7.26	8.60	9.63
FFC	0.253	35.68	-1.71	0.00	0.92	3.42	4.32	5.02	5.94	6.13	5.67	6.34	7.24	8.06
FF	0.250	36.56	-2.12	-0.92	0.00	3.03	4.11	4.83	5.71	5.88	5.46	6.08	7.08	7.92
CAPM SP500	0.229	25.67	-8.60	-3.42	-3.03	0.00	0.35	3.78	4.52	4.70	4.22	4.91	5.87	6.38
Excess Market	0.227	28.45	-9.10	-4.32	-4.11	-0.35	0.00	3.28	3.94	4.06	3.95	4.23	5.12	5.81
Return	0.174	14.39	-5.83	-5.02	-4.83	-3.78	-3.28	0.00	1.10	1.71	0.45	2.24	2.31	2.24
Excess Return	0.170	13.91	-6.96	-5.94	-5.71	-4.52	-3.94	-1.10	0.00	1.40	0.20	2.54	2.15	2.20
C-CAPM	0.167	13.73	-7.06	-6.13	-5.88	-4.70	-4.06	-1.71	-1.40	0.00	0.04	3.01	1.66	1.75
LRR 0	0.166	11.08	-6.56	-5.67	-5.46	-4.22	-3.95	-0.45	-0.20	-0.04	0.00	0.11	0.80	1.32
Habit	0.165	13.52	-7.26	-6.34	-6.08	-4.91	-4.23	-2.24	-2.54	-3.01	-0.11	0.00	1.32	1.49
LRR 90	0.154	11.97	-8.60	-7.24	-7.08	-5.87	-5.12	-2.31	-2.15	-1.66	-0.80	-1.32	0.00	0.45
LRR 70	0.151	11.52	-9.63	-8.06	-7.92	-6.38	-5.81	-2.24	-2.20	-1.75	-1.32	-1.49	-0.45	0.00

Panel C: 1 Year Horizon

Model	$\beta_{F\varepsilon}$	Univ t-stat	CAPM	FFC	FF	CAPM SP500	Ex Mkt	LRR 0	Ret	LRR 90	Ex Ret	C-CAPM	Habit	LRR 70
CAPM	0.261	30.83	0.00	0.98	1.65	8.26	8.86	3.45	4.28	5.24	5.13	5.16	5.19	6.83
FFC	0.256	31.72	-0.98	0.00	1.32	4.45	4.75	3.10	4.04	4.82	4.82	4.93	4.96	6.09
FF	0.253	31.89	-1.65	-1.32	0.00	3.82	4.34	2.94	3.87	4.62	4.61	4.71	4.74	5.80
CAPM SP500	0.225	22.25	-8.26	-4.45	-3.82	0.00	0.21	1.91	2.58	3.07	3.05	3.15	3.19	3.98
Excess Market	0.224	23.54	-8.86	-4.75	-4.34	-0.21	0.00	1.80	2.20	2.61	2.60	2.68	2.72	3.51
LRR 0	0.186	9.65	-3.45	-3.10	-2.94	-1.91	-1.80	0.00	0.34	0.43	0.43	0.52	0.56	0.83
Return	0.177	12.54	-4.28	-4.04	-3.87	-2.58	-2.20	-0.34	0.00	0.07	0.20	0.55	0.66	0.38
LRR 90	0.176	12.14	-5.24	-4.82	-4.62	-3.07	-2.61	-0.43	-0.07	0.00	0.02	0.21	0.27	0.47
Excess Return	0.176	12.65	-5.13	-4.82	-4.61	-3.05	-2.60	-0.43	-0.20	-0.02	0.00	0.92	1.19	0.37
C-CAPM	0.174	12.44	-5.16	-4.93	-4.71	-3.15	-2.68	-0.52	-0.55	-0.21	-0.92	0.00	1.03	0.18
Habit	0.173	12.29	-5.19	-4.96	-4.74	-3.19	-2.72	-0.56	-0.66	-0.27	-1.19	-1.03	0.00	0.11
LRR 70	0.171	12.05	-6.83	-6.09	-5.80	-3.98	-3.51	-0.83	-0.38	-0.47	-0.37	-0.18	-0.11	0.00

Table continues on following page ...

