Cost-Minimizing Intervention in a Market-Based Financial System

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Abstract

Is taxpayer money best spent on supporting the real or the financial sector? When the financial sector's net worth is low, limited funding liquidity in financial markets exacerbates credit rationing and depresses real activity. Public intervention can increase real investment by lending directly to the real sector or to financial institutions. Successful interventions are costly to the taxpayer because the private sector's participation constraints imply that the government overpays for its claims. Lending to financial institutions is cheaper than supporting the real sector directly because it minimizes rents by optimally sharing downside risk between the public and the private sectors.

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1 Introduction

During the recent financial crisis, both the Federal Reserve and the U.S. Treasury actively participated in a number of key financial markets. Nevertheless, an important question is still largely unanswered: why didn't U.S. policy makers spend more to directly support the real sector? In other words, is taxpayer money best spent on supporting the real or the financial sector? This question is relevant not only looking back, but also looking forward, in order to design policies that can stimulate an economy weakened by a financial meltdown. For instance, the Funding for Lending Scheme (FLS) was launched in July 2012 by the Bank of England and HM Treasury in order to, according to Mervyn King, "boost lending to the real economy... by providing funding to banks and building societies for an extended period, at a price below current market rates, so that they can make loans cheaper and more easily available." On the other hand, the French government launched a 40 bilion euro "public bank for investment" ("Banque Publique d'Investissement" or BPI) in January 2013 in order to directly support the real sector by lending funds to French small and medium-sized businesses. Whether it is more effective to support the real or the financial sector remains an open question.

In order to address this question, I present a stylized model in which the financial and the real sectors are closely linked. Entrepreneurs (or firms, or households) finance real projects by borrowing from financial institutions, which fund themselves by borrowing from each other in financial markets. Lending to the real sector carries systemic downside risk: an aggregate shock would lead to a wave of default. Moreover, there is an informational friction in the market for loans to entrepreneurs: project quality is the entrepreneurs' private information. Potential lenders are heterogenous in their ability to cope with downside risk. Natural lenders have limited cash but can borrow from other financial institutions in financial markets. However, there is an enforcement friction in these markets: all loans between financial institutions must be collateralized. I model heterogeneity in ability to cope with downside risk as heterogenous beliefs and use this modeling to solve for equilibrium financial contracts. As in Akerlof (1970), adverse selection implies that entrepreneurs with the best projects are reluctant to borrow. Natural lenders will borrow from others to lever up and buy portfolios of loans on margin. In equilibrium, credit supply to the real economy and financial markets are inextricably linked. When the aggregate net worth of the financial sector is low, limited funding liquidity exacerbates credit rationing due to adverse selection and depresses real activity. In this case, the laissez-faire allocation is inefficient and there is room for public intervention.

I focus on direct interventions in markets, in which the government (or the Central Bank) trades and contracts with private agents, rather than on regulation, taxation or conventional monetary policy. The objective of the government is to stimulate real activity at the minimum cost to taxpayers. It can directly intervene either in the market for real loans or in financial markets. Importantly, participation in a public program is voluntary and the government takes into account participation constraints of the private sector when designing an intervention. In order to support the real sector, the government can finance some of the low-quality projects and therefore reduce the amount of adverse selection faced by the market. As first shown by Philippon and Skreta (2012) and Tirole (2012), this intervention leads the market to "rebound": interest rates fall and more projects are financed. Alternatively, the government can support the financial sector by lending to financial institutions. By relaxing financial constraints, this intervention in turn allows lenders to finance more real projects. However, in both cases, participation constraints of the private sector imply that intervention is costly because the government overpays for its claims. Given that valuations are heterogeneous, this result relies on the government using taxpayer's preferences to evaluate profits. Interestingly, this result holds even for a financial sector intervention without adverse selection and therefore generalizes some insights from Philippon and Skreta (2012) and Tirole (2012).

The main result is that supporting the financial sector is cheaper than supporting the real sector. When the preferences of taxpayers diverge from those of financial institutions (or their shareholders), having the government substitute itself fully for the financial sector is excessively costly. Intuitively, the cost of an intervention depends on how much downside risk the government is exposed to. However, in any intervention that reaches a given volume of intermediation, total downside risk in the economy is fixed. Any downside risk not borne by the government must be borne by private agents. Since participation to a program is voluntary, private agents must have incentives to hold this risk. The cost-minimizing intervention therefore trades off a reduced exposure to downside risk with higher rents paid to the private sector. When taxpayers are more averse to downside risk than the marginal private lender, it is optimal to share as much of this risk as possible with the private sector. However, this risk cannot directly be shared with entrepreneurs in a real sector intervention since they default precisely in a downturn. Lending instead to financial institutions reduces public exposure to downside risk because their capital acts as a buffer. A financial sector intervention therefore allows for an optimal split of the surplus between the public and private sectors across future states of the world in a way that is both attractive to financial institutions and better aligned with taxpayers' interests. Note that public funds follow private funds: absent intervention, the taxpayer would lend to financial institutions, not directly to entrepreneurs.

Strikingly, to show that supporting the real sector is more expensive, it is not necessary to assume that the government is less efficient in making loans or less informed than the financial sector. In other words, the logic above is based on preferences, not technology nor information. The main driving force is the divergence of preferences between taxpayers and financial institutions, independent of the origin of this divergence. While belief disagreement is a convenient way to model this divergence, it could instead be modeled in other ways.

Additionally, I characterize the cost-minimizing intervention for different taxpayer preferences. While supporting the financial sector is always optimal, which financial contracts will be offered depends on these preferences. For instance, if the taxpayer resembles the marginal lender in the market, equity injections in financial institutions are optimal. On the other hand, if the government is cautious and only considers worst-case outcomes, purchasing risk-free debt from financial institutions is optimal. This intervention is in fact equivalent to subsidizing the financial sector.

This paper makes a contribution to the literature that studies public intervention when the government trades directly with private agents in a market environment, stressing the implications of participation constraints. However, in none of these papers does the government have a choice of which sector to support. In Philippon and Skreta (2012), the financial sector is frictionless and the government competes with private lenders to finance firms which own real projects. In Tirole (2012) and Philippon and Schnabl (2013), the government can only support the financial sector by assumption, even though it would prefer to finance real projects directly. Rather than assuming it, I show that supporting the financial sector is the optimal intervention. Bebchuk and Goldstein (2011) also consider different public policies aimed at preventing credit market freezes, including the choice between supporting the real or the financial sector. However, they model policy in a reduced-form way and abstract from the cost of intervention to the taxpayer. Moreover, I emphasize the role of collateral in providing funding liquidity and the endogenous choice of leverage by financial institutions. Reflecting the changing nature of the financial system, many of the most striking policy interventions during the crisis were aimed at "restoring leverage". I am able to model such policies explicitly and show that restoring leverage in the financial sector is costly but is nevertheless the cost-minimizing intervention. This paper also makes a contribution to the literature that studies the interaction of adverse selection and financial constraints. The economy features a two-way feedback between the real and financial sector through the endogenous value of collateral, in a similar spirit than in Bigio (2013).

2 Setup

There are two sectors in the economy: a real and a financial sector. The real sector consists of entrepreneurs (or households, or firms) who each own a project (housing, investment) that needs financing and who can raise funds from the financial sector in a market for real loans. This market is subject to an informational friction: project quality is the entrepreneur's private information, which leads to adverse selection. The financial sector consists of heterogenous financial institutions which

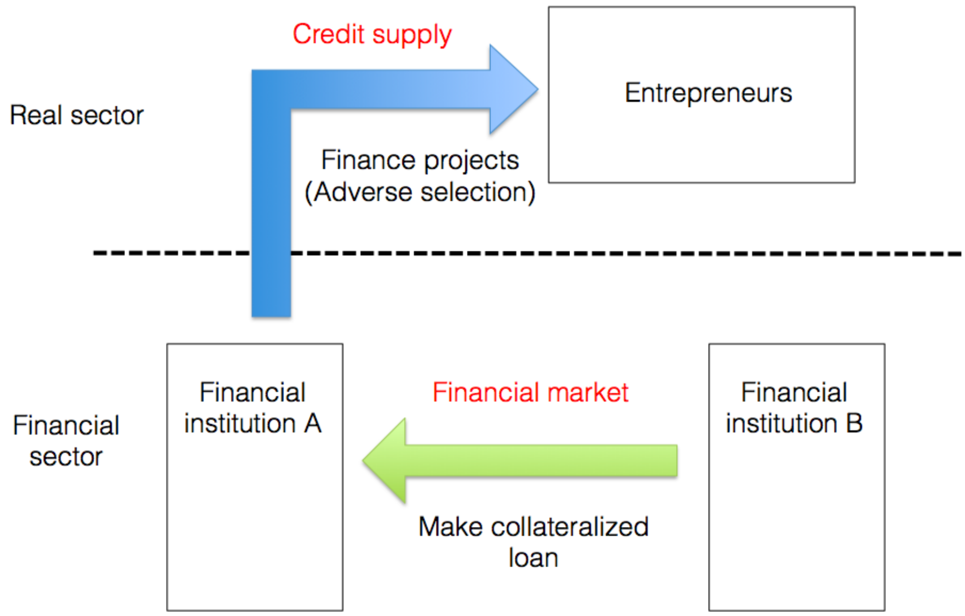

Figure 1: The economy

differ in their valuation of the loans. Natural lenders can borrow from other financial institutions in financial markets. However, these markets are subject to an enforcement friction: loans must be collateralized by claims on the real sector they are used to finance. Figure 1 summarizes the economy.

2.1 Real projects and credit supply

Each entrepreneur has one real project in need of financing and can borrow from financial institutions. For example, this project can represent the purchase of a home or a new corporate investment. It costs $x > 0$ at $t = 0$ and carries a utility benefit $\delta > 0$ that accrues to the borrower but is not pledgeable to lenders. Each entrepreneur has wealth $A < x$. I assume that financing real projects is socially efficient, the net gains from trade being strictly positive: $\delta - x > 0$. As a normalization, financial institutions offer a loan of face value 1. Whether the entrepreneur is able to repay the loan depends on the realization of its future income at $t = 1$. In order to focus solely on the systemic nature of credit supply, I assume that loans are only subject to aggregate risk.

