Performance Share Plans: Valuation, Optimal Design, and Empirical Tests*

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Abstract

Performance share plans are an increasingly important component of executive compensation. A performance share plan is an equity-based, long-term incentive plan where the number of shares to be awarded is a quasi-linear function of a performance result over a fixed time period. A special case is a performance-vested share plan, which provides a fixed number of shares whenever a performance result exceeds a threshold goal. We begin by documenting the size and importance of performance share plans and performance-vested share plans. Next, we derive closed-form formulas for the value of a performance share plan or performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). Next, in a principal-agent setting we solve for the optimal design of a performance share plan that maximizes outside shareholder wealth while accounting for the incentive effect on executive effort. We find that the optimal performance share plan is linear (has no upper bound) and that performance-vested share plans are not optimal. Next, we compare the actual plan parameters to optimal parameters. We conclude that a standard principal-agent model cannot rationalize observed performance share plans or observed performance-vested share plans. Finally, we compare the perfect foresight value of plans to our new valuation formulas, the reported values on proxy statements, and heuristic values. We find that our valuation formulas do better or tie reported value and heuristic value in matching the magnitude of perfect foresight value. We find that our valuation formulas are generaly more accurate, but not always. The policy implication is that FASB should require that grant date fair value be estimated using valuation formulas such as ours.

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1 Introduction

For several decades stock options have been the most widely-used incentive component of U.S. executive compensation (Murphy 1999; Clementi and Cooley 2010). Hall and Murphy (2003) cite tax laws enacted in 1994 (Internal Revenue Code 162(m)) as a major driver of the growth in stock options. The Financial Accounting Standards Board (FASB) regulation 123R (2004) changed the accounting treatment of stock options to require that they be expensed at the grant date fair value as estimated by one of several option pricing models. Since that time the use of stock options has declined from 99% of the Forbes 250 in 2003 to 67% in 2012, according to Frederick W. Cook, a compensation consulting firm. By contrast performance share plans, which are an alternative incentive component, have risen from 26% of the Forbes 250 in 2003 to 75% in 2012. Figure 1 illustrates the rapid rise of performance share plans to exceed stock options in recent years.[1] Given their growth, the economics of performance share plans in executive compensation is increasingly important for academics to model, optimize, and test, for practitioners to know the true costs and incentive effects, and for regulators to guide disclosure requirements.

A performance share plan is an equity-based, long-term incentive component of executive compensation in which the number of shares to be awarded is a quasi-linear function of a performance result over a fixed time period. A plan can be based on a variety of alternative performance measures, such as earnings per share, revenue, stock price, return on invested capital, return on equity, etc. Once a performance measure is chosen, then the typical plan's share payoff is determined by three points in share payoff-performance space. For example, in 2009 Coca Cola Enterprises adopted a performance share plan for its CEO based the coming year's earnings per share (EPS). If the EPS at maturity is below the threshold goal of \$1.35/share, then the CEO gets no incentive payoff. If EPS at maturity equals the

[1]The data comes from *The Top 250 Survey* by Frederick W. Cook from 1997 to 2012. Similar findings are also reported in *2011 CEO Pay Strategies Report* by Equilar.

threshold goal of $1.35/share, then CEO gets 109,900 shares. If EPS at maturity equals the target goal of $1.40/share, then the CEO gets 219,800 shares. If EPS at maturity equals or exceeds the stretch goal of $1.45/share, then the CEO gets the maximum award of 439,600 shares.

Figure **??** illustrates the Coca Cola share payoff as a function of EPS at maturity. In the incentive zone, between the threshold goal and the target goal, and between the target goal and the stretch goal, the number of shares awarded increases linearly in performance. Thus, the share payoff is completely determined by the x and y coordinates of threshold, target, and stretch points in share payoff-performance space. Specifically, the six design parameters are threshold goal, target goal, stretch goal, threshold shares, target shares, and stretch shares. The dollar payoff of a performance share plan depends on *both* performance at maturity and stock price at maturity.

Closely related are performance-vested share plans. A performance-vested share plan provides a fixed number of shares whenever performance exceeds a threshold goal and zero shares otherwise.[2] A performance-vested share plan can be thought of as a special case of a performance share plan in which the three points in share payoff-performance space are identical. That is, the threshold goal equals the target goal equals the stretch goal and the threshold shares equals the target shares equals the stretch shares.

We begin by documenting the size and importance of performance share plans and performance-vested share plans. We analyze a large sample of S&P 500 firms from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before November 30, 2012. We find that for those firms who use them, performance share plans have an average value of $3.65 million and performance-vested share plans have an average value of $3.35 million. Unconditionally on whether they are used or not, performance share plans repre-

[2]There is some inconsistency in the way that different people use the terms "performance share plans" and "performance-vested share plans." We follow the most widespread convention.

sent 16.9% of total compensation and performance-vested share plans represent 3.9% of total compensation. We find that the three most popular performance measures for performance share plans are stock return (21.6%), earnings per share (17.3%), and revenue (9.8%), but that a wide variety of other performance measures are used as well.

We derive closed-form formulas for the value of a performance share plan or a performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). Of course, the value depends on the three design parameters: the threshold goal, stretch goal, and the slope of the payoff function. But it also depends on environmental factors, such as the volatility of the performance measure, the beginning stock price, the beginning level of the performance measure, the length of the performance period, and the risk-neutral growth rate of the performance measure.[3]

Using a principal-agent model, we solve for the optimal design of performance share plans under some constraints to simplify the problem. We find that the optimal performance share plan is linear (i.e., the stretch goal goes to infinity) and that performance-vested share plans are not optimal. Intuitively, it is always optimal to provide a marginal incentive for higher performance. Capping a performance share plan eliminates the marginal incentive at high levels of performance and performance-vested share plans only incentivize one threshold, not all levels. We also find that the optimal slope of the payoff function balances the marginal incentive effect against marginal cost.

We hand collect design parameters from proxy statements for S&P 500 firms with fiscal years ending on or after December 15, 2006 to fiscal years ending on or before November 30, 2012. Comparing the actual parameters to the optimal parameters mentioned above, we

[3]For a non-traded performance measure, the risk-neutral growth rate is the nominal growth rate less the risk premium for the systematic risk of the performance measure. For a traded performance measure (e.g., a stock price), the risk neutral growth rate is the risk-free rate.

conclude that the standard principal-agent model cannot rationalize observed performance share plans or observed performance-vested share plans.

We compare values generated by our new valuation formulas versus reported values on proxy statements versus heuristic values. In all cases, we find that reported value and heuristic value have nearly the same mean and median. For performance share plans, we find that our new valuation formulas are statistically and economically different than reported value and heuristic value.

We compare the perfect foresight value of plans to our new valuation formulas, the reported values on proxy statements, and heuristic values. We find that our valuation formulas do better or at least tie reported value and heuristic value in matching the magnitude of perfect foresight value in all subsamples and the full sample. We find that our valuation formulas are more accurate in two subsamples, tie in one subsample, is less accurate in one subsample, and is better or the same in accuracy in the full sample. In most cases, these statistical differences are economically signifcant as well. The policy implication is that FASB should change the accounting treatment of performance share plans and performance-vested share plans to require that grant date fair value be estimated by valuation formulas such as ours.

Our paper is related to three streams of literature within the extensive literature on executive compensation.[4] One stream of literature analyzes the optimal design of CEO compensation in a canonical principal-agent setting. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) analyze the optimal dynamic contract including the dynamic reoptimization of effort over time. These studies have contributed to our knowledge of the optimal *unconstrained* design of CEO compensation, where compensation may take any functional form and may be renegotiated at any time. However, their great generality abstracts away from

[4]Surveys of the executive compensation literature include Abowd and Kaplan (1999), Murphy (1999), Prendergast (1999), Core, Guay, and Larcker (2003), Hall and Murphy (2003), Aggarwal (2008), Bertrand (2009), Edmans and Gabaix (2009), and Frydman and Jenter (2010).

many real-world compensation components, such as options, bonuses, performance shares, etc.

A second stream of literature takes the functional form of real-world compensation components as given and perform a theoretical analysis of the up-front executive compensation decision.[5] Representative of this approach are Hall and Murphy (2000, 2002), Dittmann and Maug (2007), and Dittmann, Maug, and Spalt (2010), which take the functional form of stock options as given and perform a theoretical analysis of the up-front executive compensation decision in the presence of stock options. Analogously, our optimal design section takes the functional form of real-world performance shares as given and performs a theoretical analysis of the up-front executive compensation decision in the presence of performance shares.

A third stream of literature analyzes performance share plans or performance-vested share plans. Martellini and Urosevic (2005) value performance share plans when the performance measure is the stock price. This is a no-arbitrage result, because the underlying asset is traded. By contrast, we expand the scope to value performance shares when the performance measure is a non-traded measure following either an Arithmetic or a Geometric Brownian Motion. These two cases are equilibrium results, precisely because the underlying assets are not traded.[6] Bettis, Bizjak, Coles, and Kalpathy (2010, 2012) empirically investigate stock or option grants with performance-based vesting provisions. They find that these provisions provide meaningful incentives to executives and document that the firms with performance-based vesting provisions significantly outperform the control firms. Bizjak, Kalpathy, and Thompson (2012), a concurrent paper to ours, develops *approximate* present value formulas for equity awards with performance-based vesting provisions. By contrast with these papers,

[5] In practice, renegotiation typically takes place when the executive contract comes up for renewal and seldom within the life of a contract.

[6] A related stream of accounting literature studies different performance measures used in executive compensation. See Lambert and Larcker (1987), Bushman and Indjejikian (1993), Kim and Suh (1993), Sloan (1993), Lambert (1993), and Core, Guay, and Verrecchia (2003).

we develop closed-form formulas for the value of performance share plans and performance-vested share plans, analyze their optimal design, and empirically test our theoretical results.

The remainder of the paper is organized as follows. Section 2 empirically documents the size and importance of performance share plans and performance-vested share plans. Section 3 derives closed-form formulas for the value of performance share plans and performance-vested share plans. Section 4 solves a principal-agent model to determine the optimal design of performance share plans. Section 5 empirically compares actual plan parameters versus optimal plan parameters and empirically compares the perfect foresight value of plans to values generated by our new valuation formulas versus reported values on proxy statements versus heuristic values. Section 6 concludes. The appendices contain proofs and numerical solution procedures.

2 The Importance of Performance Share Plans and Performance-Vested Share Plans

We begin by documenting the size and importance of performance share plans and performance-vested share plans. We collect data from proxy statements on all firms that were in the S&P 500 index as of January 2006. The sample period is from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before November 30, 2012.

Table 2 describes the compensation structure of CEOs in our sample. Panel A presents the mean and median target amount[7] conditional on that component being granted and number of firms granting each compensation component. The amount of compensation granted in the form of performance shares is sizeable. 1,238 firms used performance share plans; granting a mean of $3.65 million and a median of $2.66 million. The mean amount is

[7]For performance cash, the *target amount* represents the amount of cash award the CEO will receive when the ending performance is exactly at performance target. For performance share plans, we obtain reported grant date fair value of equity awards assuming ending performance is at the performance target.

significantly larger than average salary ($1.10 million) and performance cash ($2.23 million), which includes annual cash bonus and long-term cash incentive plans.

Panel B reports the unconditional breakdown of the compensation components (i.e, not conditioning on whether that component is offered). Performance shares represent 16.9% of total compensation. This approaches the size of stock options and performance cash, which represents 23.7% and 22.6% of total compensation, respectively.

Panel C reports on the specific performance measures that are used in performance share plans. We hand-collected this information from definitive proxy statements of S&P 500 firms from the Edgar database.[8] Out of 500 firms, 401 unique firms reported 3,039 firm-year observations of performance measures. They reported using stock return 21.6% of the time, earnings per share (EPS) 17.3%, revenue 9.8%, return on invested capital (ROIC) 6.0%, return on equity (ROE) 4.8%, and other performance measures 40.5% of the time. There are many other measures used including profit measures (e.g., operating income, net income, and profit before tax) and cash flow measures (e.g., free cash flow, cash flow from operations, and economic value added).

3 Valuation

3.1 Payoff At Maturity

When designing a performance share plan, the board of directors first chooses the performance measure, such as stock return, earnings per share, revenue, etc. Next, the board of directors designs the share reward function, which is illustrated in Figure 3(a). The x-axis is the level of performance at maturity (P_T). The y-axis is the number of shares awarded at maturity (N_T). The share reward function is the thick, five-segment line, which shows the number of shares awarded as a function of performance.

[8] Proxy statements can be found from http://edgar.sec.gov/edgar/searchedgar/companysearch.html.

On the x-axis, let L be the threshold goal, M be the target goal, and H be the stretch goal. On the y-axis, let N_L be the threshold shares, N_M be the target shares, and N_H be the stretch shares. If performance is below L, then zero shares are awarded. If the performance equals L, then N_L shares are awarded. If performance equals M, then N_M shares are awarded. Between L and M, the share reward function increases linearly with a slope $\lambda_L = (N_M - N_L)/(M - L)$. If performance equals or exceeds H, then the maximum number of shares N_H is awarded. Between M and H, the share reward function increases linearly with a slope $\lambda_H = (N_H - N_M)/(H - M)$. Typically, the two slopes are not equal ($\lambda_L \neq \lambda_H$), so there is a kink in the share reward function at M.

Based on this structure, we can express the number of shares (N_T) that are awarded at maturity under a performance share plan as

$$N_T = \begin{cases} 0 & \text{for } P_T < L \quad \text{(1a)} \\ N_L + \lambda_L (P_T - L) & \text{for } L \leq P_T < M, \quad \text{(1b)} \\ N_M + \lambda_H (P_T - M) & \text{for } M \leq P_T < H, \quad \text{(1c)} \\ N_H & \text{for } H \leq P_T. \quad \text{(1d)} \end{cases}$$

The dollar payoff of a performance share plan (PSP_T) is the number of shares awarded multiplied by the stock price at maturity (S_T)

$$PSP_T = N_T S_T. \tag{2}$$

Figure 3(b) shows that the dollar payoff of a performance share plan depends on *two* random variables: (1) the performance measure at maturity (P_T) and (2) the stock price at maturity (S_T). The influence of the performance measure at maturity is easily seen by looking at the upper-left edge, where the monetary payoff is flat below L, jumps up at L, increasing linearly between L and M, kinks at M to a new linear slope between M and

H, and then is flat above H. The influence of the stock price at maturity is easily seen by looking at the upper-right edge, where the monetary payoff increases linearly in the stock price at maturity.