Panel D: 2 Year Horizon

Model	$\beta_{F\varepsilon}$	Univ t-stat	CAPM	FF	FFC	LRR 0	Ex Mkt	CAPM SP500	Ex Ret	C-CAPM	Habit	Ret	LRR 90	LRR 70
CAPM	0.260	26.98	0.00	0.04	0.90	0.55	7.70	5.89	2.42	2.63	2.70	2.49	3.64	5.42
FF	0.259	29.76	-0.04	0.00	1.36	0.52	5.54	3.76	2.39	2.59	2.66	2.48	3.63	5.17
FFC	0.254	27.67	-0.90	-1.36	0.00	0.38	5.10	3.41	2.15	2.36	2.43	2.28	3.34	4.91
LRR 0	0.241	12.42	-0.55	-0.52	-0.38	0.00	0.67	0.67	0.88	1.04	1.08	1.14	1.68	2.66
Excess Market	0.221	21.77	-7.70	-5.54	-5.10	-0.67	0.00	0.04	0.63	0.87	0.93	0.96	1.76	3.10
CAPM SP500	0.221	18.21	-5.89	-3.76	-3.41	-0.67	-0.04	0.00	0.67	0.93	1.00	1.02	1.82	3.12
Excess Return	0.207	13.37	-2.42	-2.39	-2.15	-0.88	-0.63	-0.67	0.00	1.64	2.17	1.06	1.67	2.48
C-CAPM	0.201	12.88	-2.63	-2.59	-2.36	-1.04	-0.87	-0.93	-1.64	0.00	0.97	0.54	1.22	2.04
Habit	0.200	12.74	-2.70	-2.66	-2.43	-1.08	-0.93	-1.00	-2.17	-0.97	0.00	0.40	1.14	1.98
Return	0.196	11.97	-2.49	-2.48	-2.28	-1.14	-0.96	-1.02	-1.06	-0.54	-0.40	0.00	0.74	1.56
LRR 90	0.181	10.66	-3.64	-3.63	-3.34	-1.68	-1.76	-1.82	-1.67	-1.22	-1.14	-0.74	0.00	1.24
LRR 70	0.161	8.67	-5.42	-5.17	-4.91	-2.66	-3.10	-3.12	-2.48	-2.04	-1.98	-1.56	-1.24	0.00

Panel E: 3 Year Horizon

Model	$\beta_{F\varepsilon}$	Univ t-stat	FF	CAPM	FFC	LRR 0	Ex Ret	Ex Mkt	Habit	Return	C-CAPM	CAPM SP500	LRR 90	LRR 70
FF	0.251	25.61	0.00	0.56	1.22	0.55	1.40	4.33	1.60	1.39	1.69	4.48	2.35	4.76
CAPM	0.247	22.02	-0.56	0.00	0.15	0.46	1.21	5.80	1.40	1.22	1.48	6.35	2.16	4.58
FFC	0.246	24.88	-1.22	-0.15	0.00	0.44	1.14	3.63	1.35	1.18	1.43	3.87	2.09	4.47
LRR 0	0.229	8.16	-0.55	-0.46	-0.44	0.00	0.19	0.38	0.32	0.32	0.36	0.78	0.81	1.75
Excess Return	0.220	12.05	-1.40	-1.21	-1.14	-0.19	0.00	0.22	1.19	0.57	1.44	0.81	1.15	2.74
Excess Market	0.214	17.37	-4.33	-5.80	-3.63	-0.38	-0.22	0.00	0.03	0.05	0.09	1.70	0.86	2.58
Habit	0.213	11.17	-1.60	-1.40	-1.35	-0.32	-1.19	-0.03	0.00	0.06	0.92	0.52	0.83	2.21
Return	0.213	10.20	-1.39	-1.22	-1.18	-0.32	-0.57	-0.05	-0.06	0.00	0.07	0.45	0.70	1.86
C-CAPM	0.212	11.25	-1.69	-1.48	-1.43	-0.36	-1.44	-0.09	-0.92	-0.07	0.00	0.48	0.78	2.16
CAPM SP500	0.199	13.56	-4.48	-6.35	-3.87	-0.78	-0.81	-1.70	-0.52	-0.45	-0.48	0.00	0.27	1.70
LRR 90	0.192	10.13	-2.35	-2.16	-2.09	-0.81	-1.15	-0.86	-0.83	-0.70	-0.78	-0.27	0.00	1.26
LRR 70	0.164	8.02	-4.76	-4.58	-4.47	-1.75	-2.74	-2.58	-2.21	-1.86	-2.16	-1.70	-1.26	0.00

Panel F: 4 Year Horizon

Model	$\beta_{F\varepsilon}$	Univ t-stat	CAPM	FFC	FF	Ret	Ex Ret	C-CAPM	Habit	Ex Mkt	LRR 90	CAPM SP500	LRR 70	LRR 0
CAPM	0.247	18.28	0.00	0.82	1.09	0.93	1.24	1.21	1.30	6.23	1.63	5.51	2.81	1.15
FFC	0.240	19.79	-0.82	0.00	0.55	0.81	1.08	1.08	1.17	3.12	1.36	3.11	2.38	1.05
FF	0.237	19.12	-1.09	-0.55	0.00	0.73	1.00	1.00	1.09	2.85	1.24	2.90	2.21	1.01
Return	0.214	9.60	-0.93	-0.81	-0.73	0.00	0.13	0.20	0.35	0.14	0.33	0.55	0.77	0.60
Excess Return	0.212	10.10	-1.24	-1.08	-1.00	-0.13	0.00	0.14	0.63	0.11	0.31	0.60	0.86	0.59
C-CAPM	0.211	10.19	-1.21	-1.08	-1.00	-0.20	-0.14	0.00	1.09	0.08	0.28	0.56	0.80	0.58
Habit	0.209	10.04	-1.30	-1.17	-1.09	-0.35	-0.63	-1.09	0.00	0.00	0.22	0.49	0.73	0.55
Excess Market	0.209	14.64	-6.23	-3.12	-2.85	-0.14	-0.11	-0.08	0.00	0.00	0.27	1.40	0.94	0.61
LRR 90	0.201	9.18	-1.63	-1.36	-1.24	-0.33	-0.31	-0.28	-0.22	-0.27	0.00	0.26	0.49	0.44
CAPM SP500	0.193	10.81	-5.51	-3.11	-2.90	-0.55	-0.60	-0.56	-0.49	-1.40	-0.26	0.00	0.26	0.35
LRR 70	0.187	7.27	-2.81	-2.38	-2.21	-0.77	-0.86	-0.80	-0.73	-0.94	-0.49	-0.26	0.00	0.24
LRR 0	0.173	4.47	-1.15	-1.05	-1.01	-0.60	-0.59	-0.58	-0.55	-0.61	-0.44	-0.35	-0.24	0.00