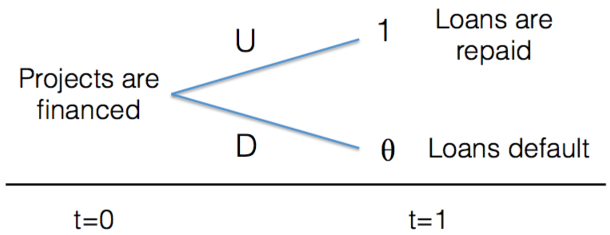

Figure 2: Loans to the real sector

More precisely, whether loans are repaid depends on the realized aggregate state labelled "up" (U) or "down" (D): the borrower has enough income to repay 1 in state U, but has income of only $\theta < 1$ in state D. Figure 2 illustrates the nature of uncertainty.

The value of θ represents how much can be recovered in the low state. This "quality" is the entrepreneur's private information, and θ belongs to [0,1]. Potential lenders cannot observe θ but have a log-concave prior F with full support that captures the distribution of entrepreneurs' types. This quality θ can be thought as affecting the value of one's house after the housing bubble burst or the value of firm's assets in bankruptcy.

Entrepreneurs are risk-neutral and there is no discounting. They can raise funds at a gross interest rate R from financial institutions by pledging their future income. Each entrepreneur can raise $\frac{1}{R}$ at $t = 0$. At this interest rate, the entrepreneur incurs an upfront cost of $x - \frac{1}{R}$ and borrowing to undertake the project yields $\delta - (x - \frac{1}{R})$. Later, I will make assumptions on the primitives that guarantee that in equilibrium $x - \frac{1}{R}$ is less that than initial entrepreneurial wealth. On the other hand, not raising funds gives an entrepreneur with quality θ an outside option of value $u(\theta)$ that depends on its type, with u strictly increasing. An entrepreneur will undertake the project if the loan terms are generous enough. Moreover, since not all types have the same incentives to borrow, there will be adverse selection in this market.

This setup is meant to capture some important features of a typical bank asset. Aggregate risk implies that financial institutions are exposed to macroeconomic shocks to cannot be diversified away across entrepreneurs. Moreover, these assets tend to pay off a given amount after a good shock. However, the amount that can be recovered in the event of a bad shock is uncertain. In other words, there is adverse selection with respect to downside risk. Note also that this market is assumed to follow a particularly simple trading mechanism in the sense that all lenders are constrained to offer a single debt contract for an interest rate R. This assumption is restrictive in the sense that, in the presence of private information, lenders generally find it optimal to offer a menu of contracts in order to screen entrepreneurs, as in Rotschild and Stiglitz (1983), Tirole (2012) or Nachman and Noe (1994). However, a general result in this literature is that allowing for screening can reduce but never totally alleviate the inefficiencies created by adverse selection. In particular, in this economy downside risk cannot be reduced by screening because entrepreneurs default precisely in a downturn. Since the most important element of the analysis is that real loans carry downside risk, I focus on a pooling debt contract for tractability.

2.2 Financial institutions

The financial sector consists of financial institutions with heterogeneous valuation for loans. All financial institutions can lend to the real sector and are equally uninformed: they share the prior F over the entrepreneurs' type. However, only the subset with a high enough valuation will decide to finance real projects. In addition, each financial institution has only $c > 0$ in cash but can raise funds from others by selling financial contracts in financial markets. However, there is an enforcement friction and to ensure repayment these contracts are collateralized by the claims on the real sector they are used to finance. In equilibrium, natural lenders will borrow from other financial institutions in order to fund their portfolio of loans. A financial contract is formally defined as follows:

Definition *A* ***financial contract*** *is a state-contingent promise* (ρ_D, ρ_U) *collateralized by a portfolio of loans* (y_D, y_U) *such that: (i)* $\rho_D \leq y_D$ *and* $\rho_U \leq y_U$*, and (ii)* $\rho_D \leq \rho_U$*.*

A financial institution can raise funds to finance a real project by selling a financial contract at $t = 0$ at a price q^i and committing to repay ρ_s^i at $t = 1$ in state s by pledging future cash-flows $\{y_s^i\}$ of its portfolio of loans as collateral. The first set of constraints reflects the fact that the threat of seizing the collateral is the only way the borrower can be made to keep its promise of repayment: the promised payment in each state must be less than or equal to the collateral payoff in this state. Otherwise, the borrower would walk away and default on the contract. Alternatively, this assumption can be restated as allowing the amount promised to differ from the actual delivery. The second constraint is a monotonicity condition about the security's payoff: the contract promises at least as much in the high state than in the low state. This condition reflects the fact that these financial contracts are profit-sharing schemes rather than insurance contracts. This second condition is important in that it prevents contracts that resemble Arrow securities to be traded for speculative motives. Which financial contracts are traded and at what price are determined in equilibrium.

Financial institutions are heterogeneous in their ability to cope with downside risk and differ in their valuation for income stream that are not constant across states U and D. These differences in marginal utilities can represents unmodeled differences in endowment of assets, goods or labor income whose values depend on the aggregate state of the economy. Agents with low aversion for downside risk are natural lenders to the real sector. However, as a modeling device it is convenient to represent this heterogeneity in valuation as difference in beliefs. This modeling choice allows to solve for equilibrium financial contracts jointly with equilibrium in the real sector, rather than assuming a particular form of financing. Moreover, solving for optimal contracts ensures that there is no trivial way in which the government can improve on the laissez-faire outcome. I follow Geanakoplos (2010), He and Xiong (2012) and Simsek (2013) and label financial institutions as optimists and pessimists, even though this terminology is not meant to be interpreted literally. Specifically, each financial institution is indexed by a probability $h^i \in [0, 1]$: its belief that the high state will occur. Beliefs $\{h^i\}_i$ are distributed uniformly on [0,1]. This disagreement is not rooted in a private signal: they agree to disagree. Optimists with a high h^i believe that loans are repaid with high probability and are therefore natural lenders to the real sector. Financial institutions are risk-neutral with respect to their belief and there is no discounting. Cash can be stored from $t = 0$ to $t = 1$ at no cost.

Note that financial institutions only disagree on the relative probabilities of aggregate states. In particular, they agree on the distribution of loan quality F. This reflects the idea that even an optimist understands that it is facing adverse selection when it finances a project. Moreover, it is convenient to use belief disagreement to characterize equilibrium in a tractable and intuitive way, and it is not necessarily meant to be interpreted literally. The important aspect of lenders' heterogeneity is that their differences in valuation also make it more difficult for them to agree on a funding arrangement. This feature would also prevail in setups where heterogenous valuations arise because of differences in risk aversion, liquidity shocks or impatience.

2.3 Inefficiencies

The assumption that $\delta - x > 0$ implies that there are gains from trade on all projects *independently both of the true probability h and of quality θ.* Any allocation that leaves some projects not financed thus entails some inefficiencies. The laissez-faire outcome is therefore inefficient if some entrepreneurs do not borrow from financial institutions to finance their project. While welfare analysis can be difficult in some models with belief disagreement, lending to entrepreneurs is an unambiguous improvement in this economy. For instance, the welfare criterion introduced in Brunnermeier, Simsek and Xiong (2012) would apply here.

The fact that there are gains from trade on all projects implies that raising the volume of credit is unambiguously socially desirable. Welfare analysis is more intricate when some projects are inherently bad, i.e. negative NPV projects. Note however that if the lowest quality projects are the ones with no gains from trade, adverse selection implies that these projects will nevertheless always be the first to be traded. In this case, conditional on all these "zombie" projects being financed, raising the volume of credit is still socially desirable. It is nevertheless important to keep in mind that this setup is more appropriate to model an economy weakened by a financial meltdown rather than a striving economy in danger of an excessive credit boom, such as in Lorenzoni (2008).

3 Laissez-faire

In order to understand the role of government intervention and its potential cost, I first solve for the laissez-faire equilibrium. The real and financial sectors are closely linked because of the endogenous quality of collateral, and credit supply and financial markets equilibrium must be solved jointly. The main result characterizes in which cases the laissez-faire economy is inefficient.

3.1 Credit supply

As in Akerlof (1970), there is a partial breakdown in the market for project financing because of adverse selection. I interpret this as credit rationing. Indeed, for a given market interest rate R, an entrepreneur with type θ is willing to undertake its project if:

$$u(\theta) \leq \delta - (x - \frac{1}{R}) \iff \theta \leq u^{-1}[\frac{1}{R} + (\delta - x)] = \theta(R) \tag{1}$$

Therefore, any interest rate R defines a quality cutoff $\theta(R)$ such that only projects below this cutoff are financed. There is adverse selection: the best qualities are not financed. For tractability, the following assumption ensures that no entrepreneur is willing to borrow at an infinite interest rate and that all entrepreneurs would accept an interest rate of one:

Assumption 1: $u^{-1}[\delta - x] < 0$ and $\lim_{R \to 0} u^{-1}[\frac{1}{R} + (\delta - x)] > 1$.

While buyers do not know θ, they recognize the entrepreneur's incentives to accept a loan. Financial institutions are thus willing to break-even on the average quality, conditional on the types that are participating in the market. Since only types below $\theta(R)$ are willing to sell, the expected loan repayment in the low state is $E[\theta|\theta \leq \theta(R)]$. The following technical assumption ensures that there is enough log-concavity for the market equilibrium with adverse selection to be unique:

Assumption 2: $\frac{\partial}{\partial \frac{1}{R}} E[\theta|\theta \leq \theta(R)] < 1$.