3.2 Decomposition

The typical share reward function of a performance share plan can be decomposed into five simplier components. One component is a performance-vested share plan and the other four components are variations of what we call a "linear performance share plan." We define a linear performance share plan as a particularly simple performance share plan, where the payoff above the treshold level is a linear function. A linear performance share plan could be seen as a special case of performance share plan, where the number threshold shares is zero ($N_L = 0$), the two slopes are equal ($\lambda_L = \lambda_H$), and in the limit the stretch goal goes to infinity ($N_H \to \infty$).

The share payoff functions of the five components are shown in Figure 4. Component 1 payoff (the solid blue line) is the payoff of a performance-vested share plan that pays N_L shares if performance equals or exceeds the threshold goal L and nothing otherwise. Component 2 payoff (the first long-dashed black line) has a threshold L and rises at the flater slope λ_L. Component 3 payoff (the second long-dashed black line) has a threshold M and rises at the flater slope λ_L. Component 4 payoff (the first short-dashed red line) has a threshold M and rises at the steeper slope λ_H. Finally, component 5 payoff (the second short-dashed red line) has a threshold H and rises as the steeper slope λ_H.

The table below shows that the share reward function of five components sum to a performance share plan. Specifically, the last four columns show the share rewards for the four regions of performance: (1) below L, (2) between L and M, (3) between M and H, and (4) above H. The first row shows the share reward function of a long position in performance-vested share plan with a threshold L and the next four rows show the share reward function for long or short positions in the other four components (linear performance share plans).

Table 1: Two linear plans sum to a performance share plan

The share reward function of a long position in an linear performance share plan with a threshold L, a short position in an linear performance share plan with a threshold H, and a long position in performance-vested share plan with a threshold L sum to the share reward function of a performance share plan with threshold L and stretch H.

	$P_T < L$	$L \leq P_T < M$	$M \leq P_T < H$	$H \leq P_T$
Long a PVSP with threshold L	0	N_L	N_L	N_L
Long an LPSP with L and λ_L	0	$\lambda_L(P_T - L)$	$\lambda_L(P_T - L)$	$\lambda_L(P_T - L)$
Short an LPSP with M and λ_L	0	0	$-\lambda_L(P_T - M)$	$-\lambda_L(P_T - M)$
Long an LPSP with M and λ_H	0	0	$\lambda_H(P_T - M)$	$\lambda_H(P_T - M)$
Short an LPSP with H and λ_H	0	0	0	$-\lambda_H(P_T - H)$
Total = PSP with L, M, H	0	$N_L + \lambda_L(P_T - L)$	$N_M + \lambda_H(P_T - M)$	N_H

The final row shows the total of these five positions.[9] It is identical to the share reward function of a performance share plan with threshold L, target M, and stretch H shown above in equation 1. The tight connection between a performance share plan and the five components will carry over their valuation.

3.3 Stochastic Processes

We observe that performance share plans are based on a wide variety of performance measures that have different attributes. Some performance measures have strictly non-negative realizations (e.g., revenue). However, other performance measures can be either positive or negative (e.g., earnings per share, return on equity). Most performance measures are not traded assets. However, an exception is when the performance measure is the firm's stock price.

To encompass all of these cases, we will value performance share plans under three alternative modeling assumptions: (1) a non-traded performance measure following an Arithmetic

[9] The total takes advantage of the identities: $N_M = N_L + \lambda_L(M - L)$ and $N_H = N_M + \lambda_H(H - M)$.

Brownian Motion, (2) a non-traded performance measure following a Geometric Brownian Motion, or (3) the price of a traded asset following a Geometric Brownian Motion. We begin by analyzing the first case and then turn to the latter two cases after that.

Let P_t be the performance measure at time t at any time during the performance period $[0, T]$. Assume that the performance measure is not traded, but evolves continuously based on an Arithmetic Brownian Motion as given by

$$dP = \alpha_P dt + \sigma_P dW_1, \tag{3}$$

where α_P is the instantaneous drift of performance, σ_P is the instantaneous standard deviation of performance, and dW_1 is the increment of a standard Wiener process. An Arithmetic Brownian Motion can have negative realizations, so this would be a good model to represent performance measures that can go negative (e.g., earnings per share, free cash flow, operating income, etc.).

Let S_t be the firm's stock price at time t. Assume that the stock price follows a Geometric Brownian Motion as given by

$$\frac{dS}{S} = hdP + \alpha_S dt + \sigma_S dW_2, \tag{4}$$

where h is the sensitivity of the stock price to the performance measure, α_S is the instantaneous drift of the stock, σ_S is the instantaneous standard deviation of the stock, and dW_2 is the increment of a standard Wiener process which is independent of dW_1. Substituting (3) into (4), we obtain

$$\frac{dS}{S} = (h\alpha_P + \alpha_S)\, dt + h\sigma_P dW_1 + \sigma_S dW_2. \tag{5}$$

We value performance shares using the risk-neutral valuation methodology of Cox and

Ross (1976).[10] To do so, we transform the processes above to their corresponding risk-neutral processes. For a non-traded asset this is done by reducing the instantaneous growth rate by the market price of risk times the corresponding instantaneous standard deviation (σ_M).[11] Let ν be the market price of risk for this particular type of risk. Let $\hat{P}_t$ be the performance measure under the following risk-neutral process

$$d\hat{P} = (\alpha_P - \nu\sigma_P)\, dt + \sigma_P dW_1. \tag{6}$$

For a traded asset, such as a stock, the instantaneous drift is adjusted to be the instantaneous riskfree rate r. Let $\hat{S}_t$ be the stock price under the following risk-neutral process

$$\frac{d\hat{S}}{\hat{S}} = rdt + \sigma dz, \tag{7}$$

where $\sigma \equiv \sqrt{h^2\sigma_P^2 + \sigma_S^2}$ and dz is an increment of a standard Wiener process. Based on these processes, the terminal value of the risk-neutral performance measure ($\hat{P}_T$) is normally distributed and the terminal value of risk-neutral stock price ($\hat{S}_T$) is log-normally distributed. For simplicity, let $\hat{Y}_T$ be the natural log of the risk-neutral stock return $\ln\left(\hat{S}_T/S_0\right)$, which is normally distributed. Thus, the distributions of $\hat{P}_T$ and $\hat{Y}_T$ are given by:

$$\hat{P}_T \sim \mathcal{N}\left(P_0 + (\alpha_P - \nu\sigma_P)\, T, \sigma_P^2 T\right) \text{ and} \tag{8}$$

$$\hat{Y}_T \sim \mathcal{N}\left(\left(r - \frac{1}{2}\sigma^2\right) T, \sigma^2 T\right). \tag{9}$$

[10] The Cox and Ross (1976) risk-neutral method values derivative securities as if agents are risk neutral, but it does not require that agents actually are risk neutral.

[11] See, for example, Hull (2012), page 767.

Finally, the correlation between $\hat{P}_T$ and $\hat{Y}_T$ is[12]

$$\rho = \frac{\sigma_P}{\sqrt{\sigma_P^2 + \frac{\sigma_S^2}{h^2}}}. \tag{10}$$

3.4 Performance share plans and performance-vested share plans

3.4.1 When A Non-Traded Measure Follows An Arithmetic Brownian Motion

Let $PSP_0^i(L, M, H, N_L, N_M, N_H)$ be the date 0 value of a performance share plan with threshold L, target M, stretch H, threshold shares N_L, target shares N_M, stretch shares N_H, and where the superscript $i \in \{A, G, S\}$ identifies one of the three types of performance measures. Let $PVS_0^i(N_L, L)$ be the date 0 value of a performance-vested share plan that pays N_L shares above the threshold L. Let $LPS_0^i(X, \lambda_j)$ be the date 0 value of an linear performance share plan, where the first argument is the threshold $X \in \{L, M, H\}$, the second argument is the slope λ_j with $j \in \{L, H\}$), and where the superscript $i \in \{A, G, S\}$ identifies one of the three types of performance measures. In this subsection, an A superscript is used to identify variables in the case when a non-traded performance measure follows an Arithmetic Brownian Motion (e.g., earnings per share).

Proposition 1 *When a non-traded performance measure follows an Arithmetic Brownian Motion, the date 0 value of a performance share plan is*

$$\begin{aligned} PSP_0^A(L, M, H, N_L, N_M, N_H) = {} & PVS_0^A(N_L, L) + LPS_0^A(L, \lambda_L) - LPS_0^A(M, \lambda_L) \\ & + LPS_0^A(M, \lambda_H) - LPS_0^A(H, \lambda_H), \end{aligned} \tag{11}$$

[12] The correlation between $\hat{P}_T$ and $\hat{Y}_T$ can be derived as follows:

$$\rho = \frac{\sigma_{\hat{P}\hat{Y}}}{\sigma_{\hat{P}}\sigma_{\hat{Y}}} = \frac{h\sigma_P^2 T}{\sigma_P\sqrt{T}\sigma\sqrt{T}} = \frac{h\sigma_P}{\sigma} = \frac{h\sigma_P}{\sqrt{h^2\sigma_P^2 + \sigma_S^2}} = \frac{\sigma_P}{\sqrt{\sigma_P^2 + \frac{\sigma_S^2}{h^2}}}.$$

the date 0 value of a performance-vested share plan is

$$PVS_0^A(N_L, L) = N_L S_0 N\left(d_1^A(L)\right), \tag{12}$$

and the date 0 value of an linear performance share plan is

$$\begin{aligned} & LPS_0^A(X, \lambda_j) \\ & = S_0\lambda_j\left[\left\{M_0 + \left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T - X\right\}N\left(d_1^A(X)\right) + \sigma_M\sqrt{T}n\left(d_1^A(X)\right)\right] \end{aligned} \tag{13}$$

where $\lambda_L = \frac{N_M - N_L}{M-L}$, $\lambda_H = \frac{N_H - N_M}{H-M}$, $d_1^A(X) = \frac{M_0 - X + \left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$ *and* $N(\cdot)$ *and* $n(\cdot)$ *are the cumulative distribution and density functions of the standard normal.*

Proof See the appendix.

Intuitively, equation (11) shows that the value of a performance share plan is sum of the value of a long position in an linear performance share plan with a threshold goal of L, the value of a short position in an linear performance share plan with a threshold goal of M, the value of a long position in an linear performance share plan with a threshold goal of M, the value of a short position in an linear performance share plan with a threshold goal of H, and the value of performance-vested share plan with a threshold goal of L. The performance-vested share plan formula has the intuitive interpretation of being the current stock price times the fixed number of shares times the probability that performance exceeds a threshold goal ($P_T > L$) under the risk neutral process. Furthermore, equation (12) shows that the value of an linear performance share plan is the product of the current stock price S_0, the slope of the payoff function λ_j, and the term in square brackets, which is the expected value of the performance measure.[13]

[13]This formula is qualitatively different from the Black-Scholes option pricing model in that it involves a performance measure component $(M_0 + (\alpha_M - \nu\sigma_M)T)$, in addition to the stock price.

Figures 5(a) and 5(b) illustrate the value of a performance share plan. In Figure 5(a), the solid curve is the ex-ante value of a performance share plan, which rises rapidly from slightly below the threshold goal, continues rising in the incentive zone, slows down as current performance (M_0) approaches the stretch goal. The value asymptotically approaches $S_0 N_H$. By analogy to the options literature, the dotted line represents the *intrinsic value* of the performance share and the vertical gap between the date 0 value of a performance share and the intrinsic value represents the *time value* of the performance share plan. The time value is positive over most of the incentive zone, but turns slightly negative near the stretch goal H. Figure 5(b) shows how the date 0 value of a performance share varies with the current level of performance (M_0) and the current stock price (S_0). On the upper-left edge, we observe an S-curve, where date 0 value increases non-linearly with current performance. On the upper-right edge, we observe that the date 0 value increases linearly with the current stock price.

3.4.2 When A Non-Traded Measure Follows A Geometric Brownian Motion

Now we consider the case in which a non-traded performance measure follows a Geometric Brownian Motion. A Geometric Brownian Motion never goes negative, so this would be a good model to represent performance measures that never go negative (e.g., revenue). Specifically, we assume that

$$\frac{dM}{M} = \alpha_M dt + \sigma_M dW_1, \tag{14}$$

and

$$\frac{dS}{S} = h\frac{dM}{M} + \alpha_S dt + \sigma_S dW_2 \tag{15}$$

$$= (h\alpha_M + \alpha_S)\, dt + h\sigma_M dW_1 + \sigma_S dW_2, \tag{16}$$

where h is the sensitivity of the stock price to change in the performance measure, and dW_1 and dW_2 are increments of independent standard Wiener processes. A G superscript is used to identify variables in the case when a non-traded performance measure follows a Geometric Brownian Motion.

Proposition 2 *When a non-traded performance measure follows a Geometric Brownian Motion, the date 0 value of a performance share plan is*

$$PSP_0^G(L, M, H, N_L, N_M, N_H) = PVS_0^G(N_L, L) + LPS_0^G(L, \lambda_L) - LPS_0^G(M, \lambda_L) + LPS_0^G(M, \lambda_H) - LPS_0^G(H, \lambda_H), \quad (17)$$

the date 0 value of a performance-vested share plan is

$$PVS_0^G(N_L, L) = N_L S_0 N\left(d_1^G(L)\right), \quad (18)$$

and the date 0 value of an linear performance share plan is

$$LPS_0^G(X, \lambda_j) = S_0 \lambda_j \left[M_0 e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T} N\left(d_1^G(X)\right) - XN\left(d_2^G(X)\right)\right] \quad (19)$$

where $\lambda_L = \frac{N_M - N_L}{M - L}$, $\lambda_H = \frac{N_H - N_M}{H - M}$, $d_1^G(X) = \frac{\ln\frac{M_0}{X} + \left(\alpha_M - \nu\sigma_M + \left(h + \frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$ *and* $d_2^G(X) = d_1^G(X) - \sigma_M\sqrt{T}$.