Table 4: **Tests of Statistical Significance:** The first two columns in the table provides the coefficient estimate and double-clustered t-statistic (see Thompson (2011) and the discussion in Petersen (2009)) of the univariate regression of signed flows on signed out-performance. The rest of the columns provide the statistical significance of the pairwise test, derived in Proposition 5, of whether the models are better approximations of the true asset pricing model. For each model in a column, the table displays the double-clustered t-statistic of the test that the model in the row is a better approximation of the true asset pricing model, that is, that β_F row $> \beta_F$ column. The rows (and columns) are ordered by $\beta_{F\varepsilon}$, with the best performing model on top. The number following the long run risk models denotes the percentage of the wealth portfolio invested in bonds.

horizon is 7.01, indicating that we can reject the hypothesis that the behavioral model is a better approximation of the true model than the CAPM. Based on these results, we can reject the hypothesis that investors just react to past returns. The next possibility is that investors are risk neutral. In an economy with risk neutral investors we would find that the excess return best explains flows, so the performance of this model can be assessed by looking at the columns labeled "Ex. Ret." Notice that all the risk models nest this model, so to conclude that a risk model better approximates the true model, the risk model must statistically outperform this model. The factor models all satisfy this criterion, allowing us to conclude that investors are not risk neutral. Unfortunately, none of the dynamic asset pricing model satisfy this criterion. Finally, one might hypothesize that investors benchmark their investments relative to the market portfolio alone, that is, they do not adjust for any risk differences (beta) between their investment and the market. The performance of this model is reported in the column labeled "Ex. Mkt." Again, all the factor models statistically significantly outperform this model — investors actions reveal that they adjust for risk using beta.

Our results also allow us to discriminate between the factor models. Recall that both the FF and FFC factor specifications nest the CAPM, so to conclude that either factor model better approximates the true model, it must statistically significantly outperform the CAPM. The test of this hypothesis is in the columns labeled "CAPM." Neither factor model statistically outperforms the CAPM at any horizon. Indeed, at all horizons other than 3 years, the CAPM actually outperforms both factor models. What this implies is that the additional factors add no more explanatory power for flows. In short, the additional factors cannot be omitted risk factors.

The relative performance of the dynamic equilibrium models is poor. We can confidently reject the hypothesis that any of these models is a better approximation of the true model than the CAPM. But this result should be interpreted with caution. These models rely on variables like consumption that are notoriously difficult for empiricists to measure, but are observed perfectly by investors.

4   Implication for Cost of Capital Calculations

Ultimately, the reason financial economists are interested in deriving the true risk model is to provide a method to calculate the cost of capital for an investment opportunity. In the 50 years since the CAPM was first derived, it has become clear that the model cannot completely explain the cross-sectional variation of asset returns. In response to this failure, researchers have attempted to derive new risk models to better explain this cross sectional

variation. The main concern with this approach is that it is hard to distinguish between extensions that represent progress towards finding a better risk model rather than just better fitting the cross section of asset returns. That is, there is no way to know whether the new factors are truly risk factors that require compensation, or just common factors in asset returns. This difference is crucial because the cost of capital is only a function of the riskiness of the investment opportunity.

By analogy, consider how astronomers reacted to the inability of the Ptolemaic theory to explain the motion of the planets. In that case each observational inconsistency was "fixed" by adding an additional epicycle to the theory. By the time Copernicus proposed the theory that the Earth revolved around the Sun, the Ptolemaic theory had been fixed so many times it actually better explained the motion of the planets than the Copernican system.[13] Similarly, although the extensions to the CAPM better explain the cross section of asset returns, it is hard to know, using traditional tests, whether these extensions represent true progress towards measuring risk or simply the asset pricing equivalent of an epicycle.

