Information Management in Banking Crises*

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Abstract

A regulator resolving a bank faces two audiences: depositors, who may run if they believe the regulator will not provide capital, and banks, which may take excess risk if they believe the regulator will provide capital. When the regulator's cost of injecting capital is private information, it manages expectations by using costly signals: (i) A regulator with a low cost of injecting capital may forbear on bad banks to signal toughness and reduce risk taking, and (ii) A regulator with a high cost of injecting capital may bail out bad banks to increase confidence and prevent runs. Regulators perform more informative stress tests when the market is pessimistic.

Keywords: bank regulation, bailouts, reputation, financial crisis, sovereign debt crisis, stress tests

JEL Codes: G01, G21, G28

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"If money isn't loosened up, this sucker could go down,"

- Statement by former President George W. Bush, quoted in the New York Times on September 26, 2008[1]

1 Introduction

In the quote above, former President George W. Bush highlights the uncertainty in the U.S. over whether funds would be released to resolve the banking crisis in 2008. Uncertainty about whether the regulator will act to stabilize shaky financial institutions has been an element of both the subprime crisis and the ongoing European sovereign debt crisis.

Two audiences pay close attention to how the regulator resolves an unhealthy bank: depositors at other banks and the other banks themselves. Depositors at troubled institutions need assurance that they will not face losses to prevent them from running. At the same time, if a bailout is likely, a troubled institution may decide to take excessive risk. In this paper, we study a theoretical model of how a regulator with private information about its cost of providing capital chooses to resolve a potential bank failure. The regulator will manage information to balance the incentives of these two audiences.

In the model, the regulator must resolve two banks in succession. The regulator learns about the health of a bank (which is unknown to the market) and can decide to inject capital, liquidate the bank, or forbear (do nothing). The regulator's decisions regarding the first bank serve two roles: to resolve the bank and to signal the regulator's cost of injecting capital. The signal is interpreted by both depositors at the second bank *and* the second bank's owners. Balancing the expectations of these two audiences presents a trade-off to the regulator, and leads to information management and multiple equilibria.

Several important results arise in this framework. First, the regulator may want to reduce its reputation of having a low cost of bailing out banks so as to minimize subsequent risk taking by banks. It can do this by forbearing on a bank it knows to be bad when its preferred choice is to bail out the bad

[1] "Talks Implode During a Day of Chaos; Fate of Bailout Plan Remains Unresolved" by David M. Herszenhorn, Carl Hulse, and Sheryl Gay Stolberg, *New York Times*, September 26, 2008. As one may notice from the title of this article, the day was rife with drama and uncertainty.

bank. This allows the regulator to build a reputation with the second bank's owners that it has high costs of capital injections - by potentially letting a bad bank fail, it is acting 'tough' and sending a costly signal that future banks may not be bailed out. This is a reputation-based explanation for a regulator acting tough to diminish moral hazard, as opposed to the commonly found assumption in the literature (discussed below) that a regulator may commit to not conduct bailouts. While it is hard to isolate the reputation effect, it was an important element of both the Lehman Brothers episode and tough talk from top German leadership about Eurozone bank bailouts in 2010 and 2011.

Second, the regulator may want to reduce the perception that it has a high cost of bailing out banks in order to prevent future runs. It can do this by bailing out a bank it knows to be bad when its preferred option is to forbear. This allows the regulator to build a reputation among depositors that it has low costs of capital injections. This appears to have been the case of Ireland, who guaranteed their banks despite the fact that the banking sector was too large to effectively do so. The guarantees did prevent runs until the Irish government was given a lifeline by the European Union.

Last, we also examine information transmission by regulators through verifiable reports, rather than through the costly actions we describe above. We call these reports 'stress tests'. We find that regulators are more likely to conduct stress tests that are informative when beliefs about the banking system are negative. Providing credible information gives them a way to prevent runs on healthy banks. This may explain why a crisis must be significantly advanced before regulators will begin credibly revealing information about the banking system.

In the model, we represent regulator types by the regulator's cost of injecting capital into banks. The cost of funding will depend on the political capital needed to establish and tap new bailout funds (as in the example above and in the European case). The funds could also come from outside sources, such as a super-regulator or another sovereign. This means that the political cost could be vis-a-vis a country's own taxpayers or other sovereign entities.[2] While the regulator may also face uncertainty about the likelihood

[2] During the eurozone crisis, countries have been resorting to scrambling for diverse means of outside support; at various times, the EU, ECB, IMF, private equity firms, and even China have entered the conversations. Hoshi and Kashyap (2010) detail how crippling it was for the Japanese government to attempt to use taxpayer funds to assist the banking sector.

that it can access sufficient funds, it will have private information about this process.

This cost of funding will incur deadweight losses. The amount of funding necessary will also depend on the size of banks, which may be very large compared to the tax base (for example, the banking crises of Ireland and Iceland had this feature).

Government funding is, of course, important for the banking sector; Demirgüç-Kunt and Huizinga (2010) show that a larger fiscal balance (government revenues minus spending) decreases bank CDS spreads. Mariathasan and Merrouche (2012) show that when banks are larger in size compared to GDP, it is more likely there will be recapitalizations. They also show that if a country is indebted, or is running a large deficit, recapitalizations are less likely. At the same time, there is also great uncertainty about what governments will do even if they have the capital - Acharya, Drechsler, and Schnabl (2011) document significant decreases in bank CDS spreads after the initial wave of bailouts[3] in the U.S. and Europe (from 9/26/2008 to 10/21/2008). Government actions then led to learning about the government's position.[4]

The closest paper to ours is Morrison and White (2013), who also study reputation management by a regulator. They argue that a regulator may choose to forbear when it knows that a bank is in danger of failing, because liquidating the bank may lead to a poor reputation about the ability of the regulator to screen which banks are healthy and trigger contagion in the banking system. We also have potential contagion through reputation, but examine the resources of a regulator rather than its skill for screening. In addition, we incorporate asymmetric information about the regulator's type.[5]

In the following subsection, we review the remaining related literature. Section 2 sets up the model. In Section 3, we examine the benchmark model of bank resolution. In the benchmark model, there is no risk-shifting. In Section 4, we add risk-shifting and study how the regulator manages information about its cost of capital injections through costly actions. In Section 5, we

[3]In their Figure 6. They state, "The bailouts typically consisted of asset purchase programs, debt guarantees, and equity injections or some combination thereof" (p.22).

[4]In that same Figure 6, Acharya, Drechsler, and Schnabl (2011) point out that sovereign CDS spreads increase right after the initial bailouts. As they correctly state, there is a transfer of risk to sovereigns. Nevertheless, our paper suggests there may have been a learning effect as well about the sovereign's resources and willingness to use them.

[5]We also assume that forbearing is preferable for some regulators to liquidation, whereas Morrison and White (2013) assume that liquidation is generally preferable to forbearing.

allow regulators to announce information through stress tests. In Section 6, we conclude. All proofs are in the appendix.

1.1 Related Literature

There is a theoretical literature that examines how regulators choose to conduct bank closures and bailouts. Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about contagion as in Morrison and White (2013). In their model, the regulator has private information about its screening ability and there are no bailouts. Cordella and Levy Yeyati (2003) focus on the moral hazard dimension, where a regulator must balance being tough and not bailing out any institution (avoiding moral hazard) with allowing for bailouts (increasing long run bank value through insurance). Keister (2012) also discusses the difference between committing to no bailouts and allowing the regulator discretion to use bailouts in the context of bank runs and liquidity. He finds that bailouts can increase social welfare by reducing the impact of private consumption shocks, and therefore will also increase confidence and reduce fragility. We also demonstrate that a bailout may reduce the likelihood of a run, but through a signaling channel. Freixas (1999) looks at optimal bailout policies and finds that if a regulator can commit, it may commit to a mixed strategy of bailouts and liquidations, which is termed "constructive ambiguity". We do not allow for commitment and add the possibility of depositor runs. Nevertheless, we do have an element of the regulator facing a tradeoff from conducting bailouts that forces it to create uncertainty to diminish moral hazard. Allen, Carletti, Goldstein, and Leonello (2013) look at the tradeoff for a guarantee scheme between stopping runs and providing banks with incentives to pay out too much to depositors who withdraw early. Mailath and Mester (1994) discuss credible bank closure policies in a model with full information and without bailouts. Acharya and Yorulmazer (2007, 2008) examine the idea of "too many to fail" and show that because a regulator will use bailouts when many banks are failing, banks will herd in their risk taking.

In our paper, the regulator is transmitting information to two audiences, depositors and banks. There is a small theoretical literature on this type of information transmission. Gertner, Gibbons, and Scharfstein (1988) examine a firm that signals in a static model to both the capital market and product market rivals through its choice of capital structure and finds that equilibria are pooling. Bouvard and Levy (2012) and Frenkel (2012) consider certifiers such as credit rating agencies, who earn profits from those being certified, but must also consider their reputation for being truthful. These papers show that intermediate reputations can be optimal. Bar-Isaac and Deb (2013) examine a more general framework where an agent can develop a reputation with two audiences who may observe the same action, which is good for the agent, or different actions, which is bad for the agent. In our paper, the regulator has private information about its type *and* about the health of the banks, and can choose from a multitude of actions which are observable to both audiences in a reputational framework.

A previous literature has examined the need for regulators to disclose information about the health of banks. DeYoung et al. (1998) and Berger and Davies (1994) find empirical evidence suggesting that banks disclose good news but look to hide bad news, which is revealed because of bank exams by regulators. Prescott (2008) develops a model to argue that too much information disclosure by a bank regulator decreases the amount of information that the regulator can gather on banks. Bouvard, Chaigneau, and de Motta (2012) have results similar to ours on stress tests. They show that transparency is better in bad times and opacity is better in good times. However, they do not consider the policy tools of the regulator to conduct bailouts, liquidate banks or forbear and therefore do not look at stress tests in conjunction with regulator responses. They instead focus on liquidity and diversification choices by financial institutions. Goldstein and Leitner (2013) also find that stress test should be more transparent when times are bad, but in a very different model. They focus on the tradeoff between the Hirshleifer effect, where transparency destroys risk sharing, and the need for transparency to maintain minimum funding levels. They do not look at regulator interventions in conjunction with stress testing. Spargoli (2012) models the reactions of banks to more transparency, highlighting the tradeoff for the regulator that exposing risk will either require capital injections or reduce the supply of credit to the economy. Peristiani, Morgan, and Savino (2010) show that markets had largely identified the distribution of weaker and stronger banks before the 2009 US stress test was conducted, but the stress test provided new information about the size of capital needs among the weaker banks. Hirtle, Schuermann, and Stiroh (2009) highlight that the 2009 US stress test was credible and stabilizing for the banking system because the standard microprudential process of analyzing individual bank loss exposures was combined with a macroprudential focus of the need for broad financial stability and the upfront commitment to provide capital to banks.

2 The Model

We consider a model with three types of risk-neutral agents: the regulator, banks, and depositors. In the model, the regulator will choose how to resolve two banks sequentially. The regulator has two types of private information: its cost of capital injections and the health of the bank it is resolving. Its choice on how to resolve the first bank sends a signal about its private information on its cost of capital to both the second bank's owners and the depositors at the second bank. The regulator may therefore strategically choose its actions to affect perceptions about its type, which we will term information management.

For the benchmark, we assume there is no moral hazard. We add this element in Section 4.

The regulator's sends signals about its type through costly actions. In Section 5, we will allow the regulator to credibly communicate information through statements (if it chooses to do so). We will call these communications "stress tests".

2.1 Banks and Depositors

For each bank t, where $t = \{1, 2\}$, there are three stages:

1. The regulator privately observes the type of bank t and decides whether and how to resolve the bank. Its choice is public.

2. Depositors at bank t decide whether to withdraw or not.

3. The state of the world is realized, and payoffs are made.

There are a mass one of depositors in each bank[6], who have each deposited 1 unit. We assume they have an outside option, a return of 1 on their savings.

[6]Depositors may be wholesale lenders or retail depositors. Martin, Skeie, and Von Thadden (2012) provide micro foundations for wholesale lending contracts.