Proof See the appendix.

3.4.3 When A Traded Asset Price Follows A Geometric Brownian Motion

Now we consider the case in which the performance measure is the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). Specifically, we assume that

$$\frac{dP}{P} = \frac{dS}{S} = \alpha_S dt + \sigma_S dW_1. \tag{20}$$

When the underlying asset (the stock) is a traded asset, a performance share plan can be valued by the no arbitrage approach of Black and Scholes (1993). A S superscript is used to identify variables in the case where the performance measure is the price of a traded asset following a Geometric Brownian Motion.

Proposition 3 *When the performance measure is the price of a traded asset following a Geometric Brownian Motion, the date 0 value of a performance share plan is*

$$\begin{aligned} PSP_0^S\left(L, M, H, N_L, N_M, N_H\right) = PVS\left(L\right) + LPS_0^S\left(L, \lambda_L\right) - LPS_0^S\left(M, \lambda_L\right) \\ + LPS_0^S\left(M, \lambda_H\right) - LPS_0^S\left(H, \lambda_H\right) \end{aligned} \tag{21}$$

the date 0 value of a performance-vested share plan is

$$PVS_0^S\left(N_L, L\right) = N_L S_0 N\left(d_1^S(L)\right), \tag{22}$$

and the date 0 value of an linear performance share plan is

$$LPS_0^S\left(X, \lambda_j\right) = S_0 \lambda_j \left[S_0 e^{\left(r+\sigma_S^2\right)T} N\left(d_1^S(X)\right) - XN\left(d_2^S(X)\right)\right],$$

where $\lambda_L = \frac{N_M - N_L}{M-L}$, $\lambda_H = \frac{N_H - N_M}{H-M}$, $d_1^S(X) = \frac{\ln\frac{S_0}{X} + \left(r+\frac{3}{2}\sigma_S^2\right)T}{\sigma_S\sqrt{T}}$ *and* $d_2^S(X) = d_1^S(X) - \sigma_S\sqrt{T}$.

Proof This follows immediately from Proposition 2 by changing performance measure values to stock price values: $P_0 = S_0$, $h = 1$, $\alpha_P = \alpha_S$, and $\sigma_P = \sigma_S$. Since the underlying asset is the price of a traded asset, the performance share plan can be valued by the no arbitrage approach. In this case, the risk neutral growth rate becomes the riskfree rate ($\alpha_P - \nu\sigma_P = r$). **Q.E.D.**

This formula is similar to that of Proposition 2, but it is fundamentally different in one respect from those of both Propositions 1 and 2. Unlike other performance measures which are not traded securities, a stock is a traded security, and thus the result in Proposition 3 is no arbitrage result and there are no *investor preference parameters*. By contrast, the results for performance measures other than the stock price are equilibrium results and these formulas involve the investor preference parameters, α_M and ν. In the special case in which $N_L = 0$ and $\lambda_L = \lambda_H$, equation 21 reduces to the Martellini and Urosevic formula.

3.4.4 Factors affecting the value of performance shares

Figures 6(a)-6(c) show how the value of a performance share plan is affected by the contractual terms of the performance share plan. Figure 6(a) shows the value of a performance share plan for different widths of incentive zone. L and H represent a narrow incentive zone and L' and H' represent a wide incentive zone. A wide incentive zone plan is more valuable at low levels of current performance and a narrow incentive zone plan is more valuable at high levels of current performance. Also, wider incentive zone smooths the value curve.

Figure 6(b) shows the value of a performance share plan for different times to maturity (or different lengths of a performance period). Analogous to a call option on a non-dividend-paying stock, more time increases the value of a performance share plan, because it increases the time value component (the extra value above the intrinsic value).

Figure 6(c) shows the value of a performance share plan when there is a convex kink at the target vs. concave kink at the target. We can see that the value of two performance share plans with different type of kinks have huge price gap around the kinks, although the gap narrows as you move away from the target. It implies that choosing the type of kink at the performance target might have value implications.

Figures 6(d)-6(f) shows how the value of a performance share plan is affected by various environmental factors. Figure 6(d) shows the value of a performance share for different volatilities of the performance measure. For high (low) values of current performance, higher volatility decreases (increases) the value of a performance share plan because there are limited potential gains (losses) on the upside (downside) and greater potential losses (gains) on the downside (upside).

Figure 6(e) shows the value of a performance share plan for different consensus estimate. If the consensus is high, that means the market forecast on the firm's performance is more optimistic. Intuitively, we can see that as the market is more optimistic, the value of a performance share plan increases.

Finally, Figure 6(f) shows how the value changes as the risk premium on non-traded assets change. Because the risk premium negatively affects the attractiveness of a plan with non-traded asset as a performance measure, we can observe that the value of a performance share plan decreases as the risk premium is higher.

3.5 A Generalized Performance Share Plan

The base model can be generalized to fit real-world payoff structures of performance share plans. While many firms offer plans with one kink at the target, many other firms have more complicated structure with multiple kinks within the incentive zone. Our model allows to value any performance share plans with multiple kinks. Suppose a performance share plan has C kinks, where X_C is the stretch goal. Let X_c be the c^{th} performance level from the threshold goal, and N_c be the number of shares awarded when the performance is at X_c, with N_1 being the jump in payoff at the threshold goal. Furthermore, assume the slope of the payoff structure between X_c and X_{c+1} be given by λ_c.

Proposition 4 (Generalized Performance Share Plan) *Under three alternative assumptions about the performance measure, the date 0 value of a generalized performance share plan is*

$$PSP_0^i = PVS_0^i\left(X_1\right) + \sum_{c=1}^{C-1}\left[LPS_0^i\left(X_c, \lambda_c\right) - LPS_0^i\left(X_{c+1}, \lambda_c\right)\right] \tag{23}$$

where $i = \{A, G, S\}$, $PVS_0^i\left(X_1\right)$, *and* $LPS_0^i\left(X_c, \lambda_c\right)$ *and corresponding* d_{2c-1} *and* d_{2c} *are calculated following Propositions 1-3.*

This model can also value a generalized performance share plan with many kinks but which is linear (i.e., without stretch goal). This is done by taking the limit as X_C goes to ∞, which will make the last term of the equation vanish.

4 Optimal Design

In this section, we embed the valuation models derived above in a principal-agent model to determine the optimal design of a performance share plan under some constraints. Our model follows the classic principal-agent model. The firm hires a manager with a compensation plan at the beginning of the period (time 0) and compensates the manager at the end of the period (time T).

Given the difficulty of numerically optimizing both levels (the manager's expected utility and outside shareholder value) over many parameters, we impose three constraints to reduce the dimensionality of the problem to managable size. Specifically, we require that $N_L = 0$, $\lambda_A = \lambda_B$, and $L \leq fP_{0,}$, where f is a constant in the neighborhood of 1.0.[14] The first constraint eliminates the parameter N_L.[15] The second constraint effectively eliminates both

[14] In the empirical section, we find that the two performance share plan subsamples have mean values for f of 0.98 and 1.05, so this seems like a reasonable representation of the real world.

[15] $N_L = 0$ in 23% of our empirical sample.

parameters M and N_M, because making the slope of the two incentive zone segments identical effectively combines them into a single long segment, whose location is uniquely pinned down by the three endpoint parameters (L, H, N_H). The third constaint merely eliminates the nuisance solution of an extreme "lottery ticket" that pays $H = \$1$ billion if performance exceeds a threshold L that is 20 standard deviations above the mean.

We assume that the manager is compensated via a fixed salary (b) paid at the end of the period and a simplified performance share plan with just three parameters (L, H, N_H). This can still include a performance-vested share plan by a simple reinterpretation of the parameters. In the special case of $L = H$, the simplified plan can be interpreted as a performance-vested share plan that pays N_H shares above the performance level H and nothing otherwise.

4.1 The Manager's Problem

During the performance period, the manager exerts effort to maximize her expected utility given the compensation contract. At the end of the period, the manager receive her compensation. The manager's initial endowment is assumed to be zero. Thus, her end-of-period compensation is her sole wealth. The manager's end-of-period compensation C_T is

$$C_T = b + N_T S_T, \tag{24}$$

where the second term is the dollar payoff of the performance share plan.

The performance of the firm is assumed to depend on both the manager's realized effort a and other random factors. Let the end-of-period performance P_T be given by

$$P_T = P_B + a + \int_0^T dP, \tag{25}$$

where P_B is the hypothetical base level of firm performance that would result if the manager put in zero effort and $\int_0^T dP$ is the influence of other random factors over time.

We assume that on date 0, immediately after the manager is hired, the compensation contract is publicly disclosed and the market forecasts the manager's effort $\bar{a}$ under the contract. Also on date 0, the market updates its forecast of firm performance by incorporating its forecast of the manager's effort $P_0 = P_B + \bar{a}$. Finally, the date 0 stock price updates to $S_0 = RP_0 = R(P_B + \bar{a})$, where R is a stock price / performance ratio.

Analogously, the terminal stock price S_T depends on the manager's realized effort a and other random factors as follows

$$S_T = R\left(P_B + a\right) + \int_0^T dS, \tag{26}$$

where $\int_0^T dS$ is the influence of other random factors over time and we assume that the stock does not pay any dividends.

The manager's utility $U_M\left(C_T(a), a\right)$ is assumed to depend on terminal compensation, which is influenced by the manager's effort, and on the disutility of effort. Specifically, we assume that the manager's utility is risk neutral in terminal compensation and suffers disutility as a cubic function of effort[16]

$$U_M\left(C_T(a), a\right) = C_T(a) - ka^3, \tag{27}$$

where k is the manager's utility cost of effort.

The manager chooses a non-negative effort level to maximize her expected utility of

[16]Examining the performance share plan formulas in Propositions 1 and 2, we see that compensation is linear in the current stock price S_0 (which is linear in effort), has a term that is linear in current performance M_0 (which is linear in effort), and includes the cumulative normal terms $N\left(\right)$ (which are influenced by effort). So overall compensation is greater than a quadratic function of effort, but less than a cubic function of effort. Thus, the disutility of effort must be at least cubic in effort in order to produce a well-defined concave function with a unique optimum.

terminal wealth given the compensation contract.

$$\max_{a\in[0,\infty)} E\left(C_T(a) - ka^3\right), \tag{28}$$

$$\text{s.t. } E\left(C_T(a) - ka^3\right) \geq \overline{U}, \tag{29}$$

where $\overline{U}$ is the reservation utility of the manager. Equation (28) is the incentive compatibility condition and equation (29) is the participation constraint of the manager. Let a^* denote optimal managerial effort, which maximizes her expected utility above. In a rational expectations equilibrium, the market's forecast of managerial effort must turn out to be correct ($\bar{a} = a^*$).

4.2 The Outside Shareholders' Problem

Shareholders are assumed to be risk neutral. Outside shareholders[17] are assumed to maximize outside shareholder value (i.e., the value of the firm net of compensation paid to the manager). The essential trade-off is that a compensation contract can create an incentive for the manager to increase effort, which increases outside shareholder value, but the cost of managerial compensation decreases outside shareholder value. Let C_0 be the date 0 value of the manager's compensation. Discounting (24) back to present, we get

$$C_0 = be^{-rT} + PSP_0^i\left(L, H, N_H\right). \tag{30}$$

where $i \in \{A, G, S\}$ represents the three alternative performance measure cases.

Let N be the number of shares outstanding. It is assumed that shareholders understand the manager's problem above and thus can correctly forecast the optimal managerial effort

[17]In practice, the board of directors designs the compensation contracts and negotiates with the manager. It is assumed that the board acts in the best interests of outside shareholders.

a^* that will result from a given compensation plan. The outside shareholders' problem is

$$\max_{L,\lambda,H} NS_0\left(a^*\right) - C_0, \tag{31}$$

$$\text{s.t. } a^* = \arg\max_{a\in[0,\infty)} E\left(U_M\left(C_T(a), a\right),\right) \tag{32}$$

$$L, H, N_H \geq 0, and \tag{33}$$

$$H \geq L. \tag{34}$$

Equation (31) has outside shareholders choosing optimal design parameters L, λ, and H, which combined with the resulting optimal managerial effort, maximizes outside shareholder value. Equation (33) is the non-negativity constraints on the three design parameters. Equation (34) states that the stretch goal needs to be at least as high as the threshold goal.

4.3 Numerical Solution

We solve the principal-agent model by dynamic programming. Since the manager moves last, the manger's problem is solved first. Given that the manager's problem is nonlinear, we solve it numerically using a standard hill-climbing technique for constrained optimization. The result is an optimal managerial effort for a given triplet of contract parameters $a^*\left(L, H, N_H\right)$.

Next, we turn to the outside shareholders' problem. Outside shareholders wish to determine the optimal contract parameters out of the set of all feasible contract parameters. In order to analyze the outside shareholders objective function we need to know the optimal managerial effort $a^*\left(L, \lambda, H\right)$ for any set of contract parameters that we wish to consider. We could use brute force to determine the optimal managerial effort and outside shareholder objective function for thousands or even tens of thousands of contract parameter triplets and then select the highest outside shareholder objective function, but that would be inefficient. Instead, we devise a method to approximate the optimal managerial effort over a reasonable range of contract parameters and then steadily improve the accuracy of the approximation to any arbitrary degree of accuracy (see Appendix B for details).

4.4 Optimal Design Results

The base parameter values that we analyze are: salary b=\$1,000,000, performance with zero effort M_B=2, mean performance change α_M=2, standard deviation of performance change σ_M=2, constraint constant f=1, mean stock return α_S=12%, standard deviation of stock return σ_S=20%, market price of risk ν=.2, sensitivity to stock price to performance h=.01, stock price to performance ratio R=10, time to maturity T=1 year, riskfree rate r=1%, and number of outstanding shares N=1,000,000. We analyze a wide range of values for the cost of effort $k \in [1,000,\ 5,000,000]$.

Let the $*$ superscript designate an optimal design parameter or the value of a performance share plan with optimal design parameters. Our first design result that the optimal threshold goal is always equal to the upper bound of the constraint $L^* = fP_0$.

