License to Spend: Consumption-Income Sensitivity and Portfolio Choice *

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Abstract – Contrary to the predictions of traditional life-cycle models, households do not engage in perfect consumption smoothing. Instead, consumption tracks current income. Similarly, weak evidence of income hedging runs against standard portfolio theory. We link these two puzzles by proposing a model in which current income is an entitlement to consume, or a license to spend. License-to-spend investors feel more entitled to consume as income rises and do not perfectly smooth consumption. Therefore, they are also less interested in the income hedging potential of financial assets. We test the license-to-spend model using data from the Panel Study of Income Dynamics and find that households whose consumption tracks current income also exhibit a weakened income hedging motive in their portfolio decisions. Overall, we show that the absence of income hedging is the portfolio choice analogue of imperfect consumption smoothing.

JEL classification: D11, D12, D14, G11.

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Abstract – Contrary to the predictions of traditional life-cycle models, households do not engage in perfect consumption smoothing. Instead, consumption tracks current income. Similarly, weak evidence of income hedging runs against standard portfolio theory. We link these two puzzles by proposing a model in which current income is an entitlement to consume, or a license to spend. License-to-spend investors feel more entitled to consume as income rises and do not perfectly smooth consumption. Therefore, they are also less interested in the income hedging potential of financial assets. We test the license-to-spend model using data from the Panel Study of Income Dynamics and find that households whose consumption tracks current income also exhibit a weakened income hedging motive in their portfolio decisions. Overall, we show that the absence of income hedging is the portfolio choice analogue of imperfect consumption smoothing.

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I. Introduction

Consumption and portfolio decisions are fundamentally interrelated because they are governed by the same preferences (e.g., Merton (1969), Samuelson (1969)). However, the empirical literatures on consumption and portfolio choice have developed in relative isolation. A common conclusion in both strands of literature is that standard economic models cannot fully explain household decisions. Traditional life-cycle models predict that household consumption depends on life-time income. Yet, empirical evidence documents that household consumption is excessively sensitive to current income.[1] Canonical portfolio choice models suggest that households should engage in income hedging when making portfolio decisions because income risk cannot be traded or insured.[2] Nevertheless, existing studies have not detected a strong income hedging motive in portfolio decisions.[3]

In this paper, we bridge the gap between these two literatures, and identify a novel connection between consumption smoothing and portfolio decisions. Our key conjecture is that households who do not smooth consumption (i.e., exhibit excess sensitivity of consumption to current income) might be less concerned about income hedging in their portfolio decisions. First, we formalize this conjecture and propose a theoretical framework in which current income is an entitlement to consume, or a license to spend. We show that license-to-spend investors do not engage in perfect consumption smoothing. As a consequence, they are less interested in the income hedging potential of financial assets. Second, we test the license-to-spend model using data from the Panel Study of Income Dynamics. We find that investors who do not engage in perfect consumption smoothing also exhibit a weakened income hedging motive in their portfolio decisions. To our knowledge, we are the first to jointly explore the implications of consumption-income

[1] Hall and Mishkin (1982), Courant et al. (1986), Shapiro and Slemrod (1995), Shea (1995), Parker (1999), Souleles (1999), Shapiro and Slemrod (2003), Agarwal et al. (2007), Johnson et al. (2009), Jappelli and Pistaferri (2010), Parker et al. (2013), Parker (2014).

[2] Viceira (2001), Campbell and Viceira (2002), Cocco et al. (2005), Gomes and Michaelides (2005).

[3] Heaton and Lucas (2000), Vissing-Jørgensen (2002b), Angerer and Lam (2009), Bonaparte et al. (2013).

sensitivity and portfolio decisions.

We begin our analysis with a model that can generate *both* weak consumption smoothing and weak income hedging. The novel feature of the model is that individuals treat income as an entitlement to consume, or as a license to spend. This feature is inspired by Akerlof (2007). Based on evidence from sociology and behavioral economics, Akerlof (2007) argues that household consumption is affected by consumption entitlements, with current income being the primary determinant of such entitlements.[4] We model the entitlement effect by including current income in the utility function so that each unit of consumption becomes more desirable when current income rises. We then embed the license-to-spend preferences in a consumption/portfolio choice model with exogenous labor supply (e.g., Viceira (2001), Campbell and Viceira (2002)). We solve the model analytically, and show that the license-to-spend effect results in a lack of perfect consumption smoothing. Specifically, a strong license-to-spend effect leads to a positive correlation between optimal consumption growth and current income growth.

We also find that in the license-to-spend model consumption smoothing is related to portfolio decisions. Specifically, in contrast to canonical portfolio choice models (e.g., Viceira (2001) and Campbell and Viceira (2002)), the optimal equity share in the model is directly affected by whether investors have a preference for consumption smoothing or not. The license-to-spend model essentially predicts that a high consumption-income sensitivity weakens the income hedging motive. The attenuation of income hedging arises naturally because investors who are not concerned with smoothing consumption do not value the relative income hedging properties of financial assets. The weakening of the income hedging motive due to consumption-income sensitivities is a novel feature of the license-to-spend model that has not been studied in the literature.

Next, we use household data from the Panel Study of Income Dynamics (PSID) and empirically test the predictions of the model. We use the PSID because it is the

[4]For example, see Keynes (1936), Weber (1958), Tversky and Kahneman (1981), Bourdieu (1984), Shefrin and Thaler (1988), and Guiso et al. (2006).

only longitudinal survey of U.S. households that includes data on both consumption and portfolio decisions. In our empirical exercise, we first confirm that on average PSID households do not smooth consumption by showing that their consumption growth tracks current labor income growth. We find that a one standard deviation increase in labor income growth leads to about a 2.6% increase in consumption growth. Interestingly, we find that consumption-income sensitivity is strong even among the wealthiest households in the sample. These households do not face borrowing constraints that can potentially explain consumption-income sensitivities (e.g., Jappelli (1990), Runkle (1991), Parker (2014)).

In our portfolio choice tests, we examine how preferences for consumption smoothing affect the portfolio decisions of PSID households. To implement the portfolio tests, we first estimate household-level regressions of consumption growth on income growth. We use the coefficient estimates on income growth to measure the consumption-income sensitivity for each household. We interpret these consumption-income sensitivities as an index of consumption smoothing, where the higher the sensitivity, the lower the desire to smooth consumption.

We use the household-level sensitivities to construct an interaction term that measures the attenuation of income hedging. The license-to-spend model suggests that consumption-income sensitivity affects portfolio decisions through an interaction with the traditional income hedging motive. A standard proxy for traditional income hedging motive is the correlation between household income growth and stock market returns (e.g., Vissing-Jørgensen (2002b), Massa and Simonov (2006)). Therefore, we interact consumption-income sensitivities with income-growth market-return correlations for each household. This interaction term is our income hedging attenuation measure and it is the main explanatory variable in our market participation (Probit) and asset allocation (Tobit and Heckman) regressions.

Consistent with model predictions, we find that the attenuation of income hedging is an important determinant of portfolio decisions. Specifically, the interaction term measuring the attenuation effect is economically and statistically significant in our portfolio choice regressions. For example, our estimates from the Tobit regressions suggest that a one standard deviation increase in the income-hedging attenuation term leads to a 3.0% increase in the portion of wealth allocated to risky assets. This effect is comparable to the impact of income risk, which is one of the most important determinants of equity allocation: a one standard deviation decrease in the volatility of income growth leads to a 4.0% increase in the equity share. Further, the attenuation effect is stronger than the traditional income hedging motive: a one standard deviation increase in the income-growth market-return correlation leads to just a 0.6% decrease in the equity share.

We continue to find a strong attenuation effect even when we focus on market participants alone and estimate Heckman (1979) regressions. The Heckman estimates suggest that a one standard deviation increase in the income hedging attenuation term leads to a 1.3% increase in the equity share. This effect is comparable to the impact of wealth which is also an important determinant of equity allocation: a one standard deviation increase in wealth leads to a 2.2% increase in the equity share. Overall, we find that income hedging motive is attenuated for PSID households with high consumption-income sensitivity.

A potential concern with our empirical results is that the household-level consumption-income sensitivities are estimated quantities, and may introduce generated regressor biases into our tests. To address this issue, we conduct a bootstrap simulation exercise, and obtain standard errors that account for potential estimation noise in the consumption-income sensitivities. We find that the attenuation of the income hedging motive remains statistically significant even when we use the bootstrapped standard errors. Moreover, the bootstrap simulations suggest that potential generated regressor biases in our estimates are economically and statistically insignificant. Therefore, our empirical results are robust to measurement error.

Overall, our findings are related to the household finance literature, where the evidence for income hedging has been mixed. On the one hand, Heaton and Lucas (2000) find weak evidence in support of the income hedging motive in the investment decisions of entrepreneurs. Vissing-Jørgensen (2002b) finds no evidence that the correlation between income growth and market returns influences portfolio decisions. Further, Massa and Simonov (2006) show that income hedging motives do not influence the portfolio decisions of Swedish investors. On the other hand, Bonaparte et al. (2013) find that Dutch and U.S. households consider the comovement between income growth and market returns when making portfolio decisions. A common feature of these studies is examining portfolio decisions in isolation from consumption decisions. Instead, we examine both decisions jointly, and show that consumption-income sensitivity attenuates the income hedging motive.

Our work is also related to the literature on the excess sensitivity of consumption to current income. A leading explanation of consumption-income sensitivity is borrowing constraints.[5] Borrowing restrictions are a reasonable explanation, especially for young individuals who have not accumulated a substantial stock of wealth. However, we find that even wealthy households in our sample choose not to smooth consumption. Another interesting feature of the license-to-spend model is that it can account for consumption-income sensitivity when changes in income are predictable. However, even recent state-of-the-art models with borrowing constraints, permanent and transitory labor income shocks, and hyperbolic discounting[6] cannot explain the responsiveness of consumption to anticipated income fluctuations.[7]

The rest of the paper is organized as follows. Section II presents the license-to-spend model. Section III describes the PSID data. Section IV presents evidence that consumers in the PSID do not smooth consumption. Section V examines how a lack of consumption smoothing affects the portfolio decisions of PSID households. Section VI concludes with a brief discussion.

[5]Zeldes (1989), Jappelli (1990), Runkle (1991), Deaton (1992), Carroll (1994), Gourinchas and Parker (2002), Parker (2014).

[6]Laibson (1997), Laibson (1998), Laibson et al. (1998), Gomes and Michaelides (2005)

[7]Poterba (1988), Wilcox (1989), Parker (1999), Souleles (1999, 2002), Browning and Collado (2001), Hsieh (2003), Johnson et al. (2006), Baker et al. (2007)).

II. Income as Consumption Entitlement

In this section we present the license-to-spend model. The model is an extension of the dynamic portfolio choice model with fixed labor supply (i.e., Viceira (2001)). The license-to-spend model does not include features like borrowing constraints or market participation costs that would hinder us from solving the model analytically. We choose to work with a simple tractable model, so that we can obtain analytical solutions for the optimal portfolio weights and clearly illustrate how the absence of perfect consumption smoothing affects portfolio decisions.[8]

II.A. Utility from Consumption and Income

We assume that income, Y_t, is an entitlement to consume or a license to spend. To make income a consumption entitlement, we assume that consumption, C_t, becomes more desirable as income increases. Formally, we assume that the marginal utility of consumption is increasing in income. In the model, we make the marginal utility dependent on income by including income in the utility function:

$$U(C_t; Y_t) = \frac{C_t^{1-\gamma}}{1-\gamma} Y_t^{\theta}. \tag{1}$$

The above utility function is an extension of the constant relative risk aversion (CRRA) model. The constant γ is the risk aversion (RA) parameter, and the coefficient θ determines the importance of the license-to-spend effect.[9] For positive θ, an extra dollar of

[8]Generally, there are no closed-form solutions for the optimal portfolio weights in multiperiod models. To gain tractability Campbell and Viceira (1999) and Jurek and Viceira (2011) assume that log returns are predictable and follow an autoregressive model with normally distributed disturbances. Brennan and Xia (2002) use numerical methods to solve the dynamic portfolio problem assuming that returns are predictable due to asset pricing anomalies. Wachter (2002) examines portfolio choice under complete markets and mean-reverting returns. Chacko and Viceira (2005) assume that expected return are constant across time but return volatility is time-varying. Liu (2007) assumes that returns are quadratic following general Markovian processes.