The bank has used the deposits to purchase one unit of an asset. If the asset is liquidated at stage 2, it provides a return of 1.[7] In stage 3, if the asset has not been liquidated, the aggregate state of the world is revealed. The state is defined as the returns on the asset, which may be either high returns, where the asset pays off $\bar{R}$, or low returns, where the asset pays off $\underline{R}_\theta$, and $\theta \in \{G, B\}$ is the type of the bank. From an ex-ante perspective, the high returns state occurs with probability q. All agents have a prior that a bank is good (G) with probability α, and is bad (B) with probability $1-\alpha$.

For a solvent bank, the exogenous return promised on deposits is $\tilde{R}$ if they are withdrawn at stage 3.[8] The promised return is 1 if deposits are withdrawn earlier (at stage 2). If the bank is insolvent at any stage, the asset return is equally divided among all withdrawing depositors at that stage. Any remaining value is paid to the owners of the bank at stage 3.

We assume the following ordering on returns:

$$\bar{R} \geq \underline{R}_G \geq \tilde{R} \geq 1 > \underline{R}_B \tag{A1}$$

The good bank can always pay depositors the promised return $\tilde{R}$ on deposits, while the bad bank won't be able to in the bad state. The return promised to depositors for keeping their money in the bank until stage 3 ($\tilde{R}$), is larger than that for withdrawing it (1) at stage 2.

At stage 2, if depositors of a bank expect not to get a return at least as much as their outside option of 1, a run occurs and they withdraw their money from the bank immediately, leaving the bank insolvent. We assume that if depositors knew a bank was bad, meaning that in the low returns state it would have a bad shock, they would run at stage 2:

$$q\tilde{R} + (1-q)\underline{R}_B < 1$$

[7] We could allow this return to be lower than 1. In that case, there would be multiple equilibria where self-fulfilling bank runs occur, but we could get similar results by focusing on the equilibria that have fundamentals-based runs.

[8] The return $\tilde{R}$ can be set optimally before the game begins. For example, in an ex-ante stage, $\tilde{R}$ can be set large enough so that the expected return to the depositors equals their outside option of 1, as in Acharya and Yorulmazer (2007, 2008). In the ex-ante stage, there is a positive probability of entering into the "crisis" game we describe here and a positive probability of entering into a game where there are no shocks that would make banks insolvent.

We assume there is no deposit insurance.[9] From condition A1, if depositors know that the bank is good, then they would not run.

In order to define the beliefs of depositors, it is useful first to define a cutoff parameter. We denote α^* as the probability that a bank is good when depositors are indifferent between a run and keeping their money in the bank. Specifically, α^* is defined by:

$$q\tilde{R} + (1-q)(\alpha^*\tilde{R} + (1-\alpha^*)\underline{R}_B) = 1 \tag{1}$$

If depositors perceive the probability that the bank is good to be smaller than α^*, they would withdraw their funds. Otherwise, they keep their money in the bank.

If there is no run but the bank cannot fully pay depositors at stage 3, the bank is insolvent. We assume there is a cost C to society per bank that is insolvent or liquidated by the regulator.[10] The cost may represent the loss of value from future intermediation the bank may perform, the cost to resolve the bank, or the cost of contagion. These costs may be heightened in a crisis.[11]

We also assume a cost $C_{run} > C$ if a bank is made insolvent by a run. This would be more costly because of the need to immediately liquidate the asset (for example, it could reduce the value of the asset for other agents holding it). Depositors may run if they believe the bank is likely to be bad and the bank is unlikely to be bailed out by the regulator. While there is the

[9] The capital injections in our model are similar to enacting unlimited deposit insurance in the case of retail depositors. Of course, for wholesale lenders, there is no deposit insurance.

[10] The insolvency cost may be different in stage 2 versus stage 3, as in stage 2 it occurs because of liquidation, while in stage 3 it occurs because of a bad shock. To simplify matters, we maintain it is the same in both stages. Mailath and Mester (1994) have a similar cost.

[11] We consider the model as capturing a crisis event because of the significantly large probability of bank failures and banks runs. These are costly enough to the economy such that the regulator may prefer to prevent them with bailouts that include capital injections. In the situation where there is no crisis, which is not modelled in the paper, we presume that the probability of the bad state, the shortfall in the bad state, or the cost of a bank default is low enough such that no type of regulator prefers to bail out bad banks. For example, in practice, deposit insurance and resolution mechanisms provided by the FDIC and the lending facility against collateral provided by the Federal Reserve's discount window are considered sufficient to handle microprudential regulation for isolated bank distress.

threat of runs in the model, in equilibrium there are no runs, so this cost is not incurred.

2.2 The Regulator

The regulator's objective function is to maximize the sum of the expected surplus of all agents minus the cost of insolvencies and potential capital injections. The regulator costlessly observes the type of a bank and then must choose how to resolve it. It has three possible actions: to liquidate the bank, do nothing (forbearance), or inject capital. We now define the expected surplus from each of these actions.

Liquidation: The surplus to the regulator if a bank is liquidated is:

$$1 - C$$

This is the value of the liquidated asset minus the insolvency cost C.

Forbearing: When depositors do not know the type of the bank, a situation may arise where a bad bank receives no capital injection, is not liquidated, and has no run. This occurs if the regulator can effectively "hide" the bad bank's type through forbearance, i.e. (i) the regulator does not pursue a course of action to prevent potential default of a bank that it knows may be bad, and (ii) depositors choose not to run. This gives the regulator a surplus of:

$$S_F = q\bar{R} + (1-q)\underline{R}_B - (1-q)C \tag{2}$$

This is equal to the expected value of the bad asset minus the insolvency cost, which is incurred only in the bad state.

We make the following assumption on the parameters throughout the paper:

$$S_F > 1 - C \tag{A2}$$

This assumption leads to the important case in which the regulator would prefer to hide the type of the bad bank rather than liquidate it. This is at the heart of the information problem, and leads to the need for information management by the regulator.

Injecting capital: The regulator can inject any amount of capital $X \in [0, \bar{X}]$ into a bank. Injecting an amount of capital X costs the regulator $\lambda_i X$.

We define the type i of the regulator in terms of the cost of injecting funds λ_i. There are two types of regulator, the low cost regulator with cost λ_L and the high cost regulator with cost λ_H, where $\lambda_H > \lambda_L$. Injecting capital will be costly and incur deadweight losses ($\lambda_i > 1$, $i \in \{L, H\}$) from raising government funds.[12] The variation in funding costs arises because some regulators may have easy access to funds, while some may face a deadlocked political system and find that the tap is dry. The access to funds could be from outside sources, such as a super-regulator or another sovereign. While it is true that some aspects of funding are observable (taxes raised), the cost of accessing these funds, especially in times of crisis, will be subject to asymmetric information.

While the regulator may inject any amount of capital $X \in [0, \bar{X}]$, we denote the specific amount of capital injection needed in order to prevent an insolvency as $X_I = \tilde{R} - \underline{R}_B$.[13] Note that preventing the insolvency also prevents a run in stage 2. The surplus to the regulator from injecting X_I is:

$$S_i(X_I) = q\bar{R} + (1-q)\underline{R}_B - (\lambda_i - 1)(\tilde{R} - \underline{R}_B) \tag{3}$$

This is equal to the expected value of the (bad) asset minus the cost of having to inject X_I. The capital injection initially costs the regulator λ_i per unit, but as each unit goes to an agent in the economy (in this case, depositors), the surplus lost from the injection is $\lambda_i - 1$ per unit. Since the injection prevents insolvency, there is no insolvency cost.[14]

We will see later that sometimes the regulator will find it worthwhile to inject amounts of capital smaller than X_I, even though the amount will not

[12] Laffont and Tirole (1993) label this the "shadow cost of public funds".

[13] We assume X_I is below the upper bound $\bar{X}$.

[14] In order to streamline our presentation, we will assume that if the regulator could stop both runs and insolvency at a bank *or* only stop a run (and permitting a possible subsequent insolvency), it would prefer to stop both runs and insolvency. This reduces the number of cases to consider and simplifies the presentation. This assumption says that the surplus from a bailout (stopping both runs and insolvency), $S_i(X_I)$ is larger than the surplus from injecting just enough to stop a run: $q\bar{R} + (1-q)\underline{R}_B - (1-q)C - (\lambda - 1)(\frac{1-q\tilde{R}-(1-q)\alpha\tilde{R}}{(1-q)(1-\alpha)} - \underline{R}_B)$. We can rewrite this relationship as:

$$C > \frac{(\lambda_i - 1)(\tilde{R} - 1)}{(1-q)^2(1-\alpha)}$$

Note that if the insolvency is prevented, the run will also be prevented, but the reverse is not true.

prevent insolvency.

The regulator's choices: The cost of injecting capital influences the regulator's choice of action. In particular, we will assume that the low cost regulator and the high cost regulator, when faced with a bad bank, will choose differently between bailouts (injecting X_I), liquidations, and forbearance.

The low cost regulator can afford to bail out banks and strictly prefers to do so, preventing the possibility of a costly future bankruptcy:

$$S_F < S_L(X_I)$$

The high cost regulator prefers hiding the type of the bad bank and forbearing to bailouts, but would rather bail a bank out than liquidate it:[15]

$$1 - C < S_H(X_I) < S_F$$

The regulator knows its own type, but during the resolution of bank t (where $t = \{1, 2\}$) , the market (depositors at bank t and bank t) has an ex-ante belief that, with probability $1 - z_t$, the regulator has a low cost of capital λ_L. With probability z_t, the regulator is believed to have a high cost of capital λ_H.

After the resolution of the first bank, these beliefs will be updated depending on the inference based on what that regulator did with the first bank and on the ex-ante beliefs z_1.

2.3 Summary of Timing

The regulator chooses whether and how to resolve the two banks sequentially. We illustrate the timing in Figure 1. For the benchmark that we analyze in Section 3, there will be no risk-shifting. In Section 4, we will define risk-shifting (which will be an option for the second bank) and examine its implications.

We assume that the regulator has a discount factor δ for the payoffs from the resolution of the second bank. For simplicity, we do not allow for discounting within the resolution of a given bank.

[15] One might imagine a third type of regulator whose costs are so high that it prefers to liquidate rather than bail out. In a previous version of the paper, we analyze this type of regulator in more detail and the results are similar. Furthermore, in the moral hazard section, it will become clear that the high cost type may become this third type of regulator if there is risk-shifting.

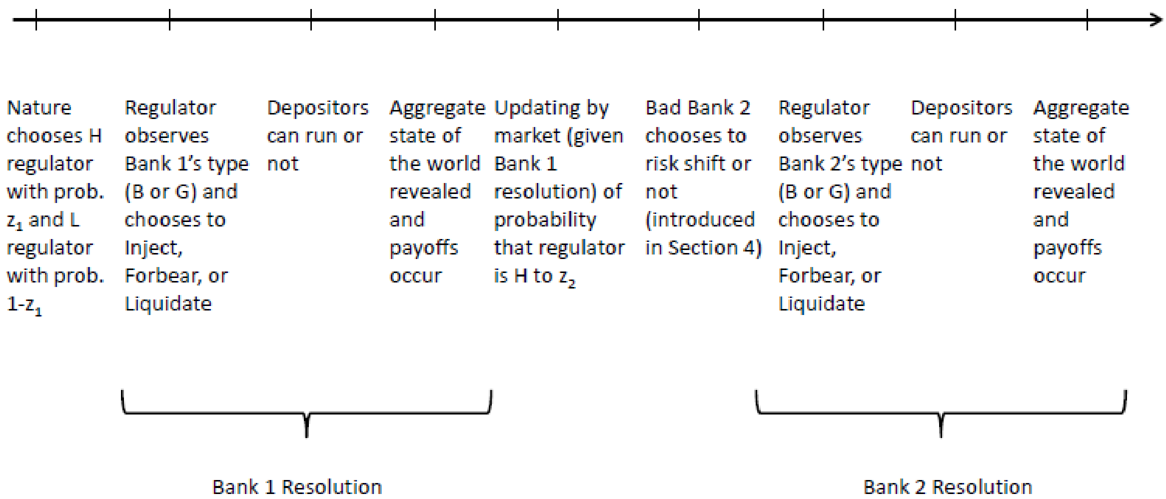

Figure 1: Timeline of events

We assume that the regulator does not know the type of the second bank when resolving the first bank and that this type is independent of the first bank's type. We further assume that the ex-ante probability of having a good bank is α for both banks and that the types of the regulator and the types of the bank are independent.[16]

We use the concept of Perfect Bayesian Equilibrium and focus on pure strategies. We use the intuitive criterion of Cho and Kreps (1987) to refine off-the-equilibrium path beliefs where possible.