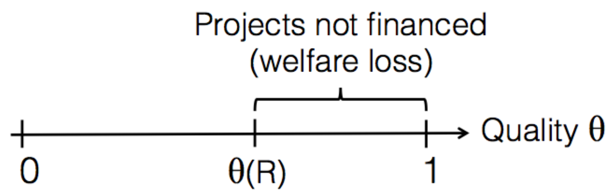

Figure 3: Credit rationing

In equilibrium, the interest rate R is determined by:

$$\frac{1}{R} = b + (1-b)E[\theta|\theta \leq \theta(R)] \tag{2}$$

where $b \in [0,1]$ indexes the belief of the marginal lender. This first equilibrium condition reflects the fact that by definition the marginal lender is indifferent between financing a project and saving in cash, and must therefore break even on its loans. Figure 3 illustrates the credit supply equilibrium. Importantly, note that if R is low enough, there is no adverse selection since all types can be convinced to finance their project. Indeed, if $u(1) < \frac{1}{R} + \delta - x$, the entrepreneur with the best quality $\theta = 1$ is willing to finance his project, implying that all types below will also do so. Furthermore, the financial institution with the highest valuation has belief $h^i = 1$ and would be ready to offer an interest rate as low as $R = 1$ for a loan. What prevents this lender from bidding down the rate to such a low value? It is because, as opposed to Akerlof (1970), lenders are financially constrained. If the most optimistic lenders do not have enough cash to finance all proects, then the marginal lender is low, pushing up the equilibrium interest rate. Moreover, because financial institutions can borrow from each other in financial markets, the amount of cash that optimists can borrow depends on the type of financial contracts traded in equilibrium. The marginal lender b is therefore determined by the amount of funding liquidity that can be raised in financial markets.

3.2 Financial markets

The type of financial contracts traded and their price are determined in equilibrium, and this section closely follows Geanakoplos (2010). Selling a promise (ρ_D, ρ_U) for a price q at $t = 0$ allows a financial institution to finance more projects by taking on some leverage. With a net worth c and a haircut $\frac{1}{R} - q$, expected profits are:

$$\frac{c}{\frac{1}{R} - q}[h^b(1 - \rho_U) + (1 - h^b)(E[\theta|\theta \leq \theta(R)] - \rho_D)] \tag{3}$$

The second term reflects the fact that the collateral (the loan to entrepreneurs) pays on average $E[\theta|\theta \leq \theta(R)]$ in the low state, i.e. financial institutions have not yet learned the quality of the project at the time when they raise financing. In addition, assume that loans across financial institutions are perfectly diversified (by making many small loans) so that so that the payoff in the low state in case of default is $E[\theta|\theta \leq \theta(R)]$ with certainty. As in Geanakoplos (2010), optimists will borrow from pessimists in order to finance more projects. Financial contracts are traded in the following principal-agent framework: sellers of promises simultaneously make a take-it-or-leave-it offer $(\rho^i_U, \rho^i_D, q^i)$ to buyers. In equilibrium, optimists (borrowers) will thus trade the contract that maximizes their expected returns given pessimists' (lenders) break-even constraints. The equivalence of this principal-agent framework with a competitive equilibrium with latent contracts can be found in Simsek (2013).

As in Geanakoplos (2010), Lemma 1 in the Appendix shows that in equilibrium only risk-free promises are traded. The face value of loans in financial markets is $E[\theta|\theta \leq \theta(R)]$ so that default never occurs. Because there is no discounting, the interest rate on these risk-free loans is zero. The logic behind this striking result is that while an optimistic borrower and a pessimistic lender can agree on a high repayment in the low state, sharing the upside is more difficult. Why can't optimists borrow a little more at $t = 0$ by promising a larger upside at $t = 1$ if the high state occurs? Because optimists are confident that the high state will realize, they ask for a relatively large increase in the $t = 0$ price in order to be compensated for the loss. However, from a pessimist point of view, this price increase is judged to be unfairly high. In other words, risky promises that appeal to optimists yield losses for pessimists. Similarly, risky promises that allow pessimists to break even are judged too expensive by optimists. This logic will be important to understand the the cost of a government intervention in financial markets. See Simsek (2013) for the appropriate generalization of this result to settings with more than two states. The novelty here is that the equilibrium amount of funding liquidity depends on the average quality of collateral, which is endogenous.

Given the equilibrium financial contract, one can write a market clearing condition for loans to entrepreneurs. Given that the haircut is $\frac{1}{R} - E[\theta|\theta \leq \theta(R)]$ and that each lender only has c in cash, the total supply of credit to the real sector is given by $\frac{(1-b)c}{\frac{1}{R}-E[\theta|\theta\leq\theta(R)]}$. Moreover, the demand for credit is given by $F(\theta(R))$. The second equilibrium condition linking R and b is thus:

$$F(\theta(R)) = \frac{(1-b)c}{\frac{1}{R} - E[\theta|\theta \leq \theta(R)]} \tag{4}$$

In order to emphasize that credit supply is driven by available funding liquidity, this market clearing condition can be rewritten as follows:

$$F(\theta(R))(\frac{1}{R} - E[\theta|\theta \leq \theta(R)]) = (1-b)c \tag{5}$$

This simply reflects the fact that the total amount of cash that financial institutions are endowed with must equal the total amount of net worth they spend on their portfolio of loans. Importantly, note that as in Geanakoplos (2010), the first equilibrium condition is still valid even when financial institutions can take on some leverage. This is because in the laissez-faire equilibrium, the marginal lender to the real sector is indifferent between lending with or without leverage. This will not always be true in the equilibrium with government intervention.

3.3 Equilibrium

A laissez-faire equilibrium (R^{lf}, b^{lf}) is defined by the two equilibrium conditions:

$$\frac{1}{R^{lf}} = b^{lf} + (1 - b^{lf})E[\theta|\theta \leq \theta(R^{lf})] \tag{6}$$

$$F(\theta(R^{lf}))(\frac{1}{R^{lf}} - E[\theta|\theta \leq \theta(R^{lf})]) = (1 - b^{lf})c \tag{7}$$

The following proposition shows that a unique laissez-faire equilibrium exists and characterizes when it is inefficient.

Proposition 1 *A laissez-faire equilibrium exists and is unique. If the aggregate financial sector net worth c or gains from trade $\delta - x$ are low enough, the laissez-faire allocation is inefficient, i.e. $\theta(R^{lf}) < 1$. As c increases, the interest rate falls and credit supply increases.*

This result shows that government intervention is only warranted in a post-crisis scenario in which the economy's fundamentals are depressed. In particular, government intervention can be warranted even long after the peak of the crisis, as long as financial institutions' balance sheets remain weak. Inefficiencies arise when net worth is low because binding financial frictions prevent high-valuation financial institutions from borrowing sufficiently to drive the interest rate low enough to convince the best entrepreneurs to fund their project. Note that because equilibrium is unique, inefficiencies are not due to private agents coordinating on a bad equilibrium.

Figure 4 illustrates how the interest rate depends on the financial sector aggregate net worth c for a particular numerical example. The lowest efficient interest rate is given by $R = \frac{1}{u(1)-\delta+x}$: all projects are financed for any rate above this level. As c increases, the laissez-faire interest rate falls until efficiency is reached and converges to one.

Financial sector net worth is the state variable that measures the impact of frictions on efficiency. Importantly, market breakdown arises because of the interaction between financial constraints and asymmetric information. Limited funding liquidity exacerbates adverse selection and leads to credit rationing. As in Bigio (2013) there is a feedback loop between funding liquidity and real

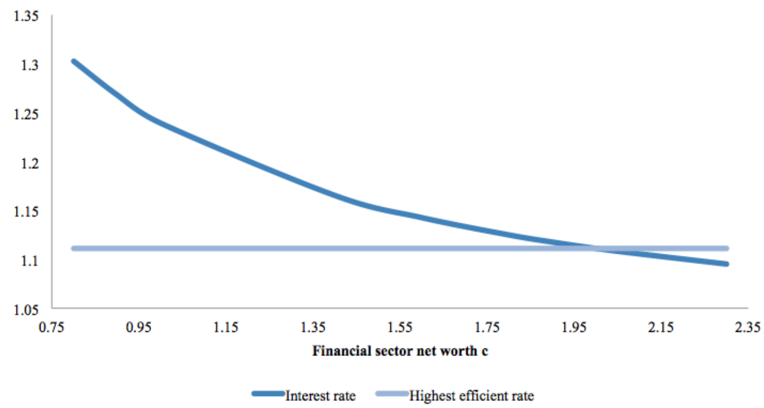

Figure 4: Laissez-faire asset price as function of financial institutions' net worth

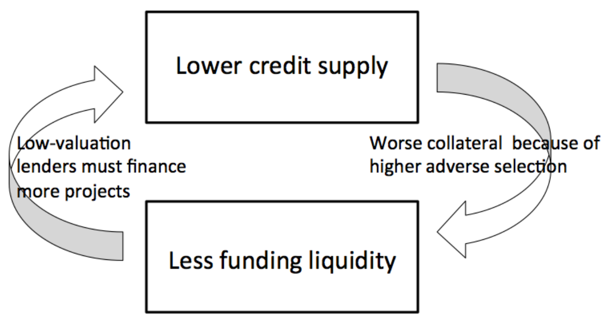

Figure 5: Amplification

investment, because of the endogenous value of collateral. Figure 5 illustrates this mechanism.

4 Public intervention

Because there are gains from trade on all projects, market breakdown is inefficient. Any increase in the cutoff $\theta(R)$ of projects that are financed unambiguously reduces inefficiencies. The questions are whether the government can mitigate these inefficiencies by intervening directly in markets, and, if so, at what cost to taxpayers. For the rest of the paper, assume that parameters are such that the laissez-faire outcome is inefficient, i.e. $\theta(R^{lf}) < 1$. The government objective is as follows: it wants to reach a given cutoff target θ^* larger than the laissez-faire cutoff, at the minimum cost to taxpayers. Any intervention is therefore indexed by an exogenous target $\theta^* \in (\theta(R^{lf}), 1)$.

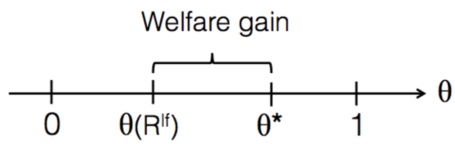

Figure 6: The government's welfare target

The government optimizes across interventions that can reach this target by finding the profit-maximizing one.