The advantage of our testing methodology is that it can differentiate between whether current extensions to the CAPM just improve the model's fit to existing data or whether they represent true progress towards a better model of risk. The extensions of the CAPM model were proposed to better fit returns, not flows. As such, flows provide a new set of moments that those models can be confronted with. Consequently, if the extension of the original model better explains mutual fund flows, this suggests that the extension does indeed represent progress towards a superior risk model. Conversely, if the extended model cannot better explain flows, then the extension is the modern equivalent of an epicycle, an arbitrary fix designed simply to ensure that the model better explains the cross section of returns. So, the fact that we find that the CAPM outperforms every extension to the model, implies that these extensions to the original CAPM do not represent progress towards the goal of deriving the risk model investors use to price capital assets.

More importantly, the fact that the CAPM does a poor job explaining the cross sectional variation in asset returns does not necessarily imply that a better, yet undiscovered, method exists to calculate the cost of capital. To conclude that a better risk model exists, one has to show that the part of the variation in asset returns not explained by the CAPM can be explained by variation in risk. This is what the flow of funds data allows us to do. If variation in asset returns that is not explained by the CAPM attracts flows, then one can conclude that this variation is not compensation for risk, and thus should not be

---

[13]Copernicus wrongly assumed that the planets followed circular orbits when in fact their orbits are ellipses.

included in the cost of capital calculation.

5 Tests of the Robustness of our Results

In this section we consider two possible alternative explanations for our results. First we look at the possibility that mutual fund fee changes might be part of the market equilibrating mechanism. Then we test the hypothesis that investors information sets contains more than what is in past and present prices.

5.1 Fee Changes

As argued in the introduction, capital flows are not the only mechanism that could equilibrate the mutual fund market. An alternative mechanism is for fund managers to adjust their fees to ensure that the fund's alpha is zero. In fact, fee changes are rare, occurring in less than 4% of our observations, making it unlikely that fee changes play any role in equilibrating the mutual fund market. Nevertheless, in this section we will run a robustness check to make sure that fee changes do not play a role in explaining our results.

The fees mutual funds charge are stable because they are specified in the fund's prospectus, so theoretically, a change to the fund's fee requires a change to the fund's prospectus, a relatively costly endeavor. However, the fee in the prospectus actually specifies the maximum fee the fund is allowed to charge because funds are allowed to (and do) rebate some of their fees to investors. Thus, funds can change their fees by giving or discontinuing rebates. To rule out these rebates as a possible explanation of our results, we repeat the above analysis by assuming that fee changes are the primary way mutual fund markets equilibrate.

We define a positive (negative) fee change as an increase (decrease) in the fees charged from the beginning to the end of the horizon. For each fund, in periods that we observe a fee change, we assume the fee change is equilibrating the market and so the flow variable takes the sign of the fee change. In periods without a fee change, we continue to use the sign of the flows. That is, define F_{it}^* as:

$$F_{it}^* \equiv \begin{cases} \Delta_{it} & \Delta_{it} \neq 0 \\ F_{it} & \Delta_{it} = 0 \end{cases}$$

where Δ_{it} is the fee change experienced by fund i at time t.

Table 5 reports the results of estimating $(\beta_{F^*\varepsilon} + 1)/2$, that is the average conditional probability using the flow variable that includes fee changes. The results are qualitatively

Model	Horizon					
	3 month	6 month	1 year	2 year	3 year	4 year
Market Models (CAPM)						
CRSP Value Weighted	**62.07**	**60.78**	**58.38**	**56.41**	**54.02**	**53.04**
S&P 500	61.06	59.52	57.44	55.26	52.54	51.07
No Model						
Return	58.27	57.72	55.60	52.16	51.05	50.85
Excess Return	58.02	57.60	55.99	53.10	51.16	51.00
Return in Excess of the Market	60.63	59.22	56.92	55.13	53.51	52.07
Multifactor Models						
FF	61.72	60.19	57.99	56.20	53.60	52.71
FFC	61.83	60.29	58.13	55.95	53.62	52.50
Dynamic Equilibrium Models						
C-CAPM	58.02	57.45	55.68	53.12	50.93	50.64
Habit	57.99	57.34	55.72	53.07	50.99	50.75
Long Run Risk 0	57.22	57.72	56.88	52.30	47.80	43.72
Long Run Risk 70	57.11	56.90	55.59	51.10	48.15	48.04
Long Run Risk 90	57.18	57.09	56.10	53.16	52.70	52.34

Table 5: **Effect of Fee Changes:** The table shows the effect of assuming that the market equilibrates through fee changes if they occur. That is, we use the sign of the fee change instead of the sign of the flow whenever we have a non-zero fee change observation. In period when there is no fee change, we use the sign of the flow as before. The table reports $(\beta_{F^*\varepsilon} + 1)/2$ in percent which is equivalent to the average conditional probability that the sign of outperformance matches the sign of this new flow viable. Each row corresponds to a different risk model. The first two rows report the results for the market model (CAPM) using the CRSP value weighted index and the S&P 500 index as the market portfolio. The next three lines report the results of using as the benchmark return, three rules of thumb: (1) the fund's actual return, (2) the fund's return in excess of the risk free rate, and (3) the fund's return in excess of the return on the market as measured by the CRSP value weighted index. The next two lines are the Fama-French (FF) and Fama-French-Carhart (FFC) factor models. The final four lines report the results for the dynamic equilibrium models: the Consumption CAPM (C-CAPM), the habit model derived by Campbell and Cochrane (1999), and the long run risk model derived by Bansal and Yaron (2004). For the long run risk model we consider three different versions, depending on the portfolio weight of bonds in the aggregate wealth portfolio. The maximum number in each column (the best performing model) is shown in bold face.

unchanged — the CAPM outperforms all the other models — and quantitatively very similar. More importantly, including fee changes in this way reduces the explanatory power of all the models (the point estimates in Table 5 are lower than in Table 3) so there is no evidence that fee changes play an important role in equilibrating the market for mutual funds.