3 Benchmark: The Model without Risk-Shifting

We begin the analysis of the model without risk-shifting by using backward induction, and studying the regulator's resolution of the second bank. Since the game ends after the second bank is resolved, there are no reputational

[16] In reality, it may be the case that the type of the regulator and the type of the bank are correlated. The regulator's function outside of times of crisis is to supervise and screen banks. If its ability to supervise and screen is related to its funding (or both are explained by the institutional framework), then it can be the case that the quality of the banking system is related to its funding.

incentives in the regulator's choice.

The resolution mechanism chosen by the regulator depends on whether depositors are likely to run or not. Depositors will run if they believe the bank is good with probability below the cutoff α^*, unless the bank is bailed out, liquidated, or they can be convinced that the bank is actually good. The depositors' belief about the bank being good depends on their prior about the bank α and the equilibrium behavior of the regulator. We will see below that their belief in equilibrium that the bank is good is equal to $\frac{\alpha}{\alpha+z_2(1-\alpha)}$, which also depends on the prior z_2 on the type of the regulator. The more likely the regulator is high cost (higher z_2), the less likely the bank is believed to be good.

In the following proposition we study how both types of regulator (L and H) resolve both types of bank (G and B) in equilibrium. This implies we have four equilibrium resolution choices when depositors are likely to run, and four when they are unlikely to run, i.e. eight in total.

Proposition 1 *For the second bank:*

1. *If $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$: There is an equilibrium where the high cost regulators of both types of bank pool with the low cost regulator of the good bank and take no action. The low cost regulator of the bad bank injects X_I.*

2. *If $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$: There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of X^{**} (where $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$), and both types of regulator of the bad bank inject X_I.*

We depict the results in Figure 2.

The proposition distinguishes between two parameter ranges. When the probability that the bank is good given that the regulator takes no action ($\frac{\alpha}{\alpha+z_2(1-\alpha)}$) is larger than the critical value for a run to take place α^*, depositors won't run when they observe no intervention by the regulator. This allows the high cost regulator to take advantage and forbear on a bad bank, hiding its weakness from depositors by pooling with the regulators who are overseeing good banks. In this sense, forbearance is in fact a form of information management, as the regulator hides the type of the bank from the market. The low cost regulator with the bad bank bails it out, which is its preferred strategy.

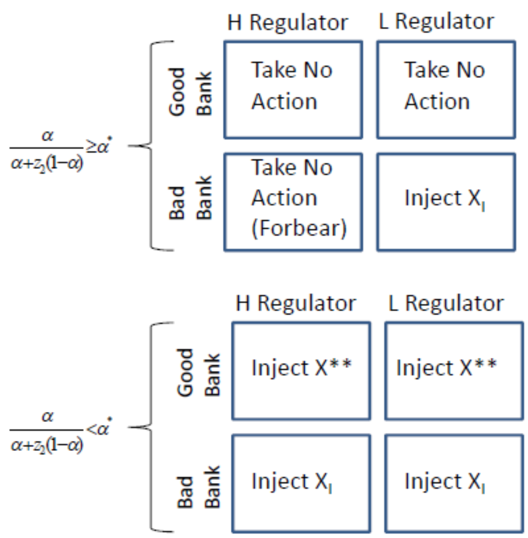

Figure 2: Results of Proposition 1 - Equilibrium Choices of the Regulator for the Second Bank in the Benchmark Model

When the probability that the bank is good given that the regulator takes no action is *lower* than the critical value, no intervention by the regulator will trigger a run by the depositors. Thus both types of regulator will bail out the bad bank. At the same time, both types of regulator will inject an amount of capital X^{**} into the good bank. This amount is positive and less than the amount needed to prevent an insolvency X_I. The role of this injection is to separate the good bank from the bad bank and uses a form of 'money burning' to accomplish this. Given the capital injection X^{**} is only given to good banks in equilibrium, the depositors decide not to run on any bank with such a capital injection. Since the high cost regulator with the bad bank would like to forbear and pretend that the bad bank was good, the capital injection X^{**} must be large enough that the high cost regulator with a bad bank would not deviate from bailing out that bank. The high cost regulator with the bad bank has two difficulties with injecting X^{**}; first, it would incur the high cost of capital injections, and second, it would not be large enough to prevent the bank from failing and incurring the insolvency cost. This ensures that a positive X^{**} exists.[17]

[17] For example, an equilibrium where both the high cost and low cost regulators with the good bank inject no capital, and both regulators bail out the bad bank does not exist.

In part 1 of the Proposition, the equilibrium is not unique. We select this equilibrium as it has many desirable properties: (i) it is the only equilibrium that holds for all off-the-equilibrium-path beliefs, (ii) it is the unique equilibrium when beliefs off-the-equilibrium-path are that the bank is good, and (iii) it is the pareto dominant one (in the sense that when comparing to other equilibria, in this equilibrium at least one type[18] is strictly better off and no types are worse off) and satisfies the undefeated criterion of Mailath, Okuno-Fujiwara, and Postlewaite (1993).[19]

There are clear inefficiencies in these results. Both regulators have to inject X^{**} of capital into a good bank when beliefs are unfavorable in order to signal the bank's type and prevent a run. This wasteful injection clearly results from asymmetric information about the bank's health.

Intriguingly, there are efficiencies from the type of the bank being unknown. This is because, in the situation where beliefs are favorable, the high cost regulator can forbear on the bad bank rather than liquidate or inject capital. Forbearing creates a larger surplus for the high cost regulator (S_F).

The presence of inefficiencies and efficiencies depends on the perception of depositors about whether the regulator is high cost (z_2), as this determines their belief about whether the bank is good ($\frac{\alpha}{\alpha+z_2(1-\alpha)}$). This will influence the behavior of the regulator when it resolves the first bank, as it may benefit from altering this perception.

The capital injections into both good and bad banks when market beliefs are pessimistic have a flavor of the initial TARP injections, where several banks received capital injections when they did not need it (e.g., J.P. Morgan)[20]. The commonly held view is that by injecting all of the largest financial institutions with capital, the U.S. regulators were trying to prevent runs on

The high cost regulator with the bad bank would deviate to no action (forbearance).

[18] Where type is defined as a pair of a regulator type (H or L) and a bank type (G or B).

[19] Other equilibria exist when off-the equilibrium path beliefs are that the bank is bad. In all such equilibria, the low cost regulator with the bad bank will still inject X_I. For instance, some equilibria have the three other regulator types injecting a small positive amount of capital. Another equilibrium takes the same form as when $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$, i.e. both regulators with the good bank inject X^{**} and the high cost regulator with the bad bank injects X_I. It is easy to see these equilibria are pareto dominated (in the sense defined above) by the equilibrium in part 1 of Proposition 1.

[20] The injections were for the nine largest U.S. banks and summed to \$250 billion (see "Drama Behind a \$250 Billion Banking Deal," by Mark Landler and Eric Dash, *New York Times*, October 14, 2008).

bad banks by hiding them in a pool with good banks. Our model provides a different perspective on the capital injections. Both good and bad banks receive capital injections in our model. However, here the reason good banks receive injections is to prevent runs on them (*not* on the bad banks). The injections signal to the market that these banks will not fail. The regulator also injects sufficient capital into the bad banks to bail them out. While the initial TARP injections were likely insufficient for bailouts, subsequent TARP injections into Citigroup and Bank of America provided substantial assistance.

Hiding the bad bank through forbearing is a strategic choice of the regulator in our model. There is little direct evidence of regulators hiding information about banks, but recent events provide indirect evidence. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.[21] Hoshi and Kashyap (2010) discuss several accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.

3.1 The Resolution of the First Bank

We now analyze the actions the regulator takes for the first bank, given how it anticipates that the second bank will be resolved. To prove the existence of an equilibrium, we must look at deviations. We define $\tilde{z}$ as the off-the-equilibrium-path belief that the regulator is high cost at the first bank, and $\tilde{\alpha}$ as the off-the-equilibrium-path belief that the first bank is good.

Using this framework, we demonstrate that there is an equilibrium where the regulators' choices for the first bank are the same as equilibrium behavior by the regulator for the resolution of the second bank.

Proposition 2 *The equilibrium regulator behavior for the second bank is an equilibrium for the first bank, i.e.,*

[21]The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appear in the *Financial Times* ("Diamond Lets Loose Over Libor," by Brooke Masters, George Parker, and Kate Burgess, *Financial Times*, July 3, 2012).

1. *If* $\frac{\alpha}{\alpha+z_1(1-\alpha)} \geq \alpha^*$*: The high cost regulators of both types of bank pool with the low cost regulator of the good bank and take no action. The low cost regulator of the bad bank injects* X_I*.*

2. *If* $\frac{\alpha}{\alpha+z_1(1-\alpha)} < \alpha^*$*: Both types of regulator of the good bank provide a capital injection of* X^{**}*(where* $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$*), and both types of regulator of the bad bank inject* X_I*.*

Sufficient conditions for this equilibrium to exist are $\frac{\alpha}{\alpha+\tilde{z}(1-\alpha)} \leq \alpha^*$ *and* $\tilde{\alpha} < \alpha^*$*.*

In the following section, we will add moral hazard by the second bank to the game. This equilibrium will remain, but two other equilibria will appear due to signaling incentives. The behavior of the regulator at the first bank has two effects beyond the resolution of the first bank. First, it sends a signal to depositors about the type of the regulator, influencing their decision on whether to run on the second bank. Second, it sends a signal to the second bank, which will subsequently decide whether to risk-shift or not. Both of these give the regulator strong incentives to manage information about its type.

4 Information Management

When we add the possibility of the second bank to risk-shift, we will see that this game bears a similarity to models of reputation-building, a la Kreps and Wilson (1982) and Milgrom and Roberts (1982), as the regulator's actions will influence perceptions about its type. Here, however, we have two sources of asymmetric information: the type of the regulator and the health of the bank. We also do not have a "behavioral" type player, as both regulator types will play rationally given their preferences. This implies that from an ex-ante point of view, it is not clear which type will be the one that the other wants to mimic (to build reputation), a feature that will play prominently in the results.

We will see that reputation can be used for very different purposes. For the depositors, the regulator would prefer to instill confidence that banks will not fail and thus prevent runs. This involves signaling that the regulator is low cost. For the second bank, the regulator would prefer to instill fear that the bank will be liquidated, which involves signaling that the regulator is high cost. These conflicting incentives lead to multiple equilibria, which we detail below.

We begin by describing the risk-shifting choice of the second bank's owners. Subsequently, we examine the multiple equilibria that arise in this framework.

4.1 Risk-Shifting

We suppose that the owners of the second bank, if it is bad, can risk-shift.[22] The risk-shifting choice takes place after the events at the first bank (so the owners of the second bank can observe and update their beliefs about the regulator), but before the regulator can take any action at the second bank. The timing is illustrated in the timeline in Figure 1. The risk-shifting is observable, but not verifiable. It can increase expected returns in the good state while reducing expected returns in the bad state. Specifically, it can increase $\bar{R}$ to $\bar{R}'$ while simultaneously reducing $\underline{R}_B$ to $\underline{R}'_B$.[23] For simplicity, we restrict this to be a discrete choice (the bank can choose between $(\bar{R}, \underline{R}_B)$ and $(\bar{R}', \underline{R}'_B)$) and make the shift mean-preserving (set $\underline{R}'_B = \underline{R}_B - \frac{q(\bar{R}'-\bar{R})}{1-q}$). The bank's owners maximize their risk-neutral payoff. As its downside is limited, the bank has a strong incentive to risk-shift. We will demonstrate that the bank's choice will depend on the regulators's actions.