[9]We require that $\gamma > 1$ so that the elasticity of intertemporal substitution $(1/\gamma)$ is lower than one. For $\gamma > 1$, we also require that $0 \leq \theta < (\gamma-1)(1-\phi_1)$ in which $(1-\phi_1)$ is the income elasticity of consumption. Since the consumption function is an isoelastic aggregator of income and wealth $(C_t = e^{\phi_0} W_t^{\phi_1} Y_t^{1-\phi_1})$, these conditions on θ guarantee that the utility function is increasing in income and concave.

consumption becomes more valuable if it is financed by an increase in current income. In other words, individuals feel happier when they finance their consumption from current income rather than tapping into their savings or taking on debt. For a detailed description of the properties of the utility function in (1), see Appendix A.

The assumption that income is an entitlement to consumption is consistent with the sociology literature. For instance, according to Akerlof (2007), there is ample evidence that consumption decisions are affected by norms about what people think they are entitled to consume.[10] He argues that

> "First, sociology gives motivations for consumption that are very different from the reasons for it in the life-cycle hypothesis. A major determinant of consumption is what people think they *should* consume. Second, what people think they should consume can often be viewed either as an entitlement or as obligations. Finally, in turn, current income is one of the major determinants of these entitlements, and obligations." (Akerlof (2007), p. 15).

Inspired by Akerlof's arguments, we include current income in the utility function so that as income rises, marginal utility also rises, and consumers feel more entitled to increase their consumption spending.

Several reasons guide our choice to include current income in the utility function instead of life-time income or wealth. First, based on Akerlof's (2007) arguments, life-time income does not affect consumption entitlements, which are mainly driven by current income. Second, including income in the utility function allows us to solve analytically for the consumption function and optimal portfolio weights, which is not possible with wealth in the utility function. More importantly, labor income seems to be a more plausible proxy for consumption entitlements than wealth. This is because consumption entitlements essentially reflect internal and external influences on the proper level of consumption. Labor income can capture internal consumption entitlements better than

[10] For example, see Tversky and Kahneman (1981), Bourdieu (1984), Shefrin and Thaler (1988), and Guiso et al. (2006).

wealth because labor income represents compensation for the provision of effort. In other words, labor income, as opposed to wealth, can capture preferences related to a "work hard, play hard" mentality, which is related to the internal license to spend. Labor income can also capture external consumption entitlements better than wealth since individuals are usually better informed about the labor income of their peers than they are about the wealth of their peers.

Including income in the utility function is also consistent with evidence from behavioral economics. To better illustrate the link with the behavioral literature, we can rewrite the utility function by replacing Y_t/C_t with $1/(1-s_{y,t})$, where $s_{y,t} = (Y_t - C_t)/Y_t$ is the savings rate out of income:

$$U(C_t; Y_t) = \frac{C_t^{1-\gamma+\theta}}{1-\gamma}\Big(\frac{1}{1-s_{y,t}}\Big)^{\theta}.$$

This alternative specification of the license-to-spend model suggests that investors derive utility from consumption as well as savings. This is consistent with findings that consumers view savings as a separate decision, instead of simply a residual to consumption (e.g., Furnham and Argyle (1998)).

A utility function defined over consumption as well as savings is also consistent with the debt aversion model of Prelec and Loewenstein (1998). They argue that the process of spending involves an immediate pain of paying that can reduce the pleasure of consuming. Thus, the utility process should be the *sum* of the happiness from consuming and the grief from paying instead of saving. The disutility of paying is the highest when spending is financed by borrowing.

Thaler (1985) also proposes a transaction utility theory in which transactions involve *both* acquisition utility and transaction utility. Thaler's analysis is similar to the debt aversion framework of Prelec and Loewenstein (1998). Both papers stress that the process of buying a good has two dimensions: acquisition and transaction. In our framework, the transaction utility is related to the reward from saving.

II.B. Life-Cycle Consumption-Portfolio Model

We embed the license-to-spend effect in the dynamic portfolio choice model of Viceira (2001) in which investors have access to a risky and a risk-free asset. In our model, investors can either be employed or retired. When investors are employed, they receive a non-tradeable endowment Y_t (labor income). Retirement can occur with probability $\pi_r > 0$, while $\pi_e = 1 - \pi_r$ is the probability of staying employed. Retirement is an absorbing state and is independent of income growth shocks or asset returns. During retirement, investors receive a constant pension $\bar{Y}$, which is equal to the last pre-retirement income payment.

To close the model, we follow Viceira (2001) and assume that income growth during employment is an i.i.d. process with constant volatility given by

$$\Delta y_{t+1} = \mu_y + \sigma_{\Delta y} \epsilon_{y,t+1},$$

where $\epsilon_{y,t+1}$ are i.i.d. $N(0,1)$ random variables. We also assume that the risk-free rate is constant, and that the log return on the risky asset r_m is normally distributed with constant mean and volatility:

$$r_{m,t+1} = \mu_m + \sigma_m \epsilon_{m,t+1},$$

where $\epsilon_{m,t+1}$ are i.i.d. $N(0,1)$ shocks. Finally, the correlation between income growth shocks (ϵ_y) and asset return shocks (ϵ_m) is $\rho_{y,m}$.

Next, we derive the optimal consumption and portfolio rules. We first consider the investor's decisions post-retirement. Then, conditional on the post-retirement decisions, we use backward induction to solve for optimal consumption and portfolio rules pre-retirement. The behavior of the investor prior to retirement is the basis for our empirical analysis.

II.C. Consumption and Portfolio Rules During Retirement

During retirement, the investor chooses consumption and portfolio weights to maximize his lifetime utility by solving the following maximization problem:

$$\max_{\{C_\tau\}_{\tau=t}^{\infty},\{\alpha_\tau\}_{\tau=t}^{\infty}} \mathbf{E}_t\Big[\sum_{\tau=t}^{\infty}\beta^{\tau-t}\frac{C_\tau^{1-\gamma}}{1-\gamma}\bar{Y}^{\theta}\Big], \text{ subject to} \tag{2}$$
$$W_{t+1} = (W_t - C_t + \bar{Y})[a_t(e^{r_{m,t+1}} - e^{r_{f,t+1}}) + e^{r_{f,t+1}}] \; \forall \; t,$$

where $\beta \in (0,1)$ is the rate of time preference, W_t is wealth, and a_t is the portfolio weight on the risky asset.

Proposition 1: During retirement, the optimal consumption rule is linear in wealth

$$c_t^r = \phi_0^r + \phi_1^r w_t^r, \text{ with } \phi_1^r = 1, \tag{3}$$

the optimal portfolio rule is

$$a_t^r = a^r = \frac{\mu_m - r_f + 0.5\sigma_m^2}{\gamma\sigma_m^2}, \tag{4}$$

and the Euler equation for the risk-free rate is

$$\mathbf{E}_t[\beta e^{-\gamma\Delta c_{t+1}^r}] = e^{-r_{f,t+1}}. \tag{5}$$

Proof: See Appendix B.

Lower-case c^r and w^r respectively represent log consumption and log wealth during retirement.[11] The parameter ϕ_1^r in equation (3) is the elasticity of consumption to wealth, which is equal to one during retirement. The constant ϕ_0^r is an endogenous, negative parameter that captures the precautionary savings motive.

The optimal decision rules after retirement are very similar to traditional life-cycle models. Specifically, based on the Euler equation for the risk-free rate in (5), we find

[11] The superscript r denotes retirement and the superscript e denotes employment.

that consumption growth does not depend on income growth because income is constant. Further, according to the expression in (4), the optimal equity share of the risky asset during retirement depends on the risk-return trade-off of the risky asset and the investor's risk aversion, as in Merton (1969) and Samuelson (1969).

II.D. Consumption and Portfolio Rules During Employment

Based on the optimal decisions during retirement, the investor solves for the optimal consumption and portfolio rules during employment. During employment, the consumption-portfolio problem becomes:

$$\max_{C_t, a_t} V_t^e = \frac{C_t^{1-\gamma}}{1-\gamma} Y_t^\theta + \beta \mathbf{E}_t \Big[\pi_e V_{t+1}^e + \pi_r V_{t+1}^r \Big], \text{ subject to} \tag{6}$$
$$W_{t+1} = (W_t - C_t + Y_t)[a_t(e^{r_{m,t+1}} - e^{r_{f,t+1}}) + e^{r_{f,t+1}}],$$

where V_t^e is lifetime utility while employed and V_t^r is lifetime utility when retired.

Proposition 2: During employment, the optimal log consumption-income difference ($c_t^e - y_t$) is affine in the log wealth-income difference ($w_t^e - y_t$),

$$c_t^e - y_t = \phi_0 + \phi_1 (w_t^e - y_t), \text{ with } 0 < \phi_1 < 1, \tag{7}$$

the optimal portfolio rule is

$$a^e = \frac{\mu_m - r_f + 0.5\sigma_m^2}{\gamma(\pi_r + \pi_e \phi_1)\sigma_m^2} - \Big(1 - \phi_1 - \frac{\theta}{\gamma}\Big) \frac{\pi_e \sigma_{\Delta y} \sigma_m}{(\pi_r + \pi_e \phi_1)\sigma_m^2} \rho_{y,m}, \tag{8}$$

and the Euler equation for the risk-free rate between two consecutive employment periods is

$$\mathbf{E}_t[\beta e^{-\gamma \Delta c_{t+1}^e + \theta \Delta y_{t+1}}] = e^{-r_{f,t+1}}. \tag{9}$$

Proof: See Appendix C.

According to the consumption function in (7), the parameter ϕ_1 is the elasticity of consumption to wealth, and $(1 - \phi_1)$ is the elasticity of consumption to income. The consumption-wealth elasticity depends on π_e and the log-linearization constants in (23). The elasticity ϕ_1 is always less than one, and it does not depend on the consumption-income sensitivity parameter (θ) because income growth is i.i.d.

The parameter ϕ_0 in (7) captures precautionary savings during employment. ϕ_0 is constant because we assume that asset returns and income growth are unpredictable processes with constant volatility. In Appendix C (equation (28)), we show that consumption-income sensitivity affects ϕ_0, which may lead to consumption that is higher, lower, or unchanged consumption relative to the traditional life-cycle model. The strength and direction of the relative change in consumption depends on the interaction between θ and the rest of the parameters.

II.E. Theoretical Predictions: Consumption-Income Sensitivity

The assumption that income is an entitlement to consumption affects the optimal consumption and portfolio decisions during employment, a period during which income is stochastic. According to the Euler equation (9) in *Proposition 2*, consumption growth depends on income growth. In other words, consumption growth is sensitive to current income changes. This prediction arises because income is an entitlement to consumption and thus, the marginal utility of consumption rises with income. Therefore, the license-to-spend model can rationalize the evidence that household consumption growth is excessively sensitive to current income growth.

II.F. Theoretical Predictions: Attenuation of Income Hedging

The optimal weight on the risky asset according to the license-to-spend model in equation (8) is different from models that ignore the entitlement effect. Specifically, the optimal equity share is determined by both the risk-return term, similar to Merton (1969) and

Samuelson (1969), and by an income hedging term. The novel feature of our model is that the importance of income hedging depends on two confounding effects: the traditional income hedging effect driven by the consumption-income elasticity $(1-\phi_1)$, and the preference for consumption smoothing driven by the new consumption-income sensitivity effect (θ/γ).

To identify the traditional income hedging motive, assume that the investor does not exhibit consumption-income sensitivity (i.e., $\theta = 0$). In this case, our model reduces to that of Viceira (2001), and the hedging term in equation (8) becomes

$$-(1-\phi_1)\frac{\pi_e \sigma_{\Delta y}\sigma_m}{(\pi_r+\phi_1\pi_e)\sigma_m^2}\rho_{y,m}.$$

Because the consumption-income elasticity $(1-\phi_1)$ does not depend on θ and it is always less than 1, the sign of the traditional hedging term depends on the correlation between income growth and the return of the risky asset, $\rho_{y,m}$. When $\rho_{y,m}$ is positive, investors have a disincentive to allocate much of their wealth to the risky asset because such an investment will magnify their total risk exposure. However, when $\rho_{y,m}$ is negative, the risky asset has income hedging benefits and investors should allocate a significant portion of their savings to the risky asset.