Our second design result is that there is an unique optimal slope parameter λ^*. The intuition for this result is shown in Figure 8. It shows the outside shareholder value for different values of the slope λ, where the optimal managerial effort $a^*(L, \lambda, H)$ is updated for each value of λ. In this figure, the other two design parameters (L and H) are kept at fixed values. At one extreme as the slope λ is reduced down to zero, then optimal managerial effort drops to zero. In this case, even though the cost of performance-based compensation drops to zero, outside shareholder value drops to a low level because there is zero managerial effort. At the other extreme, as λ is increased to a high level, optimal managerial effort rises, but outside shareholders are worse off on a net basis. This is because the terms of the compensation contract become so costly that the entire value of the firm is paid to the manager and outside shareholder value drops to zero. In between these two extremes, outside shareholder value is hump-shaped. This leads to a unique interior optimum λ^* that precisely balances the marginal incentive effect against marginal cost.

Our third design result is that the optimal stretch goal H^* is unbounded. Said differently, the optimal performance share plan is an *linear* performance share plan.[18] Intuitively, the reason for this result is that it is always optimal to provide a marginal incentive for higher performance. Even at very high levels of performance, it is optimal to incentivize even higher performance.

Recall that the simplified performance-vested share plans can be reinterpreted as a performance share plan when $L = H$. A direct implication of L^* having a finite value and H^* being unbounded is that $L^* \neq H^*$ and thus, performance-vested share plans are *not* optimal. Intuitively, this is because performance-vested share plans are a step-function that focus on incentivizing a single threshold value, whereas it is optimal to provide a marginal incentive at all levels.

Incorporating the results above, the following proposition shows that the optimal performance share plan formulas under the three constraints that we impose are much simpler than the general formulas.

Proposition 5 *Given three constraints, the date 0 value of a optimal performance share plan under three alternative assumptions about the performance measure are*

$$PSP_0^{A*} = LPS_0^A\left(fM_0, \lambda^*\right), \tag{35}$$

$$PSP_0^{G*} = LPS_0^G\left(fM_0, \lambda^*\right), \tag{36}$$

$$PSP_0^{S*} = LPS_0^S\left(fM_0, \lambda^*\right), \tag{37}$$

[18] At each iteration of the numerical solution process, the provisional stretch goal becomes larger. By a certain point in the process, the provisional stretch goal is hundreds of standard deviations above the mean of the performance measure. As a result the value of a performance share plan under the provision optimal parameters becomes identical (down to penny accuracy) to the value of an linear performance share plan.

where

$$LPS_0^A\left(fM_0,\lambda^*\right) = S_0\lambda^*\left[\left\{(1-f)M_0+\left(\alpha_M-\nu\sigma_M+h\sigma_M^2\right)T\right\}N\left(d_1^A(fM_0)\right)\right.$$
$$\left.+\,\sigma_M\sqrt{T}n\left(d_1^A(fM_0)\right)\right], \tag{38}$$
$$LPS_0^G\left(fM_0,\lambda^*\right) = S_0M_0\lambda^*\left[e^{\left(\alpha_M-\nu\sigma_M+h\sigma_M^2\right)T}N\left(d_1^G(fM_0)\right)-fN\left(d_2^G(fM_0)\right)\right], \tag{39}$$
$$LPS_0^S\left(fM_0,\lambda^*\right) = (S_0)^2\lambda^*\left[e^{\left(r+\sigma_S^2\right)T}N\left(d_1^S(fM_0)\right)-fN\left(d_2^S(fM_0)\right)\right], \tag{40}$$

and where $d_1^A(fM_0) = \frac{(1-f)M_0+\left(\alpha_M-\nu\sigma_M+h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_1^G(fM_0) = \frac{\ln\frac{1}{f}+\left(\alpha_M-\nu\sigma_M+\left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_2^G(fM_0) = d_1^G(fM_0)-\sigma_M\sqrt{T}$, $d_1^S(fM_0) = \frac{\ln\frac{1}{f}+\left(r+\frac{3}{2}\sigma_S^2\right)T}{\sigma\sqrt{T}}$, *and* $d_2^S(fM_0) = d_1^S(fM_0)-\sigma\sqrt{T}$.

Proof The optimal performance share plan formulas are obtained by starting with the analogous performance share plan formula, substituting $N_L = 0$, $L = fM_0$ and $\lambda_L = \lambda_H = \lambda = \lambda^*$, and then evaluating the limit as $H \to +\infty$. Substituting $N_L = 0$ causes $PVS_0^i\left(N_L, L\right) = 0$ in all three cases ($i \in \{A, G, S\}$). Substituting $\lambda_L = \lambda_H = \lambda = \lambda^*$ causes the third adn fourth terms $-LPS_0^A\left(M,\lambda^*\right) + LPS_0^A\left(M,\lambda^*\right)$ to cancel each other out in all three cases ($i \in \{A, G, S\}$). Taking the limits $\lim_{H\to+\infty} N(d_1^i(H)) = 0$ and $\lim_{H\to+\infty} n(d_1^i(H)) = 0$, so $LPS_0^i\left(H,\lambda^*\right) = 0$ in all three cases ($i \in \{A, G, S\}$). The sole remaining term $LPS_0^A\left(L,\lambda_L\right)$ is evaluated at $L = fM_0$ and $\lambda_L = \lambda^*$ in all three cases ($i \in \{A, G, S\}$). **Q.E.D.**

4.4.1 Comparative Statics

Next, we examine some comparative statics for the optimal constrained performance share plan. Figures 9(a)-9(d) show the impact of the cost of effort on optimal compensation design parameters. Figure 9(a) shows the very intuitive result that a higher cost of effort leads to lower optimal managerial effort a^*. Figure 9(b) shows that firms with a higher cost of effort choose a lower optimal threshold goal L^*. The intuition is that a higher cost of effort leads to lower effort, which leads to a lower date 0 value of performance P_0 (incorporating the market's effort forecast) and this constrains L^* to be lower. Figure 9(c) shows that firms with a higher cost of effort have a lower current stock price S_0, which follows immediately from the lower date 0 value of performance P_0. Figure 9(d) shows a non-monotonic, humped-shape relationship between the cost of effort and the optimal slope λ^*. So the optimal slope λ^* is sometimes increasing and sometimes decreasing in the cost of effort.

These comparative static results are for a non-traded performance measure following a Geometric Brownian Motion. Qualitatively we get the same results as Figures 9(a)-9(c) in the other two performance measure cases. However, the optimal slope λ is strictly decreasing in the cost of effort in the other two performance measure cases.

5 Empirical Tests

In this section, we empirically test our valuation formulas compared to reported value on proxy statements, heuristic value, and perfect foresight value. We hand-collect plan parameters whenever they are reported on the firms' definitive proxy statements. We collect data on all firms that were in the S&P 500 index as of January 2006. The sample period is from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before November 30, 2012. We limit the scope to performance measures whose definitions are standard across firms, which includes earnings per share (EPS) and revenue. By contrast, the definitions of ROIC and ROE are quite different across firms. To value the plans, we need to have six design parameters for performance share plans or two parameters (threshold goal and threshold shares) for performance-vested share plans. We are able to identify 255 firm-years of performance share plans and 39 firm-years of performance-vested share plans with all required data.

Table 3 reports the design parameters of performance share plans and performance-vested share plans based on earnings per share (EPS) or revenue. The first three columns report the threshold goal (L), target goal (M), and stretch goal (H) divided by the performance level in the previous year (P_0). It also reports the slope (λ) of the plans. The second three columns report the threshold shares, target shares, and stretch shares. Panels A and B are the subsamples of performance share plans using Earnings Per Share and Revenue, respectively. As discussed above, the ratio of the threshold goal / prior performance has a mean value of 0.987 for EPS plans and 1.031 for revenue plans. This ratio can be viewed as a proxy for the constraint constant f and it illustrates the property of being in the neighborhood of 1. The ratio of stretch goal / prior performance has a mean value of 1.126 for EPS plans and 1.26 for revenue plans. These values for the stretch goal are nowhere close to optimal stretch goal / prior performance H^*/P_0 of infinity. There is significant chance of producing a 12.6% increase in EPS or a 12.6% increase in revenue, which implies that these caps are seriously binding. Therefore, we conclude that the standard principal-agent model cannot rationalize observed performance share plans.

Panels C and D report the threshold goal divided by prior performance and threshold shares for performance-vested share plans using Earnings Per Share and Revenue, respectively. We find that the ratio of the threshold goal / prior performance has a mean value of 0.885 for EPS plans and 1.275 for revenue plans. Again, these performance-vested share plans focus the incentive on a single threshold, not all levels. Therefore, we conclude that the standard principal-agent model cannot rationalize observed performance-vested share plans.

Next, we compare our new valuation formulas versus the reported value on proxy statements versus heuristic value. Firms report the grant date fair value of equity awards in the *Grants of Plan-based Award Table* in the annual proxy statements. In the majority of cases, the reported value is the same as heuristic value, which is defined below.[19]

For performance share plans, heuristic value is arrived at by supposing that the perfor-

[19] In recent year, a small number of firms have began to use Monte Carlo simulation to estimate the grant date fair value.

mance outcome at maturity will exactly equal the target value ($P_T = M$). In this case, the number of shares that will be awarded will be N_M. Then, heuristic value values that number of shares at S_0, the current stock price at time 0. Let PSP_0^H be the heuristic value of a performance share plan at time 0, which is given by

$$PSP_0^H = N_M S_0. \tag{41}$$

For performance-vested share plans, recall that the manager receives the threshold shares (N_L) when the performance threshold goal is achieved. Heuristic value assumes that the performance will equal or exceed the threshold goal *for certain* and then values the fixed number of shares at S_0. Let PVS_0^H be the heuristic value of a performance-vested share plan at time 0, which is given by

$$PVS_0^H = N_H S_0. \tag{42}$$

Table 4 compares the formula value based on Propositions 1, and 2 versus the reported value on proxy statements versus the heuristic value for performance share plans and performance-vested share plans based on EPS or revenue. For EPS measure firms, we use price-to-earnings ratio as the sensitivity of stock price to performance improvement (h), and for revenue measures, we use return-to-revenue growth (previous year's stock return divided by previous year's revenue growth) as h. We calibrate α_P to the historical average of EPS increments or revenue growth rate, σ_P to the standard deviation of historical EPS increments or revenue growth rates, ν to 0.4, r_f to the yield on 3-month US Treasury Bills.

Panels A and B cover performance share plans using EPS or revenue, respectively. We find that reported value and heuristic value have nearly the same mean and median. For plans using EPS, we find that our new formula value has a 7.8% lower mean and a 14.3% lower median than reported value, similar differences relative to heuristic value, and all four differences are signficant. For plans using revenue, our formula value has a 6.3% lower mean and a 39.7% lower median, similar differences relative to heuristic value, and all four differences are signficant. The large magnitude of the differences are economically significant, as well as being statistically signficant. Intuitively, the differences in valuation are driven by the fact that reported values (typically based on heuristic value) and heuristic value make the counterfactual assumption that hitting the performance target is the most likely outcome. By contrast, our new valuation formulas accounts for the true distribution of the performance measure.

Panels C and D cover performance-vested share plans using EPS or revenue, respectively. We find that reported value and heuristic value have nearly the same mean and median. For plans using EPS, we find that our new valuation formulas have a 39.1% lower mean and 57.0% lower median compared to reported value, similar differences relative to heuristic value, and all four differences are signficant. For plans using revenue, we find that our valuation formulas have a 49.9% lower mean and a 48.1% lower median, similar differences relative to heuristic value, and all four differences are signficant.

For a most, but not all of our sample, we are able to determine the realized value of the performance measure at the plan maturity date. In the spirit of Shiller (1980) who compares stock prices to a "perfect foresight dividend series," we compare three candidate measures of date 0 plan value to a "perfect foresight value." Specifically, we define the perfect foresight value as $N_T S_0$. Intuitively, it is what a performance share plan would be worth at date 0 if you had perfect foresight of what the future performance measure P_T would be and thus could perfectly predict the future share payoff N_T, but had no information about the future stock price realization S_T and ignored the correlation between N_T and S_T.

Perfect foresight value is a useful benchmark to compare with date 0 measures of plan value, because we can see which date 0 measure does the best job of predicting the share payoff, but without any distortions due to the stock price noise. In a large sample, the stock price noise would vanish, but in our small sample it wouldn't vanish and including it in the empirical test would hurt the accuracy of assessing our three candidates.

Based on this rational, Table 5 reports the magnitude and accuracy compared to perfect foresight value of formula value based on Propositions 1 and 2, reported value on proxy statements, and heuristic value. The sample is a subset of performance share plans and performance-vested share plans for which we can obtain the realized value of the performance measure at the plan maturity date. We lose some 2011 and 2012 plans whose realized value is based on 2013 or 2014 performance.

The first four columns report perfect foresight value, formual value, reported value, and heuristic value in millions of dollars. The next three columns report the percent difference between perfect foresight value and the three candidate measures in order to judge the difference in magnitude. The last three column report the *absolute* percent difference between perfect foresight value and the three candidate measures in order to judge their forecast accuracy.

Starting with the magnitude columns, we find that formula value is closer in magnitude to perfect foresight value than reported value in all subsamples and for the full sample, although some of the differences are not significant. We find that formula value is closer in magnitude to perfect foresight value than heuristic value in all subsamples and for the full sample, although some of the differences are not significant. Many of the differences are large in economic significance. For example looking at the full sample mean, formula value is 3.1% different vs. 16.2% and 12.5% for the other two measures. For the full sample mediam, formula value is 1.7% different in magnitude vs. 43.2% and 39.2% for the other two measures.

Turning to the accuracy columns, we find that for performance share plans using EPS (Panel A) formula value is essentially insignificantly different in accuracy compared to the other two measures. For revenue plans (Panel B), formula value significantly more accurate than the other two measures. For performance-vested share plans using EPS (Panel C), formula value is significantly less accurate than the other two measures. For revenue plans (Panel D), formula value is significantly more accurate than the other two measures. For the full sample mean (Panel E), formula value is essentially insignificantly different in accuracy than the other measures. For full sample median (also Panel E), formula value is significantly more accurate than the other two measures. Many of the differences are large in economic significance. For example looking at the full sample mediam, formula value is 43.4% accurate vs. 52.5% and 58.3% for the other two measures.