This type of intervention reduces inefficiencies compared to laissez-faire, since entrepreneurs with project quality between $\theta(R^{lf})$ and θ^* are now funding their project. Figure 6 illustrates the welfare gain of government intervention. The desire to avoid putting public money at risk and generally minimizing taxpayer losses was evident at the peak of the recent financial crisis, as well as in its aftermath. For instance, in his 2008 speech defending the Troubled Asset Relief Program (TARP) former Secretary of Treasury Henry Paulson told Congress that: "[TARP] has to be properly designed... to have maximum impact... It must also protect the taxpayer to the maximum extent possible". The current strain on public finances in many Western countries points in the same direction. Moreover, this government's objective is not specific to deep crisis scenario where drastic policies must be put in place in emergency. This analysis also applies to post-crisis periods in which the government aims to stimulate an economy weakened by previous shocks, at the minimum taxpayer cost.

The government's objective here nevertheless relies on a reduced-form welfare criterion. However, as argued in Philippon and Skreta (2012), this objective can be rationalized as the second step of a more general welfare maximization problem with a shadow cost of public funds. In a first step, the government determines the optimal cutoff θ^*, and in the second step finds the intervention that reaches this cutoff at the minimum cost. Appendix B formalizes this claim. Importantly, the government is not a social planner. As any other private agent, the government can trade in private markets. The focus of this paper is on direct intervention, and it therefore considers how to stimulate the real economy solely by trading in markets. Moreover, I assume that participation in a government program is voluntary: the government cannot expropriate property or force any agent to accept a particular trade. The government's problem is not equivalent to a planner's problem. In fact, private markets act as a constraint on what the government can offer. Furthermore, since markets are open, prices are well defined and one can compute government's profits and the cost of intervention.

The only advantage that the government has over the private sector is that it is not financially constrained. Moreover, I do not assume that the government is less efficient at making loans to entrepreneurs, although such direct cost could be included in a straightforward manner. However, the taxpayer has potentially different preferences with respect to downside risk than the financial sector, captured by a government's belief h^g in $[0, 1]$. As before, this belief is a modeling device and need not be interpreted literaly.

The government can support either the real sector or the financial sector. More precisely, it can either lend to entrepreneurs or buy financial contracts in financial markets. I will describe these two in turn, and then compare them to determine the optimal intervention. Intuitively, the trade-off behind the optimal intervention is as follows. The government aims to minimize its exposure to downside risk. Because the intervention reaches target θ^*, the interest rate is given by:

$$\theta(R^*) = \theta^* \iff \frac{1}{R^*} = u(\theta^*) - (\delta - x) \tag{8}$$

so that type θ^* is indifferent between financing its project and its outside option. Since the interest rate must be R^*, the entrepreneurs' surplus is fixed, and the government and financial institutions split the aggregate surplus (or profits) that remains. In state U, loans are repaid, and aggregate surplus is $S^U = \int_0^{\theta^*} 1 - \frac{1}{R^*} dF$, while in the state D aggregate surplus is $S^D = \int_0^{\theta^*} \theta - \frac{1}{R^*} dF$. Necessarily, government's profits in each state equals aggregate surplus minus total private buyers' profits. Any downside risk that is not borne by the government must therefore be borne by the private sector. The cost of a specific intervention is therefore related to how much profits it leaves to private buyers in each state of the world. Participation constraints imply that the government cannot extract all surplus from the private sector. The cost-minimizing intervention is therefore such that it distributes private rents across states of the world in the optimal way.

A potential concern is that it is necessary to take a stance on government's beliefs h^g in order to compute its expected profits. However, in this paper the welfare analysis is unambiguous: gains from trade exist for any belief h^g in $[0, 1]$. Moreover, the ranking of interventions is unambiguous: supporting the financial sector is cheaper than supporting the real sector for any belief h^g in $[0, 1]$. Furthermore, being agnostic about the government's preferences allows for a characterization of the optimal financial sector intervention for different levels of the taxpayers' taste for risk-taking.

5 Supporting the real sector

In this type of intervention, the government lends directly to entrepreneurs. In terms of interpretation, this intervention is meant to capture different ways it can directly support the real sector, for example by backing General Motors, buying commercial paper or sponsoring residential mortgages through Fannie Mae and Freddie Mac. The government is equally uninformed, as it shares the same belief F regarding the distribution of entrepreneurs' types.

As a private lender, the government is restricted to offering a single debt contract with interest rate R^g, potentially different from the market rate. Participation in the program is voluntary and entrepreneurs can refuse to trade with the government. A key assumption is that entrepreneurs who have rejected the government offer have the option to borrow from the private sector. The following analysis borrows extensively from Tirole (2012) and Philippon and Skreta (2012), which analyzed public intervention in markets with adverse selection. A public lending program (or a debt guarantee program) reduces the amount of adverse selection faced by the market, which therefore increases the volume of credit at a lower interest rate. This intervention is therefore inherently a dynamic game, as an entrepreneur can decide whether to accept the government offer or to wait to borrow in a rejuvenated market. Figure 7 illustrates the decision tree for an entrepreneur. The government's problem is non-standard in the sense that entrepreneurs' participation constraints

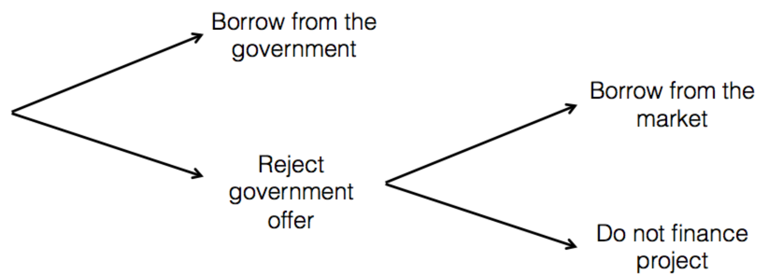

Figure 7: Decision tree for an entrepreneur in a real sector intervention

are endogenous. More precisely, they are mechanism-dependent, meaning that they depend on the government's offer. Indeed, if the government finances low quality projects, the subsequent market interest rate will be low, reflecting the quality of the types left in the market. However, a lower market interest rate increases the rents that must be paid to participating entrepreneurs, in this sense the government creates its own competition. At the heart of the analysis is the two way interaction between the optimal intervention and market equilibrium. In the context of this paper, intervention stimulates the real economy by two channels. The *fundamental channel* reflects the fact that by financing low quality projects, the government directly reduces the lemons problem. The *collateral channel* reflects the fact that a successful intervention raises the value of loans held by private lenders and therefore the average quality of collateral, thus enhancing funding liquidity. This second channel distinguishes this paper from Tirole (2012) and Philippon and Skreta (2012).

Mechanism-dependent participation constraints put some limits on what type of interventions the government can design. Strikingly, the government interest rate must match the rate that will prevail in the rejuvenated market. If R^m denotes the subsequent market price for entrepreneurs who turned down the government offer, Lemma 2 in the Appendix shows that $R^g = R^m$. This implies that the government cannot appropriate any of the surplus created by an increase in trading volume and that the existence of a subsequent market constrains the type of offers that the government can make. Intuitively, the government cannot offer a rate higher than the market rate, otherwise each entrepreneur would prefer to turn down the offer and trade with private lenders. At the same time, if the government offers a lower rate than the market, all type with $\theta < \theta(R^g)$ will accept

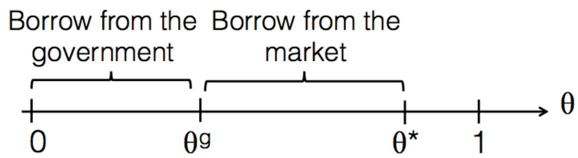

Figure 8: Real sector intervention

this offer. However, in this case financial institutions have incentives to undercut the market to attract some of the high types left, contradicting the fact that R^m is the equilibrium market rate. An immediate consequence is that it is optimal to set $R^g = R^*$ such that $\theta(R^*) = \theta^*$, so that the target is reached exactly. This implies an interest rate strictly lower than in laissez-faire. In equilibrium, the market and the government post the same interest rate R^*.

This suggests that a successful intervention resembles a bailout of the real sector, as illustrated in Figure 8. The government purchases the lowest quality projects, and the market rebounds by lending to better types at a low rate. Denoting by θ^g the cutoff below which entrepreneurs trade with the government, the average quality left to the market is $E[\theta|\theta^g \leq \theta \leq \theta^*]$. The market is willing to lend at a rate R^* precisely because it faces less adverse selection. The cutoff θ^g is determined in equilibrium so that the marginal financial institution with belief b^R breaks even on its loans:

$$\frac{1}{R^*} = b^R + (1 - b^R)E[\theta|\theta^g \leq \theta \leq \theta^*] \tag{9}$$

This first equilibrium condition reflects the fundamental channel: government intervention stimulates real activity by directly reducing the lemon problem faced by the market. The marginal buyer b^R in this real sector intervention is determined by the amount of funding liquidity available in financial markets. Note that the logic behind the financial market equilibrium in laissez-faire is still valid, but now accounts for the fact that the market faces a distribution of types truncated from below; only types with $\theta \geq \theta^g$ are left. The financial contract traded in equilibrium is still risk-free debt but with a higher face value of $E[\theta|\theta^g \leq \theta \leq \theta^*]$, since this is now the collateral payoff in the low state. This reflects the collateral channel. The second equilibrium condition is

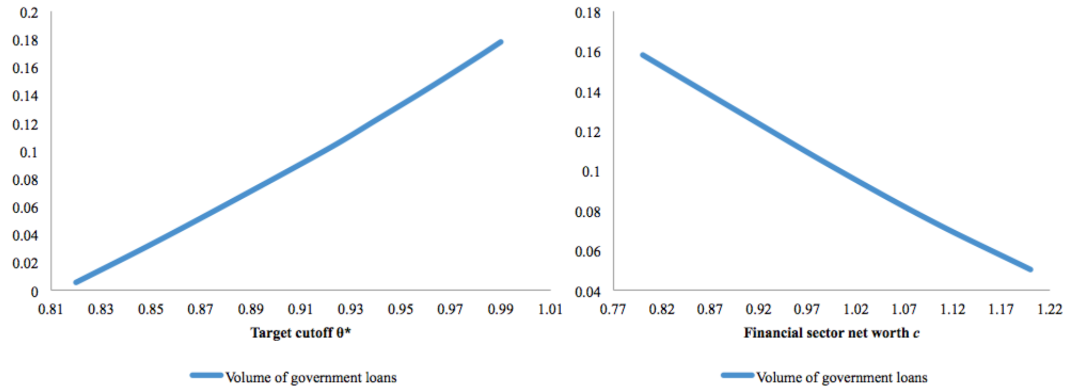

Figure 9: Real sector intervention

market clearing:

$$(F(\theta^*) - F(\theta^g))(\frac{1}{R^*} - E[\theta|\theta^g \leq \theta \leq \theta^*]) = (1 - b^R)c \tag{10}$$

The policy parameter θ^g is uniquely defined by these two equilibrium conditions. Different patterns of participation are nevertheless consistent with equilibrium but considering a simple cutoff rule is without loss of generality and in the appendix, I show that a "fleeting opportunity" refinement implies that all successful real sector interventions have this form.