When consumption is sensitive to current income (i.e., $\theta > 0$), the hedging term in (8) is given by

$$-\left(1-\phi_1-\frac{\theta}{\gamma}\right)\frac{\pi_e \sigma_{\Delta y}\sigma_m}{(\pi_r+\phi_1\pi_e)\sigma_m^2}\rho_{y,m}.$$

In this case, if an investor exhibits strong consumption-income sensitivity, the term $1-\phi_1-\theta/\gamma$ is much smaller than the traditional hedging parameter $1-\phi_1$. The model then predicts that investors with positive correlation $\rho_{y,m}$ who are also highly consumption-income sensitive should not fully hedge their income risk.

The intuition for this prediction is simple: investors with strong consumption-income sensitivity are not concerned with smoothing consumption. Consequently, they do not value risky assets as vehicles for mitigating income shocks and reducing consumption volatility. Instead, when they exhibit strong consumption-income sensitivity (i.e., $1-\phi_1 \approx \theta/\gamma$), they want to invest a lot in assets that pay well when their income is rising, thus enhancing the consumption-entitlement effects of income. Overall, the model confirms our intuition that a strong consumption-income sensitivity confounds and attenuates the traditional income hedging motive.

In our empirical study, we disentangle the traditional hedging component of risky asset demand from the component that is affected by the consumption-income sensitivity. To estimate our model, we decompose the optimal equity share into three components:

$$a^e = \frac{\mu_m - r_f + 0.5\sigma_m^2}{\gamma(\pi_r + \pi_e\phi_1)\sigma_m^2} - (1-\phi_1)\frac{\pi_e\sigma_{\Delta y}\sigma_m}{(\pi_r + \pi_e\phi_1)\sigma_m^2}\rho_{y,m} + \frac{\pi_e\sigma_{\Delta y}\sigma_m}{(\pi_r + \pi_e\phi_1)\sigma_m^2}\frac{\theta}{\gamma}\rho_{y,m}. \quad (10)$$

The above decomposition breaks the optimal equity share into a risk-return ratio term, a traditional income hedging term, and an income sensitivity hedging term.

To identify each of the two hedging terms, we include two hedging control variables in our portfolio choice regressions (i.e., Probit, Tobit and Heckman regressions). The first is the correlation $\rho_{y,m}$, which captures the traditional income hedging motive and depends on the consumption-income elasticity. The second is the interaction between the consumption-income sensitivity term θ/γ and the correlation $\rho_{y,m}$. Our model predicts that the estimate on $\rho_{y,m}$ should be negative and the estimate on the interaction term should be positive.

III. Data and Summary Statistics

In this section we describe the data, and present summary statistics of the main variables used in our empirical analysis.

III.A. Panel Study of Income Dynamics

We use data from the Panel Study of Income Dynamics (PSID) because, to our knowledge, it is the only longitudinal survey that includes both consumption and portfolio decisions for a large sample of U.S. households.[12] The long panel nature of the PSID allows us to estimate household-level consumption growth regressions to obtain estimates for consumption-income sensitivities.[13]

In the first part of our empirical exercise we estimate regressions of consumption growth on income growth. For these regressions, we collect consumption and income data for all available survey years between 1978 and 2009. Our measure of consumption is total food expenditures, the sum of expenditures on food consumed at and away from home. As in many prior studies using the PSID, we treat food consumption as a proxy for total consumption (e.g., Zeldes (1989), Mankiw and Zeldes (1991), Runkle (1991), Lusardi (1996)).

We also collect income and wealth data. Our income measure is total household labor income. Wealth is measured as the household's net worth. A large component of wealth is financial wealth that includes holdings in equities, IRA's, and bonds, as well as checking and savings accounts.[14] We define stock market participants as households that hold equity directly or indirectly through IRA holdings in stocks. We also collect various demographic variables such as age, employment status, number of children, and education. Finally, we use the U.S. stock market return index and the risk-free rate from Kenneth French's data library. Further, we deflate all asset returns, income, and consumption using the consumer price index provided by the Bureau of Labor Statistics.

We compute consumption growth and income growth to estimate consumption-income sensitivities. We also compute several household income-growth moments that we use in

[12]The PSID is partly supported by the National Institutes of Health under grant number R01 HD069609 and the National Science Foundation under award number 1157698.

[13]The long panel nature of the PSID has made it a frequent data source for studies of consumption and, more recently, asset allocation (e.g., Mankiw and Zeldes (1991), Shea (1995), Dynan (2000), Brunnermeier and Nagel (2008)).

[14]Wealth information in the PSID is available only every five years between 1984 and 1999. Thereafter, it is available every two years.

our portfolio choice regressions. Similar to Guiso et al. (1996) and Heaton and Lucas (2000), we define income risk as the standard deviation of income growth. To measure the traditional income hedging motive, we compute the correlation between household labor income growth and stock market returns. We compute one correlation for each household using all available data for the household, which is consistent with Vissing-Jørgensen (2002b), Massa and Simonov (2006), and Bonaparte et al. (2013).

Following the literature (e.g., Runkle (1991), Vissing-Jørgensen (2002a), Angerer and Lam (2009)), we impose various sample filters. We delete household-year observations in which annual consumption growth or income growth is higher than 300% or lower than -70%, or where these quantities are missing. We also delete observations with income less than $100, and households whose income growth has a standard deviation greater than 110%.

One issue with the PSID is that surveys were administered annually prior to 1997 and only biannually after 1997. To maximize our sample size, we combine data from the annual and biannual waves. We annualize growth rates from the biannual observations by first computing the 2-year income and consumption log-growth rates, and then dividing the 2-year growth rates by 2.

Following Zeldes (1989) and Mankiw and Zeldes (1991), we interpret the PSID question on consumption as a measure of consumption during the first quarter of the survey year t.[15] Therefore, to match the timing of consumption with the timing of the risk-free rate in the consumption growth regressions, we measure the risk free rate between the first quarter of the survey year t ($Q1_t$) and the first quarter of the subsequent year $t+1$ ($Q1_{t+1}$). We then compute the $Q1_t$-to-$Q1_{t+1}$ risk-free rate, $r_{t,t+1}$, by compounding monthly risk-free rates as in Mankiw and Zeldes (1991). After 1997, when the PSID became biannual, we compute an annualized 2-year risk-free rate by compounding the $r_{t-1,t}$ and $r_{t,t+1}$ rates, and then dividing by 2.

[15] This survey question is administered in the first quarter of the following year and refers to recent food consumption.

The existing literature also finds that the timing of the survey question regarding income is ambiguous (e.g., Zeldes (1989)). The literature typically interprets the income reported in survey year t as the average income between years t and $t-1$. We adopt the same timing convention. Therefore, when we compute the correlation between income growth and market returns, we ensure that the timing of the market return follows the same convention. For example, the market return for survey year t is the average of the annual market return in year t and year $t-1$.

III.B. Summary Statistics

Summary statistics for the full sample are presented in Table I. In Table III, we present summary statistics for the portfolio choice sample. The latter sample, which is smaller than the full sample, focuses on households that do not have missing information for wealth, stock market participation, and education.

The statistics in Panel A of Table I show that consumption growth is on average about 1% whereas income growth is on average 2.5%. Both income and consumption growth are highly volatile; their standard deviation is higher than 30%. Consistent with the hypersensitivity literature, consumption and current income growth are also positively correlated; their correlation coefficient is 7.5% and it is statistically significant. The statistics in Panel B show that the average age in the full sample is 41, and only 3% of the households are retired. Finally, about 83% of households have financial assets such as savings or retirement accounts.

Next, we present the regression-based evidence of consumption-income sensitivity among households in the PSID.

IV. Consumption-Income Sensitivity Evidence

In this section, we provide evidence that PSID households, on average, do not choose to smooth consumption, and that consumption growth depends on current income growth.

IV.A. Empirical Specification

Following existing studies of consumer behavior (e.g., Zeldes (1989), Vissing-Jørgensen (2002b)), our estimation of consumption-income sensitivities is based on the Euler equation for the risk-free rate during two consecutive employment dates which, according to *Proposition 2*, is:

$$\mathbf{E}_t[\beta_i e^{-\gamma \Delta c_{i,t+1} + \theta \Delta y_{i,t+1}}] = e^{-r_{f,t+1}}.$$

In the above specification, cross-sectional heterogeneity across households is captured by differences in the rate of time preference β_i. To obtain the empirical regression, we rewrite the Euler equation by replacing the conditional expectation $\mathbf{E}_t$ with a multiplicative error term $e^{\epsilon_{i,t+1}}$. Then, we take logs and solve for consumption growth $\Delta c_{i,t+1}$. The resulting expression is the following:

$$\Delta c_{i,t+1} = log\beta_i + \frac{1}{\gamma} r_{f,t+1} + \frac{\theta}{\gamma} \Delta y_{i,t+1} + \epsilon_{i,t+1}. \tag{11}$$

The term $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution (EIS), and shows how consumption growth reacts to changes in interest rates. The parameter $\frac{\theta}{\gamma}$ captures consumption-income sensitivity, or the propensity of the household to not smooth consumption over time. Since the EIS is positive, a positive θ implies that an increase in income growth will also lead to an increase in consumption growth. When θ is zero, we obtain the Euler equation for the traditional life-cycle model.

A reduced-form specification for the Euler equation in (11) is given by

$$\Delta c_{i,t+1} = \alpha_0 + \boldsymbol{\alpha}_{0,x} \mathbf{X}_{i,t+1} + \alpha_{r_f} r_{f,t+1} + \alpha_y \Delta y_{i,t+1} + \epsilon_{i,t+1}, \tag{12}$$

in which α_{r_f} captures the EIS, α_y is the interaction between consumption-income sensitivity and the EIS. The rate of time preference $log\beta_i$ is replaced by a constant, α_0, and a vector of household control variables, $\mathbf{X}_{i,t+1}$, that capture cross-sectional heterogeneity. These variables include age, age^2, number of children, as well as indicators for college education and unemployment of the household head. Since total consumption, $C_{i,t}$, is proxied by total food expenditures, $\tilde{C}_{i,t}$, at and away from home, the above specification may suffer from measurement error. However, if the measurement error in consumption is multiplicative and independent of all other variables, α_{r_f} and α_y can be consistently estimated with ordinary least squares (Vissing-Jørgensen (2002a)).

IV.B. Estimation of Consumption-Income Sensitivities

We estimate the consumption growth regressions with OLS, and present the results in Table II. For the full sample case in column (2), we find that the estimate on income growth is 0.081. This estimate is statistically significant (t-statistic = 17.69) even in the presence of additional control variables (i.e., age, age^2, number of children, and indicators for college education and unemployed household head). The consumption-income sensitivity is also economically significant. A one standard deviation increase in income growth (0.32) leads to a 2.6% ($0.32 \times 0.081 \times 100$) increase in consumption growth.

We also find that consumption growth is responsive to interest rate changes. Consistent with the results in Vissing-Jørgensen (2002a), the estimate of the EIS is positive, significant, and less than one (estimate = 0.162, t-statistic = 3.98). This estimate implies that a one standard deviation increase in the interest rate (0.029) leads to an increase in consumption growth of 0.47% ($0.162 \times 0.029 \times 100$). In unreported results, we also find that based on the estimates of the consumption-income sensitivity α_y and the EIS, the consumption entitlement parameter θ in equation (1) is 0.466 ($0.081 \div 0.162$), with a t-statistic of 4.50.[16] This estimate is consistent with our assumption that θ is positive and less than 1.

The significance of the consumption-income sensitivities is robust and present in various subsamples. First, the results in columns (3) and (4) indicate that consumption is

[16] The standard error for θ is calculated using the delta-method.

sensitive to current income irrespective of the retirement status of the head-of-household. However, consistent with our model, consumption of retired households is less sensitive to income. Further, consumption-income sensitivity is also present in age-based subsamples. For example, the consumption-income sensitivity for the youngest households is 0.091 (t-statistic = 9.69) in column (5) and 0.041 (t-statistic = 5.04) for the oldest households in column (6).