Expected payoffs to the bank and the regulator when there is risk-shifting to $(\bar{R}', \underline{R}'_B)$ are summarized in the following table:

[22] The owners of a good bank could potentially use this tactic as well, but as long as it does not impact regulatory decision making (driving the low return for a good bank $\underline{R}_G$ below $\tilde{R}$), it will not affect our results. Of course, this may be less likely to occur at a good bank because of the better governance and monitoring in place.

[23] Banks can risk-shift in several ways. For example, they can change the composition of their loans towards risky borrowers. Dam and Koetter (2012) show a positive link for German banks between bailout expectations and the fraction of nonperforming loans. Gropp, Gruendl, and Guettler (2013) show that when government guarantees are removed for some German banks, the banks tighten their lending standards and shift their liabilities away from risk-sensitive debt. Jiminez, Ongena, and Peydro (2013) show that in Spain, reducing capital requirements may have led the least well capitalized banks to lend to riskier firms. Diamond and Rajan (2011) describe another type of risk-shifting: banks hoard illiquid assets gambling that they will recover, rather than selling them off and potentially keeping the firm solvent. They cite evidence from He, Khang, and Krishnamurthy (2010) that, "while hedge funds and broker-dealers...were reducing their holding of (illiquid) securitized assets by approximately \$800 billion...commercial banks were increasing their holdings by close to \$550 billion."

Regulator Action	Payoff to Bad Bank	Surplus for Type i Regulator
Bailout	$\gamma q(\bar{R}' - \tilde{R})$	$S_i(X'_I)$
Liquidation	0	$1 - C$
Forbearance	$q(\bar{R}' - \tilde{R})$	S_F

where $X'_I = X_I + (\underline{R}_B - \underline{R}'_B)$. We also assume that in a bailout the bank's owners get a positive stake γ of the upside, where $\gamma \in (0, 1]$. The stake can be strictly lower than one, indicating that the bank's owners may be forced to take losses from the capital injection.

It is worth noting here that, by our definition of surplus, the fact that the bank risk-shifts alone does not affect our measure of surplus (it is just a transfer of wealth), except for the fact that it induces the regulator to wastefully pump in more money in a bailout.

The decision of the bank to shift risk will impact the expectations of depositors. This implies a different cutoff for when depositors decide to run. We denote the cutoff when the bank risk-shifts as α'^*, which is defined by:

$$q\tilde{R} + (1 - q)(\alpha'^* \tilde{R} + (1 - \alpha'^*)\underline{R}'_B) = 1$$

It is obvious from the above that $\alpha'^* > \alpha^*$. Therefore risk-shifting not only requires more wasteful capital injections, it increases the threat of runs and their associated welfare losses.

A key condition for moral hazard to have bite is:

$$S_H(X'_I) < 1 - C \tag{A3}$$

This condition says that diversion of cash flows increases the cost of a bailout so much that the high cost regulator now prefers liquidation to a bailout. The bad bank's choice therefore changes the behavior of the high cost regulator. This creates a risk for a bad bank, as it would strictly prefer to be bailed out rather than liquidated.[24]

[24] For the low cost regulator, we assume that:

$$S_L(X'_I) > S_F \tag{A4}$$

This implies that the low cost regulator still prefers to bail out a bad bank when there is risk-shifting. In an earlier version of this paper, we consider the case where $S_F > S_L(X'_I) > 1 - C$. The qualitative results were similar. We do not include this case for

This risk is made explicit when we consider the bad bank's risk-shifting choice. When the probability that the bank is good is below the cutoff ($\frac{\alpha}{\alpha+z_2(1-\alpha)} \leq \alpha'^*$), the threat of depositor runs induce the regulator to intervene, and the expected payoffs for the bad bank are:[25]

$$\begin{aligned} \text{no risk-shifting} &: \gamma q(\bar{R}-\tilde{R}) \\ \text{risk-shifting} &: (1-z_2)\gamma q(\bar{R}'-\tilde{R}) \end{aligned}$$

The bad bank would prefer no risk-shifting if $z_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and risk-shifting otherwise.[26] This makes sense as the bad bank is only willing to risk-shift if there is a high probability that the regulator is low cost, as the bank will be liquidated if the regulator is high cost. The actions of the regulator at the first bank influence beliefs about its type, and consequently directly affect the risk-shifting decision.

In order to describe how risk-shifting affects the behavior of the regulator at the second bank, we derive an analogue to Proposition 1 in the Appendix (Section 7.4). This analogue is identical to Proposition 1, except that when beliefs are unfavorable, the high cost regulator liquidates the bad bank instead of bailing it out. This changes the capital injections for good banks to X^* (where X^* is defined by $S_F - (\lambda_H - 1)X^* = 1 - C$). And, of course, the capital injections needed for bailouts are larger (X_I').

4.2 Information Management to Mitigate Risk-Shifting

Moral hazard is a key risk discussed by policymakers when bailouts are considered.[27] The argument is that saving a bank today may imply that banks in the future will likely be saved, which will encourage those banks to take

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brevity.

[25]Note that the payoffs are the same for the interval $[0, \alpha^*]$ and $[\alpha^*, \alpha'^*]$.

[26]Note that when the probability that the bank is good is *above* the cutoff ($\frac{\alpha}{\alpha+z_2(1-\alpha)} > \alpha'^*$), there is no threat of depositor runs, and the expected payoffs are:

$$\begin{aligned} \text{no risk-shifting} &: \gamma q(\bar{R}-\tilde{R}) \\ \text{risk-shifting} &: \gamma q(\bar{R}'-\tilde{R}) \end{aligned}$$

Choosing risk-shifting clearly dominates.

[27]Keister (2012) summarizes and adds to the discussion.

excessive risks. This suggests that a commitment device that prevents the regulator from discretionary bailouts may be needed to prevent moral hazard. However, we will now show that such a commitment device is not needed, as the regulator can prevent moral hazard by creating uncertainty about its cost of conducting bailouts.

In the model, if the cost of a bailout is too high, bad banks would be liquidated. Therefore a bank will not risk-shift if that act makes it much less likely to be saved. In this case, a low cost regulator may want to pretend to be a high cost regulator in order to reduce moral hazard at bad banks. The low cost regulator thus does not need commitment power and can use information management to mitigate the moral hazard problem.

In the following proposition, we demonstrate that there is another equilibrium for the regulator at the first bank besides that of Proposition 2. In this equilibrium, the low cost regulator reduces the perception that it is low cost by forbearing on the first bad bank when priors about the bank's health are favorable.

Proposition 3 *At the first bank, there is an equilibrium where the regulators of the bad bank pool at choosing to forbear when beliefs are favorable. Specifically,*

1. *If $\alpha \geq \alpha^*$: Both types of regulator of the good bank take no action and both types of regulator of the bad bank forbear.*

2. *If $\alpha < \alpha^*$: Both types of regulator of the good bank provide a capital injection of X^{**}(where $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$), and both types of regulator of the bad bank inject X_I.*

Sufficient conditions for this equilibrium to exist are that $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\frac{\alpha}{\alpha+\tilde{z}(1-\alpha)} \leq \alpha'^*$, $\tilde{\alpha} < \alpha^*$, and δ, α, and C are large.

We depict the equilibrium in Figure 3. Comparing with the equilibrium in Proposition 2, the difference is the highlighted box, the choice of the *low* cost regulator with the bad bank when priors are favorable.

In Proposition 2, the low cost regulator with the bad bank and favorable priors fully identified itself by separating and bailing out the bad bank. Bailing out the bad bank was its dominant choice, so choosing to forbear is costly. However, by now forbearing, the low cost regulator of the bad bank is able to increase the perception that it is a high cost regulator by pooling with it.

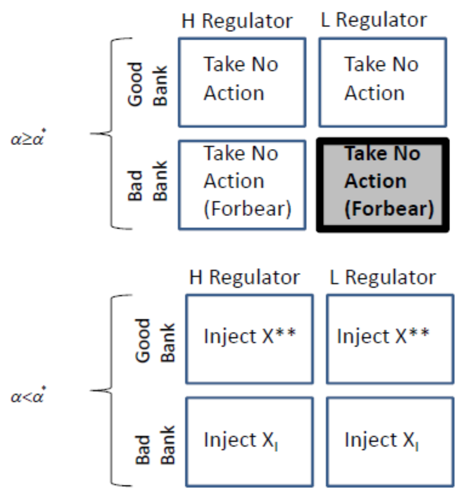

Figure 3: The Equilibrium in Proposition 3

This is useful to the low cost regulator if it prevents risk-shifting. In order to do so, it must satisfy two conditions. First, the belief of the second bank that the regulator is high cost, z_1, must be sufficiently large (this belief z_1 is both the ex-ante and ex-post belief given the equilibrium pooling choice of the regulator types). Second, if the low cost regulator deviates to a bailout, the second bank must interpret this choice as likely having come from the low cost regulator, and then decide to risk-shift. This implies a restriction on the off-the-equilibrium-path beliefs, which we detail in the sufficient conditions and in the appendix.

The result states that, with the possibility of risk-shifting, we are more likely to see bad financial institutions left to the markets at the beginning of a crisis, rather than receiving injections. While there were many things going on at the time, certainly future risk-taking factored into the U.S. government's decision to not save Lehman Brothers. Furthermore, the Lehman decision was seen as critical for the market to learn about the government's willingness to inject capital. Subsequently, the Dodd-Frank Act took concrete steps to make bailouts more costly; it amends Section 13(3) of the Federal Reserve Act to (i) require approval of the Treasury Secretary and additional congressional oversight for any emergency measures taken by the Fed, and (ii) use emergency lending for liquidity purposes only, i.e., not for insolvent firms.

Similarly, there has been less intervention in Europe than one might expect. The popular media has designated the position of the European leadership as trying to "muddle" through. While Germany seems like it could act, it repeatedly mentioned legal restraints on itself, the eurozone, and the European Central Bank. One might wonder whether this tough talk was a play to reduce future moral hazard.

4.3 Information Management to Prevent Runs

Consider the incentives of the high cost regulator. The benefit for the high cost regulator of revealing its type is that it may stop risk-shifting by the second bank. The cost for the high cost regulator of revealing its type is that depositors become more wary since the high cost regulator prefers to hide bad banks, making it more likely that a run will occur. In the following proposition, we demonstrate that there is another equilibrium for the regulator at the first bank besides that of Propositions 2 and 3. In this equilibrium, the high cost regulator mimics the low cost regulator to reduce the perception that it is high cost. It does this by paying a cost - bailing out the first bad bank when beliefs about the bank's type are favorable:

Proposition 4 *At the first bank, there is an equilibrium where the regulators of the bad bank pool and inject X_I into the bad bank when beliefs are favorable. Specifically,*

1. *If $\alpha \geq \alpha^*$: Both types of regulator of the good bank take no action and both types of regulator of the bad bank inject X_I.*

2. *If $\alpha < \alpha^*$: Both types of regulator of the good bank provide a capital injection of X^{**}(where $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$), and both types of regulator of the bad bank inject X_I.*

Sufficient conditions for this equilibrium to exist are that $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\frac{\alpha}{\alpha+\tilde{z}(1-\alpha)} \leq \alpha'^*$, $\tilde{\alpha} < \alpha^*$, and $S_H(X_I)$ is close to S_F.

We depict the equilibrium in Proposition 4 in Figure 4. Comparing with the equilibrium in Proposition 2, the difference is the highlighted box, the choice of the *high* cost regulator with the bad bank when priors are favorable.