5.2   Other Information Sets

Conceivably, the poor performance of some of the models reported in the last section could result because the assumption that the information set for most investors does not include any more information than past and present prices is incorrect. If this assumption is false and the information set of most investors includes information in addition to what is communicated by prices, what appears to us as a positive NPV investment might actually be zero NPV when viewed from the perspective of the actual information available at the time.

If information is indeed the explanation and if investors are right in their decision to allocate or withdraw money, the alpha must be zero even when the flow has the opposite sign to the outperformance. We test this Null hypothesis by double sorting firms into terciles based on their past alpha as well as their past flows. Going forward, over a specified measurement horizon, we test to see whether funds in the highest alpha tercile and the lowest flow tercile outperform funds in the lowest alpha tercile and the highest flow tercile.[14] Put differently, we investigate whether previously outperforming funds that nevertheless experience an outflow of funds outperform previously underperforming funds that experience an inflow. Under the Null that the asset pricing model under consideration holds, these two portfolios should perform equally well going forward (both should have a zero net alpha in the measurement horizon).

The main difficulty with implementing this test is uncertainty in the estimate of the fund's betas for the factor models. When estimation error in the sorting period is positively correlated to the error in the measurement horizon, as would occur if we would estimate the betas only once over the full sample, a researcher could falsely conclude that evidence of persistence exists when there is no persistence. To avoid this bias we do not use information from the sorting period to estimate the betas in the measurement horizon. This means that we require a measurement horizon of sufficient length to produce reliable beta estimates, so the shortest measurement horizon we consider is two years.

---

[14]The sorts we do are unconditional sorts, meaning that we independently sort on flows and alpha. The advantage of this is that our results are not influenced by the ordering of our sorts. The downside is that the nine "portfolios" do not have the same number of funds in them.

| | Horizon (years) | | |
Model	2	3	4
CAPM (b.p./month)	0.00	1.57	-1.66
t-statistic	0.00	0.25	-0.26
Fama-French (b.p./month)	18.06	19.65	20.84
t-statistic	3.32	3.62	3.83
Fama-French-Cahart (b.p./month)	13.50	14.83	16.23
t-statistic	2.81	3.08	3.38
C-CAPM (b.p./month)	9.90	9.90	7.21
t-statistic	1.18	1.18	0.86
Habit (b.p./month)	10.22	9.86	7.96
t-statistic	1.21	1.17	0.95
Long Run Risk – 0% Bonds (%/month)	-13.60	-13.49	-13.58
t-statistic	-1.20	-1.19	-1.20
Long Run Risk – 70% Bonds (b.p./month)	-9.81	-19.43	-20.67
t-statistic	-0.65	-1.28	-1.36
Long Run Risk – 90% Bonds (b.p./month)	1.28	3.37	-5.32
t-statistic	0.16	0.43	-0.69

Table 6: **Out of Sample Persistence:** The table shows by how much the top alpha/bottom flow tercile outperforms the bottom alpha/top flow tercile, where outperformance is the realized alpha under the given model. At time τ, we use all the information until that point in time to calculate the fund's information ratio. We also calculate the fund's capital flow over the number of years equal to the specified horizon. We then sort firms into 9 flow performance terciles based on the information ratio and measured capital flow and then measure outperformance over the specified future measurement horizon. At the end of the measurement horizon we then sort again and repeat the process as many times as the data allows. By the end of the process we have a time series of monthly outperformance measurements for each of the 9 portfolios. We then subtract the bottom information ratio /top flow from the top information ratio/bottom flow and the table reports the mean and t-statistic of this time series.

At time τ, we use all the information until that point in time to calculate the fund's information ratio, that is, we estimate the fund's alpha using all of its return data up to time τ and divide this by the standard error of the estimate. We then calculate the fund's capital flow over the prior h years. We sort firms into 9 flow performance terciles based on the estimated information ratio and measured capital flow. We require a fund to have at least three years of historical data to be included in the sort. Because we need at least 6 months to estimate the fund's betas in the measurement horizon, we drop all funds with less than 6 observations in the measurement horizon. To remove the obvious selection bias, we estimate the betas over the full measurement horizon, but then calculate ε_{it} by dropping the first 6 observations, that is, we only use $\{\varepsilon_{i,\tau+6}, \ldots, \varepsilon_{i,\tau+h}\}$ when we measure future performance. At the end of the measurement horizon we then sort again and repeat the process as many times as the data allows. By the end of the process we have a time series of monthly outperformance measurements for each of the 9 portfolios. We then subtract the bottom information ratio/top flow portfolio from the top information ratio/bottom flow portfolio. Table 6 reports the mean and t-statistic of this time series for horizons $h = 2, 3$ and 4 years.