One potential explanation for the consumption-income sensitivity is the presence of borrowing constraints. The inability to borrow might prohibit consumers from smoothing consumption, forcing consumption expenditures to track income (e.g., Runkle (1991)). To ensure that our estimates of consumption-income sensitivity do not reflect borrowing constraints, we estimate the consumption growth regressions on subgroups of households that should not face difficulty borrowing. Drawing on Jappelli (1990), we examine households with the highest net worth (top quartile) and income (top quartile). The estimates in columns (8) and (10) of Table II show that consumption tracks current income even for the wealthiest households (estimate = 0.076, t-statistic = 5.15) and top income earners (estimate = 0.086, t-statistic = 8.55). This finding is consistent with prior evidence in the literature (e.g., Parker (2014)).

Vissing-Jørgensen (2002a) argues that the Euler equation used in the consumption-income sensitivity literature is only valid for households that hold some form of financial assets. Following her work, we estimate the consumption growth regressions for holders and non-holders of financial assets, and report the results in columns (11) and (12) of Table II. Similar to Vissing-Jørgensen (2002a), we find that consumption growth is more responsive to interest rate changes in the subsample of financial asset holders. More importantly, the consumption-income sensitivity is significant for both groups, though it is stronger among financial asset holders.

Overall, our findings from the consumption growth regressions indicate that consumption is overly sensitive to income, and that this sensitivity cannot be entirely explained by borrowing constraints. Next, we use this finding, along with the predictions of our license-to-spend model, to study the relationship between consumption-income sensitivity and investment decisions.

V. Portfolio Choice and Attenuation of Income Hedging

In this section, we examine the impact of consumption-income sensitivity on portfolio decisions, and provide evidence that strong consumption-income sensitivities weaken the income hedging motive in portfolio decisions. Our model predicts that the optimal allocation to the risky asset of an employed investor is given by

$$a_i^e = \frac{\mu_m - r_f + 0.5\sigma_r^2}{(\pi_r + \pi_e \phi_1)\sigma_m^2}\frac{1}{\gamma_i} - (1-\phi_1)\frac{\pi_e \sigma_{\Delta y_i}\sigma_m}{(\pi_r + \pi_e \phi_1)\sigma_m^2} \times \rho_{y_i,m} + \frac{\pi_e \sigma_{\Delta y_i}\sigma_m}{(\pi_r + \pi_e \phi_1)\sigma_m^2} \times \frac{\theta_i}{\gamma_i} \times \rho_{y_i,m}, \tag{13}$$

or equivalently,

$$a_i^e = b_{i,0} + b_{i,1}\rho_{y_i,m} + b_{i,2}(\alpha_{i,y} \times \rho_{y_i,m}), \tag{14}$$

where the parameter $\alpha_{i,y}$ is the consumption-income sensitivity.

The main prediction of the model is that consumption-income sensitivity attenuates the traditional hedging motive. If the correlation $\rho_{y_i,m}$ is positive, the traditional hedging motive predicts that the equity share a_i^e should be low. However, when the consumption-income sensitivity effect is strong (i.e., $\alpha_{i,y} > 0$), the magnitude of the traditional hedging motive on the optimal equity share, a_i^e, is offset by the consumption-income sensitivity term, $b_2\alpha_{i,y}$. Testing this prediction is the main focus of our empirical analysis.

V.A. Household-Level Consumption-Income Sensitivities

In our portfolio analysis we estimate pooled Probit, Tobit, and two-stage Heckman regressions. The main independent variables are household-level estimates of the correlation,

$\rho_{y_i,m}$, and of the interaction term between the correlation and the consumption-income sensitivity, $\alpha_{i,y}$.

A novel feature of our work is estimating household-level consumption-income sensitivities $\alpha_{i,y}$. Specifically, for each household we estimate a consumption growth regression,

$$\Delta c_{i,t+1} = \alpha_{i,0} + \alpha_{i,r_f} r_{f,t+1} + \alpha_{i,y} \Delta y_{i,t+1} + \epsilon_{i,t+1}, \tag{15}$$

and obtain estimates of $\alpha_{i,y}$. To ensure precision in our estimates of $\alpha_{i,y}$, we focus on households that have at least 12 valid (i.e., non-missing) consumption growth observations. Also, we exclude all observations with a retired head.

Table III reports the average of the $\alpha_{i,y}$ estimates, which is 0.082. The average of the estimated $\alpha_{i,y}$ is reasonable because it is almost identical to the full-sample sensitivity estimate from our pooled consumption growth regressions in column (2) of Table II. In Table III, we also report summary statistics related to all of the variables we use in our portfolio choice regressions. In our sample, about 47% of respondents own stocks directly or indirectly through mutual funds and retirement accounts. On average, stockholders allocate 57% of their financial wealth to risky assets. Further, about one third of the sample is college educated, with an average age of 47.

To test the asset allocation prediction of the model, we develop the empirical counterpart of equation (14). We first proxy $\rho_{y_i,m}$, $\alpha_{y_i,m}$, and $\sigma_{\Delta y_i}$ with their respective estimates $\hat{\rho}_{y_i,m}$, $\hat{\alpha}_{y_i,m}$, and $\hat{\sigma}_{\Delta y_i}$, and add an error term u_i. To control for investor heterogeneity, we then include a group of control variables $\mathbf{Z}_i$, which have been found to be significant in the prior literature. These control variables are income, wealth, the standard deviation of income growth ($\sigma_{\Delta y_i}$), age, age^2, number of children, an unemployment indicator, and a college graduate indicator. Finally, we include a set of year indicators (time fixed effects) to capture aggregate economic conditions as well as any time-variation in the Sharpe ratio, which affects the optimal portfolio weight in (13). The reduced-form expression for the optimal equity share reads

$$a^e_{i,t} = b_t + \mathbf{b}_{0,z}\mathbf{Z}_{i,t} + b_1\hat{\rho}_{y_i,m} + b_2(\hat{\alpha}_{i,y} \times \hat{\rho}_{y_i,m}) + u_{i,t}. \tag{16}$$

The above relation is the basis for our empirical asset allocation regressions.

V.B. Stock Market Participation: Probit Estimates

We first examine whether consumption-income sensitivity affects the decision to participate in the stock market. Even if our model does not allow for participation costs, and thus all households should participate in the market, there is ample evidence that many households do not invest in risky stocks (e.g., Mankiw and Zeldes (1991), Haliassos and Bertaut (1995), Campbell (2006)). It is therefore worthwhile examining whether consumption-income sensitivity is related to the participation decision. We present estimates from market participation probit regressions in columns (1) and (2) of Table IV.

In the first probit regression in column (1), we include only the control variables. The estimates from this regression indicate that wealthy college graduates with high income and low income-growth volatility participate the most. In the second probit regression in column (2), we add the interaction term between the income-growth market-return correlation and the consumption-income sensitivity. We find that the estimate of the interaction term is positive and statistically significant (estimate = 0.075, t-statistic = 2.82). The information in the interaction term is also not related to any of the control variables because its inclusion in the probit regression does not affect their estimates and statistical significance.

The effect of the consumption-income sensitivity on participation is economically significant. For instance, consider two households with the same positive correlation $\rho_{y_i,m}$, equal to 0.50. However, the first household is more consumption-income sensitive. If their consumption-income sensitivities differ by one standard deviation (0.559, see Table

III), then the first household will be about 2.1% ($0.075 \times 0.559 \times 0.50 \times 100$) more likely to own stocks. This effect is close to the economic effect of income risk on participation; our estimates suggest that a one-standard deviation increase in the volatility of income growth (0.115, see Table III) leads to a decrease in participation of about 2.6% ($-0.231 \times 0.115 \times 100$).

The positive effect of the $\alpha_{i,y} \times \rho_{y_i,m}$ term on participation is consistent with our model, which predicts that consumption-income sensitivity should attenuate the traditional income-hedging motive. However, our model does not allow for limited stock-market participation, and its predictions are largely related to the asset allocation decision, which we examine next.

V.C. Asset Allocation: Tobit Estimates

According to the license-to-spend model, consumption-income sensitivity should affect the share of financial wealth invested in risky assets. Next, we estimate Tobit regressions, and present the estimation results in columns (3) and (4) of Table IV. In these regressions the dependent variable is the percentage of financial wealth invested in stocks held directly or indirectly through retirement accounts.

The Tobit regression in column (3) includes only the control variables, while the Tobit regression in column (4) adds the consumption-income sensitivity term $\alpha_{i,y} \times \rho_{y_i,m}$. Consistent with previous evidence, we find that wealthy, middle-aged college graduates with high income invest the most in risky assets (e.g., Campbell (2006)). Further, we find that households with low income-growth volatility allocate more to risky assets, in line with the findings of Angerer and Lam (2009) and Betermier et al. (2012). Although not significant, we also find that the estimate on the correlation $\rho_{y_i,m}$ is negative, which is consistent with the traditional income-hedging motive. In column (4), the estimate on the correlation is -2.558 and its t-statistic is -1.02.

The results in Table IV provide evidence that consumption-income sensitivity attenuates the traditional income-hedging motive. The estimate on the interaction term

$\alpha_{i,y} \times \rho_{y_i,m}$ is positive and statistically significant (estimate = 15.467, t-statistic = 3.63). This estimate implies a strong economic effect of consumption-income sensitivity on the asset allocation decision. Consider again the two households with the same positive correlation $\rho_{y_i,m}$, equal to 0.50, and the first household being more consumption-income sensitive. If their consumption-income sensitivities differ by one standard deviation, 0.559 from Table III, then the first household will invest about 4.3% ($15.467 \times 0.50 \times 0.559$) more in risky assets.

Similarly, our estimates suggest that a one standard deviation increase in the income-hedging attenuation term (0.189, see Table III) leads to about a 3.0% (15.467×0.189) increase in the allocation to stocks. This effect is economically important, and its magnitude is comparable to the effect of income risk on equity allocation. Our estimation suggests that a one standard deviation increase in the volatility of income growth (0.115) will lead to about a 4.4% (-38.401×0.115) decrease in the equity share. This result is notable since income risk has been shown to be one of the most important determinants of equity allocation (e.g., Vissing-Jørgensen (2002b), Angerer and Lam (2009), Bonaparte et al. (2013)).

V.D. Asset Allocation: Heckman Estimates

The estimates from the Tobit regressions suggest that consumption-income sensitivity weakens the traditional income hedging motive. However, the Tobit results are based on a sample that includes both stockholders and non-stockholders. To ensure that the Tobit results are not driven entirely by the participation decision, we estimate Heckman (1979) regressions that simultaneously consider the participation and asset allocation decisions.

As in Vissing-Jørgensen (2002b), we estimate a system of two equations. The first is the participation equation estimated with data on both stockholders and non-stockholders. The first stage regression provides an estimate for the probability of participating, which is used in the second stage estimation of the equity share regression. The equity share regression is estimated using data for the stockholders only. We present the results of the joint estimation of the participation and asset allocation regressions in Table V.

Table V reports estimates from two Heckman specifications. For the first specification in columns (1) and (2), we exclude the $\alpha_{i,y} \times \rho_{y_i,m}$ interaction term. For the second in columns (3) and (4), we include the interaction term in order to capture the effects of consumption-income sensitivity on portfolio allocation.

V.D.1. Heckman Participation Estimates

Consistent with our previous findings, the participation regressions for both specifications in columns (1) and (3) show that wealthy college educated households with higher income and lower income growth volatility tend to participate more. Also, for the participation equation in column (3), the interaction term $\alpha_{i,y} \times \rho_{y_i,m}$ has a positive and statistically significant estimate (estimate = 0.075, t-statistic = 2.81). Therefore, households with strong consumption-income sensitivity and positive income-growth market-return correlation have a higher propensity to own stocks. For these households, the perceived costs of participation might be lower because they do not care much about consumption smoothing, and thus do not need to incur any costs to uncover the hedging potential of financial assets. However, our model does not explicitly include participation costs, and, therefore, we cannot precisely pin down the mechanism by which consumption-income sensitivity affects the participation decision.