The high cost regulator's ideal choice was to forbear on the bad bank when priors are favorable and there is no chance of a run. By choosing to bail out the bad bank instead, it is incurring a cost. The benefit of doing

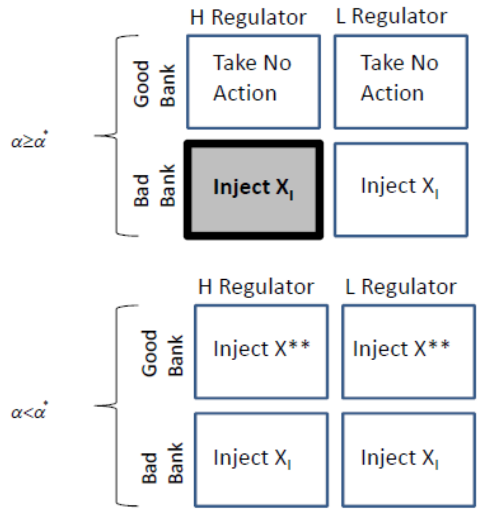

Figure 4: The Equilibrium in Proposition 4

so comes from the fact that the high cost regulator is now pooling with the low cost regulator, increasing the perception that it is low cost. This reduces the threat of runs at the second bank, saving both on the cost of bailing out a second bad bank and the cost of injecting capital into a good bank. Essentially, it is building a reputation for its willingness to bail out banks so that depositors will trust that it will do the right thing with the second bank.

This behavior is reminiscent of the actions taken by Ireland in its banking crisis. Ireland guaranteed its banks even though the banking sector was too large for it to effectively do so. The guarantees did prevent runs until the Irish government was given a bailout package by the European Union.

Another recent example is from October 2011, when it seemed like most European countries (including Germany) wanted to recapitalize their banks. This was likely because either they had the capital to inject into their banks or perhaps they wanted to build their reputation for action. However, France protested against a coordinated action and recapitalizing in general.[28] The French may not only have had larger costs of injecting capital into banks

[28] *The Economist* ("Banks Face New European Stress Tests," October 5, 2011) writes that, "The French government signalled it was uncomfortable with the accelerating talk of recapitalisation, insisting its banks did not need help...any state recapitalisation could threaten France's triple A soverign debt rating."

(there was some discussion of France losing its AAA rating), but they especially did not want to establish this as a precedent going forward because of their banks' exposure to Italy and Spain. In this sense, it seems like France could not afford to build its reputation.

5 Stress Tests

The results above have highlighted that the regulator may use costly actions to transmit information. In this section, we will allow the regulator to convey information through verifiable reports, which we will call 'stress tests'.

Stress tests have been recently adopted as a tool by regulators to communicate information about the health of banks. However, the informativeness and the timing of adoption of the stress tests have varied considerably. In particular, the first few stress tests conducted by the European Union were roundly criticized for being uninformative.

We add an initial stage to the game where the regulator may commit to doing stress tests for both banks. In the initial stage, we will assume the regulator does not know the types of either of the banks. A stress test, when performed, has a tiny cost and will perfectly reveal the type of the bank to the public. We will interpret this perfect revelation as an effective stress test and the lack of a stress test as either simply that or an ineffective stress test.[29]

When the high cost regulator and the low cost regulator make different decisions (i.e., one chooses to do a stress test and the other does not), this is a separating equilibrium and both depositors and the banks learn the type of the regulator. When they make the same decision, nothing is learned about types from this pooling. For simplicity, we will assume that $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, which implies that the second bank will not risk-shift if its belief that the regulator is high cost is equal to or larger than the initial prior.

The low cost regulator has a trade-off when doing a stress test. As the stress test reveals the quality of the bank, the low cost regulator benefits by being able to choose its preferred action: bail out a bad bank and take no action with a good bank. This avoids the cost of asymmetric information, which is having to inject capital into the good bank when depositors' beliefs are negative. There is a cost of doing the stress test: if the low cost regulator

[29] While stress tests by their nature are inherently noisy, there is not much to be gained in this model by having a stress test that is not on the extreme ends of full or no revelation.

does the stress test but the high cost regulator does not, the low cost regulator will reveal its type perfectly, triggering moral hazard by the second bank. This trade-off is represented by the following condition, in which the benefit of the stress test of avoiding wasteful injections into the good bank outweighs the cost of risk-shifting.

$$(1+\delta)(1-I_{\alpha>\alpha^*})(\lambda_L-1)X^{**} > \delta(1-\alpha)(S_L(X_I)-S_L(X_I')) \tag{C1}$$

Where I is the indicator function.

The high cost regulator faces a different trade-off. By doing a stress test, the regulator credibly reveals the type of the good bank, and thus saves having to inject capital into the good bank when depositors' beliefs about the bank's health are negative. However, the stress test also reveals the type of the bad bank, which the high cost regulator prefers to hide when depositors have positive beliefs. This forces the high cost regulator to deal with the problem and bail out the bad bank rather than forbear on it. This trade-off is evident in the following condition, which compares the expected cost of injections into good banks with the expected benefits of hiding the bad banks:

$$(1-I_{\alpha>\alpha^*})\alpha(\lambda_H-1)X^{**} > I_{\alpha>\alpha^*}(1-\alpha)(S_F-S_H(X_I)) \tag{C2}$$

We now look at equilibrium choices. We restrict the regulators to pure strategies. When looking at the two conditions, it is clear that C1 can only hold if $\alpha \leq \alpha^*$. Condtion C2 holds automatically if $\alpha \leq \alpha^*$. This means that C1 holding implies that C2 holds. This eliminates the possibility of a pure strategy separating equilibrium where the low cost regulator does a stress test, but the high cost regulator does not. In fact, there is only one pure strategy equilibrium, and it exists when C2 holds. In this equilibrium, both types of regulator perform stress tests.[30]

[30] As we have not considered yet in the paper the situation where there is full information about the bank types (due to both regulators doing the stress test) but private information about the regulator types, we solve for an equilibrium of this game in the appendix. As in Proposition 1 part 1, this is not the unique equilibrium, but is the only equilibrium that holds for any beliefs off-the-equilibrium-path and is the the pareto dominant equilibrium (in the sense that in this equilibrium, at least one of the regulator-bank types has a higher payoff and no types have lower payoffs than other equilibria).

Proposition 5 *When C2 holds, both types of regulator will perform a stress test.*

Consider condition C2. As the benefit of the stress test for the high cost regulator is felt only when beliefs are negative and the cost of the stress test is felt only when beliefs are positive, this condition depends on the beliefs about the banks' health. In good times, the high cost regulator prefers not to perform stress tests, while in bad times, it needs to do them to save the good banks. While the initial European stress tests were not informative, they have been improving[31], which may be in part due to a deteriorating situation in Europe.

6 Conclusion

In the subprime crisis and European sovereign debt crisis, a recurrent theme is the uncertainty about whether the regulator could and/or would support banks at risk. We model this uncertainty about the regulator to be about its cost of bailing out banks. We demonstrate that regulators can take advantage of this uncertainty by managing information. The regulator faces two audiences when it takes an action: depositors, who are likely to run if their bank will not receive support, and the banks themselves, who may take excess risk if they are likely to be supported. The regulator's attempt to balance the audiences' wishes leads to multiple equilibria. A regulator with a low cost of injecting capital may forbear on a bad bank rather than bail it out in order to act tough and thus eliminate moral hazard. A regulator with a high cost of injecting capital may bail out a bad bank rather than forbear on it in order to give confidence to the market and stop runs. We also show that regulators can do credible stress tests if the market has negative beliefs about the health of the banks.

It would be interesting to extend the model to allow for a richer set of instruments available to the regulator, such as forcing banks to raise outside equity or merge. Examining further the regulator's budget constraint would

[31] For example, "the Irish central bank asked asset-management group Blackrock to come up with the worst numbers it could realistically posit, hired BCG to make sure Blackrock was doing its work properly - then added another 28% for good measure to come up with its total estimated capital shortfall."(from "EU Banking Waits on a Knife-Edge," by Geoffrey T. Smith, *Wall Street Journal Online*, April 7, 2011). Spain has recently discussed emulating the Irish approach.

also be worthwhile. One might imagine that with a hard budget constraint, our signaling process may be reversed as the regulator uses up its capital on the first bank. Finally, elaborating on the political economy of the regulator's decision process and allowing for correlation between regulator funding and bank quality would also be worth pursuing.

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7 Appendix

We begin the appendix by defining some notation that will be useful for the proofs.

1. Expected surplus of the regulator at a good bank when no action is taken:
$$S_G = (q\bar{R} + (1-q)\underline{R}_G)$$

2. The decision of a bad bank to risk shift: The bad bank would prefer no risk-shifting if $z_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and risk-shift otherwise. To capture this decision, we define the indicator function $\eta(z_2) = I_{z_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}}$. The function η is equal to 1 (and zero otherwise) when the belief that the regulator is high cost, z_2, is larger than $\frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$.

3. The beliefs of the depositors that the bank is good: Define the function $p_1(z_2) = I_{\frac{\alpha}{\alpha+z_2(1-\alpha)} > \alpha'^*}$. This function p_1 is equal to 1 (and zero otherwise) when the market belief that the bank is good is above α'^*, the threshold for a run when there is risk-shifting. This conditions on the pooling of the regulator types (both regulators with the good bank, the high cost regulator with the bad bank) and the belief that the regulator is high cost (z_2). Similarly, define the function $p_2(z_2) = I_{\alpha^* < \frac{\alpha}{\alpha+z_2(1-\alpha)} \leq \alpha'^*}$. The function p_2 is equal to 1 (and zero otherwise) when the belief is between α'^* and α^*.

Lastly, for clarity we will refer to the events involving bank 1 as taking place in period 1 and the events involving bank 2 as taking place in period 2.

7.1 Proof of Proposition 1

Given that there are no reputation considerations (this is the final period), the low cost regulator has a dominant strategy to inject X_I in the bad bank. We will therefore consider the actions of the high cost regulator of both types of bank and the low cost regulator of the good bank. Furthermore, using the Cho-Kreps Intuitive Criterion, any off-the-equilibrium-path beliefs must place a probability of zero that a deviation comes from the low cost regulator with the bad bank.

A. The parameter space where $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$:

Note that under the proposed equilibrium strategy of the regulator, the probability that a bank is good given no action is taken by the regulator is $\frac{\alpha}{\alpha+z_2(1-\alpha)}$. Thus, depositors will not run when they see no capital injection given that $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$.

There is a semi-pooling equilibrium where the high cost regulator injects no capital for both bank types and the low cost regulator injects no capital for the good bank. None of these types would deviate for any beliefs off-the-equilibrium path (to liquidate or capital injection of size $X < X_I$) or on the equilibrium path (to a capital injection of X_I).

For all other potential equilibria, we consider off-the-equilibrium-path beliefs where the probability that a bank is good is above α^*. This is consistent with the restriction on off-the-equilibrium-path beliefs above. Consider other semi-pooling equilibria where the three types of regulator inject an amount X of capital or liquidate the bank. Each type of regulator would deviate to taking no action. Any possible equilibrium where there is more separation (two types pool or there is no pooling) has a similar profitable deviation for the regulator to take no action. If there was a possible equilibrium with the high cost regulator of the bad bank as the only regulator type taking no action, this would provoke a run, meaning that the regulator would deviate to the lowest cost action that another regulator type was taking.[32]

The semi-pooling equilibrium we found is therefore unique when beliefs

[32] Therefore there is no equilibrium for any off-the-equilibrium path-beliefs where the high cost regulator with the bad bank does not pool.

off-the-equilibrium-path are that the bank is good. It also exists when beliefs off-the-equilibrium-path are that the bank is bad. It also satisfies the undefeated criterion of Mailath, Okuno-Fujiwara, and Postlewaite (1993) and is the highest surplus for all of the regulator-bank types (and therefore is pareto dominant).

B. The parameter space where $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$: Define X^{**} such that $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$. Given that $S_F > S_H(X_I)$, the injection X^{**} is smaller than X_I, i.e., it is not large enough to prevent insolvency.

As the high cost regulator with the bad bank would never deviate to $X \in [\, X^{**}, X_I)$ for any off-the-equilibrium-path beliefs, the intuitive criterion allows us to set the beliefs such that an X in that range comes from the high cost or low cost regulator with the good bank.

The proposed equilibrium is that both regulators of the good bank inject X^{**} and both regulators of the bad bank inject X_I. Given the definition of X^{**}, the high cost regulator with the bad bank prefers to inject X_I rather than choose X^{**} or liquidate. The regulators of the good bank strictly prefer to inject X^{**} than to inject X_I or liquidate. As long as off-the-equilibrium-path beliefs are such that the probability that a deviation of an injection $X < X^{**}$ (including $X = 0$) comes from a regulator with a good bank are below α^*, this is an equilibrium.