In summary, we find that our valuation formulas do better or at least tie reported value and heuristic value in matching the magnitude of perfect foresight value in all subsamples and the full sample. We find that our valuation formulas are more accurate in two subsamples, tie in one subsample, is less accurate in one subsample, and is better or the same in accuracy in the full sample. In most cases, these statistical differences are economically signifcant as well.

The policy implication of finding economically significant differences between our valuation formulas and reported values in most cases is that FASB should change the accounting treatment of performance share plans and performance-vested share plans to require that grant date fair value be estimated by valuation formulas such as ours. This is analogous to the way that FASB 123R requires that stock options be valued on the grant date by one of several option pricing models. This change would provide shareholders with a more accurate assessment of the true cost of these plans. An extensive accounting literature establishes that more accurate accounting disclosure yields real economic benefits, including the more efficient allocation of resources (for example, see Healy and Wahlen 1999).

6 Conclusion

We document the size and importance of performance share plans and performance-vested share plans. Next, we derive closed-form formulas for the value of a performance share plan or performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). Next, in a principal-agent setting we solve for the optimal design of a performance share plan that maximizes outside shareholder wealth while accounting for the incentive effect on executive effort. We find that the optimal performance share plan is linear (has no upper bound) and that performance-vested share plans are not optimal. Next, we compare the actual plan parameters to optimal parameters. We conclude that a standard principal-agent model cannot rationalize observed performance share plans or observed performance-vested share plans. Finally, we compare the perfect foresight value of plans to our new valuation formulas, the reported values on proxy statements, and heuristic values. We find that our valuation formulas do better or at least tie reported value and heuristic value in matching the magnitude of perfect foresight value in all subsamples and the full sample. We find that our valuation formulas are generaly more accurate, but not always. The policy implication is that FASB should require that grant date fair value be estimated using valuation formulas such as ours.

A Proofs

A.1 Proof of Proposition 1

Table 1 shows that the five components have the same payoff at maturity as a performance share plan. Therefore, as shown in equation (11), the date 0 value of a performance share plan must be equal to the date 0 value of the five components in the absence of arbitrage.

The date 0 value of a performance-vested share plan is the expected value of the payoff at maturity under the risk-neutral growth rate discounted back to date 0 at the riskfree rate as given by

$$PVS_0^A = e^{-rT} \int_{-\infty}^{L} \int_{-\infty}^{+\infty} 0 \times S_0 e^{Y_T} f(\cdot) dY_T dP_T$$
$$+ e^{-rT} \int_{L}^{+\infty} \int_{-\infty}^{+\infty} N_L S_0 e^{Y_T} f(\cdot) dY_T dP_T \tag{A.1.1}$$
$$= N_L S_0 e^{-rT} \int_{L}^{+\infty} \int_{-\infty}^{+\infty} e^{Y_T} f(\cdot) dY_T dP_T. \tag{A.1.2}$$

Denote the PDF of conditional distribution of Y_T given P_T as $f(Y_T|P_T)$ and the PDF of P_T as $f(P_T)$. Then the value of a performance-vested share plan is given by

$$PVS_0^A = N_L S_0 e^{-rT} \lambda \int_{L}^{+\infty} \int_{-\infty}^{+\infty} e^{Y_T} f(P_T) f(Y_T|P_T) dY_T dP_T. \tag{A.1.3}$$

Conditional distribution of Y given M is

$$Y_T|P_T \sim \mathcal{N}\left(\mu_Y + \frac{\sigma_Y}{\sigma_M}\rho\left(P_T - \mu_M\right), \left(1-\rho^2\right)\sigma_Y^2\right), \tag{A.1.4}$$

where ρ is the correlation coefficient between Y_T and P_T given in Equation (10).[20]

$$e^{Y_T} f(Y_T|P_T) = \frac{1}{\sqrt{2\pi\left(1-\rho^2\right)\sigma_Y^2}} \exp\left(Y_T - \frac{\left(Y_T - \left(\mu_Y + \frac{\sigma_Y}{\sigma_M}\rho\left(P_T - \mu_M\right)\right)\right)^2}{2\left(1-\rho^2\right)\sigma_Y^2}\right) \tag{A.1.5}$$
$$= \exp\left(\frac{\left(1-\rho^2\right)\sigma_Y^2}{2} + \mu_Y - h\mu_M\right) \exp\left(hP_T\right) f(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T). \tag{A.1.6}$$

[20]Please see Greene (2003, pp. 868).

where $h = \frac{\sigma_Y}{\sigma_M}\rho$. Thus,

$$PVS_0^A = N_L S_0 e^{-rT} \lambda C_1 \int_L^{+\infty} e^{hP_T} f(P_T) \int_{-\infty}^{+\infty} f(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T) dY_T dP_T \quad \text{(A.1.7)}$$

$$= N_L S_0 e^{-rT} \lambda C_1 e^{\left(h\mu_M + \frac{1}{2}h^2\sigma_M^2\right)} \int_L^{+\infty} f(P_T - h\sigma_M^2 T) dP_T, \quad \text{(A.1.8)}$$

because $\int_{-\infty}^{+\infty} PDF = CDF(+\infty) = 1$. Plugging in $\mu_Y = rT - \frac{1}{2}\sigma_Y^2$, we have $C_1 e^{\left(h\mu_M + \frac{1}{2}h^2\sigma_M^2\right)} = 1$.

$$PVS_0^A = N_L S_0 e^{-rT} \int_L^{+\infty} f(P_T - h\sigma_M^2 T) dP_T \quad \text{(A.1.9)}$$

$$= N_L S_0 \frac{1}{\sqrt{2\pi}\sigma_M} \int_L^{+\infty} \exp\left[-\frac{\left(P_T - h\sigma_M^2 T - \mu_M\right)^2}{2\sigma_M^2}\right] dP_T. \quad \text{(A.1.10)}$$

Let $H_T = \frac{P_T - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}$. Then $dH_T = \frac{dP_T}{\sigma_{P_T}}$, and $dP_T = \sigma_{P_T} dH_T$.

$$PVS_0^A = N_L S_0 \frac{1}{\sqrt{2\pi}} \int_{\frac{L - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}}^{+\infty} \exp\left[-\frac{1}{2}H_T^2\right] dH_T. \quad \text{(A.1.11)}$$

Using $\int_c^{\infty} n(x)dx = N(-c)$ and $n(c) = n(-c)$, when $n(x)$ and $N(x)$ are the PDF and CDF of a standard normal variable x, we can rewrite the equation as

$$PVS_0^A = N_L S_0 N\left(\frac{\mu_{P_T} - L + h\sigma_{P_T}^2}{\sigma_{P_T}}\right), \quad \text{(A.1.12)}$$

where $d_1 = \frac{M_0 + \left(\alpha - \nu\sigma_M + h\sigma_M^2\right)T - L}{\sigma_M\sqrt{T}}$ after plugging in $\mu_{P_T} = M_0 + (\alpha - \nu\sigma_M)T$ and $\sigma_{P_T} = \sigma_M\sqrt{T}$. We finally have equation (12)

$$PVS_0^A = N_L S_0 N\left(d_1\right). \quad \text{(A.1.13)}$$

The value of an linear performance share plan with strike level L and slope of payoff λ_L

can be split into two terms

$$LPS_0^A(L,\lambda_L) = S_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} P_T e^{Y_T} f(\cdot) dY_T dP_T - LS_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(\cdot) dY_T dP_T. \quad \text{(A.1.14)}$$

Denote the PDF of conditional distribution of Y_T given P_T as $f(Y_T|P_T)$ and the PDF of P_T as $f(P_T)$. Then the value of an linear performance share plan is given by

$$LPS_0^A(L,\lambda_L) = S_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} P_T e^{Y_T} f(P_T) f(Y_T|P_T) dY_T dP_T - LS_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(P_T) f(Y_T|P_T) dY_T dP_T. \quad \text{(A.1.15)}$$

Conditional distribution of Y given M is

$$Y_T|P_T \sim \mathcal{N}\left(\mu_Y + \frac{\sigma_Y}{\sigma_M}\rho\left(P_T - \mu_M\right), \left(1-\rho^2\right)\sigma_Y^2\right), \quad \text{(A.1.16)}$$

where ρ is the correlation coefficient between Y_T and P_T given in Equation (10).[21]

$$e^{Y_T} f(Y_T|P_T) = \frac{1}{\sqrt{2\pi\left(1-\rho^2\right)\sigma_Y^2}} \exp\left(Y_T - \frac{\left(Y_T - \left(\mu_Y + \frac{\sigma_Y}{\sigma_M}\rho\left(P_T-\mu_M\right)\right)\right)^2}{2\left(1-\rho^2\right)\sigma_Y^2}\right) \quad \text{(A.1.17)}$$

$$= \exp\left(\frac{\left(1-\rho^2\right)\sigma_Y^2}{2} + \mu_Y - h\mu_M\right)\exp\left(hP_T\right) f(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T). \quad \text{(A.1.18)}$$

[21]Please see Greene (2003, pp. 868).

where $h = \frac{\sigma_Y}{\sigma_M}\rho$. Thus,

$$
\begin{aligned}
LPS_0^A(L,\lambda_L) = {} & S_0 e^{-rT}\lambda_L C_1 \int_L^{+\infty} P_T e^{hP_T} f(P_T) \int_{-\infty}^{+\infty} f(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T) dY_T dP_T \\
& - L S_0 e^{-rT}\lambda_L C_1 \int_L^{+\infty} e^{hP_T} f(P_T) \int_{-\infty}^{+\infty} f(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T) dY_T dP_T
\end{aligned} \tag{A.1.19}
$$

$$
\begin{aligned}
& = S_0 e^{-rT}\lambda_L C_1 e^{\left(h\mu_M + \frac{1}{2}h^2\sigma_M^2\right)} \int_L^{+\infty} P_T f(P_T - h\sigma_M^2 T) dP_T \\
& - L S_0 e^{-rT}\lambda_L C_1 e^{\left(h\mu_M + \frac{1}{2}h^2\sigma_M^2\right)} \int_L^{+\infty} f(P_T - h\sigma_M^2 T) dP_T,
\end{aligned} \tag{A.1.20}
$$

because $\int_{-\infty}^{+\infty} PDF = CDF(+\infty) = 1$.[22]

Plugging in $\mu_Y = rT - \frac{1}{2}\sigma_Y^2$, we have $C_1 e^{\left(h\mu_M + \frac{1}{2}h^2\sigma_M^2\right)} = 1$. Therefore

$$
\begin{aligned}
LPS_0^A(L,\lambda_L) = {} & S_0 e^{-rT}\lambda_L \int_L^{+\infty} P_T f(P_T - h\sigma_M^2 T) dP_T \\
& - L S_0 e^{-rT}\lambda_L \int_L^{+\infty} f(P_T - h\sigma_M^2 T) dP_T
\end{aligned} \tag{A.1.21}
$$

$$
\begin{aligned}
& = S_0\lambda_L \frac{1}{\sqrt{2\pi}\sigma_M} \int_L^{+\infty} P_T \exp\left[-\frac{\left(P_T - h\sigma_M^2 T - \mu_M\right)^2}{2\sigma_M^2}\right] dP_T \\
& - L S_0\lambda_L \frac{1}{\sqrt{2\pi}\sigma_M} \int_L^{+\infty} \exp\left[-\frac{\left(P_T - h\sigma_M^2 T - \mu_M\right)^2}{2\sigma_M^2}\right] dP_T.
\end{aligned} \tag{A.1.22}
$$

[22] We followed the same steps from Equations (A.1.17) to (A.1.18) to derive (A.1.20).

Let $H_T = \frac{P_T - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}$. Then $dH_T = \frac{dP_T}{\sigma_{P_T}}$, and $dP_T = \sigma_{P_T} dH_T$.

$$\begin{aligned} LPS_0^A(L, \lambda_L) &= S_0 \lambda_L \frac{\sigma_{P_T}}{\sqrt{2\pi}} \int_{\frac{L - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}}^{+\infty} H_T \exp\left[-\frac{1}{2} H_T^2\right] dH_T \\ &+ S_0 \lambda_L \left(\mu_{P_T} + h\sigma_{P_T}^2 - L\right) N\left(\frac{\mu_{P_T} - L + h\sigma_{P_T}^2}{\sigma_{P_T}}\right) \quad \text{(A.1.23)} \\ &= S_0 \lambda_L \frac{\sigma_{P_T}}{\sqrt{2\pi}} \left[-\exp\left[-\infty\right] + \exp\left[-\frac{\left(\mu_{P_T} - L + h\sigma_{P_T}^2\right)^2}{2\sigma_{P_T}^2}\right]\right] \\ &+ S_0 \lambda_L \left(\mu_{P_T} + h\sigma_{P_T}^2 - L\right) N\left(\frac{\mu_{P_T} - L + h\sigma_{P_T}^2}{\sigma_{P_T}}\right) \quad \text{(A.1.24)} \\ &= S_0 \lambda_L \left[\left(\mu_{P_T} + h\sigma_{P_T}^2 - L\right) N\left(d_1\right) + \sigma_{P_T} n\left(d_1\right)\right], \quad \text{(A.1.25)} \end{aligned}$$

where $d_1 = \frac{M_0 + \left(\alpha - \nu\sigma_M + h\sigma_M^2\right)T - L}{\sigma_M \sqrt{T}}$ after plugging in $\mu_{P_T} = M_0 + (\alpha - \nu\sigma_M)T$ and $\sigma_{P_T} = \sigma_M \sqrt{T}$. We finally have equation (13)

$$LPS_0^A(L, \lambda_L) = S_0 \lambda_L \left[\left(M_0 + \left(\alpha - \nu\sigma_M + h\sigma_M^2\right)T - L\right) N\left(d_1\right) + \sigma_M \sqrt{T} n\left(d_1\right)\right]. \quad \text{(A.1.26)}$$

By combination $\{L, H\}$ and $\{\lambda_L, \lambda_H\}$, we can obtain the values of other linear performance share components.