Figure 9 illustrates the optimal real sector intervention for a particular numerical example. The left-hand panel shows that the volume of government's loans $F(\theta^g)$ increases with the target θ^*; more ambitious interventions are larger. Moreover, the right-hand panel shows that aggregate financial net worth c determines the size of the intervention.

If the financial sector's balance sheets are strong, then small interventions can have large effects, through the collateral channel. With weaker balance sheets, larger interventions are required to relax financial constraints. Financial sector net worth is the state variable that not only quantify inefficiencies of the laissez-faire outcome, but also determines the size of the public intervention and its cost to taxpayers.

Denoting the government's beliefs by h^g, the government's profits on a real sector intervention are given by:

$$\Pi_R^g = F(\theta^g)[h^g + (1-h^g)E[\theta|\theta \leq \theta^g] - \frac{1}{R^*}] \tag{11}$$

$F(\theta^g)$ is the volume of the government's loans and $h^g + (1-h^g)E[\theta|\theta \leq \theta^g]$ is the expected payoff per loan made to entrepreneurs. While supporting the real sector can successfully stimulate real investment, a first striking result is that a real sector intervention is costly to the taxpayer as long as the taxpayer is less optimistic than the marginal private lender. To see this, note that an interest rate R^* is just enough to let the marginal buyer break even on its loans with the better types. Since in equilibrium the government must also charge R^* because of private sector participation constraints, this implies that the government overpays for its claims.

Moreover, there are reasons to believe that this cost might be excessive. To see this, it is useful to remember how the government convinces the market to lend at a lower rate. Financial institutions are willing to finance projects at a rate R^* if the expected surplus they receive across future states of the world allows them to break even. The entrepreneurs' surplus is fixed since the interest rate must be R^* in order to reach the target θ^*. In state U, loans are repaid and the aggregate surplus that remains to be shared between the government and financial institutions is $S^U = \int_0^{\theta^*} 1 - \frac{1}{R^*} dF$. In state D, aggregate surplus is $S^D = \int_0^{\theta^*} \theta - \frac{1}{R^*} dF$. A real sector intervention splits this surplus in a coarse way. Specifically, in each state, the surplus from types in $[0, \theta^g]$ goes to the government and the surplus from types in $[\theta^g, \theta^*]$ goes to private lenders. The taxpayer is exposed to a given amount of downside risk. However, if the taxpayer is less optimistic than the marginal private lender, the government could reduce its losses by exchanging some of its surplus in the high state for a larger share in the low state. Moreover, financial institutions would be happy to make this trade. In this sense, having the government fully substitute itself for financial institutions is excessively costly. In the next section, I show that writing state-contingent funding arrangements directly with the financial sector improves how the surplus is shared between the private and public sector, and therefore minimizes the cost of intervention. The following proposition summarizes the main results of this section:

Proposition 2 *(i) A real sector intervention* θ^{g*} *achieving target* θ^* *exists and is unique. (ii) With the fleeting opportunity refinement, there exists a unique cutoff* θ^g *such that types in* $[0, \theta^g]$ *borrow from the government, while types in* $[\theta^g, \theta^*]$ *borrow from the market. (iii) In any real sector intervention achieving target* θ^**, the government loses money as long as it is less optimistic than the marginal lender.*

6 Supporting the financial sector

The government can also stimulate real activity by injecting liquidity into financial markets. It can raise the amount of funding liquidity available by buying financial contracts from financial institutions. In a financial sector intervention, all loans to the real economy are made by private lenders and a successful intervention injects enough liquidity to reduce the interest rate to $\frac{1}{R^*} = u(\theta^*) - (\delta - x)$, so that the target is reached. In terms of interpretation, this intervention is meant to capture the wide range of liquidities facilities or "liquidity support" put in place by the Federal Reserve or the Treasury in order to "alleviate the strains associated with shrinking balance sheets of intermediaries" (Adrian and Shin, 2010). Prominent examples include TALF or TARP in the U.S. and the FLS in the U.K.. Importantly, because participation to a public program is voluntary, the government must offer more generous terms than the market.

At an abstract level, the government can offer an arbitrary financial contract, subject to some constraints. This contract trades at a price q and can ask for upside ρ in the high state. It is never optimal to ask for less than $E[\theta|\theta \leq \theta^*]$ in the low state when the government is less optimistic than the marginal lender. Contrary to a real sector intervention, the government leaves no surplus to financial institutions in the low state and therefore faces the minimal amount of downside risk. How the surplus is shared in the high state is determined by ρ. Moreover, for the intervention to relax financial constraints, it must be that $q > E[\theta|\theta \leq \theta^*]$, i.e. that the size of the loan is larger than what the market would offer. In addition, the government's contract (q, ρ) must satisfy three other constraints, two of which may or may not be binding:

$$\begin{aligned} \rho &\geq E[\theta|\theta \leq \theta^*] && (12) \\ \frac{1-\rho}{\frac{1}{R^*}-q} &\geq \frac{1-E[\theta|\theta \leq \theta^*]}{\frac{1}{R^*}-E[\theta|\theta \leq \theta^*]} && (13) \\ F(\theta^*)(\frac{1}{R^*}-q) &= (1-\frac{\frac{1}{R^*}-q}{1-\rho})c && (14) \end{aligned}$$

The first constraint is the restriction that the government contract is monotonic. The second is the participation constraints of financial institutions. Specifically, leveraging with a government loan must yield higher returns than with a market loan. Note that b drops from this equation because of risk-neutrality. Therefore, the marginal lender b^F solves:

$$\frac{b^F(1-\rho)}{\frac{1}{R^*}-q} = 1 \iff b^F = \frac{\frac{1}{R^*}-q}{1-\rho} \tag{15}$$

The last constraint is the market clearing condition, guaranteeing that the government injects enough liquidity to reach the target θ^*. Indeed, the total amount of cash that lenders are endowed with is $(1-b^F)c = (1-\frac{\frac{1}{R^*}-q}{1-\rho})c$. Importantly, this immediately implies that in any successful intervention the government must face downside risk. Indeed, risk-free debt only provide enough liquidity to reach the laissez-faire volume of intermediation.

Many common forms of financial sector intervention can be mapped into a particular choice of (q, ρ). Consider for instance an *interest rate intervention* in which the government also offers risk-free debt contracts but at a lower interest rate than the market. This maps into $\rho^I = E[\theta|\theta \leq \theta^*]$, so that the face value is the same as the market loan. Since the market offers an interest rate of zero, the government loan must trade at a negative interest rate, i.e. for a price q^I above its face value. Therefore, buyers strictly prefer the government loans over the market loans. In an interest rate intervention, the monotonicity constraint faced by the government binds but the participation constraints of the private sector is slack. The Appendix shows that there is a unique interest rate intervention (q^I, ρ^I) that reaches the target θ^*.

Alternatively, the government can inject liquidity by offering the same interest rate as the market but with a higher collateral rate. This intervention is meant to capture public programs in the spirit of the Term Securities Lending Facility (TSLF) or the Primary Dealer Credit Facility (PDCF). The TSLF and PDCF were introduced in march 2008 and allowed market participants to swap relatively illiquid collateral with liquid Treasury collateral via the triparty repo market, effectively reducing haircuts. See Fleming, et al. (2009) and Adrian, et al. (2009) for a more detailed description of TSLF and PDCF. The government can offer to lend for a higher face value $\rho = \varphi$ but at the same interest rate of zero, i.e. $q = \varphi$. This loan raises funding liquidity only if $\varphi > E[\theta|\theta \leq \theta^*]$, implying that buyers strictly prefer it to the market loan. This implies that this loan is risky, as the government receives only $E[\theta|\theta \leq \theta^*]$ in the low state. The Appendix shows that there is a unique collateral rate intervention (φ, φ) that reaches the target θ^*. In such a collateral rate intervention, both the monotonicity and participation constraints are slack.

In the two previous interventions, the government leaves rent to the private sector in the sense that financial institutions strictly prefer the government loan to the market loan. The government can extract this rent by offering a non-debt contract that asks for an upside. I interpret these contracts aimed at extracting this upside as *equity injections*, and in particular as purchases of preferred stock in financial institutions. This intervention is meant to capture a program such as the Trouble Asset Relief Program (TARP) which was used by the Treasury to inject $125 billion in equity in nine of the largest American financial institutions in October 2008. An equity contract is backed by one unit of loans to the real sector, trades at price q^E and has a state-contingent payoff. It pays ρ^E in the high state and $E[\theta|\theta \leq \theta^*]$ in the low state, with $\rho^E > E[\theta|\theta \leq \theta^*]$. I define an equity injection as an intervention (ρ^E, q^E) that leaves the private sector indifferent between public and market loans. The appendix shows that there is a unique equity injection (q^E, ρ^E) that reaches the target θ^*. In such an intervention, the monotonicity constraint is slack but the participation constraints of financial institutions are binding.