There is no other equilibrium where both the high cost and low cost regulator with the good bank pool (and the high cost regulator with the bad bank separates), as the types with the good bank must inject $X \in [\, X^{**}, X_I)$ in capital to keep the high cost regulator with the bad bank from deviating. From the beliefs established by the intuitive criterion, these regulators would deviate to inject X^{**}.

Consider a potential equilibrium where the high and low cost regulators with good banks pool with the high cost regulator with a bad bank. If they pool at a capital injection less than X_I, there will be a run, and they would have been better off injecting X_I. However, if they pool at X_I, the regulators with the good bank would deviate to X^{**}.

There are also no equilibria where only the high cost regulator with both the good and bad bank pool (as they would want to emulate the low cost regulator with the good bank) or where the low cost regulator with the good bank and the high cost regulator with the bad bank pool (as they would want to emulate the high cost regulator with the good bank). There are also no pure separating equilibria, since the regulator types with the good bank would have an incentive to mimic whoever is taking the lowest cost action.

Lastly, consider a possible equilibrium where all four regulator types pool and inject X_I into the bank. The regulators with the good bank would deviate to X^{**}.

Therefore the proposed equilibrium is unique.

7.2 Proof of Part 1 of Proposition 2

We will examine possible deviations for the four types of regulators when $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^*$.[33] In subsection 8.3, we will examine possible deviations for the four types of regulators when $\frac{\alpha}{\alpha+z_1(1-\alpha)} \leq \alpha^*$.[34] We examine each type of regulator in succession. For each, we will show that the sufficient condition in the proposition imply there is no beneficial deviation.[35] We define $\hat{z}_2 \equiv \frac{z_1(\alpha+(1-\alpha)q)}{z_1(\alpha+(1-\alpha)q)+(1-z_1)\alpha}$, the probability that the regulator is high cost given the regulator has taken no action, when three of the four regulator types (both high cost types, the low cost type with the good bank) pool at taking no action.

High cost regulator with the bad bank: If $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^*$ and the bad bank had no injection and was not liquidated (the static strategy of forbearing), the bank would go insolvent with probability $1-q$. If it goes insolvent, the depositors realize that the regulator has high costs with probability $z_2 = 1$. Otherwise, $z_2 = \hat{z}_2$, which is greater than z_1. Therefore its expected payoff from using the static strategy in period 1 is:

$$\begin{aligned}
&S_F + \delta\{qp_1(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
&+(1-q)(p_1(1)+p_2(1))(\alpha S_G + (1-\alpha)S_F) \\
&+qp_2(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
&+q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))(\alpha(S_G-(\lambda_H-1)X^{**})+(1-\alpha)S_H(X_I)) \\
&+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G-(\lambda_L-1)X^{**})+(1-\alpha)S_H(X_I))\}
\end{aligned} \tag{4}$$

Consider a deviation to injecting X_I (and mimicking the low cost regu-

[33] We will show in subsequent propositions that other equilibria may exist.

[34] We analyze this in a separate subsection as this proof will be relevant to Propositions 3 and 4.

[35] As the conditions are sufficient, there may be other conditions for which this equilibrium exists. A more general derivation that demonstrates which conditions work and which don't is available upon request.

lator with the bad bank). In this case, $z_2 = 0$. Therefore its payoff from deviating in period 1 is:

$$S_H(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{5}$$

Is the deviation profitable? We write the payoff from not deviating as the difference between equations 4 and 5 :

$$(S_F - S_H(X_I))(1 - \delta[q(1-p_2(\hat{z}_2)) + (1-q)(1-p_1(1)-p_2(1))]) \tag{6}$$

We simplified using the fact that $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$ and $\hat{z}_2 > z_1$. This expression is positive.

Consider another possible deviation that yields surplus S_{dev}, with off-the-equilibrium-path beliefs $\tilde{z}$ and $\tilde{\alpha}$. This give a payoff:

$$\begin{aligned} &S_{dev} + \delta\{qp_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ &+(1-q)(p_1(1)+p_2(1))(\alpha S_G + (1-\alpha)S_F) \\ &+qp_2(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ &+q(1-p_1(\tilde{z})-p_2(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I)) \\ &+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_H(X_I))\} \end{aligned} \tag{7}$$

It is obvious that if $\tilde{z} = \hat{z}_2$, there would be no profitable deviation. Using the sufficient conditions in the proposition ($p_1(\tilde{z}) + p_2(\tilde{z}) = 0$, $\tilde{\alpha} < \alpha^*$), the payoff to not deviating is:

$$S_F - S_{dev} + \delta q(p_1(\hat{z}_2) + p_2(\hat{z}_2))(S_F - S_H(X_I)) \tag{8}$$

Given that $\tilde{\alpha} < \alpha^*$, $S_{dev} < S_F$. Therefore this expression is positive.

Low cost regulator with bad bank: Consider a deviation by the low cost regulator with a bad bank when $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$.

The low cost regulator with the bad bank has the static strategy of injecting X_I. In this case $z_2 = 0$ and the payoff is:

$$S_L(X_I) + \delta(\alpha S_G + (1-\alpha)S_L(X_I)) \tag{9}$$

Consider a deviation to forbearing (and pooling with the high cost regulator with the bad bank). In this case, if the bad bank does not go insolvent, which occurs with probability q, $z_2 = \hat{z}_2$ as defined in the proposition. If the bank goes insolvent, with probability $1-q$, the depositors believe that the regulator has high costs with probability $z_2 = 1$. Therefore its payoff from deviating in period 1 is:

$$\begin{aligned}
&S_F + \delta\{qp_1(\hat{z}_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&+(1-q)(p_1(1)+p_2(1))(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&+qp_2(\hat{z}_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&+q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))(\alpha(S_G-(\lambda_L-1)X^{**}) + (1-\alpha)S_L(X_I)) \\
&+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G-(\lambda_L-1)X^{**}) + (1-\alpha)S_L(X_I))\}
\end{aligned} \tag{10}$$

Is the deviation profitable? The payoff to not deviating is the difference between equations 9 and 10. Simplifying that expression, we get:

$$S_L(X_I) - S_F + \delta[q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2)) + (1-q)(1-p_1(1)-p_2(1))]\alpha(\lambda_L-1)X^{**} \tag{11}$$

This expression is always positive.

Now consider a deviation to a surplus level S_{dev}, with off-the-equilibrium-path beliefs set to $\tilde{z}$. Given our sufficient condition of $\tilde{\alpha} < \alpha^*$, $S_{dev} < S_L(X_I)$. If we assume that if there is a default, the beliefs will be equal to $z_2 = 1$, the above results still hold, i.e., there are no profitable deviations.

High cost regulator with a good bank: Consider a deviation by the high cost regulator with a good bank when $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$. The payoff from using the static strategy in period 1 is:

$$\begin{aligned}
&S_G + \delta\{(p_1(\hat{z}_2)+p_2(\hat{z}_2))(\alpha S_G + (1-\alpha)S_F) \\
&+(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))(\alpha(S_G-(\lambda_H-1)X^{**}) + (1-\alpha)S_H(X_I))\}
\end{aligned} \tag{12}$$

Consider a deviation to injecting X_I (and mimicking the low cost regulator with the bad bank). In this case, $z_2 = 0$. Therefore its payoff from deviating in period 1 is:

$$S_H(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{13}$$

Is the deviation profitable? The payoff to not deviating is:

$$S_G - S_H(X_I) + \delta(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))(S_H(X_I)-S_F) \tag{14}$$

This is strictly positive.

Another possible deviation would be to a different surplus level S_{dev}, with off-the-equilibrium-path beliefs $\tilde{z}$. This gives a payoff:

$$\begin{aligned}&S_{dev} + \delta\{(p_1(\tilde{z}) + p_2(\tilde{z}))(\alpha S_G + (1-\alpha)S_F)\\&+(1 - p_1(\tilde{z}) - p_2(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I))\}\end{aligned} \quad (15)$$

It is obvious that if $\tilde{z} = \hat{z}_2$, there would be no profitable deviation. Now using the sufficient conditions in the proposition ($p_1(\tilde{z}) + p_2(\tilde{z}) = 0$, $\tilde{\alpha} < \alpha^*$), the payoff to not deviating is:

$$S_G - S_{dev} + \delta\{(p_1(\hat{z}_2) + p_2(\hat{z}_2))(S_F - S_H(X_I))\} \quad (16)$$

This is positive given $S_{dev} < S_G$.

Low cost regulator with a good bank: Consider a deviation by the low cost regulator with a good bank when $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$.

The low cost regulator with the good bank has the static strategy of injecting no capital. In this case, $z_2 = \hat{z}_2$ and the payoff is:

$$\begin{aligned}&S_G + \delta\{(p_1(\hat{z}_2) + p_2(\hat{z}_2))(\alpha S_G + (1-\alpha)S_L(X_I))\\&+(1 - p_1(\hat{z}_2) - p_2(\hat{z}_2))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I))\}\end{aligned} \quad (17)$$

Consider a deviation to a surplus level of S_{dev}. Its payoff from deviating in period 1 is:

$$\begin{aligned}&S_{dev} + \delta\{(p_1(\tilde{z}) + p_2(\tilde{z}))(\alpha S_G + (1-\alpha)S_L(X_I))\\&+(1 - p_1(\tilde{z}) - p_2(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I))\}\end{aligned} \quad (18)$$

Is the deviation profitable? Given the sufficient conditions ($p_1(\tilde{z})+p_2(\tilde{z}) = 0$, $\tilde{\alpha} < \alpha^*$), then we write the payoff from not deviating as the difference between equations 17 and 18 :

$$S_G - S_{dev} + \delta\{(p_1(\hat{z}_2) + p_2(\hat{z}_2))(\alpha(\lambda_L - 1)X^{**})\} \quad (19)$$

This is positive given that $S_{dev} < S_G$.

7.3 Proof of Part 2 of Proposition 2

We now examine the case where $\frac{\alpha}{\alpha + z_1(1-\alpha)} \leq \alpha^*$.

High cost regulator with the bad bank: On the equilibrium path, the regulator chooses to inject X_I which gives a payoff of:

$$\begin{aligned}&S_H(X_I) + \delta\{(p_1(z_1) + p_2(z_1))(\alpha S_G + (1-\alpha)S_F)\\&+(1 - p_1(z_1) - p_2(z_1))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I))\}\end{aligned} \tag{20}$$

One possible deviation is to inject X^{**} into the bank. By the definition of X^{**}, the regulator would have a lower surplus with the first bank. As this is on the equilibrium path, the beliefs about the probability that the regulator is high cost after the injection would still be z_1 (due to pooling), and therefore the second-period surplus would be equal. This is not a beneficial deviation.

Now suppose there is a possible deviation that yields a surplus of S_{dev}, with off-the-equilibrium-path beliefs $\tilde{z}$ (we will assume beliefs remain $\tilde{z}$ if there is a default) and $\tilde{\alpha}$. Its payoff is then:

$$\begin{aligned}&S_{dev} + \delta\{(p_1(\tilde{z}) + p_2(\tilde{z}))(\alpha S_G + (1-\alpha)S_F)\\&+(1 - p_1(\tilde{z}) - p_2(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I))\}\end{aligned} \tag{21}$$

There is no profitable deviation given the sufficient conditions of $p_1(\tilde{z}) + p_2(\tilde{z}) = 0$, $\tilde{\alpha} < \alpha^*$. The payoff from not deviating is then:

$$S_H(X_I) - S_{dev} + \delta\{(p_1(z_1) + p_2(z_1))(S_F - S_H(X_I))\} \tag{22}$$

Since $\tilde{\alpha} < \alpha^*$, the maximum surplus for S_{dev} is $S_H(X_I)$. This expression is therefore positive.