The main takeaway from the results reported in Table 6 is that outperformance relative to the CAPM shows no evidence of persistence while outperformance relative to the other factor models is highly persistent and economically large. Consequently, we can confidently reject the Null hypothesis that the differential information set explains the poor performance of the factor models relative to the CAPM.

We find no evidence of predictability for the dynamic equilibrium models. In this case the likelihood that investors are have better information is higher because they observe their own consumption. So the lack of predictability is consistent with the possibility that the poor performance of these models is due to the fact that the empiricist measures consumption with error.[15]

6   Conclusion

The field of asset pricing is primarily concerned with the question of how to compute the cost of capital for investment opportunities. Because the net present value of a long-dated investment opportunity is very sensitive to assumptions regarding the cost of

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[15]Note that the outperformance point estimate for the long run risk model when the wealth portfolio consists entirely of stocks is four orders of magnitude higher than all other models, despite the fact that it is still statistically indistinguishable from zero. As we have already pointed out, given the volatility of stocks, the SDF of this model is extremely volatile leading to highly volatile estimates of outperformance for this model.

capital, computing this cost of capital correctly is of first order importance. Since the initial development of the Capital Asset Pricing Model, a large number of potential return anomalies relative to that model have been uncovered. These anomalies have motivated researchers to develop improved models that "explain" each anomaly as risk factor. As a consequence, in many (if not most) research studies these factors and their exposures are included as part of the cost of capital calculation. In this paper we examine the validity of this approach to calculating the cost of capital.

We propose a new way of testing the validity of an asset pricing model. Instead of following the common practice in the literature which relies on moment conditions related to returns, we use mutual fund capital flow data. Our study is motivated by revealed preference theory: if the asset pricing model under consideration correctly prices risk, then investors must be using it, and must be allocating their money based on that risk model. Consistent with this theory, we find that investors' capital flows in and out of mutual funds does reliably distinguish between asset pricing models. We find that the CAPM outperforms all extensions to model, which implies, given our current level of knowledge, that it is the best method to use to compute the cost of capital of an investment opportunity.

Perhaps the most important implication of our paper is that it highlights the usefulness and power of mutual fund data when addressing general asset pricing questions. Mutual fund data provides insights into questions that stock market data cannot. Because the market for mutual funds equilibrates through capital flows instead of prices we can directly observe investors' investment decisions. That allows us to infer their risk preferences from their actions. The observability of these choices and what this implies for investor preferences has remained largely unexplored in the literature.

Appendix

A Proofs

A.1 Proof of Proposition 2

The denominator of (6) is positive so we need to show that the numerator is positive as well. Conditioning on the information set at each point in time gives the following expression for the numerator:

$$cov(\phi(F_{it+1}), \phi(\varepsilon_{it+1})) =$$
$$E[E[\phi(F_{it+1})\phi(\varepsilon_{it+1})) \mid I_t] - E[\phi(F_{it+1}) \mid I_t]E[\phi(\varepsilon_{it+1}) \mid I_t]]. \tag{13}$$

Taking each term separately,

$$
\begin{aligned}
E[\phi(F_{it+1})\phi(\varepsilon_{it+1}) \mid I_t] &= E[\phi(\varepsilon_{it+1})\phi(\alpha_{it+1}(q_{it})) \mid I_t] \\
&= E[\phi(\varepsilon_{it+1})\phi(\alpha_{it+1}(q_{it})) \mid \theta_i > \bar{\theta}_{it},\ I_t] \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] \\
&\quad + E[\phi(\varepsilon_{it+1})\phi(\alpha_{it+1}(q_{it})) \mid \theta_i \le \bar{\theta}_{it},\ I_t] \Pr[\theta_i \le \bar{\theta}_{it} \mid I_t] \\
&= E[\phi(\varepsilon_{it+1}) \mid \theta_i > \bar{\theta}_{it},\ I_t] \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] \\
&\quad - E[\phi(\varepsilon_{it+1}) \mid \theta_i \le \bar{\theta}_{it},\ I_t] \Pr[\theta_i \le \bar{\theta}_{it} \mid I_t],
\end{aligned}
$$

where the first equality follows from Lemma 1 and the last equality follows from (2) and (3) because when $\theta_i > \bar{\theta}_{it}$ then $\alpha_{it+1}(q_{it}) > 0$ and similarly for $\theta_i \le \bar{\theta}_{it}$. Using similar logic

$$
\begin{aligned}
E[\phi(F_{it+1}) \mid I_t] &= E[\phi(\alpha_{it+1}(q_{it})) \mid \theta_i > \bar{\theta}_{it},\ I_t] \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] \\
&\quad + E[\phi(\alpha_{it+1}(q_{it})) \mid \theta_i \le \bar{\theta}_{it},\ I_t] \Pr[\theta_i \le \bar{\theta}_{it} \mid I_t] \\
&= \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] - \Pr[\theta_i \le \bar{\theta}_{it} \mid I_t],
\end{aligned}
$$

and

$$
\begin{aligned}
E[\phi(\varepsilon_{it+1}) \mid I_t] &= E[\phi(\varepsilon_{it+1}) \mid \theta_i > \bar{\theta}_{it},\ I_t] \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] \\
&\quad + E[\phi(\varepsilon_{it+1}) \mid \theta_i \le \bar{\theta}_{it},\ I_t] \Pr[\theta_i \le \bar{\theta}_{it} \mid I_t].
\end{aligned}
$$