A.2 Proof of Proposition 2

Table 1 shows that the five components have the same payoff at maturity as a performance share plan. Therefore, as shown in equation (17), the date 0 value of a performance share plan must be equal to the date 0 value of the five components in the absence of arbitrage.

When the performance measure follows geometric motion, above derivation of performance-vested share plan can be slightly modified.

$$PVS_0^G = N_L S_0 e^{-rT} \lambda \int_L^{+\infty} \int_{-\infty}^{+\infty} e^{Y_T} f(P_T) f(Y_T | P_T) dY_T dP_T. \quad \text{(A.2.1)}$$

Introduce a change of variables for the terminal value $\hat{M}_T$. Specifically, define $\hat{X}_T = ln\left(\frac{\hat{M}_T}{M_0}\right)$. Then $\hat{X}_T$ is normally distributed as follows

$$\hat{X}_T \sim \mathcal{N}\left(\left(\alpha_M - \nu\sigma_M - \frac{1}{2}\sigma_M^2\right)T, \sigma_M^2 T\right). \tag{A.2.2}$$

Denote the PDF of conditional distribution of Y_T given X_T as $f(Y_T|X_T)$ and the PDF of X_T as $f(X_T)$. Then the value of an linear performance share plan is given by

$$PVS_0^G = N_L S_0 e^{-rT}\lambda \int_{\ln\frac{L}{M_0}}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(X_T) f(Y_T|X_T) dY_T dX_T.. \tag{A.2.3}$$

Following same steps from Equations (A.1.5)-(A.1.10), we obtain equation (18)

$$PVS_0^G = LS_0 N\left(d_2^G\right), \tag{A.2.4}$$

where $d_2^G = \frac{\ln\frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h - \frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$.

Introduce a change of variables for the terminal value $\hat{M}_T$. Specifically, define $\hat{X}_T = ln\left(\frac{\hat{M}_T}{M_0}\right)$. Then $\hat{X}_T$ is normally distributed as follows

$$\hat{X}_T \sim \mathcal{N}\left(\left(\alpha_M - \nu\sigma_M - \frac{1}{2}\sigma_M^2\right)T, \sigma_M^2 T\right). \tag{A.2.5}$$

Denote the PDF of conditional distribution of Y_T given X_T as $f(Y_T|X_T)$ and the PDF of X_T as $f(X_T)$. Then the value of an linear performance share plan with strike level L and slope of payoff λ_L is given by

$$\begin{aligned} LPS_0^G(L, \lambda_L) = {} & S_0 M_0 e^{-rT}\lambda_L \int_{\ln\frac{L}{M_0}}^{+\infty}\int_{-\infty}^{+\infty} e^{X_T} e^{Y_T} f(X_T) f(Y_T|X_T) dY_T dX_T \\ & - LS_0 e^{-rT}\lambda_L \int_{\ln\frac{L}{M_0}}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(X_T) f(Y_T|X_T) dY_T dX_T. \end{aligned}$$

We know that $\mu_X = \left(\alpha_M - \nu\sigma_M - \frac{1}{2}\sigma_M^2\right)T$, so $e^{\left(h+\frac{1}{2}\right)\sigma_M^2 T + \mu_X} = e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}$, and by standardizing $X_T + (h+1)\,\sigma_X^2$, it becomes

$$LPS_0^G(L,\lambda_L) = S_0 M_0 \lambda_L \int_{\ln\frac{L}{M_0}}^{+\infty} e^{X_T} f(X_T - h\sigma_X^2) dX_T \tag{A.2.6}$$

$$- LS_0\lambda_L \int_{\ln\frac{L}{M_0}}^{+\infty} f(X_T - h\sigma_X^2) dX_T \tag{A.2.7}$$

$$= S_0 M_0 e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T} \lambda_L N\left(d_1^G\right) \tag{A.2.8}$$

$$- LS_0\lambda_L N\left(d_2^G\right), \tag{A.2.9}$$

where $d_1^G = \frac{\ln\frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$ and $d_2^G = \frac{\ln\frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h-\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}} = d_1^G - \sigma_M\sqrt{T}$.

Thus, the value of an linear performance share plan is:

$$LPS_0^G(L,\lambda_L) = S_0\lambda_L\left[M_0 e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T} N\left(d_1^G\right) - LN\left(d_2^G\right)\right], \tag{A.2.10}$$

where $d_1^G = \frac{\ln\frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$ and $d_2^G = \frac{\ln\frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h-\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$. This is the equation (19). **Q.E.D.**

B Numerical solution technique

We devise a method to approximate the optimal managerial effort over a reasonable range of contract parameters and then steadily improve the accuracy of the approximation to any arbitrary degree of accuracy.

Here are the steps:

1. Select an upper bound (subscript U) and a lower bound (subscript L) for each of the three contract parameters. Figure 7 shows how these bounds specify a three-dimensional space in the shape of a cube, where the triplet of parameters (L, λ, H) are

all within the corresponding bounds $L \in [L_L, L_U]$, $\lambda \in [\lambda_L, \lambda_U]$, and $H \in [H_L, H_U]$.

2. Solve the manager's problem using a standard hill-climbing technique for constrained optimization to obtain the optimal managerial effort for the eight corners of the cube. Figure 7 shows the optimal managerial effort a^* as a function of the triplet of contract parameters at each corner: $a^*(L_L, \lambda_L, H_L)$, $a^*(L_U, \lambda_L, H_L)$, ..., $a^*(L_U, \lambda_U, H_U)$.

3. In the outside shareholder's problem, approximate the optimal managerial effort for any point in the cube space using a weighted-average of the eight corners as given by $a^*(L, \lambda, H) = \sum_{i=L,U} \sum_{j=L,U} \sum_{k=L,U} w_i^L w_j^\lambda w_k^H a^*(L_i, \lambda_j, H_k)$, where the weights in the λ dimension are $w_L^\lambda = \frac{\lambda_U - \lambda}{\lambda_U - \lambda_L}$, and $w_U^\lambda = 1 - w_L^\lambda$ and the weights in the L and H dimensions are analogous. For any point in the cube space, this function *interpolates* an approximate optimal effort from the precise optimal effort values of the eight corners.

4. Solve the outside shareholders' problem, subject to the constraints in step 1, using a standard hill-climbing technique for constrained optimization to obtain a provisional optimal contract triplet (L^*, λ^*, H^*).

5. Branch depend the following conditions:

   (a) If a provisional optimal contract parameter is on an upper (lower) bound, then raise (lower) that upper (lower) bound and repeat from step 2.

   (b) If all three provisional optimal contract parameters are on the interior of their respective ranges, then shrink both the upper and lower bounds towards the interior point (usually about halfway on each side) on one or more dimensions and repeat from step 2.

   (c) When the upper and lower bounds have become sufficiently close to the interior point on all three dimensions such that the optimal effort values for the eight corners are identical to an arbitrary number of digits (i.e., the approximation in step 3 achieves an arbitrary degree of accuracy), then stop.

The beauty of this approach is that the solution to the outside shareholders' problem is on the interior of the cube, so the upper and lower bound constraints are *not* binding, and the cube becomes arbitrarily small, so the solution to the manager's problem can achieve any arbitrary degree of accuracy. In other words, both problems are solved at the same time without any binding constraints and to any degree of accuracy.

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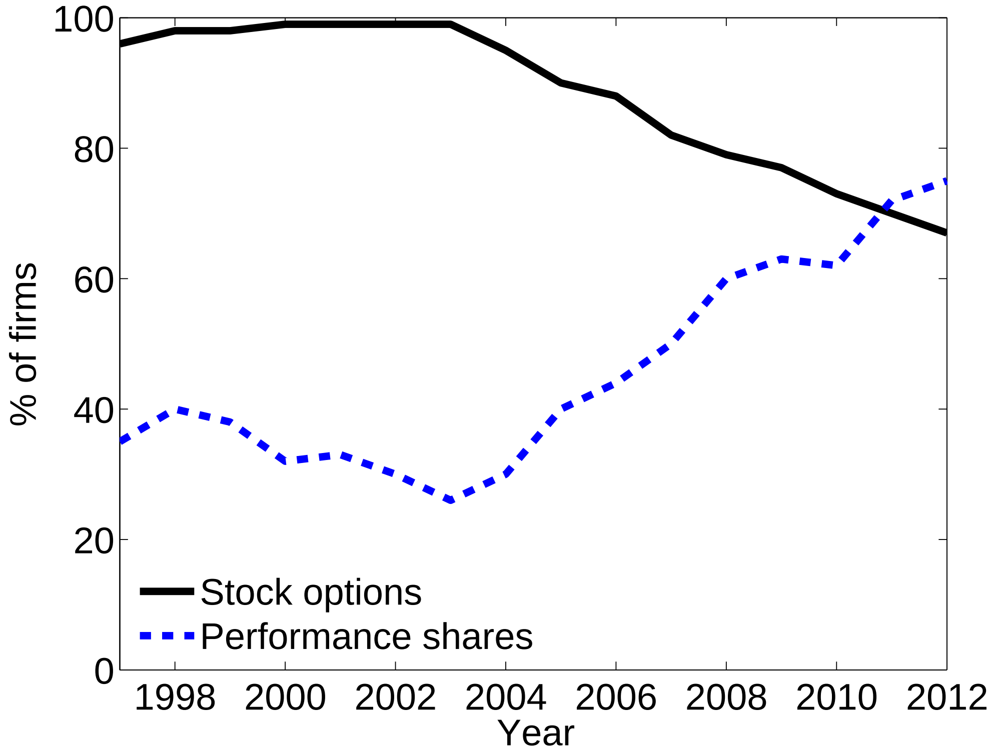
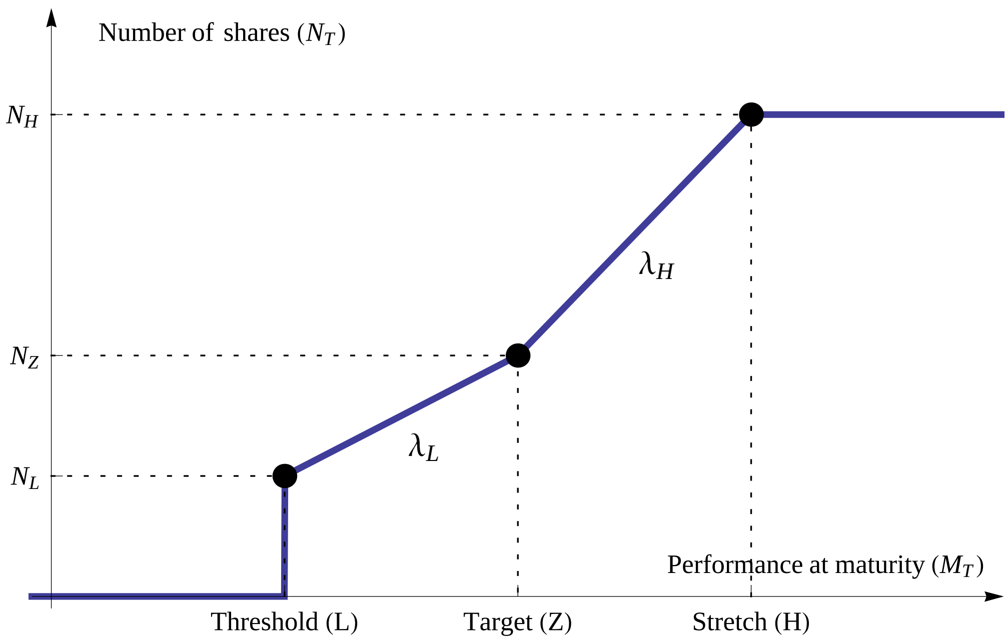


Figure 1: Firms granting performance share plans

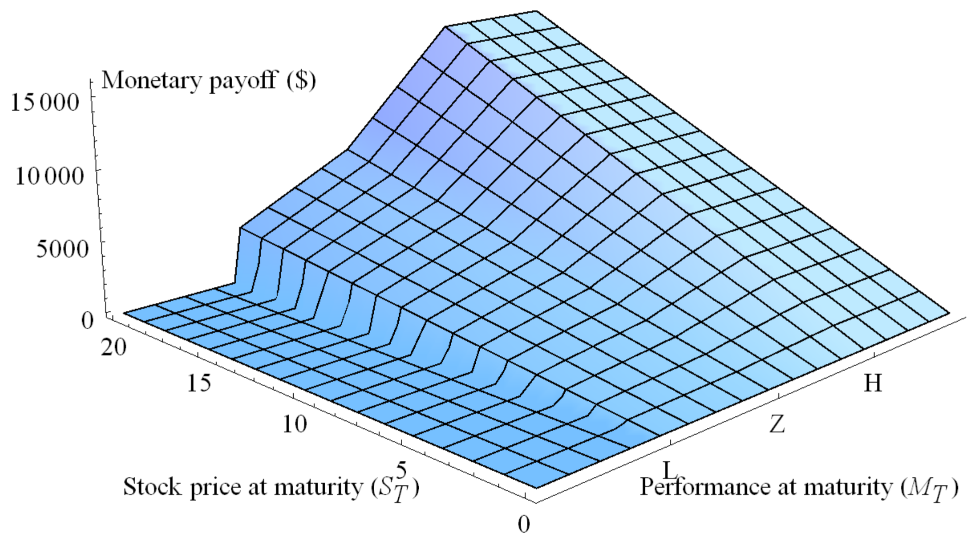
This figure shows the percentage of firms that grant performance share plans to their executives. The sample period is from 1997 to 2010 and the top 250 Forbes firms are included. The data is collected from annual *The Top 250 Survey* by C. King of Frederick W. Cook.

Figure 2: 2009 performance share plan's share reward to Coca Cola CEO.

This figure shows the 2009 performance share plan's share reward to the Coca Cola CEO as a function of 2009 Coca Cola earnings per share.



(a) Share reward function for a performance share plan



(b) Monetary payoff of a performance share plan

Figure 3: Payoff at maturity of a performance share plan

These figures show the payoff at maturity of a performance share plan. Figure 3(a) shows the number of shares awarded under a performance share plan by performance at maturity. Figure 3(b) shows the monetary payoff of a performance share by performance at maturity and by stock price at maturity.