Low cost regulator with bad bank: The low cost regulator with the bad bank injects X_I. In this case $z_2 = z_1$ and the payoff is:

$$\begin{aligned}&S_L(X_I) + \delta\{(p_1(z_1) + p_2(z_1))(\alpha S_G + (1-\alpha)S_L(X_I))\\&+(1 - p_1(z_1) - p_2(z_1))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_L(X_I))\}\end{aligned} \tag{23}$$

As in the case with the high cost regulator with the bad bank, there is no benefit to deviating to the on-the-equilibrium path choice of X^{**}. For the regulator, there is no benefit at the first bank to deviating from injecting X_I. At the second bank, there would be no benefits from the deviation if $p_1(\tilde{z}) + p_2(\tilde{z}) = 0$.

High cost regulator with a good bank: The payoff from injecting

X^{**} is:

$$\begin{aligned}&S_G - (\lambda_H - 1)X^{**} + \delta\{(p_1(z_1) + p_2(z_1))(\alpha S_G + (1-\alpha)S_F) \\ &+(1 - p_1(z_1) - p_2(z_1))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I))\}\end{aligned} \tag{24}$$

A deviation to injecting X_I yields a lower surplus, as the current surplus is smaller and the future surplus is the same since z_2 will still equal z_1.

Consider another possible deviation to a surplus S_{dev}, with off-the-equilibrium-path beliefs $\tilde{z}$ and $\tilde{\alpha}$. This give a payoff:

$$\begin{aligned}&S_{dev} + \delta\{(p_1(\tilde{z}) + p_2(\tilde{z}))(\alpha S_G + (1-\alpha)S_F) \\ &+(1 - p_1(\tilde{z}) - p_2(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I))\}\end{aligned} \tag{25}$$

Given the sufficient conditions of $p_1(\tilde{z}) + p_2(\tilde{z}) = 0$ and $\tilde{\alpha} < \alpha^*$, the payoff to not deviating would be:

$$S_G - (\lambda_H - 1)X^{**} - S_{dev} + \delta\{(p_1(z_1) + p_2(z_1))(S_F - S_H(X_I))\} \tag{26}$$

Given that $\tilde{\alpha} < \alpha^*$, the maximum surplus for S_{dev} is $S_H(X_I)$. This expression is therefore positive.

Low cost regulator with a good bank: The low cost regulator with the good bank injects X^{**} and has a payoff of:

$$\begin{aligned}&S_G - (\lambda_L - 1)X^{**} + \delta\{(p_1(z_1) + p_2(z_1))(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1 - p_1(z_1) - p_2(z_1))(\eta(z_1)(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I))\}\end{aligned} \tag{27}$$

Consider a possible deviation that gives surplus S_{dev}. Its payoff from deviating in period 1 is:

$$\begin{aligned}&S_{dev} + \delta\{(p_1(\tilde{z}) + p_2(\tilde{z}))(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1 - p_1(\tilde{z}) - p_2(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I))\}\end{aligned} \tag{28}$$

Is the deviation profitable? If $p_1(\tilde{z}) + p_2(\tilde{z}) = 0$ and $\tilde{\alpha} < \alpha^*$, then we write the payoff from not deviating as:

$$S_G - (\lambda_L - 1)X^{**} - S_{dev} + \delta\{(p_1(z_1) + p_2(z_1))(\lambda_L - 1)X^{**}\} \tag{29}$$

Since $\tilde{\alpha} < \alpha^*$, $S_{dev} < S_L(X_I) < S_G - (\lambda_L - 1)X^{**}$. Therefore this expression is positive.

7.4 Analyzing the Second Bank when there is risk-shifting

When there is risk-shifting the regulators' payoffs change. In the following Proposition, we demonstrate that the equilibrium at the second bank condition on risk-shifting is quite similar to that in Proposition 1. The only difference is that when beliefs are unfavorable about the health of the bank, the high cost regulator will liquidate the bad bank (rather than bail it out as in Proposition 1). This also induces one other change - the amount of capital injected by both regulators at the good bank when priors are unfavorable now makes the high cost regulator indifferent between injecting that amount and liquidation (rather than a bail out). We define this amount at X^*.

Proposition 6 *If there was risk-shifting by the bad second bank, the second bank equilibrium is:*

1. *If* $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha'^*$*: There is an equilibrium where the high cost regulators of both types of bank pool with the low cost regulator of the good bank and take no action. The low cost regulator of the bad bank injects* X_I'.

2. *If* $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha'^*$*: There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of* X^* *(where* $S_F - (\lambda_H - 1)X^* = 1 - C$*), the high cost regulator of the bad bank liquidates the bank, and the low cost regulator of the bad bank injects* X_I'.

The choices of the high cost regulator change when there is risk-shifting, as it now prefers to liquidate rather than conduct a bailout. We assumed in A4 that the low cost regulator's choices don't change, as it still prefers to conduct a bailout than forbear. Therefore the proof and actions for the equilibrium when $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha'^*$ are the same as in Proposition 1 Part 1 (except that in Part 1, the parameter space was $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$). Similarly, the proof for the parameters where $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha'^*$ is analogous to the proof in Part 2 where $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$. The only difference is the change in the high cost regulator's preferences. Therefore we define X^* such that $S_F - (\lambda_H - 1)X^* = 1 - C$.

7.5 Proof of Proposition 3

We check if there are deviations from the proposed equilibrium when $\alpha \geq \alpha^*$ in the first period. We don't explicitly check for deviations from the equilibrium when $\alpha < \alpha^*$, since this part of the equilibrium is very similar to the second part of the equilibrium in Proposition 2, and that proof is given in subsection 8.3. We examine each type of regulator in succession. For each, we will show that the sufficient conditions in the proposition imply there is no beneficial deviation.

Also, now that risk-shifting is possible, we use both the results from both Propositions 1 and 6 as the equilibrium actions for the regulator at the second bank, conditional on whether risk-shifting is expected to take place or not.

Low cost regulator with bad bank: We examine whether the low cost regulator with a bad bank would prefer to deviate to injecting X_I. Once again, we represent beliefs off-the-equilibrium-path by the probability the regulator is high cost $\tilde{z}$. Refinements do not allow us to specify or pin down beliefs off-the-equilibrium-path here. Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z) + p_2(z) = 1$.

Consider the payoff on the equilibrium-path of forbearing. This implies $z_2 = z_1$, and, if there is a default, it still remains the case that $z_2 = z_1$ as both types of regulator are forbearing on the bad bank. The payoff is:

$$\begin{aligned} & S_F + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ & +(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ & +p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ & +(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\} \end{aligned} \tag{30}$$

Deviating to an injection of X_I yields the payoff of:

$$\begin{aligned} & S_L(X_I) + \delta\{p_1(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ & +(1-\eta(\tilde{z}))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ & +p_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ & +(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\} \end{aligned} \tag{31}$$

The continuation payoffs in equations 30 and 31 are essentially the same except for the fact that the depositors' beliefs are different. Using the sufficient conditions $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\underline{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\underline{R}}$, and $p_1(\tilde{z}) = 0$, the condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$S_F - S_L(X_I) + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) + \alpha(\lambda_L - 1)X^*\}$$

This can be positive if δ, α, and X^* are large (S_F and/or C large).

High cost regulator with the bad bank: Now let us ask if it is possible that the high cost regulator with a bad bank when $\alpha \geq \alpha^*$ in period 1 would not deviate from a situation where it pools at forbearing with the low cost regulator. We again use the sufficient conditions that $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $p_1(\tilde{z}) = 0$, and $\tilde{\alpha} < \alpha^*$. As forbearing is the best first-period choice for an H regulator with a bad bank, any deviation gives less utility. We denote the H regulator's best deviation by S_{dev}. This gives a payoff of:

$$S_{dev} + \delta\{\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)\} \tag{32}$$

This can be derived from applying the sufficient conditions to equation 7. Keeping the equilibrium strategy of forbearing gives a payoff of:

$$S_F + \delta\{\alpha S_G + (1-\alpha)S_F\} \tag{33}$$

This can be derived from applying the sufficient conditions to equation 4 (while substituting z_1 for $\hat{z}$), and using the fact that $p_1(z_1) + p_2(z_1) = 1$.

The H regulator with the bad bank will not deviate if the difference between equations 33 and 32 is positive:

$$S_F - S_{dev} + \delta(S_F - (1-C))$$

This is positive given that $\tilde{\alpha} < \alpha^*$ implies $S_{dev} < S_F$.

High cost regulator with the good bank: Now consider the high cost regulator with a good bank when $\alpha \geq \alpha^*$ in period 1. We show it would not deviate from a situation where it pools at forbearing with the low cost regulator. We use the sufficient conditions that $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $p_1(\tilde{z}) = 0$, and $\tilde{\alpha} < \alpha^*$. As taking no action is the best first-period choice for an H regulator with a good bank, any deviation gives less utility. The H regulator's best deviation is denoted again by S_{dev}. By definition, $S_{dev} < S_G$. After simplifying (applying these conditions to equation 15), this gives a payoff of:

$$S_{dev} + \delta\{\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)\} \tag{34}$$

By maintaining the strategy of doing nothing, the regulator gets a payoff of:

$$S_G + \delta\{\alpha S_G + (1-\alpha)S_F\} \tag{35}$$

where we simplify equation 12 (substituting z_1 for $\hat{z}$) using the sufficient conditions and the fact that $p_1(z_1) + p_2(z_1) = 1$.

If $z_1 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, then the benefit from not deviating is:

$$S_G - S_{dev} + \delta(S_F - (1 - C)) \tag{36}$$

This is positive.

Low cost regulator with the good bank: The last deviation to check is from the L regulator of the good bank.

Consider the payoff on the equilibrium path of taking no action. In this case $z_2 = z_1$. The payoff is:

$$\begin{aligned} & S_G + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1 - \alpha)S_L(X_I)) \\ & +(1 - \eta(z_1))(\alpha S_G + (1 - \alpha)S_F)] \\ & +p_2(z_1)[\eta(z_1)(\alpha S_G + (1 - \alpha)S_L(X_I)) \\ & +(1 - \eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1 - \alpha)S_L(X_I')))] \end{aligned} \tag{37}$$

Notice that, as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z_1) + p_2(z_1) = 1$.

A deviation to S_{dev}, where $S_{dev} < S_G$ by definition, yields the payoff of:

$$\begin{aligned} & S_{dev} + \delta\{p_1(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1 - \alpha)S_L(X_I)) \\ & +(1 - \eta(\tilde{z}))(\alpha S_G + (1 - \alpha)S_F)] \\ & +p_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1 - \alpha)S_L(X_I)) \\ & +(1 - \eta(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1 - \alpha)S_L(X_I')))] \end{aligned} \tag{38}$$

Similarly, $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

We use the sufficient conditions that $z_1 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, $p_1(\tilde{z}) = 0$, and $\tilde{\alpha} < \alpha^*$. The condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$S_G - S_{dev} + \delta(\alpha(\lambda_L - 1)X^* + (1 - \alpha)(S_L(X_I) - S_L(X_I')))$$

This is positive.

7.6 Proof of Proposition 4

We check if there are deviations from the proposed equilibrium when $\alpha \geq \alpha^*$ in the first period. We don't explicitly check for deviations from the equilibrium when $\alpha < \alpha^*$, since this part of the equilibrium is very similar to the second part of the equilibrium in Proposition 2, and that proof is given in subsection 8.3.

Also, now that risk-shifting is possible, we use both the results from both Propositions 1 and 6 as the equilibrium actions for the regulator at the second bank, conditional on whether risk-shifting is expected to take place or not.

High cost regulator with the bad bank: We begin by seeing whether the high cost regulator with a bad bank would prefer to deviate to forbearing. Once again, we represent beliefs off-the-equilibrium path by the probability the regulator is high cost $\tilde{z}$. Refinements do not allow us to specify or pin down beliefs off-the-equilibrium path here.

Consider the payoff on the equilibrium path of injecting X_I. In this case $z_2 = z_1$. The payoff is thus:

$$\begin{aligned} &S_H(X_I) + \delta\{p_1(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\} \end{aligned} \tag{39}$$

Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z_1) + p_2(z_1) = 1$.