Using these three expressions we have

$$
\begin{aligned}
E[\phi(F_{it+1})\phi(\varepsilon_{it+1})) \mid I_t] &- E[\phi(F_{it+1}) \mid I_t]E[\phi(\varepsilon_{it+1}) \mid I_t] = \\
&E[\phi(\varepsilon_{it+1}) \mid \theta_i > \bar{\theta}_{it},\ I_t]\Pr[\theta_i > \bar{\theta}_{it} \mid I_t] - E[\phi(\varepsilon_{it+1}) \mid \theta_i \leq \bar{\theta}_{it},\ I_t]\Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t] \\
&-E[\phi(\varepsilon_{it+1}) \mid \theta_i > \bar{\theta}_{it},\ I_t]\Pr[\theta_i > \bar{\theta}_{it} \mid I_t]\,(\Pr[\theta_i > \bar{\theta}_{it} \mid I_t] - \Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t]) \\
&-E[\phi(\varepsilon_{it+1}) \mid \theta_i \leq \bar{\theta}_{it},\ I_t]\Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t]\,(\Pr[\theta_i > \bar{\theta}_{it} \mid I_t] - \Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t]) \\
=\ &E[\phi(\varepsilon_{it+1}) \mid \theta_i > \bar{\theta}_{it},\ I_t]\Pr[\theta_i > \bar{\theta}_{it} \mid I_t]\,(1 - \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] + \Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t]) \\
&+E[-\phi(\varepsilon_{it+1}) \mid \theta_i \leq \bar{\theta}_{it},\ I_t]\Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t]\,(1 + \Pr[\theta_i > \bar{\theta}_{it} \mid I_t] - \Pr[\theta_i \leq \bar{\theta}_{it} \mid I_t]) \\
>\ &0
\end{aligned}
$$

because every term in the last equation is positive. Substituting the above expression into (13) completes the proof.

A.2 Proof of Lemma 2

First, by using Bayes' law and by rearranging terms we have:

$$
\begin{aligned}
\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right] \\
= \frac{\Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(F_{it}\right) = 1\right]\Pr\left[\phi\left(F_{it}\right) = 1\right]}{\Pr\left[\phi\left(\varepsilon_{it}\right) = -1\right]} \\
= \frac{\left(1 - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(F_{it}\right) = 1\right]\right)\Pr\left[\phi\left(F_{it}\right) = 1\right]}{1 - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1\right]} \\
= \frac{\Pr\left[\phi\left(F_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right]\Pr\left[\phi\left(\varepsilon_{it}\right) = 1\right]}{1 - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1\right]}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] &- \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right] \\
&= \frac{\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1\right]}{1 - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1\right]}.
\end{aligned} \tag{14}
$$

Now note that without loss of generality, we can rescale the sign variables to take values of 0 and 1 by dividing by 2 and adding 1. Because rescaling both the left and right hand side variables does not change the slope coefficient in a linear regression, we can simply

write out the OLS regression coefficient as if the variables are rescaled:

$$
\begin{aligned}
\beta_{F\varepsilon} &= \frac{cov\left(\phi\left(F_{it}\right),\phi\left(\varepsilon_{it}\right)\right)}{var\left(\phi\left(\varepsilon_{it}\right)\right)}\\
&= \frac{\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=1\right]-\Pr\left[\phi\left(F_{it}\right)=1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=1\right]}{\Pr\left[\phi\left(\varepsilon_{it}\right)=1\right]\left(1-\Pr\left[\phi\left(\varepsilon_{it}\right)=1\right]\right)}\\
&= \frac{\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]-\Pr\left[\phi\left(F_{it}\right)=1\right]}{1-\Pr\left[\phi\left(\varepsilon_{it}\right)=1\right]},
\end{aligned}
$$

which is (14).