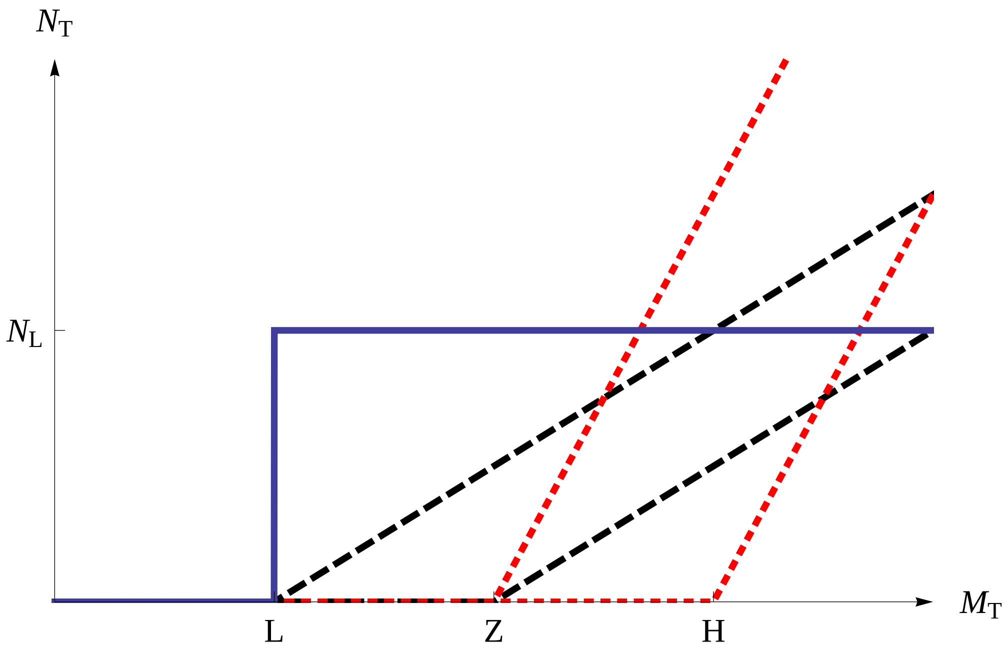
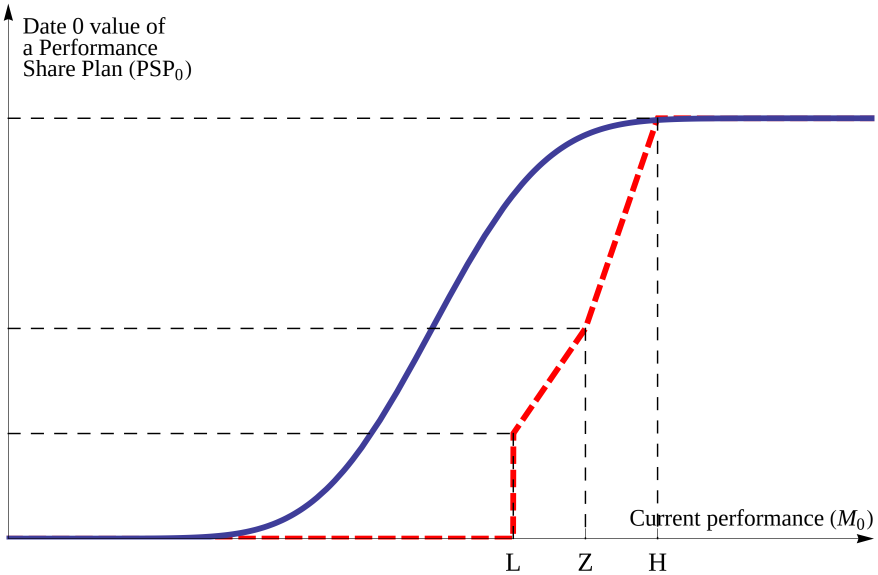
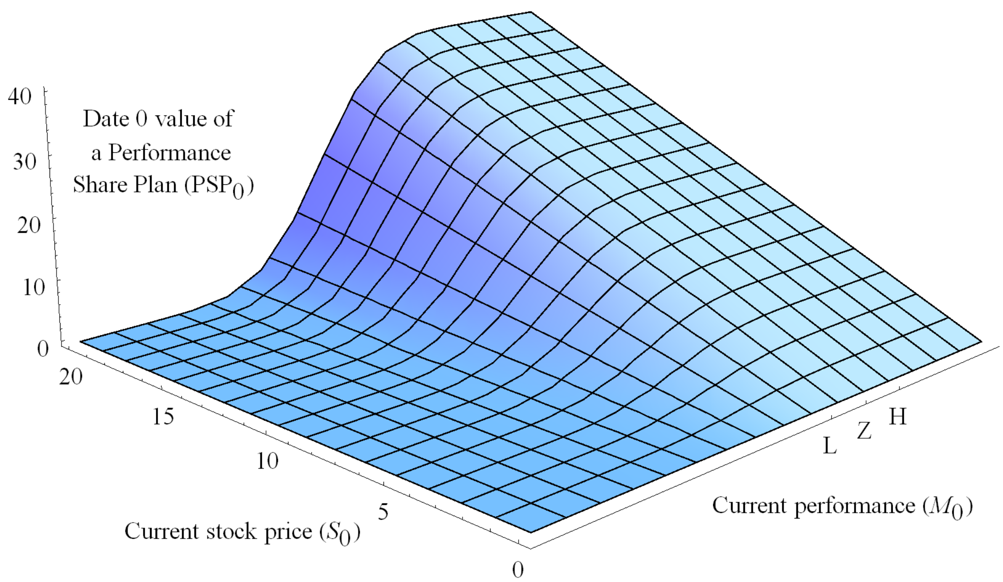


Figure 4: Payoff of five components of performance share plan

This figure shows the decomposed share reward function for a performance share plan with a jump at the threshold performance goal (L) and convex kink at the performance target (M). The blue real line with jump at L is the payoff of a performance-vested share plan with strike level at L with the payoff of N_L shares when the performance is at or above L. The two black dashed lines illustrate the share reward function of linear performance share plans with slope of λ_1 and the strike levels at L and M, respectively. The two red dotted lines depicts the share reward function of linear performance share plans with slope of λ_2 and the strike levels at M and H, respectively.



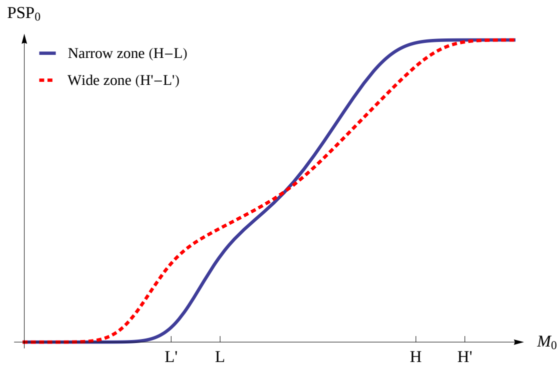
(a) Date 0 value vs. intrinsic value of a performance share plan by current performance measure



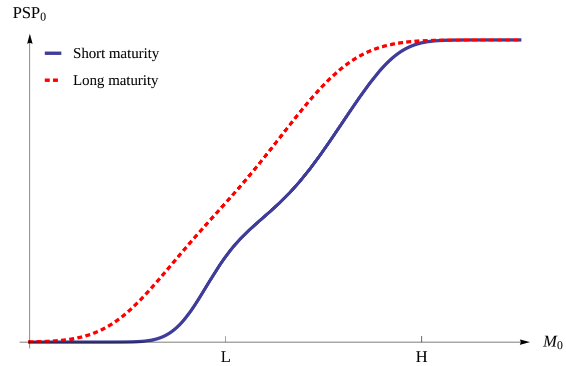
(b) Date 0 value of a performance share plan by current performance measure and current stock price

Figure 5: Value of a performance share plan

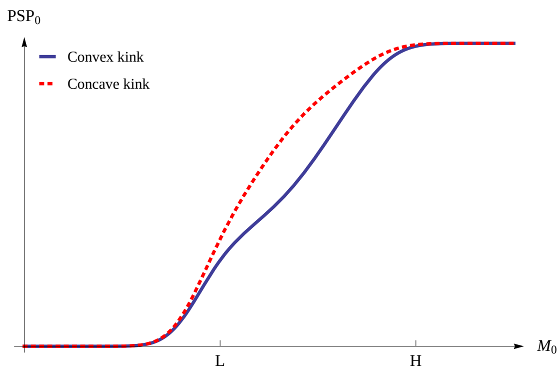
These figures show the date 0 value of a performance share. The bold line in Figure 5(a) shows how the date 0 value of a performance share changes with the current level of performance. The dotted line represents the intrinsic value of the performance share plan and the vertical gap between the date 0 value of a performance share plan and the intrinsic value represents the time value of the performance share plan. Figure 5(b) shows how the value changes with the current performance measure and the current stock price.



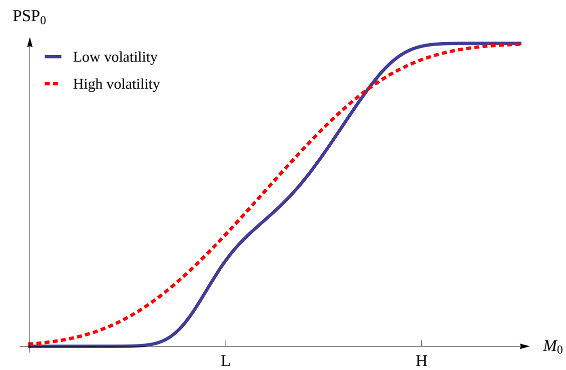
(a) Width of incentive zone $(H - L)$



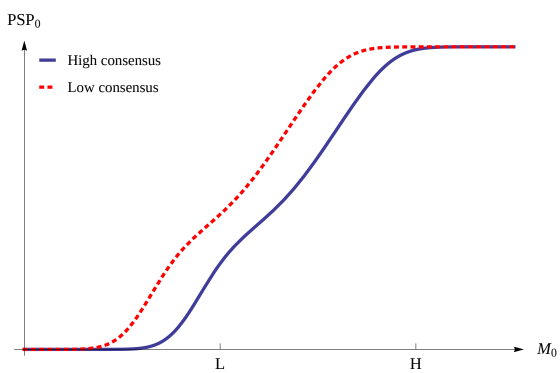
(b) Performance period (T)



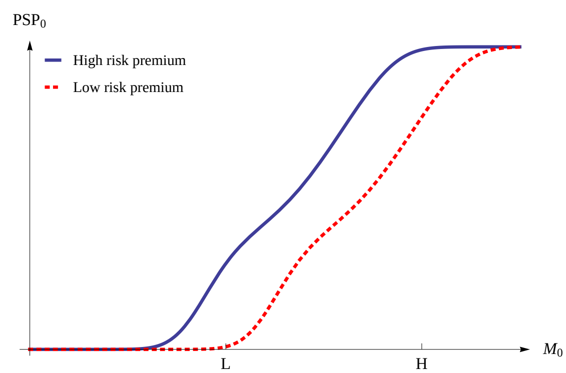
(c) Convex vs. concave kink



(d) Volatility of performance measure (σ_M)



(e) Consensus performance (α)



(f) Risk premium (ν)

Figure 6: Factors affecting the value of a performance share plan

These figures show how different factors affect the value of a performance share plan. The figures show the value for current level of performance measure (M_0). Figures 6(a), 6(b), and 6(c) show the value of a performance share plan (PSP_0) by the contractual terms: the width of incentive zone $(H - L)$, performance period (T), and whether the payoff structure has convex kink or concave kink. Figures 6(d), 6(e), and 6(f) show the value of a performance share plan (PSP_0) by various environmental factors: the volatility of performance measure (σ_M), the consensus estimate on performance (α), and the risk premium on non-traded performance measures (ν).

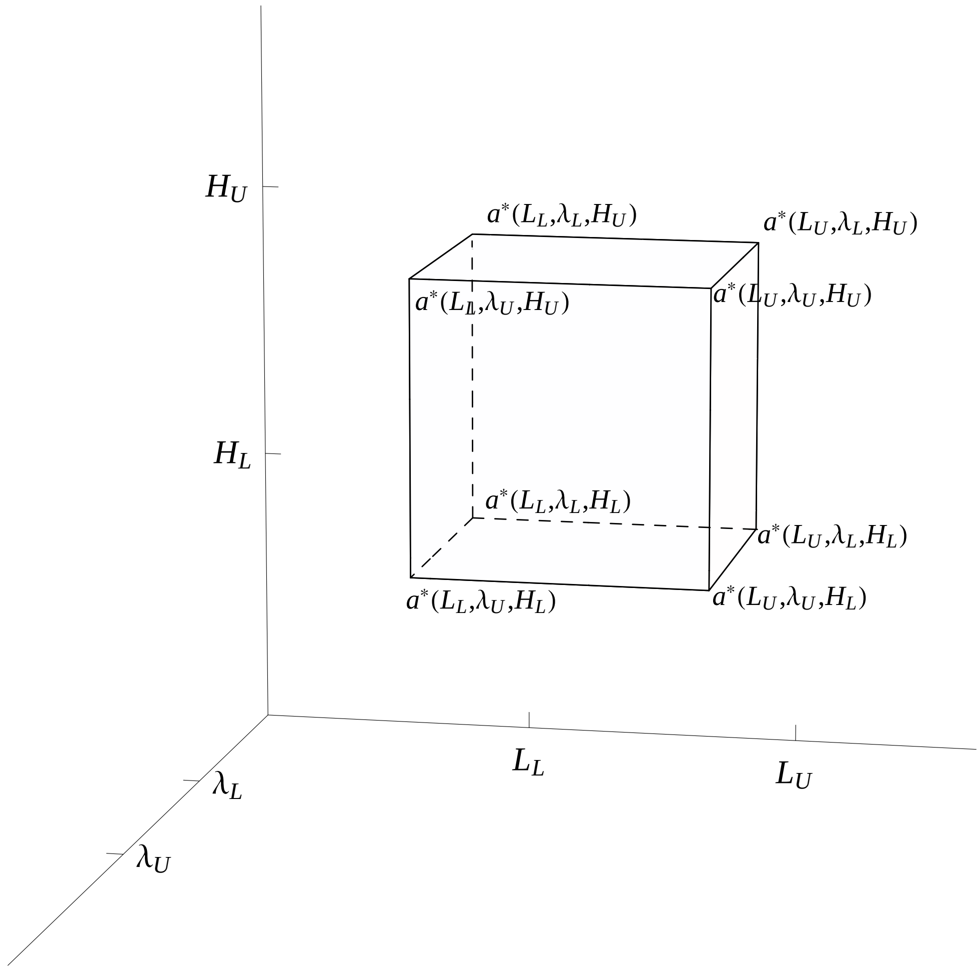


Figure 7: Upper and lower bounds specify a three-dimensional cube space

This figure shows shows how upper and lower bounds for each of the three contract parameters specify a three-dimensional space in the shape of a cube. In other words, at every point in the cube the triplet of contract parameters (L, λ, H) are within the corresponding bounds $L \in [L_L, L_U]$, $\lambda \in [\lambda_L, \lambda_U]$, and $H \in [H_L, H_U]$. We find the optimal managerial effort a^* as a function of the triplet of contract parameters at each of the eight corners.