V.D.2. Heckman Asset Allocation Estimates

The most interesting results from the Heckman system of equations are those related to the asset allocation decision. Consistent with prior evidence, older, wealthier, college educated stockholders with low income growth volatility tend to allocate more of their wealth to risky assets. Also, stockholders with income growth that has a low correlation with market returns tend to allocate more of their wealth to risky assets. For example, in column (4), the estimate of $\rho_{y_i,m}$ is -3.998, with a t-statistic of -2.55. This significant negative estimate on the correlation term is evidence of the traditional income hedging motive.

Consistent with our model featuring consumption-income entitlements, the traditional income hedging motive is attenuated by the strength of the consumption-income sensitivity. For instance, the estimate of the interaction term $\alpha_{i,y} \times \rho_{y_i,m}$ in column (4) of Table V is positive (6.641) and statistically significant (t-statistic $= 2.47$). This is the strongest evidence of the attenuation effect because it is based solely on households that own risky assets.

The attenuation effect is also economically significant. Consider once again the two households with the same positive correlation $\rho_{y_i,m}$ of 0.50. If the consumption-income sensitivity of the first household is one standard deviation larger than that of the second (0.559), then the first household should allocate more of its wealth to risky assets. The Heckman estimation suggests that the first household will invest about 1.9% ($6.641 \times 0.50 \times 0.559$) more in risky assets.

Similarly, the Heckman estimates suggest that a one standard deviation increase in the income-hedging attenuation term (0.189) leads to a 1.3% (6.641×0.189) increase in the equity share. The change of the equity share is economically significant, and its magnitude is close to the effect of income on asset allocation. Specifically, our estimates suggest that a one standard deviation increase in wealth (0.220) leads to about a 2.8% (12.667×0.220) increase in the proportion of wealth allocated to risky assets. Once again, this result is notable since wealth is one of the most important determinants of equity allocation (e.g., Vissing-Jørgensen (2002b), Campbell (2006)).

V.E. Measurement Error in Explanatory Variables

Overall, our findings suggest that the traditional income hedging motive is strongly attenuated in the presence of consumption-income sensitivity. A potential concern with these results is that our main variables are generated regressors. For instance, in equation (16), the consumption-income sensitivities, $\hat{\alpha}_{i,y}$, and the income growth-market return correlations, $\hat{\rho}_{y_i,m}$, are estimated quantities. These generated regressors could affect the consistency and statistical significance of our estimates. Therefore, we examine the impact of measurement error on our results by re-estimating the Probit, Tobit, and Heckman regressions using the block-bootstrap approach of Kunsch (1989).

We perform the bootstrap simulations by exploiting the panel structure of the PSID. Specifically, we conduct a cross-sectional bootstrap simulation in which we successively sample households with replacement. We perform one thousand bootstrap replications to compute the estimation biases and bootstrapped standard errors for the coefficient estimates in our baseline Probit, Tobit, and Heckman regressions.[17] The estimation bias for an estimate x is defined as the difference between the average of the bootstrap estimates, $\hat{E}^{(b)}[\hat{x}^{(b)}]$, and the original estimate, $\hat{x}$. The bootstrapped standard error is the standard error of the bootstrap distribution.

We report the findings from the bootstrap exercise in Table VI. For each coefficient estimate, we report the t-statistic calculated using bootstrapped standard errors, the estimation bias, and the t-statistic of the estimation bias, testing whether the bias differs significantly from zero.[18] We report these quantities for the coefficient estimates of the Probit, Tobit, and Heckman regressions.

The results in Table VI suggest that our inference is only minimally affected by generated regressor biases. While the bootstrapped t-statistics of the coefficient estimates are smaller than those reported in Tables IV and V, we continue to find statistically significant attenuation of the income hedging motive due to consumption-income sensitivity. Further, in all cases, the bootstrap estimates of the biases do not differ statistically from zero. Therefore, we cannot reject the hypothesis that our original estimates are consistent. Overall, the estimation results in Table VI are similar to those reported in Tables IV and V. Thus, we conclude that potential measurement error in $\alpha_{i,y}$ and $\rho_{y_i,m}$ affects neither the consistency nor the significance of our baseline results.

[17] Our calculations are based on 2,000 bootstrap replications. Efron and Tibshirani (1993) suggest that 1,000 bootstrap replications is adequate for calculating biases and standard errors.

[18] The t-statistic for the estimation bias is the bias divided by the bootstrapped standard error.

VI. Summary and Conclusions

Consumption and portfolio decisions are interrelated but seldom studied together. We take a first step towards jointly examining the observed consumption and portfolio decisions of a sample of U.S. households. We find that the tendency of households to consume more when current income rises affects portfolio decisions directly. We document that consumption-income sensitivity attenuates the income hedging motive in portfolio decisions.

We formalize our empirical findings in a life-cycle model in which income is an entitlement to consume. The model is inspired by evidence from the sociology literature summarized by Akerlof (2007), as well as the behavioral economics literature on consumer behavior (e.g., Thaler (1985), Furnham and Argyle (1998), Prelec and Loewenstein (1998)). In the license-to-spend model, consumption-income entitlements undo some of the desire for consumption smoothing. Because investors are not concerned about smoothing consumption, they have a weaker incentive to hedge income fluctuations using the available menu of financial assets. Hence, the effect of the income hedging motive on their financial decisions is attenuated. Using consumption and portfolio data from the PSID, we find strong support for the attenuation effect.

We acknowledge that our theoretical model is simple in many dimensions. For example, it does not include any market participation costs or borrowing constraints. However, its simplicity allows us to analytically illustrate how consumption-income sensitivity attenuates the hedging motive. Having taken the first step to connect observed consumption decisions to portfolio decisions, we leave examination of more elaborate models of consumption entitlements for future research.

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TABLE I
Summary Statistics

Panel A: Consumption Growth, Income Growth and the Risk-Free Rate

	Moments ($\times 100$)				Correlations		
	$\Delta c_{i,t}$	$\Delta y_{i,t}$	$r_{f,t}$		$\Delta c_{i,t}$	$\Delta y_{i,t}$	$r_{f,t}$
Mean	1.004	2.479	2.395	$\Delta c_{i,t}$	1		
Median	0.000	1.751	2.337	$\Delta y_{i,t}$	0.075**	1	
Std. Dev.	37.092	31.966	2.960	$r_{f,t}$	0.017**	0.024**	1
N	93264	93264	93264				

Panel B: Income, Consumption, Retirement, Age and Wealth

	Income	Consumption	Retired Ind.	Age	Wealth	Fin. Assets Ind.
Mean	147315	2369	0.034	41	118435	0.834
Median	24037	2060	0.000	39	44500	1.000
Std. Dev.	1299734	2445	0.182	12	186139	0.371
N	93264	93264	93264	93264	19004	19004

This table shows summary statistics for key variables in this study. $\Delta c_{i,t}$ is consumption growth, $\Delta y_{i,t}$ is income growth, $r_{f,t}$ is the annual log risk-free rate. Panel A shows pooled moment and correlation estimates for the entire sample. ** shows significance at the 1% confidence level. Panel B shows summary statistics for consumption, income, retirement, age, and wealth. *Consumption* is measured by food at home and food out. *Income* is total household labor income. *Retired Ind.* is an indicator function depending on whether household head has retired. *Wealth* is household net worth, and *Fin. Assets Ind.* is an indicator depending on whether households hold some type of financial asset.

TABLE II

Consumption-Income Sensitivity Regression Estimates

	Full Sample		Retirement		Age		Wealth		Income		Fin. Assets	
			Non-ret.	Ret.	1st Quart.	4th Quart.	1st Quart.	4th Quart.	1st Quart.	4th Quart.	Non-holders	Holders
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
α_{r_f}	0.178	0.162	0.164	0.093	0.177	0.083	0.456	0.025	0.104	0.211	0.023	0.160
	(4.37)	(3.98)	(3.94)	(0.47)	(1.70)	(1.12)	(2.39)	(0.18)	(1.06)	(3.14)	(0.08)	(1.43)
α_y		0.081	0.085	0.027	0.091	0.041	0.113	0.076	0.058	0.086	0.076	0.099
		(17.69)	(17.76)	(1.66)	(9.69)	(5.04)	(6.14)	(5.15)	(7.62)	(8.55)	(2.98)	(8.04)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R^2	0.004	0.009	0.009	0.000	0.008	0.002	0.017	0.009	0.006	0.008	0.009	0.015
N	93264	93264	90061	3203	21237	24420	6713	7167	22822	23624	3144	15680

This table shows pooled OLS regression results for the reduced-form consumption Euler equation:

$$\Delta c_{i,t} = \alpha_0 + \alpha_{r_f} r_{f,t} + \alpha_y \Delta y_{i,t} + \mathbf{a}_{0,x}\mathbf{X} + \epsilon_{i,t},$$

where $\Delta c_{i,t}$ is consumption growth, $r_{f,t}$ is the risk-free rate, and $\Delta y_{i,t}$ is income growth. α_{r_f} is the EIS, and α_y captures consumption-income sensitivities. The vector of control variables $\mathbf{X}$ includes age, demeaned-age-square, female indicator for household head, number of children, unemployment indicator for household head, and an indicator for college or graduate studies. Wealth is household net worth. Financial assets include household holdings in mutual funds, IRA's, equities, bonds, savings or checking accounts. Income is total household labor income. t-statistics are shown in parenthesis based on robust standard errors.

TABLE III
Summary Statistics for Stock-Market Participation Variables

	Mean	Median	Std. Dev.	N
Participation Indicator	0.468	0	0.499	7030
Equity Allocation	0.265	0	0.349	7030
Participants' Equity Allocation	0.567	0.544	0.298	3291
Correlation $\hat{\rho}_{y_i,m}$	−0.027	−0.033	0.316	7030
Cons-Inc Sensitivity $\hat{\alpha}_{i,y}$	0.082	0.100	0.559	7030
Participants' $\hat{\alpha}_{i,y}$	0.081	0.090	0.537	3291
Interaction $\hat{\alpha}_{i,y} \times \hat{\rho}_{y_i,m}$	−0.008	−0.000	0.189	7030
Income Growth Volatility $\hat{\sigma}_{\Delta y_i}$	0.249	0.233	0.115	7030
Log Income $y_{i,t}$	10.887	10.945	0.774	7030
Wealth	0.174	0.092	0.220	7030
Age	46.487	47	10.152	7030
Number of Children	0.862	0	1.088	7031
Unemployment Indicator	0.016	0	0.128	7031
College or Graduate School	0.337	0	0.472	7031

This table shows summary statistics for key variables in the stock market participation model. *Participation Indicator* is an indicator for participating in the stock market. *Equity Allocation* is the fraction of wealth invested in the stock market, while *Participants' Equity Allocation* is the fraction of wealth invested in the stock market conditional on having positive equity holdings. $\hat{\rho}_{y_i,m}$ is the correlation coefficient between income growth for household i and stock market returns. $\hat{\alpha}_{i,y}$ captures consumption-income sensitivity for household i. $\hat{\alpha}_{i,y}$ are OLS estimates of the expression in (15) for households with more than 12 time-series observations. $\hat{\sigma}_{\Delta y_i}$ is income growth volatility, and $y_{i,t}$ is labor income for household i. *Wealth* is household net worth in millions of dollars.