Note also that the names previously used to described interventions have been used in order to facilitate interpretation only. Indeed, because of the binary state setting, all financial contracts can be alternatively framed as debt contracts, with a face vale f and interest rate r. With $f = \rho$ and $r = \frac{\rho}{q}$, this debt contract is equivalent to the financial contract (q, ρ) described above. Moreover, the intervention can be implemented as a debt guarantee program, as shown by Philippon and Schnabl (2013) in a related setting. This equivalence result is useful if there are restrictions on the government's cash outlays at $t = 0$.

The three constraints faced by the government can be written in a way that is both more intuitive and easier to solve. Using the third constraint to isolate q, the second constraint is equivalent to $\rho \leq \rho^E$, where ρ^E is the upside corresponding to an equity injection. This implies that the set of constraints is equivalent to $E[\theta|\theta \leq \theta^*] \leq \rho \leq \rho^E$. Moreover, this implies that $q^I \leq q \leq q^E$, highlighting the fact that an interest intervention and an equity injection are at the opposite ends of the spectrum. In fact, the first two constraints above cannot bind at the same time. If the monotonicity constraint binds, then the intervention is the interest rate intervention previously described. If the participation constraint binds, then the intervention is the equity injection. If both are slack, the intervention is somewhere in the middle, such as in the collateral rate intervention. Furthermore, as in the previous section, the aggregate net worth of the financial sector is a state variable that determines the size of the intervention for any given target θ^*. When balance sheets are weaker, a larger intervention is required to stimulate the real economy.

As in a real sector intervention, the government loses money in any successful intervention as long as it is less optimistic than the marginal lender. Indeed, if the government offers a risk-free loan, participation constraints imply that it must trade at a negative interest rate. If the the government loan is risky, the analysis of laissez-faire showed that risky promises which allow pessimists to break even are judged too expensive by optimists. Why does the government lose money on a contract that asks for an upside? The answer lies again in the participation constraint of financial institutions. If the government asks an optimistic lender to share the upside at $t = 1$, it must be offered a high price q at $t = 0$ in order to be convinced to participate. However, because these lenders consider the high state to be very likely, the compensation they require is unfairly high from the point of view of the government. This is precisely why risky promises that resemble equity are not traded in laissez-faire.

Proposition 3 *In any financial sector intervention* (q^*, ρ^*) *achieving target* θ^**, the government loses money as long as it is less optimistic than the marginal lender.*

As seen above, there are multiple financial sector interventions (q^*, ρ^*) achieving target θ^*. The optimal intervention minimizes taxpayer losses, and depends on the government's belief h^g. Recall that the government faces the following problem:

$$\max_{\rho,q} F(\theta^*)(h^g\rho + (1-h^g)E[\theta|\theta \leq \theta^*] - q) \tag{16}$$

subject to

$$\begin{aligned} E[\theta|\theta \leq \theta^*] \quad &\leq \quad \rho \leq \rho^E & (17) \\ F(\theta^*)(\frac{1}{R^*} - q) \quad &= \quad (1 - \frac{\frac{1}{R^*} - q}{1-\rho})c & (18) \end{aligned}$$

The first set of constraints is a convenient way to express monotonicity and participation constraints, and the last constraint ensures market clearing. The following proposition gives a complete characterization of the optimal financial sector intervention as a function of the government's belief h^g:

Proposition 4 *For any target* θ^**, the optimal financial intervention* $(q^*(h^g), \rho^*(h^g))$ *is unique, continuous and increasing in* h^g*. There exist* h_1 *and* h_2 *such that (i) for* $h^g < h_1$*, an interest rate intervention is optimal, (ii) for* $h^g > h_2$*, equity injections are optimal, and (iii) for* $h_1 < h^g < h_2$*, the optimal intervention is given by:*

$$\begin{aligned} q^*(h^g) \quad &= \quad \frac{1}{R^*} - \frac{c}{F(\theta^*)}(1 - \sqrt{h^g}) & (19) \\ \rho^*(h^g) \quad &= \quad 1 - \frac{c}{F(\theta^*)}(\sqrt{\frac{1}{h^g}} - 1) & (20) \end{aligned}$$

Figure 10 illustrates the optimal financial sector intervention. If the government's belief is sufficiently low, an interest rate intervention is optimal. If it is sufficiently high, an equity injection is optimal. There is an intermediate range of beliefs in which the government asks for an upside, but not enough to make participation constraints bind. The optimal upside ρ^F increases as the government becomes more optimistic, and as ρ^F increases, q^F also increases. When the two curves meet, $q^F = \rho^F = \varphi$ and the optimal intervention is a collateral rate intervention. Intuitively, an optimistic government is willing to pay a higher price q^F at $t = 0$ in order to ask for an upside ρ^F in the high state. It is also apparent that an interest rate intervention is the most conservative type of intervention, while an equity injection is the most risky.

The logic behind this result is as follows. Note first that these interventions all give the same payoff in the low state, namely $E[\theta|\theta \leq \theta^*]$. They differ along two dimensions: the price q paid to borrowers at $t = 0$ for each unit of collateral, and how much is paid back in the high state at $t = 1$ ρ (the "upside"). The price and the upside are positively related through the borrower's participation constraint. Indeed, an optimist is willing to share the upside, which he considers likely, only if he can sell his promises at a high price. Therefore, government interventions which ask for a larger upside cost more at $t = 0$. If the government's belief h^g is sufficiently high, interventions asking for an upside are cheapest. On the other hand, for lower h^g, the government can reduce its costs by asking for little upside. A financial sector intervention optimally splits the surplus across states of the world between the public and the private sectors in order to minimize the cost to taxpayers.

Being agnostic about the government's preferences allows this proposition to relate to some of the debate surrounding public intervention during the recent financial crisis. Some commentators argue that a conservative intervention, for example by making overcollateralized short-term loans, is the best course of action. On the other hand, others have urged policy makers to be more bold and take risks in order to profit from the opportunity. These arguments reflect the trade-off behind the optimal financial sector intervention. It could help explain why the Treasury pushed for programs such as TARP that involved more risk-taking than the liquidity facilities put in place

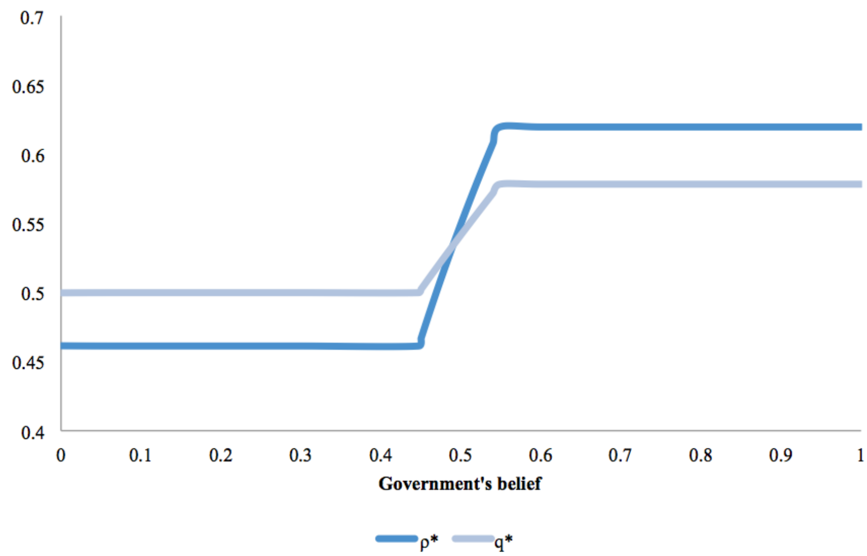

Figure 10: Optimal financial sector intervention

by the Federal Reserve at the time.

Note finally that an interest rate intervention is actually equivalent to subsidizing the financial sector. Indeed, suppose that the government does not trade in financial markets but instead introduces a lending subsidy $s > 0$: each financial institution financing a real project receives a transfer s per project. This effectively reduces the margin faced by financial institutions: while the market collateral rate is still $E[\theta|\theta \leq \theta^*]$ the effective haircut is only $\frac{1}{R^*} - E[\theta|\theta \leq \theta^*] - s$. With $s = q^I - E[\theta|\theta \leq \theta^*]$, this is isomorphic to an interest rate intervention. Indeed, an interest rate intervention can be interpreted as the purchase of market claims at an inflated price, and this is equivalent to a subsidy. This result suggests that the distinction between fiscal bailouts and unconventional monetary tools is tenuous. Moreover, if subsidies are politically unfeasible, the government can decide to phrase its policy in terms of a purchase of financial securities.

7 Optimal intervention

A recurring question in debates surrounding government intervention is whether taxpayer money is best spent supporting the financial sector or the real sector. While the ultimate aim of the government is to stimulate the real economy, I show that supporting the real sector is more expensive than intervening in financial markets in this economy. This result does not depend on specific taxpayer preferences. Supporting the financial sector is cheaper for an arbitrary h^g, although the optimal intervention is different in each case. Note that this result implies that it is optimal to subsidize the financial sector if the government is conservative enough.

Proposition 5 *For any target θ^* and any government's belief $h^g \in [0,1]$, supporting the financial sector is cheaper than supporting the real sector.*

Intuitively, the cost of an intervention depends on how much downside risk the government is exposed to. However, because the total amount of downside risk in the economy is fixed by target θ^*, private agents must bear any of this risk that the government does not bear. The optimal intervention therefore shares downside risk in the best possible way between the public and private sectors. In a real sector intervention, the government must take on a significant share of downside risk. Moreover, this risk is split in a coarse way: the taxpayer faces all the risk from projects financed by the government. This is not simply an artifact of the contracting environment, it reflects the fact that downside risk cannot be shared with entrepreneurs since they default precisely in the bad aggregate state. On the other hand, lending to financial institutions allows for a more flexible sharing of risk through the term of the loans (q and ρ in the language of section 6). This is because financial institutions have capital than can act as a buffer. In fact, there is a financial sector intervention which replicates exactly the optimal real sector intervention described in section 5, but it is only optimal in a knife-edge case. The existence of this extra flexibility is not surprising, the main point is that the government should make use of this flexibility in the interest of the taxpayer. In particular, this result suggests that public funds should follow private funds: absent intervention, the taxpayer would lend to financial institutions, not directly to entrepreneurs.