Deviating to forbearing implies $z_2 = z_1$ as both regulator types are forbearing on the good bank in equilibrium. However, with probability q the bad bank fails. This is out-of-equilibrium, so we place beliefs $z_2 = \tilde{z}$. The payoff is:

$$\begin{aligned} &S_F + \delta\{(qp_1(z_1) + (1-q)p_1(\tilde{z}))(\alpha S_G + (1-\alpha)S_F) \\ &+q(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\ &+(1-q)(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ &+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\} \end{aligned} \tag{40}$$

As before, $p_1(z_1) + p_2(z_1) = 1$ and $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

The condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$\begin{aligned}&S_H(X_I) - S_F + \delta(1-q)\{(p_1(z_1) - p_1(\tilde{z}))(\alpha S_G + (1-\alpha)S_F)\\&+(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F)\\&+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\\&-(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F)\\&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}\end{aligned} \tag{41}$$

Applying the sufficient conditions of $z_1 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, and $p_1(\tilde{z}) = 0$, the expression equals:

$$S_H(X_I) - S_F + \delta(1-q)\{S_F - (1-C)\} \tag{42}$$

This is positive when $S_H(X_I) - S_F$ is small. This would also be positive if δ was large and q was small.

We look at another deviation to a surplus level S_{dev}, where we assign beliefs off-the-equilibrium path for no default and for defaults to be $\tilde{z}$. The payoff from deviating would be:

$$\begin{aligned}&S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F)\\&+(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F)\\&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}\end{aligned} \tag{43}$$

Applying the sufficient conditions of $z_1 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, $p_1(\tilde{z}) = 0$, and $\tilde{\alpha} < \alpha^*$ the condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$S_H(X_I) - S_{dev} + \delta\{\alpha(\lambda_H - 1)X^* + (1-\alpha)(S_F - (1-C))\} \tag{44}$$

Given that $\tilde{\alpha} < \alpha^*$, S_{dev} is less than $S_H(X_I)$ and this expression is positive.

Low cost regulator with the bad bank: Now let us ask if it is possible that the low cost regulator with a bad bank when $\alpha \geq \alpha^*$ in period 1 would deviate from a situation where it pools at injecting X_I with the high cost regulator. We will consider the best deviation S_{dev}, which gives rise to off-the-equilibrium path beliefs $\tilde{z}$.[36] We also assume that if there is a default, off-the-equilibrium path beliefs remain $\tilde{z}$. As injecting X_I is the best first

[36] For example, we could set $S_{dev} = S_F$ and $\tilde{z} = z_1$.

period choice for an L regulator with a bad bank, any deviation gives less utility. This gives a payoff of:

$$S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I')) \\ +(1-p_1(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))\} \tag{45}$$

We have simplified by applying the sufficient condition $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ into equation 18.

Keeping the equilibrium strategy of injecting X_I gives a payoff of:

$$S_L(X_I) + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ +p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\} \tag{46}$$

Using the other sufficient condition $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, the payoff to not deviating is:

$$S_L(X_I) - S_{dev} + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) + (1-p_1(\tilde{z}))(\alpha(\lambda_L - 1)X^*)\} \tag{47}$$

which is strictly positive.

Low cost regulator with the good bank: Now consider the low cost regulator with a good bank when $\alpha \geq \alpha^*$ in period 1. We show it would not deviate from a situation where it pools at taking no action with the high cost regulator. We again use beliefs off- the-equilibrium path that $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. As taking no action is the best first period choice for an L regulator with a good bank, any deviation gives less utility. The L regulator's best deviation is denoted again by S_{dev}. By definition, $S_{dev} < S_G$. The payoff from deviating is given by equation 45.

Keeping the equilibrium strategy of taking no action gives a payoff of:

$$S_G + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ +p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\} \tag{48}$$

The L regulator with the good bank will not deviate if the difference between equations 48 and 45 is positive. If $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$,

$$S_G - S_{dev} + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) + (1-p_1(\tilde{z}))(\alpha(\lambda_L - 1)X^*)\} \quad (49)$$

which is strictly positive.

High cost regulator with the good bank: The last deviation to check is from the H regulator of the good bank when $\alpha \geq \alpha^*$ in period 1.

We again represent beliefs off-the-equilibrium path on the probability that the regulator is high cost as $\tilde{z}$.

Consider the payoff on the equilibrium path of taking no action. In this case, $z_2 = z_1$. The payoff is thus modified:

$$\begin{aligned}&S_G + \delta\{p_1(z_1)(\alpha S_G + (1-\alpha)S_F)\\&+(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F)\\&+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}\end{aligned} \quad (50)$$

Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z_1) + p_2(z_1) = 1$.

A deviation to S_{dev}, where $S_{dev} < S_G$ by definition, yields the payoff of:

$$\begin{aligned}&S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F)\\&+(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F)\\&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}\end{aligned} \quad (51)$$

Similarly, $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

Using the sufficient conditions $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, and $p_1(\tilde{z}) = 0$, the condition for not deviating is:

$$S_G - S_{dev} + \delta(S_F - (1-C)) \quad (52)$$

This is strictly positive.

7.7 Proofs of Proposition 5 (Stress Tests)

We will analyze the full game where each type of regulator can choose to perform a stress test or not. We will look only at pure strategy equilibria.

I. The low cost regulator does a stress test, but the high cost regulator does not

We begin with the low cost regulator's payoff:

$$\alpha S_G + (1-\alpha)S_L(X_I) + \delta(\alpha S_G + (1-\alpha)S_L(X_I')) \tag{53}$$

Since the low cost regulator separates by doing the stress test, its type is recognized and the second bad bank risk-shifts. If the low cost regulator were to deviate to not doing a stress test, it would be perceived to be a high cost regulator. Its payoff would then be:

$$\begin{aligned}&(1+\delta)\{(p_1(1)+p_2(1))[\alpha S_G + (1-\alpha)S_L(X_I)] \\ &+(1-p_1(1)-p_2(1))[\alpha(S_G-(\lambda_L-1)X^{**}) + (1-\alpha)S_L(X_I)]\}\end{aligned} \tag{54}$$

Therefore the condition for the low cost regulator not wanting to deviate is:

$$(1+\delta)(1-p_1(1)-p_2(1))(\lambda_L-1)X^{**} > \delta(1-\alpha)(S_L(X_I)-S_L(X_I')) \tag{55}$$

This is the condition labeled as C1 in the text.

The high cost regulator's payoff in this scenario is:

$$\begin{aligned}&(1+\delta)\{(p_1(1)+p_2(1))[\alpha S_G + (1-\alpha)S_F] \\ &+(1-p_1(1)-p_2(1))[\alpha(S_G-(\lambda_H-1)X^{**}) + (1-\alpha)S_H(X_I)]\}\end{aligned} \tag{56}$$

The high cost regulator does not do the stress test, but its type is recognized. It is able to forbear and hide the bad bank when the beliefs about the banks are favorable, but has to inject capital into the good bank to save it from a run when beliefs are negative.

If the high cost regulator deviated to doing a stress test, its payoff would be:

$$(1+\delta)(\alpha S_G + (1-\alpha)S_H(X_I)) \tag{57}$$

It would therefore deviate if the following condition held:

$$\alpha(\lambda_H-1)X^{**} > \frac{p_1(1)+p_2(1)}{1-p_1(1)-p_2(1)}(1-\alpha)(S_F-S_H(X_I)) \tag{58}$$

This is the condition labeled as C2 in the text.

Condition C1 implies condition C2. This can be seen as follows. Consider the case where $\alpha > \alpha^*$. Then neither condition holds. When $\alpha \leq \alpha^*$, C2 holds and C1 may hold for some parameters. This link between the conditions arises because both types of regulators get the same benefit from stress tests, which is avoiding wastefully injecting capital into good banks when priors are negative.

This implies that an equilibrium where the low cost regulator does a stress test and the high cost regulator does not do a stress test does not exist, as it would need C1 to hold and C2 to be violated.

II. The high cost regulator does a stress test, but the low cost Regulator does not

The high cost regulator's payoff is the same as in equation 57. If it were to deviate and not do a stress test, it would be thought of as a low cost type. Its payoff then would be:

$$(1+\delta)[\alpha S_G + (1-\alpha)S_F] \tag{59}$$

where we assume, in the case of a first-period bad bank that defaults, the investors do not update the type of the regulator (which is consistent with Perfect Bayesian Equilibrium). In this case, the deviation is profitable.

III. Both types do the stress test

When both types of regulator do the stress test, they reveal the type of the bank, but not necessarily their own cost of funding. Therefore, their actions may still be signals. We focus on one equilibrium (we discuss the selection of this equilibrium below) of this game, detailed in the following lemma.

Lemma 1 *When both regulators commit to stress tests, there is an equilibrium that has both types of regulator taking no action with good banks and injecting X_I into bad banks for both the first and second banks.*

In this case, the identities of the banks are revealed, but the identities of the regulators are not. It is easy to see this is an equilibrium. Given that the type of the bank is known, the regulators each choose their preferred action. There is no risk-shifting, as we assumed above that $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. There are other equilibria sustained by beliefs off-the-equilibrium-path that the regulator is low cost with probability one. This equilibrium, however, is the one that (i) maximizes surplus for all regulator-bank types (and is therefore pareto dominant in the sense that at least one regulator-bank type is strictly better off in this equilibrium and none are worse off), (ii) is undefeated in the sense of Mailath, Okuno-Fujiwara, and Postlewaite (1993) and (iii) and exists for any beliefs off-the-equilibrium-path. We will therefore focus on it. The payoff for the low cost regulator in this equilibrium is:

$$(1+\delta)(\alpha S_G + (1-\alpha) S_L(X_I)) \tag{60}$$

Obviously, the low cost regulator would not deviate for any beliefs off-of-the-equilibrium-path.

The payoff for the high cost regulator is the same as in equation 57. If the high cost regulator were to deviate and not do a stress test, using the intuitive criterion, it would be recognized as a high cost regulator. In this case, its payoff would be that of equation 56. It wouldn't deviate if the condition from equation 58 (Condition C2) held.

IV. Neither type does the stress test

Here, we will use the equilibrium found in Proposition 2, where the first bank equilibrium is the same as the equilibrium for the second bank when there is no risk-shifting. This builds many of the main intuitions that are also present in applying stress tests to the other two equilibria (Propositions 3 and 4). The payoff for the high cost regulator in this equilibrium is:

$$\begin{aligned}
&[(p_1(z_1)+p_2(z_1))(1+\delta(p_1(\hat{z})+p_2(\hat{z}))) \\
&+(1-p_1(z_1)-p_2(z_1))\delta(p_1(z_1)+p_2(z_1))](\alpha S_G+(1-\alpha)S_F) \\
&+[(p_1(z_1)+p_2(z_1))\delta(1-p_1(\hat{z})-p_2(\hat{z}))+(1-p_1(z_1)-p_2(z_1))(1+\delta(1-p_1(z_1)-p_2(z_1))] \\
&\cdot(\alpha(S_G-(\lambda_H-1)X^{**})+(1-\alpha)S_H(X_I))
\end{aligned} \tag{61}$$

where we define $\hat{z}_2 \equiv \frac{z_1(\alpha+(1-\alpha)q)}{z_1(\alpha+(1-\alpha)q)+(1-z_1)\alpha}$ as in Proposition 2.

The payoff for the low cost regulator is:

$$\begin{aligned}
&(1+\delta)[\alpha S_G+(1-\alpha)S_L(X_I)] \\
&-\alpha(\lambda_L-1)X^{**}\{(p_1(z_1)+p_2(z_1))\alpha\delta(1-p_1(\hat{z})-p_2(\hat{z})) \\
&+(1-p_1(z_1)-p_2(z_1))(1+\delta(1-p_1(z_1)-p_2(z_1)))\} \\
&-\delta(1-\alpha)^2(S_L(X_I)-S_L(X_I'))(p_1(z_1)+p_2(z_1))
\end{aligned} \tag{62}$$

where we have rearranged terms. The last line represents the fact that if there are favorable beliefs and a bad bank in the first period, the low cost regulator bails it out and reveals itself to be low cost. This reveals the low cost regulator's type, which leads to risk-shifting.

Consider off-the-equilibrium-path beliefs where the regulator is believed to be high cost for sure. In this case, the low cost regulator will deviate as it will be able to take no action with a good bank and bail out a bad bank, without risking risk-shifting.