TABLE IV
Consumption-Income Sensitivity and Stock Market Participation:
Probit and Tobit Specifications

	Probit		Tobit	
	(1)	(2)	(3)	(4)
Correlation $\hat{\rho}_{y_i,m}$	0.009	0.003	-1.375	-2.558
	(0.56)	(0.24)	(-0.56)	(-1.02)
$\hat{\alpha}_{i,y} \times \hat{\rho}_{y_i,m}$		0.075		15.467
		(2.82)		(3.63)
Income Growth Volatility $\hat{\sigma}_{\Delta y_i}$	-0.231	-0.238	-36.941	-38.401
	(-4.94)	(-5.08)	(-5.23)	(-5.41)
Log Income $y_{i,t}$	0.093	0.092	11.210	11.112
	(9.53)	(9.50)	(7.86)	(7.81)
Wealth	0.775	0.776	79.007	79.386
	(20.41)	(20.46)	(21.45)	(21.55)
Age	-0.000	-0.000	0.115	0.116
	(-0.70)	(-0.69)	(0.89)	(0.89)
Age^2	0.009	0.009	0.489	0.480
	(2.41)	(2.39)	(0.80)	(0.78)
Number of Children	-0.000	-0.000	0.375	0.339
	(-0.01)	(-0.03)	(0.47)	(0.43)
Unemployment Indicator	0.003	0.001	0.329	0.022
	(0.08)	(0.04)	(0.05)	(0.00)
College or Graduate School	0.157	0.158	23.753	23.807
	(13.94)	(13.99)	(14.22)	(14.27)
Constant	-3.718	-3.702	-154.645	-153.139
	(-10.73)	(-10.69)	(-9.76)	(-9.69)
year FE	yes	yes	yes	yes
Pseudo-R^2	18.99%	19.07%	3.93%	3.96%
N	7030	7030	7030	7030

This table shows Probit regressions estimates for stock market participation in columns (1) and (2). Columns (3) and (4) present estimates from Tobit regressions where the dependent variable is the portion of wealth in risky assets. For the Probit specification, we report estimates for the marginal effects. The dependent variable is an indicator function for stock market participation. $\hat{\rho}_{y_i,m}$ is the correlation coefficient between income growth for household i and stock market returns. $\hat{\alpha}_{i,y}$ captures consumption-income sensitivity for household i. $\hat{\alpha}_{i,y}$ are OLS estimates of the expression in (15) for households with more than 12 time-series observations. $\hat{\sigma}_{\Delta y_i}$ is income growth volatility, and $y_{i,t}$ is labor income for household i. *Wealth* is household net worth in millions of dollars. t-statistics are shown in parenthesis and are based on robust standard errors.

TABLE V
Consumption-Income Sensitivity and Stock Market Participation:
Two-Stage Heckman Specification (Heckman 1979)

	No Consumption-Income Sensitivity		Consumption-Income Sensitivity	
	Participation	Equity Allocation	Participation	Equity Allocation
	(1)	(2)	(3)	(4)
Correlation $\hat{\rho}_{y_i,m}$	0.009	−3.531	0.004	−3.998
	(0.58)	(−2.27)	(0.25)	(−2.55)
$\hat{\alpha}_{i,y} \times \hat{\rho}_{y_i,m}$			0.075	6.641
			(2.81)	(2.47)
Income Growth Volatility $\hat{\sigma}_{\Delta y_i}$	−0.233	−5.199	−0.239	−5.748
	(−4.96)	(−1.12)	(−5.10)	(−1.23)
Log Income $y_{i,t}$	0.093	−2.010	0.092	−2.060
	(9.52)	(−2.08)	(9.49)	(−2.13)
Wealth	0.775	12.493	0.776	12.667
	(20.60)	(4.30)	(20.65)	(4.30)
Age	−0.000	0.120	−0.000	0.120
	(−0.69)	(1.23)	(−0.69)	(0.12)
Age^2	0.009	−0.919	0.009	−0.927
	(2.41)	(−1.94)	(2.39)	(−1.96)
Number of Children	0.000	0.199	−0.000	0.164
	(0.00)	(0.38)	(−0.02)	(0.31)
Unemployment Indicator	0.003	3.366	0.001	3.135
	(0.08)	(0.94)	(0.05)	(0.88)
College or Graduate School	0.158	4.716	0.158	4.719
	(13.96)	(3.79)	(14.00)	(3.77)
Constant	−3.715	56.489	−3.699	57.326
	(−10.73)	(4.61)	(−10.69)	(4.67)
year FE	yes	yes	yes	yes
N	7030	3291	7030	3291

This table shows two-stage Heckman (Heckman 1979) regressions for stock market participation based on the expression for optimal portfolio weights in (16). In the selection equation, the dependent variable is an indicator function for stock market participation, while in the equity allocation equation, the dependent variable is the percentage of total wealth allocated to risky assets. For the Participation equation, we report estimates for the marginal effects. $\hat{\rho}_{y_i,m}$ is the correlation coefficient between income growth for household i and stock market returns. $\hat{\alpha}_{i,y}$ captures consumption-income sensitivity for household i. $\hat{\alpha}_{i,y}$ are OLS estimates of the expression in (15) for households with more than 12 time-series observations. $\hat{\sigma}_{\Delta y_i}$ is income growth volatility, and $y_{i,t}$ is labor income for household i. *Wealth* is household net worth in millions of dollars. t-statistics are shown in parenthesis and are based robust standard errors.

TABLE VI

Consumption-Income Sensitivity and Stock Market Participation: Block-Bootstrapped Standard Errors

	Probit		Tobit		Two-Stage Heckman			
					Participation		Equity Allocation	
	(1)		(2)		(3)		(4)	
	Estimates	Bias	Estimates	Bias	Estimates	Bias	Estimates	Bias
Correlation $\hat{\rho}_{y_i,m}$	0.003	0.001	−2.558	0.006	0.004	0.001	−3.998	−0.070
	(0.17)	(0.01)	(−0.70)	(0.001)	(0.001)	(0.01)	(−1.91)	(−0.03)
$\hat{\alpha}_{i,y} \times \hat{\rho}_{y_i,m}$	0.075	−0.000	15.468	0.210	0.075	−0.000	6.641	0.111
	(2.14)	(−0.003)	(2.85)	(0.03)	(2.12)	(−0.004)	(1.97)	(0.03)
Income Growth Volatility $\hat{\sigma}_{\Delta y_i}$	−0.238	0.003	−38.401	0.097	−0.239	0.003	−5.748	−0.153
	(−3.69)	(0.014)	(−3.80)	(0.009)	(−3.70)	(0.01)	(−0.93)	(−0.02)
Log Income $y_{i,t}$	0.092	0.001	11.112	0.070	0.092	0.001	−2.060	0.019
	(7.61)	(0.05)	(5.82)	(0.03)	(7.60)	(0.04)	(−1.67)	(0.01)
Wealth	0.776	0.007	79.386	−0.061	0.776	0.009	12.667	−0.103
	(15.70)	(0.04)	(15.02)	(−0.01)	(15.69)	(0.05)	(3.24)	(−0.02)
Age	−0.000	0.000	0.116	0.010	−0.000	0.000	0.120	0.003
	(−0.54)	(0.04)	(0.71)	(0.06)	(−0.54)	(0.04)	(1.08)	(0.03)
Age^2	0.009	−0.001	0.480	−0.066	0.009	−0.001	−0.927	−0.008
	(1.89)	(−0.07)	(0.62)	(−0.08)	(1.89)	(−0.06)	(−1.68)	(−0.01)
Number of Children	−0.000	−0.000	0.339	−0.017	−0.000	−0.000	0.164	−0.006
	(−0.03)	(−0.01)	(0.34)	(−0.01)	(−0.01)	(−0.01)	(0.27)	(−0.009)
Unemployment Indicator	0.001	−0.003	0.022	−0.218	0.001	−0.003	3.135	0.038
	(0.04)	(−0.02)	(0.00)	(−0.03)	(0.04)	(−0.02)	(0.86)	(0.01)
College or Graduate School	0.158	0.002	23.807	0.003	0.158	0.002	4.719	−0.051
	(10.08)	(0.04)	(9.91)	(0.001)	(10.08)	(0.04)	(2.87)	(−0.03)
Constant	−3.702	−0.028	−153.139	−1.088	−3.699	−0.027	57.326	−0.199
	(−8.45)	(−0.06)	(−7.18)	(−0.05)	(−8.43)	(−0.03)	(3.65)	(0.01)
year FE	yes		yes		yes		yes	

This tables shows bootstrapped results for the stock participation and allocation regressions in Tables IV and V. *Estimates* are the initial estimates. *Bias* is the bootstrapped estimate for the bias which is defined as the difference between the average estimate of the bootstrap distribution and the initial estimate. For the Probit specification and the Participation equation, we report estimates for the marginal effects, while the corresponding biases refer to parameter estimates. t-statistics are shown in parentheses, and are based on block-bootstrapped standard errors with resampling at the household-level.

Appendix For Online Publication

A Properties of Consumption-Income Utility Function

In this section, we show that the utility function is increasing and concave in C_t, and that the Inada conditions hold. The first derivative of the utility function in (1) with respect to C_t is

$$\frac{\partial U(C_t; Y_t)}{\partial C_t} = C_t^{-\gamma} Y_t^{\theta}. \tag{17}$$

Because $0 < Y_t < +\infty$ and $\gamma > 1$, the marginal utility of consumption is bounded:

$$\lim_{C_t \downarrow 0} \frac{\partial U(C_t; Y_t)}{\partial C_t} = +\infty,$$

and

$$\lim_{C_t \uparrow +\infty} \frac{\partial U(C_t; Y_t)}{\partial C_t} = 0.$$

The above conditions imply that individuals always consume part of their income, and that consumption is strictly positive. Therefore, the derivative in (17) is also positive, and the inverse consumption ratio Y_t/C_t is well defined.

Moreover, marginal utility is an increasing function of Y_t because the cross-derivative,

$$\frac{\partial^2 U(C_t; Y_t)}{\partial C_t \partial Y_t} = \theta C_t^{-\gamma} Y_t^{\theta-1},$$

is positive for $\theta > 0$. Finally, the second derivative with respect to C_t is

$$\frac{\partial^2 U(C_t; Y_t)}{\partial C_t^2} = -\gamma C_t^{-\gamma-1} Y_t^{\theta},$$

which is negative for $\gamma > 0$.

B Proof of Proposition 1

We prove *Proposition 1* by first log-linearizing the budget constraint and the Euler equations of the problem. Then, we follow the method of undetermined coefficients to solve for the optimal consumption and portfolio rules. Specifically, we make a guess about the optimal consumption and equity share policy functions, we verify our guesses, and derive the optimal portfolio rules.

First, consider the budget constraint

$$W_{t+1} = (W_t - C_t + \bar{Y})R_{p,t+1}.$$

in which $R_{p,t+1}$ are portflio returns. After dividing both sides by $W_t + \bar{Y}$, the log-linearized version of the budget constraint around $\bar{c}$ and $\bar{w}$ reads

$$w_{t+1} - \tilde{\lambda}_0^r - \tilde{\lambda}_1^r w_t = \tilde{\kappa}_0^r - \tilde{\kappa}_1^r c_t + \tilde{\kappa}_2^r w_t + r_{p,t+1},$$

where $\tilde{\kappa}_0^r = log(1 - \frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}) + \tilde{\kappa}_1^r \bar{c} - \tilde{\kappa}_2^r \bar{w}$, $\tilde{\kappa}_1^r = \frac{1}{1-\frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}} \frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}$, and $\tilde{\kappa}_2^r = \frac{1}{1-\frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}} \frac{e^{\bar{c}}}{(e^{\bar{w}}+\bar{Y})^2} e^{\bar{w}}$. Also, $\tilde{\lambda}_0^r = log(e^{\bar{w}} + \bar{Y}) - \tilde{\lambda}_1^r \bar{w}$ and $\tilde{\lambda}_1^r = \frac{e^{\bar{w}}}{e^{\bar{w}}+\bar{Y}}$.

Setting $e^{\bar{w}}$ to be much larger than $\bar{Y}$, then $\tilde{\lambda}_1^r = \frac{e^{\bar{w}}}{e^{\bar{w}}+\bar{Y}} \approx 1$, $\tilde{\kappa}_1^r \approx \tilde{\kappa}_2^r$, and the log-linearized budget constraint simplifies to

$$w_{t+1} - w_t = \kappa_0^r - \kappa_1^r (c_t - w_t) + r_{p,t+1}, \tag{18}$$

where $\kappa_0^r = log(1 - \frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}) + \kappa_1^r(\bar{c} - \bar{w}) + log(e^{\bar{w}} + \bar{Y}) - \bar{w}$, and $\kappa_1^r = \frac{1}{1-\frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}} \frac{e^{\bar{c}}}{e^{\bar{w}}+\bar{Y}}$.