8 Conclusion

Frictions have an impact on market outcomes and policy decisions. This paper shows that laissez-faire can be inefficient because of the interaction between adverse selection and financial constraints. In times where financial institutions' balance sheets are weak, public intervention can stimulate real activity. However, any successful intervention is costly to taxpayers. Moreover, even if the ultimate aim is to stimulate real activity, supporting the real sector directly is excessively costly because financial institutions are better able to bear the risk of intermediation. Lending instead to the financial sector is cheaper because it prevents taxpayers from being exposed to excessive downside risk. Modeling the government not as a social planner but as an agent that contracts with the private sector allows inquiry into the difficult question of how to best use taxpayer money. This paper shows that the existence of participation constraints for the private sector can generate new results, and this line of reasoning can be applied to the design of public interventions in settings other than financial markets.

This paper considered a stylized economy, and there are many important directions in which it could be extended. A first extension could introduce dynamics. There are at least two mechanisms that can lead to dynamic balance sheet amplification in this economy. The first mechanism is akin to the "leverage cycle" described in Geanakoplos (2010): the highest-valuation lenders take on more leverage and are therefore hit harder by bad fundamental shocks. The second effect comes from the fact that weak balance sheets today imply that the best projects are not financed, weakening balance sheets in the future. Another extension could incorporate secondary asset markets. In this setting, the government can relax financial constraints by purchasing legacy assets from financial institutions, as in Tirole (2012). Whether this type of intervention is optimal is likely to depend on the particular frictions that affect these secondary markets. Finally, an important assumption is that the government can make public funding to financial institutions conditional on them lending to the real sector. Even though some real world interventions like the FLS are designed in this way, recent work such as Benmelech and Bergman (2013) suggests that public money may remain trapped in the financial sector.

Appendix A: Omitted proofs

Lemma 1 *The only financial contract traded in equilibrium is a risk-free debt contract with face value $E[\theta|\theta \leq \theta(R)]$ per unit of collateral.*

Proof of Lemma 1: An optimist with belief h can sell a promise (ρ_D, ρ_U) to a pessimist with belief $h' < h$ in order to finance a project on margin. The seller's expected utility is proportional to its (subjective) return on the asset:

$$r^h = \frac{E^h[y(s) - \rho(s)]}{\frac{1}{R} - E^{h'}[\rho(s)]} = \frac{h(1-\rho_U) + (1-h)(E[\theta|\theta \leq \theta(R)] - \rho_D)}{\frac{1}{R} - \rho_D - h'(\rho_U - \rho_D)} \tag{21}$$

Note first that in equilibrium this return must be larger than 1, the return on holding cash. Therefore, $A = h(1-\rho_U) + (1-h)(E[\theta|\theta \leq \theta(R)] - \rho_D) - \frac{1}{R} + \rho_D + h'(\rho_U - \rho_D) \geq 0$. This implies that r^h increases with ρ_D. Indeed,

$$\frac{\partial r^h}{\partial \rho_D} = \frac{(1-h)A + h(h-h')(1-\rho_U) + (1-h)(h-h')(E[\theta|\theta \leq \theta(R)] - \rho_D)}{[\frac{1}{R} - \rho_D - h'(\rho_U - \rho_D)]^2} \geq 0 \tag{22}$$

It is therefore optimal to set $\rho_D = E[\theta|\theta \leq \theta(R)]$. Note then that for this value of ρ_D, r^h decreases with ρ_U, implying that it is optimal to set $\rho_U = \rho_D$. Indeed, r^h rewrites as $r^h = \frac{h}{h' + \frac{\frac{1}{R} - E[\theta|\theta \leq \theta(R)] - h'(1 - E[\theta|\theta \leq \theta(R)])}{1-\rho_U}}$. Because h' does not finance projects with cash, $\frac{1}{R} - E[\theta|\theta \leq \theta(R)] - h'(1 - E[\theta|\theta \leq \theta(R)]) > 0$, and r^h decreases with ρ_U. $\square$

Proof of Proposition 1: The first equilibrium condition rewrites as $b = \frac{\frac{1}{R} - E[\theta|\theta \leq \theta(R)]}{1 - E[\theta|\theta \leq \theta(R)]}$. Since $R \in [1, \infty)$, assumptions 1 and 2 imply that $b \in [0,1]$. Moreover, any interest rate R defines a unique marginal buyer b. Eliminating b in the second equilibrium condition yields:

$$(\frac{1}{R} - E[\theta|\theta \leq \theta(R)])F(\theta(R)) = \frac{1 - \frac{1}{R}}{1 - E[\theta|\theta \leq \theta(R)]}c \tag{23}$$

For $R = \infty$, the LHS is zero, while the RHS is $c > 0$. For $R = 1$, the LHS is $1 - E[\theta] > 0$, while the RHS is zero. By continuity, there exists a solution to this equation. Assumption 2 implies that this solution is unique. The laissez-faire allocation is efficient only if $\theta(R) \geq 1$. The laissez-faire interest rate must therefore be low enough: $\theta(R) \geq 1 \iff R \leq \frac{1}{u(1)-\delta+x}$. As $\delta - x \to 0$, the interest rate must be arbitrarily close to one, which means that financial institutions with beliefs close enough to $h = 1$ must finance all projects. However, for any given level of net worth c, this is impossible, and thus proves the first part of the inefficiency result. For the second part, for any given $\delta - x > 0$, $R \leq \frac{1}{u(1)-\delta+x}$ and the first equilibrium equation imply that $b \geq \frac{u(1)-\delta+x-E[\theta]}{1-E[\theta]}$. The second equilibrium equation implies that $b \leq 1 - \frac{u(1)-\delta+x-E[\theta]}{c}$. This interval is non-empty if $\frac{u(1)-\delta+x-E[\theta]}{1-E[\theta]} \leq 1 - \frac{u(1)-\delta+x-E[\theta]}{c} \iff c \geq \frac{(u(1)-\delta+x-E[\theta])(1-E[\theta])}{\delta-x}$. Finally, differentiating the expression above with respect to c shows that $\frac{dR^{lf}}{dc} < 0$. $\square$

Lemma 2 *In any real sector intervention reaching target θ^*, $R^g = R^m$.*

Proof of Lemma 2: Assume by way of contradiction that $R^g > R^m$. Then no entrepreneur accepts the government offer, but then the intervention fails to reach its target. If $R^g < R^m$ in equilibrium, then no entrepreneur accepts the market offer and all types below $\theta(R^g)$ borrow from the government. Consider a possible deviation in the market game: a financial institution with belief h offers an interest rate R' such that $\frac{1}{R'} = \frac{1}{R^g} + \varepsilon$, for a small $\varepsilon > 0$. Since $R' < R^g < R^m$, this offer is accepted by some types above $\theta(R^g)$. The profits from this deviation are thus at least $h + (1-h)\theta(R^g) - \frac{1}{R'}$. For ε small enough, there is an h close enough to one such that $h + (1-h)\theta(R^g) - \frac{1}{R'} > 0$, a contradiction. $\square$

Proof of Proposition 2: (i) The first equilibrium condition implies that $b = \frac{\frac{1}{R^*} - E[\theta|\theta^g \leq \theta \leq \theta^*]}{1-E[\theta|\theta^g \leq \theta \leq \theta^*]}$. Since F is log-concave, any θ^g defines a unique b. Eliminating b in the second equilibrium condition yields:

$$(F(\theta^*) - F(\theta^g))(\frac{1}{R^*} - E[\theta|\theta^g \leq \theta \leq \theta^*]) = \frac{(1-\frac{1}{R^*})c}{1-E[\theta|\theta^g \leq \theta \leq \theta^*]} \tag{24}$$

For $\theta^g = \theta^*$, the LHS is zero, while the RHS is $(1-\frac{1}{R^*})c > 0$. For $\theta^g = 0$, the LHS is strictly larger than the RHS, otherwise θ^* would be attainable under laissez-faire. By continuity, a solution exists. Since the LHS is strictly decreasing in θ^g while the RHS is strictly increasing, this solution is unique. $\square$

(ii) "Fleeting opportunity refinement": Suppose that for a small $\varepsilon > 0$, the entrepreneur's project disappears just before the private market open with probability $\varepsilon(1-\theta)$. Accepting the government's offer yields $\frac{1}{R^g} + (\delta - x)$, while rejecting it in favor of the market offer yields $[\frac{1}{R^m} + (\delta - x)][1 - \varepsilon(1-\theta)]$. Higher types a have a relative preference for rejecting the government offer, and types sort according to a cutoff. As $\varepsilon \to 0$, this cutoff converges to some θ^g. $\square$

(iii) For $h^g \leq b$, the government's profits are given by:

$$\begin{aligned}
\Pi^g &= \int_0^{\theta^g} h^g + (1-h^g)\theta - \frac{1}{R^*} dF && (25)\\
&= F(\theta^g)[h^g + (1-h^g)E[\theta|\theta \leq \theta^g] - \frac{1}{R^*}] && (26)\\
&< F(\theta^g)[h^g + (1-h^g)E[\theta|\theta^g \leq \theta \leq \theta^*] - \frac{1}{R^*}] && (27)\\
&\leq F(\theta^g)[b + (1-b)E[\theta|\theta^g \leq \theta \leq \theta^*] - \frac{1}{R^*}] && (28)\\
&= 0 && (29)
\end{aligned}$$

The government therefore looses money. $\square$

Lemma 3 *Target θ^* can be reached by (i) a unique interest rate intervention, (ii) a unique collateral rate intervention, or (iii) a unique equity injection.*