A.3 Proof of Proposition 3

From Lemma 2, all we need to prove is that:

$$
\begin{aligned}
&\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]+\Pr\left[\phi\left(F_{it}\right)=-1\mid\phi\left(\varepsilon_{it}\right)=-1\right]\\
>\ &\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]+\Pr\left[\phi\left(F_{it}\right)=-1\mid\phi\left(\varepsilon_{it}^{c}\right)=-1\right]
\end{aligned}
$$

Taking each term separately,

$$
\begin{aligned}
&\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]\\
=\ &\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1,\phi\left(\varepsilon_{it}\right)=1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]+\\
&\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1,\phi\left(\varepsilon_{it}\right)=-1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=-1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]\\
=\ &\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]+\\
&\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=-1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=-1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]\\
=\ &\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]+\\
&\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=-1\right]\left(1-\Pr\left[\phi\left(\varepsilon_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]\right)\\
<\ &\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]\Pr\left[\phi\left(\varepsilon_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]+\\
&\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]\left(1-\Pr\left[\phi\left(\varepsilon_{it}\right)=1\mid\phi\left(\varepsilon_{it}^{c}\right)=1\right]\right)\\
=\ &\Pr\left[\phi\left(F_{it}\right)=1\mid\phi\left(\varepsilon_{it}\right)=1\right]. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(15)}
\end{aligned}
$$

where the second equality follows from (7) and the inequality follows from Lemma 2 and $\beta_{F\varepsilon} > 0$ (from Proposition 2). Similarly,

$$
\begin{aligned}
&\Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
&= \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1, \phi\left(\varepsilon_{it}\right) = 1\right] \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] + \\
&\quad \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1, \phi\left(\varepsilon_{it}\right) = -1\right] \Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
&= \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] + \\
&\quad \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}\right) = -1\right] \Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
&< \Pr\left[\phi\left(F_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]
\end{aligned}
$$

which completes the proof.

A.4 Lemma 3

Lemma 3 *Condition (8) is equivalent to*

$$
\begin{aligned}
&\Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
&> \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^d\right) = 1\right] - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^d\right) = -1\right]
\end{aligned}
$$

which is also equivalent to

$$
\frac{cov\left(\phi\left(\varepsilon_{it}\right), \phi\left(\varepsilon_{it}^c\right)\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} > \frac{cov\left(\phi\left(\varepsilon_{it}\right), \phi\left(\varepsilon_{it}^d\right)\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)}
$$

PROOF: The proof follows identical logic as the proof of Lemma 2. ∎

A.5 Proof of Proposition 4

First define

$$
\begin{aligned}
\pi_c &= \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
\pi_d &= \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^d\right) = 1\right] - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^d\right) = -1\right].
\end{aligned}
$$

Using Lemma 2 and (7), β_{Fc} can be rewritten in terms of π_c:

$$\begin{aligned}
\beta_{Fc} &= \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
&= \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right]\Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] + \\
&\quad \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] - \\
&\quad \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right]\Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] - \\
&\quad \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right] \\
&= \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right]\left(\Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] - \Pr\left[\phi\left(\varepsilon_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right]\right) + \\
&\quad \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\left(\Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = 1\right] - \Pr\left[\phi\left(\varepsilon_{it}\right) = -1 \mid \phi\left(\varepsilon_{it}^c\right) = -1\right]\right) \\
&= \pi_c\left(\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\right)
\end{aligned}$$

Note that, from Proposition 2 and Lemma 2, the term in parenthesis is positive, that is,

$$\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right] > 0. \tag{16}$$

Assume that model c is a better approximation of the true asset pricing model than model d, that is,

$$\frac{cov\left(\phi\left(\varepsilon_{it}\right), \phi\left(\varepsilon_{it}^c\right)\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} > \frac{cov\left(\phi\left(\varepsilon_{it}\right), \phi\left(\varepsilon_{it}^d\right)\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)}.$$

By Lemma 3, this relation implies that

$$\pi_c > \pi_d,$$

which means that

$$\begin{aligned}
&\pi_c\left(\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\right) \\
&> \pi_d\left(\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\right),
\end{aligned}$$

so

$$\beta_{Fc} > \beta_{Fd}.$$

Let us now prove the reverse. Assume that $\beta_{Fc} > \beta_{Fd}$. This means that

$$\begin{aligned}
&\pi_c\left(\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\right) \\
&> \pi_d\left(\Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = 1\right] - \Pr\left[\phi\left(F_{it}\right) = 1 \mid \phi\left(\varepsilon_{it}\right) = -1\right]\right),
\end{aligned}$$

which by (16) implies that $\pi_c > \pi_d$.

A.6   Proof of Proposition 5

$$\gamma_1 = \frac{cov\left(\phi\left(F_{it}\right), \frac{\phi\left(\varepsilon_{it}^c\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} - \frac{\phi\left(\varepsilon_{it}^d\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)}\right)}{var\left(\frac{\phi\left(\varepsilon_{it}^c\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} - \frac{\phi\left(\varepsilon_{it}^d\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)}\right)}$$

$$= \frac{\beta_{Fc} - \beta_{Fd}}{var\left(\frac{\phi\left(\varepsilon_{it}^c\right)}{var\left(\phi\left(\varepsilon_{it}^c\right)\right)} - \frac{\phi\left(\varepsilon_{it}^d\right)}{var\left(\phi\left(\varepsilon_{it}^d\right)\right)}\right)}.$$

By Proposition 4, $\beta_{Fc} > \beta_{Fd}$ if and only if model c is better than model d. It then follows immediately that $\gamma_1 > 0$ because the strict inequality $\beta_{Fc} > \beta_{Fd}$ rules out the possibility that the denominator is zero.

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