Next, we log-linearize the expression for the excess portfolio returns, which are defined as

$$e^{r_{p,t+1}} - e^{r_{f,t+1}} = a_t(e^{r_{m,t+1}} - e^{r_{f,t+1}}). \tag{19}$$

In order to simplify the expression (19), we use a log-normal approximation for the excess portfolio returns $(e^{r_{p,t+1}} - e^{r_{f,t+1}})$ and the excess returns for the risky asset $(e^{r_{m,t+1}} - e^{r_{f,t+1}})$

to obtain

$$r_{p,t+1} - r_{f,t+1} = a_t(r_{m,t+1} - r_{f,t+1}) + 0.5a_t(1 - a_t)\sigma_m^2.$$

Next, we log-linearize the Euler equations of the three assets during retirement. From the problem in (2), we know that the Euler equations are:

$$\mathbf{E}_t\left[\beta e^{-\gamma\Delta c^r_{t+1}+r_{i,t+1}}\right] = 1, i \in \{p, m, f\}.$$

The log-linearised Euler equations for portfolio p and the risky asset m are:

$$log\beta - \gamma\mathbf{E}_t[\Delta c^r_{t+1}] + \mathbf{E}_t[r_{i,t+1}] + 0.5Var_t[r_{i,t+1} - \gamma\Delta c^r_{t+1}] = 0,\ i \in \{p, m\},$$

while the log-linearised Euler equation for the risk-free asset is:

$$log\beta - \gamma\mathbf{E}_t[\Delta c^r_{t+1}] + r_f + 0.5Var_t[-\gamma\Delta c^r_{t+1}] = 0.$$

We use the log-linearized budget constraint and the log-linearized Euler equations to solve for the optimal consumption and optimal equity share. To find the optimal consumption and portfolio policies, we follow a guess and verify approach where we make a guess about the optimal policy rule, and then use the log-linearized budget constraint and Euler equations to verify our guess.

Suppose that the optimal consumption policy is given by $c^r_t = \phi^r_0 + w^r_t$, and that the optimal portfolio rule is constant across time, i.e., $a^r_t = a^r$. Our goal is to pin down the parameter ϕ_0, so that our guesses satisfy the log-linearized budget constraint and the Euler equations. Under our two guesses, consumption growth is related to wealth growth, i.e., $\Delta c^r_{t+1} = \Delta w^r_{t+1}$, and hence $\mathbf{E}_t[\Delta c^r_{t+1}] = \mathbf{E}_t[\Delta w^r_{t+1}]$. Using the log-linearized budget constraint in (18) and our guess for the optimal portfolio rule ($a^r_t = a^r$), we also obtain that

$$\mathbf{E}_t[\Delta c^r_{t+1}] = a^r(\mu_m - r_f) + r_f + 0.5a^r(1 - a^r)\sigma_m^2 + \kappa^r_0 - \kappa^r_1\phi^r_0. \quad (20)$$

On the other hand, the Euler equation for portfolio returns implies that

$$\mathbf{E}_t[\Delta c^r_{t+1}] = \frac{1}{\gamma}\Big[log\beta + \mathbf{E}_t[r^r_{p,t+1}] + 0.5Var_t[r^r_{p,t+1} - \gamma\Delta c^r_{t+1}]\Big].$$

Since $\Delta c^r_{t+1} = \Delta w^r_{t+1}$, we have that

$$Var_t[r^r_{p,t+1} - \gamma\Delta c^r_{t+1}] = Var_t[r^r_{p,t+1} - \gamma\Delta w^r_{t+1}].$$

Using the log-linearized budget constraint in (18) and the guess that $a^r_t = a^r$, we can write the right-hand side of the above expression as:

$$Var_t[r^r_{p,t+1} - \gamma\Delta w^r_{t+1}] = \big(1-\gamma\big)^2 (a^r)^2\sigma^2_m.$$

Thus, the Euler equation implies that expected consumption growth is equal to

$$\mathbf{E}_t[\Delta c^r_{t+1}] = \frac{1}{\gamma}\Big[log\beta + a^r(\mu_m - r_f) + r_f + 0.5a^r(1-a^r)\sigma^2_m + 0.5\big(1-\gamma\big)^2(a^r)^2\sigma^2_m\Big]. \quad (21)$$

Equalizing the two expressions in (20) and (21), we obtain the solution for ϕ^r_0

$$\begin{aligned}\phi^r_0 &= \frac{a^r(\mu_m - r_f) + r_f + 0.5a^r(1-a^r)\sigma^2_m + \kappa^r_0}{\kappa^r_1} \\ &- \frac{1}{\kappa^r_1\gamma}\Big[log\beta + a^r(\mu_m - r_f) + r_f + 0.5a^r(1-a^r)\sigma^2_m + 0.5\big(1-\gamma\big)^2(a^r)^2\sigma^2_m\Big].\end{aligned}$$

To derive the optimal portfolio weight a^r, subtract the log-linearized Euler equation for the risk-free asset from the Euler equation for the risky asset:

$$\mu_m - r_f + 0.5\sigma^2_m = \gamma Cov_t\big(r_{m,t+1}, \Delta c^r_{t+1}\big).$$

Using our guess for optimal consumption rules ($c^r_t = \phi^r_0 + w^r_t$), the log-linearized budget constraint in (18), and our guess for the optimal portfolio rule ($a_t^r = a^r$), we obtain that

$$\mu_m - r_f + 0.5\sigma_m^2 = \gamma Cov_t\big(r_{m,t+1}, a^r r_{m,t+1}\big) \Leftrightarrow a^r = \frac{\mu_m - r_f + 0.5\sigma_m^2}{\gamma\sigma_m^2}.$$

C Proof of Proposition 2

We prove *Proposition 2* is a similar manner as *Proposition 1.* That is, we log-linearize the budget constraint and the Euler equations. Then, we follow a guess-and-verify approach to derive the optimal consumption and equity share rules.

First, we log-linearize the pre-retirement budget constraint:

$$W_{t+1} = (W_t - C_t + Y_t)R_{p,t+1}. \tag{22}$$

Dividing both sides of the budget constraint by Y_t, and multiplying by Y_t/Y_{t+1}, the log-linearized version of the budget constraint around $\overline{w-y}$ and $\overline{c-y}$ is

$$w_{t+1} - y_{t+1} = \kappa_0 + \kappa_1(w_t - y_t) - \kappa_2(c_t - y_t) - \Delta y_{t+1} + r_{p,t+1}, \tag{23}$$

where

$$\begin{aligned} \kappa_1 &= \frac{e^{\overline{w-y}}}{1 + e^{\overline{w-y}} - e^{\overline{c-y}}},\ \kappa_2 = \frac{e^{\overline{c-y}}}{1 + e^{\overline{w-y}} - e^{\overline{c-y}}} \text{ and} \\ \kappa_0 &= log[1 + e^{\overline{w-y}} - e^{\overline{c-y}}] - \kappa_1(\overline{w-y}) + \kappa_2(\overline{c-y}). \end{aligned} \tag{24}$$

Next, we simplify the Euler equations. From problem (6), the pre-retirement Euler equations are

$$\pi_e \mathbf{E}_t\big[\beta e^{-\gamma\Delta c^e_{t+1} + \theta\Delta y_{t+1} + r_{i,t+1}}\big] + \pi_r \mathbf{E}_t\big[\beta e^{-\gamma\Delta c^r_{t+1} + \theta(\bar{y} - y_t) + r_{i,t+1}}\big] = 1,\ i \in \{p, m, f\}.$$

Because we assumed that pension income $\bar{y}$ is constant and equal to the last pre-retirement income payment, the difference $\bar{y} - y_t$ in the above expression is zero. Therefore,

$$\pi_e \mathbf{E}_t \Big[\beta e^{-\gamma \Delta c^e_{t+1} + \theta \Delta y_{t+1} + r_{i,t+1}}\Big] + \pi_r \mathbf{E}_t \Big[\beta e^{-\gamma \Delta c^r_{t+1} + r_{i,t+1}}\Big] = 1,\ i \in \{p, m, f\}. \tag{25}$$

Based on equation (25) above, the second-order Taylor approximations of the Euler equations for the portfolio p and the risky asset m are

$$\pi_e \Big\{ log\beta - \gamma \mathbf{E}_t[\Delta c^e_{t+1}] + \theta \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r_{i,t+1}] + 0.5Var_t[-\gamma \Delta c^e_{t+1} + \theta \Delta y_{t+1} + r_{i,t+1}] \Big\} +$$
$$\pi_r \Big\{ log\beta - \gamma \mathbf{E}_t[\Delta c^r_{t+1}] + \mathbf{E}_t[r_{i,t+1}] + 0.5Var_t[-\gamma \Delta c^r_{t+1} + r_{i,t+1}] \Big\} = 0,\ i \in \{p, m\}.$$

Also, the log-linearised Euler equation for the risk-free asset yields

$$\pi_e \Big\{ log\beta - \gamma \mathbf{E}_t[\Delta c^e_{t+1}] + \theta \mathbf{E}_t[\Delta y_{t+1}] + r_f + 0.5Var_t[-\gamma \Delta c^e_{t+1} + \theta \Delta y_{t+1}] \Big\} +$$
$$\pi_r \Big\{ log\beta - \gamma \mathbf{E}_t[\Delta c^r_{t+1}] + r_f + 0.5Var_t[-\gamma \Delta c^r_{t+1}] \Big\} = 0.$$

Using the identity: $\Delta c^s_{t+1} = (c^s_{t+1} - y_{t+1}) - (c^e_t - y_t) + \Delta y_{t+1},\ s \in \{e, r\}$, the Euler equation for portfolio returns p becomes

$$log\beta - \gamma \sum_{s=e,r} \pi_s \mathbf{E}_t[c^s_{t+1} - y_{t+1}] + \gamma(c^e_t - y_t) - \gamma \mathbf{E}_t[\Delta y_{t+1}] + \theta \pi_e \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r^e_{p,t+1}] +$$
$$0.5\pi_e Var_t[-\gamma(c^e_{t+1} - y_{t+1}) + \gamma(c^e_t - y_t) - \gamma \Delta y_{t+1} + \theta \Delta y_{t+1} + r^e_{p,t+1}] +$$
$$0.5\pi_r Var_t[-\gamma(c^r_{t+1} - y_{t+1}) + \gamma(c^e_t - y_t) - \gamma \Delta y_{t+1} + r^e_{p,t+1}] = 0.$$

Finally, we use the guess-and-verify method to obtain the optimal policy rules. In particular, we guess that portfolio weights are constant, i.e., $a^e_t = a^e$, and that the log consumption-income ratio is linear in wealth and income

$$c^e_{t+1} - y_{t+1} = \phi_0 + \phi_1(w^e_{t+1} - y_{t+1}).$$

We can also rewrite the optimal consumption policy during retirement as

$$c^r_{t+1} - y_{t+1} = \phi^r_0 + \phi^r_1(w^e_{t+1} - y_{t+1}), \tag{26}$$

with $\phi^r_1 = 1$. Note that even if our investor retires at time $t+1$, equation (22) still describes the evolution of her wealth from time t to time $t+1$. This is why we use w^e_{t+1} in (26) rather than w^r_{t+1}. Plugging the above guesses into the Euler equation for portfolio p, we get

$$\begin{aligned}
&0 = log\beta - \gamma\Big[\pi_e\Big(\phi_0 + \phi_1\mathbf{E}_t[w^e_{t+1} - y_{t+1}]\Big) + \pi_r\Big(\phi^r_0 + \mathbf{E}_t[w^e_{t+1} - y_{t+1}]\Big)\Big] + \\
&\gamma[\phi_0 + \phi_1(w^e_t - y_t)] - \gamma\mathbf{E}_t[\Delta y_{t+1}] + \theta\pi_e\mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r^e_{p,t+1}] + \\
&0.5\pi_e Var_t\big[\gamma\big([\phi_0 + \phi_1(w^e_{t+1} - y_{t+1})] - [\phi_0 + \phi_1(w^e_t - y_t)] + \Delta y_{t+1}\big) - \theta\Delta y_{t+1} - r^e_{p,t+1}\big] + \\
&0.5\pi_r Var_t\big[\gamma\big([\phi^r_0 + (w^e_{t+1} - y_{t+1})] - [\phi_0 + \phi_1(w^e_t - y_t)] + \Delta y_{t+1}\big) - r^e_{p,t+1}\big].
\end{aligned}$$