Proof of Lemma 3: (i) The first equilibrium condition implies that $b = \frac{\frac{1}{R^*} - q}{1 - E[\theta|\theta \leq \theta^*]}$. Any q defines an unique b. Eliminating b in the second equilibrium condition yields:

$$F(\theta^*)(\frac{1}{R^*} - q) = (1 - \frac{\frac{1}{R^*} - q}{1 - E[\theta|\theta \leq \theta^*]})c \tag{30}$$

In any successful intervention, $q \in (E[\theta|\theta \leq \theta^*], \frac{1}{R^*})$. For $q = \frac{1}{R^*}$, the LHS is zero, while the RHS is $c > 0$. For $q = E[\theta|\theta \leq \theta^*]$, the LHS is strictly larger than the RHS, otherwise θ^* would be attainable under laissez-faire. By continuity, a solution exists. Since the LHS is strictly decreasing in q while the RHS is strictly increasing, this solution is unique. $\square$

(ii) The first equilibrium condition implies that $b = \frac{\frac{1}{R^*}-\varphi}{1-\varphi}$. Any φ defines an unique b. Eliminating b in the second equilibrium condition yields:

$$F(\theta^*)(\frac{1}{R^*} - \varphi) = \frac{(1-\frac{1}{R^*})c}{1-\varphi} \tag{31}$$

In any successful intervention, $\varphi \in (E[\theta|\theta \leq \theta^*], \frac{1}{R^*})$. For $\varphi = p^*$, the LHS is zero, while the RHS is $c > 0$. For $\varphi = E[\theta|\theta \leq \theta^*]$, the LHS is strictly larger than the RHS, otherwise θ^* would be attainable under laissez-faire. By continuity, a solution exists. Since the LHS is strictly decreasing in φ while the RHS is strictly increasing, this solution is unique. $\square$

(iii) Combining the first and the third equilibrium conditions implies that b only depends on θ^* and R^*: $b = \frac{\frac{1}{R^*}-E[\theta|\theta\leq\theta^*]}{1-E[\theta|\theta\leq\theta^*]} \in [0,1]$. The second equation therefore implies that $q^* = \frac{1}{R^*} - \frac{(1-b)c}{F(\theta^*)}$, which is in $(E[\theta|\theta \leq \theta^*], \frac{1}{R^*})$, otherwise θ^* would be achievable under laissez-faire. The last equation implies that $\rho = 1 - \frac{\frac{1}{R^*}-q^*}{b}$, which is in $(E[\theta|\theta \leq \theta^*], 1)$. $\square$

Proof of Proposition 3: The government's profits are given by $\Pi^g = F(\theta^*)(h^g\rho^* + (1 - h^g)E[\theta|\theta \leq \theta^*] - q^*)$. These profits are increasing in h^g. For $h^g = b = \frac{\frac{1}{R^*}-q}{1-\rho}$, the participation constraint implies that these profits are bounded by:

$$\begin{aligned} F(\theta^*)(b\rho^* + (1-b)E[\theta|\theta \leq \theta^*] - q^*) &\leq F(\theta^*)(b\rho^* + \frac{1}{R^*} - b - q^*) && (32)\\ &= F(\theta^*)(-b(1-\rho^*) + \frac{1}{R^*} - q^*) && (33)\\ &= 0 && (34) \end{aligned}$$

Therefore, for $h^g < b$, $\Pi^g < 0$. $\square$

Proof of Proposition 4: The equilibrium cash-in-the-market pricing condition implies that

$q = \frac{1}{R^*} - \frac{1-\rho}{1+\frac{F(\theta^*)}{c}(1-\rho)}$. Therefore, the government's problem can be written as:

$$\max_{\rho} F(\theta^*)(h^g\rho + (1-h^g)E[\theta|\theta \leq \theta^*] \quad - \quad \frac{1}{R^*} + \frac{1-\rho}{1+\frac{F(\theta^*)}{c}(1-\rho)}) \tag{35}$$

$$s.t. E[\theta|\theta \leq \theta^*] \leq \rho \quad \leq \quad \rho^E \tag{36}$$

The Lagragean is given by $\mathcal{L} = h^g\rho + (1-h^g)E[\theta|\theta \leq \theta^*] - \frac{1}{R^*} + \frac{1-\rho}{1+\frac{F(\theta^*)}{c}(1-\rho)} + \lambda_1(\rho^E - \rho) + \lambda_2(\rho - E[\theta|\theta \leq \theta^*])$. The following Kuhn-Tucker conditions are necessary and sufficient for this problem:

$$h^g - \lambda_1 + \lambda_2 - \frac{1}{[1+\frac{F(\theta^*)}{c}(1-\rho)]^2} \quad = \quad 0 \tag{37}$$

$$\lambda_1(\rho^E - \rho) \quad = \quad 0 \tag{38}$$

$$\lambda_2(\rho - E[\theta|\theta \leq \theta^*]) \quad = \quad 0 \tag{39}$$

Note first that the case $\lambda_1 > 0$ and $\lambda_2 > 0$ is impossible.

Consider next the case $\lambda_1 = 0$ and $\lambda_2 > 0$. Therefore, $\rho^* = E[\theta|\theta \leq \theta^*]$ and $q^* = q^I$. The first K-T condition implies that $\lambda_2 = \frac{1}{[1+\frac{F(\theta^*)}{c}(1-E[\theta|\theta \leq \theta^*]]^2} - h^g$. Therefore, $\lambda_2 > 0 \iff h^g < \frac{1}{[1+\frac{F(\theta^*)}{c}(1-E[\theta|\theta \leq \theta^*])]^2} = (b^I)^2 = h_1$.

Consider next the case $\lambda_1 > 0$ and $\lambda_2 = 0$. Therefore, $\rho^* = \rho^E$ and $q^* = q^E$. The first K-T condition implies that $\lambda_1 = h^g - \frac{1}{[1+\frac{F(\theta^*)}{c}(1-\rho^E)]^2}$. Therefore, $\lambda_1 > 0 \iff h^g > \frac{1}{[1+\frac{F(\theta^*)}{c}(1-\rho^E)]^2} = (b^E)^2 = h_2$.

Finally, consider the case $\lambda_1 = 0$ and $\lambda_2 = 0$. The first K-T condition implies that $\rho^* = 1 - \frac{c}{F(\theta^*)}(\sqrt{\frac{1}{h^g}} - 1)$. This implies that $q^* = \frac{1}{R^*} - \frac{c}{F(\theta^*)}(1 - \sqrt{h^g})$. Moreover, $E[\theta|\theta \leq \theta^*] \leq \rho^* \leq \rho^E \iff h_1 \leq h^g \leq h_2$. $\square$

Proof of Proposition 5: For a given belief $h^g \in [0,1]$, the government's profits from a real and financial sector intervention are, respectively:

$$\Pi^R \quad = \quad F(\theta^g)(h^g + (1-h^g)E[\theta|\theta \leq \theta^g] - \frac{1}{R^*}) \tag{40}$$

$$\Pi^F \quad = \quad F(\theta^*)(h^g\rho^* + (1-h^g)E[\theta|\theta \leq \theta^*] - q^*) \tag{41}$$

Denote by b^F the marginal buyer in the financial sector intervention. By manipulation of equilibrium conditions, the difference in profits can be written as:

$$\begin{aligned}\Delta\Pi = \Pi^F - \Pi^R &= (b^R - b^F)c - h^g \int_0^{\theta^*} 1 - \rho^* dF + h^g \int_{\theta^g}^{\theta^*} 1 - \theta dF &(42)\\ &= (b^R - b^F)c - h^g c\left(\frac{1-b^F}{b^F} - \frac{1-b^R}{b^R}\right) &(43)\end{aligned}$$

Therefore, a financial sector intervention is optimal if:

$$\Delta\Pi \geq 0 \iff (b^R - b^F)(1 - \frac{h^g}{b^R b^F}) \geq 0 \tag{44}$$

The optimal intervention depends on the government's beliefs h^g. Consider each case in turn. If $h^g \leq h_1 = (b^I)^2$, the optimal financial sector intervention is an interest rate intervention and $b^F = b^I$. Since $b^R > b^I$, the expression above implies that $\Delta\Pi > 0$. Similarly, if $h^g \geq h_2 = (b^E)^2$, the optimal financial sector intervention is an equity injection and $b^F = b^E$. Since $b^R < b^E$, the expression above implies that $\Delta\Pi > 0$. On the intermediate range $h^g \in (h_1, h_2)$, the optimal intervention is such that $b^F = \sqrt{h^g}$. The expression above therefore rewrites as:

$$(b^R - b^F)(1 - \frac{h^g}{b^R b^F}) \geq 0 \iff (b^R - \sqrt{h^g})^2 \geq 0 \tag{45}$$

Therefore, $\Delta\Pi \geq 0$ on this range as well. $\square$

Appendix B

Consider for instance that the government's objective V^g is to find a cutoff θ^* and an intervention $\mathcal{I}$ to maximize private sector surplus, but it faces a shadow cost of public fund $\lambda > 0$:

$$V^g = \max_{\theta^*, \mathcal{I}} E^h[U^E + U^F - (1+\lambda)D(\theta^*, \mathcal{I})] \tag{46}$$

where $D(\theta^*, \mathcal{I})$ is the associated government's deficit. Given that total private sector utility is the sum of the total gains from trade $W(\theta^*)$ and total government transfers $D(\theta^*, \mathcal{I})$, this objective rewrites:

$$\begin{aligned} V^g &= \max_{\theta^*,\mathcal{I}} W(\theta^*) - \lambda D(\theta^*,\mathcal{I}) && (47) \\ &= \max_{\theta^*} W(\theta^*) - \lambda \min_{\mathcal{I}} D(\theta^*,\mathcal{I}) && (48) \end{aligned}$$

The last term $\min_{\mathcal{I}} D(\theta^*,\mathcal{I})$ is the government's cost function for a given target cutoff θ^*. The reduced form government's objective assumed in this paper represents this cost function.

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