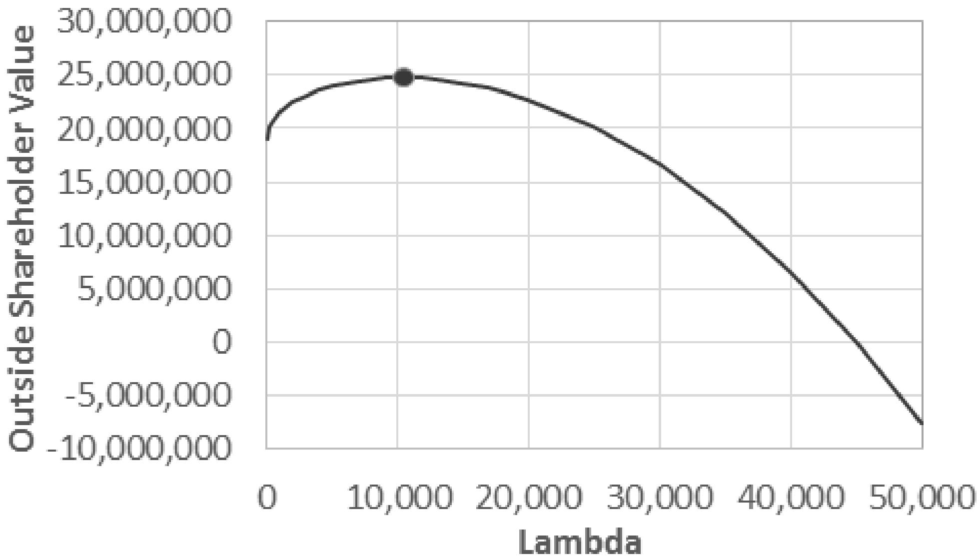
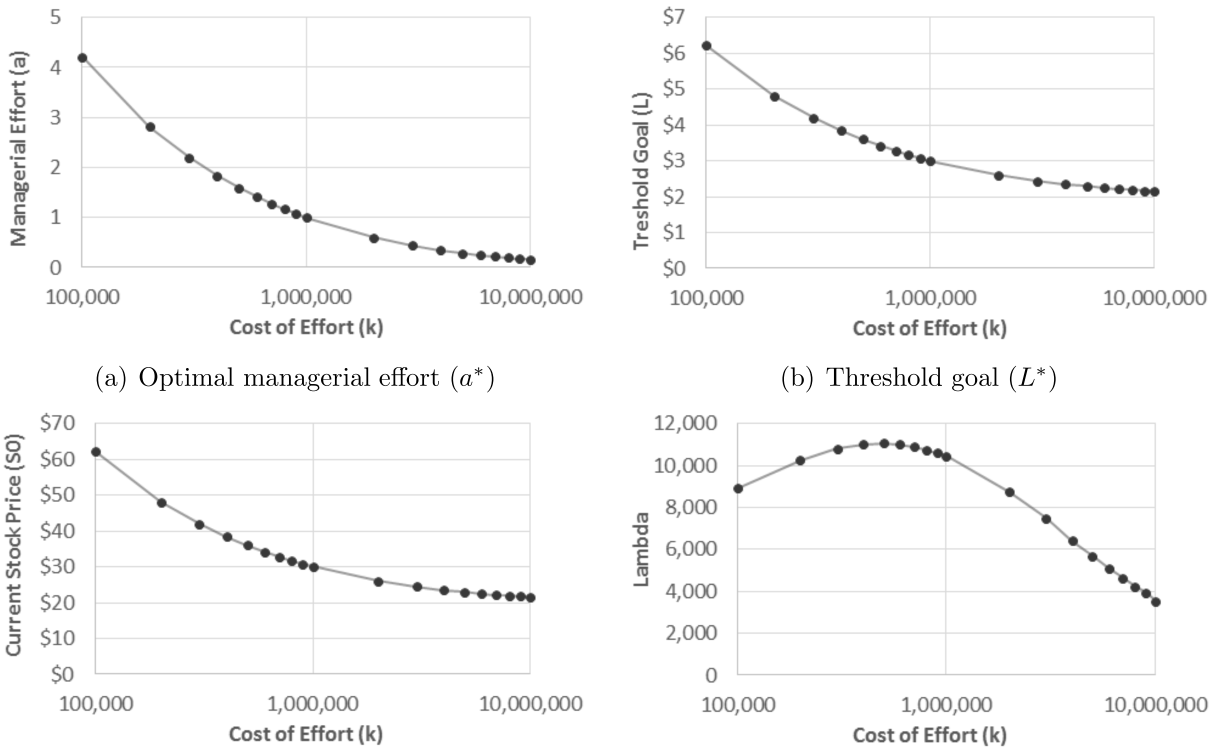


Figure 8: Outside shareholder value by lambda

This figure shows outside shareholder value (the value of the firm net of compensation to the CEO) by the slope of the performance share plan (λ), where the optimal managerial effort (a^*) varies as λ varies. The other two design parameters (L and H) are kept at fixed values.



(a) Optimal managerial effort (a^*)

(b) Threshold goal (L^*)

(c) Stock price (S_0)

(d) Slope of payoff structure (λ^*)

Figure 9: Optimal design parameters by the cost of effort

These figures show optimal design parameters by the cost of effort. Figures 9(a)-9(d) show the optimal managerial effort (a^*), optimal threshold goal (L^*), current stock price (S_0), and the optimal slope (λ^*) by the cost of effort.

Table 2: CEO Compensation Components and Performance Measures

This table shows components of CEO compensation and performance measures used in performance measures plans for CEOs at S&P 500 firms. The data comes from proxy statements. The sample spans fiscal years ending from December 15^{th}, 2006 to November 30^{th}, 2012. *Performance Share Plans*, *Performance-Vested Share Plans*, and *Performance Options* are equity-based compensation plans in which the number of shares or options awarded is tied to the performance of pre-specified measures. *Performance Cash* includes cash-based annual and long-term incentive pay.

Components	Mean	Median	Number
Panel A. Mean and median target amount conditional on that component being granted ($ MM)			
Salary	1.10	1.00	2621
Restricted Stocks	2.85	1.92	1260
Incentive Pay			
Performance Share Plans	3.67	2.68	1243
Performance Cash	2.26	1.56	2263
Performance Options	2.86	2.10	40
Performance-Vested Share Plans	3.30	2.50	292
Stock Options	3.43	2.43	1848
Panel B. Unconditional breakdown by compensation component - 2,629 observations			
Salary	18.5	13.9	
Restricted Stocks	13.9	0.0	
Incentive Pay			
Performance Share Plans	17.0	0.0	
Performance Cash	22.7	19.0	
Performance Options	0.4	0.0	
Performance-Vested Share Plans	3.9	0.0	
Stock Options	23.6	23.1	

Panel C. Performance measures used in performance share plans

Performance measures	% of total	# of obs.
Stock Return	21.6	658
Earnings Per share (EPS)	17.5	535
Revenue	9.8	299
Return on Invested Capital	5.9	181
Return on Equity	4.8	147
Other Performance Measures	40.3	1230
Total	100.0	3050

Table 3: Design parameters

This table reports the design parameters of performance share plans and performance-vested share plans with earnings per share (EPS) or revenue as a performance measure. The data is hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15^{th}, 2006 to November 30^{th}, 2012. This results in six parameters for 255 firm-years of performance share plans and two parameters for 39 firm-years of performance-vested share plans. Threshold Goal L, Target Goal M, and Stretch Goal H are divided by the previous year's performance level P_0.

	Threshold Goal (L/P_0)	Target Goal (M/P_0)	Stretch Goal (H/P_0)	Threshold Shares	Target Shares	Stretch Shares
Panel A. Performance Share Plans using Earnings Per Share (181 observations)						
Mean	0.987	1.062	1.126	23,875	89,444	140,233
Median	1.020	1.090	1.138	9,701	62,548	80,388
Panel B. Performance Share Plans using Revenue (74 observations)						
Mean	1.031	1.080	1.126	15,057	65,931	85,670
Median	1.026	1.063	1.100	5,744	41,705	36,270
Panel C. Performance-Vested Share Plans using Earnings Per Share (29 observations)						
Mean	0.885			109,704		
Median	1.080			89,969		
Panel D. Performance-Vested Share Plans using Revenue (10 observations)						
Mean	1.273			160,704		
Median	1.060			50,000		

Table 4: Formula Value vs. Reported Value vs. Heuristic Value

This table compares the formula value based on Propositions 1 and 2 vs. the reported value on proxy statements vs. the heuristic value for a set of performance share plans and performance-vested share plans with earnings per share (EPS) or revenue as the performance measure. The input values are hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15^{th}, 2006 to November 30^{th}, 2012. Formula values, reported values, and heuristic values in millions of dollars are reported for 255 firm-years of performance share plans and 39 firm-years of performance-vested share plans. * means statistically significant at the 5% level based on the t-test for the difference in means and the Wilcoxon test for the difference in medians.

	Formula Value	Reported Value	Heuristic Value	% Difference (Formula -Reported)	% Difference (Formula -Heuristic)	% Difference (Reported -Heuristic)
Panel A. Performance Share Plans using EPS (181 observations)						
Mean	3.029	3.284	3.139	-7.8%	-3.5%	4.6%
Median	2.014	2.350	2.356	-14.3%*	-14.5%*	-0.3%
Panel B. Performance Share Plans using Revenue (74 observations)						
Mean	2.198	2.346	2.337	-6.3%	-6.0%	0.4%
Median	0.914	1.518	1.591	-39.7%*	-42.5%*	-4.6%*
Panel C. Performance-Vested Share Plans using EPS (29 observations)						
Mean	1.860	3.055	3.069	-39.1%*	-39.4%*	-0.5%
Median	1.336	3.109	3.111	-57.0%*	-57.0%*	0.0%
Panel D. Performance-Vested Share Plans using Revenue (10 observations)						
Mean	1.435	2.866	2.927	-49.9%*	-51.0%*	-2.1%
Median	1.606	3.093	3.119	-48.1%*	-48.5%*	-0.9%
Panel E. Full Sample (294 observations)						
Mean	2.650	3.011	2.923	-12.0%*	-9.3%	3.0%
Median	1.580	2.227	2.222	-29.0%*	-28.9%*	0.2%

Table 5: Magnitude and Accuracy Compared to Perfect Foresight Value

This table reports the magnitude and accuracy compared to perfect foresight value of formula value based on Propositions 1 and 2, reported value on proxy statements, and heuristic value for a set of performance share plans and performance-vested share plans with earnings per share (EPS) or revenue as the performance measure. The input values are hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15^{th}, 2006 to November 30^{th}, 2012. Perfect foresight values, formula values, reported values, and heuristic values in millions of dollars are reported for 214 firm-years of performance share plans and 34 firm-years of performance-vested share plans. * means statistically significant at the 5% level based on the t-test for the difference in means and the Wilcoxon test for the difference in medians.

					Magnitude			Accuracy		
	Perfect Foresight Value	Formula Value	Reported Value	Heuristic Value	% Dif (Formula -Perfect	% Dif (Reported -Perfect	% Dif (Heuristic -Perfect	Abs % Dif (Formula -Perfect	Abs % Dif (Reported -Perfect	Abs % Dif (Heuristic -Perfect
Panel A. Performance Share Plans using Earnings Per Share (148 observations)										
Mean	3.134	3.139	3.369	3.195	0.2%	7.5%	2.0%	63.6%*	58.8%*	62.9%*
Median	1.842	2.160	2.455	2.385	17.2%*	33.3%*	29.5%*	51.3%*	52.5%*	67.9%*
Panel B. Performance Share Plans using Revenue (66 observations)										
Mean	1.750	2.306	2.395	2.411	31.7%*	36.8%*	37.8%*	47.5%*	67.3%*	65.8%*
Median	0.535	0.845	1.461	1.524	57.8%*	173.0%*	184.7%*	37.3%*	62.5%*	64.3%*
Panel C. Performance-Vested Share Plans using Earnings Per Share (26 observations)										
Mean	2.188	1.595	2.801	2.806	-27.1%*	28.0%*	28.3%*	42.5%*	19.8%*	19.2%*
Median	2.095	1.295	2.979	2.980	-38.2%*	42.2%*	42.2%*	40.4%*	0.6%	0.0%
Panel D. Performance-Vested Share Plans using Revenue (8 observations)										
Mean	1.406	1.129	2.844	2.920	-19.8%	102.2%*	107.6%*	16.2%*	52.8%*	50.0%*
Median	0.800	0.875	3.322	3.331	9.4%	315.2%*	316.3%*	7.2%	58.3%*	50.0%*
Panel E. Full Sample (248 observations)										
Mean	2.611	2.690	3.033	2.937	3.1%	16.2%*	12.5%*	55.5%*	56.5%*	58.7%*
Median	1.502	1.528	2.152	2.091	1.7%	43.2%*	39.2%*	43.4%*	52.5%*	58.3%*