Using the log-linearized budget constraint in (23),

$$\begin{aligned}
&0 = log\beta - \gamma\Big[\pi_e\Big(\phi_0 + \phi_1\mathbf{E}_t[\kappa_0 + \kappa_1(w^e_t - y_t) - \kappa_2(c^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}]\Big) + \\
&\pi_r\Big(\phi^r_0 + \mathbf{E}_t[\kappa_0 + \kappa_1(w^e_t - y_t) - \kappa_2(c^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}]\Big)\Big] + \\
&\gamma[\phi_0 + \phi_1(w^e_t - y_t)] - \gamma\mathbf{E}_t[\Delta y_{t+1}] + \theta\pi_e\mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r^e_{p,t+1}] + \\
&0.5\pi_e Var_t\Big[\gamma\big[\phi_0 + \phi_1[\kappa_0 + \kappa_1(w^e_t - y_t) - \kappa_2(c^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}]\big] + \\
&\gamma\Delta y_{t+1} - \theta\Delta y_{t+1} - r^e_{p,t+1}\Big] + \\
&0.5\pi_r Var_t\Big[\gamma[\phi^r_0 + \kappa_0 + \kappa_1(w^e_t - y_t) - \kappa_2(c^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}] + \gamma\Delta y_{t+1} - r^e_{p,t+1}\Big].
\end{aligned}$$

Once more, our guess for the optimal consumption-income ratio implies that

$$
\begin{aligned}
&0 = log\beta - \gamma\Big[\pi_e\Big(\phi_0 + \phi_1\big[\kappa_0 + \kappa_1(w_t^e - y_t) - \kappa_2(\phi_0 + \phi_1(w_t^e - y_t)) - \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r_{p,t+1}^e]\big]\Big) + \\
&\pi_r\Big(\phi_0^r + \kappa_0 + \kappa_1(w_t^e - y_t) - \kappa_2(\phi_0 + \phi_1(w_t^e - y_t)) - \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r_{p,t+1}^e]\Big)\Big] + \\
&\gamma[\phi_0 + \phi_1(w_t^e - y_t)] - \gamma\mathbf{E}_t[\Delta y_{t+1}] + \theta\pi_e\mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r_{p,t+1}^e] + \\
&0.5\pi_e Var_t\Big[\gamma\big[\phi_0 + \phi_1[\kappa_0 + \kappa_1(w_t^e - y_t) - \kappa_2(\phi_0 + \phi_1(w_t^e - y_t)) - \Delta y_{t+1} + r_{p,t+1}^e]\big] + \\
&\gamma\Delta y_{t+1} - \theta\Delta y_{t+1} - r_{p,t+1}^e\Big] + \\
&0.5\pi_r Var_t\Big[\gamma[\phi_0 + \kappa_0 + \kappa_1(w_t^e - y_t) - \kappa_2(\phi_0 + \phi_1(w_t^e - y_t)) - \Delta y_{t+1} + r_{p,t+1}^e] + \gamma\Delta y_{t+1} - r_{p,t+1}^e\Big].
\end{aligned}
$$

Since $a_t^e = a^e$ and Δy_{t+1} is an i.i.d. process, the Euler equation becomes

$$
\begin{aligned}
&log\beta - \gamma\Big[\pi_e\Big(\phi_0 + \phi_1\big[\kappa_0 + \kappa_1(w_t^e - y_t) - \kappa_2\big(\phi_0 + \phi_1(w_t^e - y_t)\big) - \\
&\mu_y + a^e(\mu_m - r_f) + r_f + 0.5a^e(1 - a^e)\sigma_m^2\big]\Big) + \\
&\pi_r\Big(\phi_0^r + \kappa_0 + \kappa_1(w_t^e - y_t) - \kappa_2\big(\phi_0 + \phi_1(w_t^e - y_t)\big) - \\
&\mu_y + a^e(\mu_m - r_f) + r_f + 0.5a^e(1 - a^e)\sigma_m^2\Big)\Big] + \\
&\gamma[\phi_0 + \phi_1(w_t^e - y_t)] - \gamma\mu_y + \theta\pi_e\mu_y + a^e(\mu_m - r_f) + r_f + 0.5a^e(1 - a^e)\sigma_m^2 + \\
&0.5\pi_e Var_t\Big[\gamma\big[\phi_1[-\Delta y_{t+1} + r_{p,t+1}^e]\big] + \gamma\Delta y_{t+1} - \theta\Delta y_{t+1} - r_{p,t+1}^e\Big] + \\
&0.5\pi_r Var_t\Big[\gamma[-\Delta y_{t+1} + r_{p,t+1}^e] + \gamma\Delta y_{t+1} - r_{p,t+1}^e\Big] = 0.
\end{aligned} \tag{27}
$$

Collecting $w_t^e - y_t$ terms, the following equation in ϕ_1 must hold

$$-\gamma\pi_e\phi_1\kappa_1 + \gamma\pi_e\kappa_2(\phi_1)^2 - \gamma\pi_r\kappa_1 + \gamma\pi_r\kappa_2\phi_1 + \gamma\phi_1 = 0.$$

Both solutions for the quadratic equation above are real and have opposite signs because the constant term $(-\frac{\pi_r\kappa_1}{\pi_e\kappa_2})$ is negative. Since ϕ_1 is the elasticity of consumption to wealth, it has to be positive. Therefore, we choose the positive solution which makes intuitive and economic sense, and conclude that

$$\phi_1 = \frac{(\pi_e\kappa_1 - \pi_r\kappa_2 - 1) + \sqrt{(1 + \pi_r\kappa_2 - \pi_e\kappa_1)^2 + 4\pi_e\kappa_2\pi_r\kappa_1}}{2\pi_e\kappa_2}.$$

Finally, ϕ_0 depends on all the remaining constant terms in (27)

$$\begin{aligned}
&log\beta - \gamma\pi_e\phi_0 - \gamma\pi_e\phi_1\kappa_0 + \gamma\pi_e\phi_1[\mu_y + a^e(\mu_m - r_f) + r_f + 0.5a^e(1-a^e)\sigma_m^2] \\
&-\gamma\pi_r\phi_0^r - \gamma\pi_r\kappa_0 + \gamma\pi_r[\mu_y + a^e(\mu_m - r_f) + r_f + 0.5a^e(1-a^e)\sigma_m^2] + \gamma\phi_0 - \gamma\mu_y + \theta\pi_e\mu_y \\
&+a^e(\mu_m - r_f) + r_f + 0.5a^e(1-a^e)\sigma_m^2 + 0.5\pi_e(1-\gamma\phi_1)^2(a^e)^2\sigma_m^2 + 0.5\pi_e\gamma^2\big(1 - \phi_1 - \frac{\theta}{\gamma}\big)^2\sigma_{\Delta y}^2 \\
&-\pi_e(1-\gamma\phi_1)a^e\gamma\big(1 - \phi_1 - \frac{\theta}{\gamma}\big)\rho_{y,m}\sigma_m\sigma_{\Delta y} + 0.5\pi_r(1-\gamma)^2(a^e)^2\sigma_m^2 = 0.
\end{aligned} \tag{28}$$

Returning to optimal portfolio weights, subtract the log-linearized Euler equation for the risk-free asset from the log-linearized Euler equation for the risky asset to get

$$\mu_m - r_f + 0.5\sigma_m^2 = \gamma\big[\pi_e Cov_t\big(r_{m,t+1}, \Delta c_{t+1}^e\big) + \pi_r Cov_t\big(r_{m,t+1}, \Delta c_{t+1}^r\big)\big] - \theta\pi_e Cov_t\big(r_{m,t+1}, \Delta y_{t+1}\big).$$

Using the identity: $c_{t+1}^s - c_t^e = (c_{t+1}^s - y_{t+1}) - (c_t^e - y_t) + \Delta y_{t+1},\ s \in \{r, e\}$, we obtain that

$$\begin{aligned}
&\mu_m - r_f + 0.5\sigma_m^2 = \gamma\big[\pi_e Cov_t\big(r_{m,t+1}, (c_{t+1}^e - y_{t+1}) - (c_t^e - y_t) + \Delta y_{t+1}\big) + \\
&\pi_r Cov_t\big(r_{m,t+1}, (c_{t+1}^r - y_{t+1}) - (c_t^e - y_t) + \Delta y_{t+1}\big)\big] - \theta\pi_e Cov_t\big(r_{m,t+1}, \Delta y_{t+1}\big).
\end{aligned}$$

Our guess about optimal consumption policy implies that

$$\begin{aligned}
&\mu_m - r_f + 0.5\sigma_m^2 = \gamma\big[\pi_e Cov_t\big(r_{m,t+1}, \phi_1(w_{t+1}^e - y_{t+1}) + \Delta y_{t+1}\big) + \\
&\pi_r Cov_t\big(r_{m,t+1}, \phi_1^r(w_{t+1}^e - y_{t+1}) + \Delta y_{t+1}\big)\big] - \theta\pi_e Cov_t\big(r_{m,t+1}, \Delta y_{t+1}\big).
\end{aligned}$$

Using the log-linearized budget constraint in (23), we obtain that

$$\begin{aligned}
&\mu_m - r_f + 0.5\sigma_m^2 = \gamma\big[\pi_e Cov_t\big(r_{m,t+1}, \phi_1(r_{p,t+1} - \Delta y_{t+1}) + \Delta y_{t+1}\big) + \\
&\pi_r Cov_t\big(r_{m,t+1}, \phi_1^r(r_{p,t+1} - \Delta y_{t+1}) + \Delta y_{t+1}\big)\big] - \theta\pi_e Cov_t\big(r_{m,t+1}, \Delta y_{t+1}\big).
\end{aligned}$$

Since $a^e_t = a^e$ and $\phi^r_1 = 1$, the previous expression becomes

$$\mu_m - r_f + 0.5\sigma^2_m = \gamma\big[\pi_e Cov_t\big(r_{m,t+1}, \phi_1(a^e r_{m,t+1} - \Delta y_{t+1}) + \Delta y_{t+1}\big) +$$
$$\pi_r Cov_t\big(r_{m,t+1}, (a^e r_{m,t+1} - \Delta y_{t+1}) + \Delta y_{t+1}\big)\big] - \theta\pi_e Cov_t\big(r_{m,t+1}, \Delta y_{t+1}\big).$$

We can solve the above expression with respect to a^e, and find that

$$a^e = \frac{\mu_m - r_f + 0.5\sigma^2_m}{\gamma(\pi_r + \pi_e\phi_1)\sigma^2_m} - \Big(1 - \phi_1 - \frac{\theta}{\gamma}\Big)\frac{\pi_e\rho_{y,m}\sigma_{\Delta y}\sigma_m}{(\pi_r + \pi_e\phi_1)\sigma^2_m}.$$

Lastly, we show that $\phi_1 < 1$ such that the term $1 - \phi_1$ in the portfolio hedging motive is positive. The proof is by contradiction. Suppose that $\phi_1 \geq 1$, then

$$\phi_1 = \frac{(\pi_e\kappa_1 - \pi_r\kappa_2 - 1) + \sqrt{(1 + \pi_r\kappa_2 - \pi_e\kappa_1)^2 + 4\pi_e\kappa_2\pi_r\kappa_1}}{2\pi_e\kappa_2} \geq 1 \Leftrightarrow$$
$$\sqrt{(\pi_e\kappa_1 - \pi_r\kappa_2 - 1)^2 + 4\pi_e\kappa_2\pi_r\kappa_1} \geq 2\pi_e\kappa_2 + (1 + \pi_r\kappa_2 - \pi_e\kappa_1) \Leftrightarrow$$
$$(1 + \pi_r\kappa_2 - \pi_e\kappa_1)^2 + 4\pi_e\kappa_2\pi_r\kappa_1 \geq 4\pi^2_e\kappa^2_2 + (1 + \pi_r\kappa_2 - \pi_e\kappa_1)^2 + 4\pi_e\kappa_2(1 + \pi_r\kappa_2 - \pi_e\kappa_1) \Leftrightarrow$$
$$\pi_r\kappa_1 \geq \pi_e\kappa_2 + (1 + \pi_r\kappa_2 - \pi_e\kappa_1) \Leftrightarrow 0 \geq 1 + \kappa_2 - \kappa_1.$$

The last inequality is false, since the definition of κ_1 and κ_2 in (24) implies that $1+\kappa_2-\kappa_1 > 0$.