Stages of Diversification and Industry Productivity Differences

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Abstract

Economies tend to diversify and then re-specialize as they develop. In an economy with many industries that experience different rates of productivity growth, these "stages of diversification" may obtain if initial conditions are skewed away from the industries that dominate economic structure in the long run as a matter of productivity-driven structural change. A calibrated multi-industry growth model with many countries replicates the main features of "stages of diversification". We also present evidence that countries shift resources towards high-TFP growth manufacturing industries, and towards low-TFP growth sectors, consistent with the calibrated model.

Keywords: Stages of diversification, structural change, productivity differences, development.

JEL Codes: O11 O14 O33 O41.

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1 Introduction

Economic development is a joint process of economic growth and economic restructuring.[1] It is well known that economic development tends to involve a shift of resources away from agriculture and towards services. In addition, Imbs and Wacziarg (2003, hereafter IW) show that there exist "stages of diversification": along the development path: countries appear to start out with employment concentrated in a few industries and sectors, diversifying until reaching a certain threshold in income per capita, after which they begin to re-specialize. In other words, industrial specialization is U-shaped along the development path.

Persistent differences in income per capita across place and across time can be largely accounted for by differences in productivity – see Barro (1998) and Prescott (1998). The question in this paper is: can persistent productivity differences across *industries* account for observed patterns of economic *restructuring* along the development path? This is an important question: a positive answer implies that a productivity-driven theory of development may account jointly for economic growth and for the evolution of economic structure.

We address this question using a multi-sector model that highlights TFP growth differences across sectors and also across manufacturing industries. We show that the pattern of diversification followed by specialization can be accounted for simply by the dynamics of industry structure resulting from these differences.

Consider the following intuition. Suppose that markets are competitive, and that there are two goods that are substitutes in consumption. Then, persistent differences in TFP growth rates will lead to an increase in the GDP share of the industry with the most rapid productivity growth, as the good it produces will register a decline in its relative price.[2] However, if the economy starts out being specialized in the *other* industry, then the economy will diversify until half of resources are devoted to each industry, after which it will appear

[1] The World Bank (2012) defines economic development as:

> Qualitative change and restructuring in a country's economy in connection with technological and social progress. The main indicator of economic development is increasing GNP per capita (or GDP per capita), reflecting an increase in the economic productivity and average material wellbeing of a country's population. Economic development is closely linked with economic growth.

Note the emphasis on productivity as a driver of economic development.

[2] Conversely, suppose the goods are complements. Then, persistent differences in TFP growth rates will lead to an increase in the GDP share of the industry with the *slowest* productivity growth. As we shall see, both cases turn out to be empirically relevant.

to specialize again. The economy will show a "U" shaped pattern of specialization along the development path.

We develop a multi-industry growth model in which productivity growth rates differ across industries, and calibrate initial productivity levels so as to reproduce the composition of manufacturing and the sectorial makeup of each of the countries in the IW dataset in 1963. Then, we allow the structure of the model economies to evolve over time based on persistent productivity growth differences across industries, calibrated to US data. Along the development path, the labor shares of different industries evolve due to disparities between their TFP growth rates. Applying the same non-parametric method as IW to the model-generated series of industrial diversification, the calibrated model generates the U-shaped stages of diversification found in IW. Our results hold both within manufacturing and across broad sectors, and are robust to a number of variations in the calibration procedure. We conclude that differences in TFP growth across industries can indeed lead to the stages of diversification, so that productivity growth differences can account for differences in economic structure along the development path. Thus, an important characteristic of the process of economic development is the reallocation of resources among industries with different rates of productivity growth.

Our results do not imply that no factors other than productivity differences might account for differences in economic structure. However, we show that these alternatives are not *required* to generate the observed stylized facts. Future work may sort out the relative contribution of one or other factor to patterns of economic structure along the development path. At the same time, we also provide evidence that, as they grow, countries do indeed shift resources towards manufacturing industries that display more rapid TFP growth, underlining the empirical relevance of the mechanisms in the paper for understanding the process of economic development. Notably, our results suggest that goods within manufacturing are substitutes but that across sectors they are complements, so that within manufacturing resources should shift towards high-TFP growth industries, whereas across sectors resources should shift towards low-TFP growth sectors. This is exactly what we find in the data. Furthermore, the model economy closely matches the link in the data between the shares of agriculture and services and the level of development.

Our model is close to Ngai and Pissarides (2007, hereafter NP). NP show that persistent productivity differences across sectors can result in structural change, and study conditions under which this may occur along a balanced growth path. However, they focus on the behavior of agriculture and services (as do most studies of structural change), and do not study "stages of diversification." We build on their work by performing a rigorous quantitative analysis of the implications of productivity-driven structural change for a large set of countries. In addition, we find that the restrictions NP identify that are required for a balanced growth path with structural change do not appear to hold empirically – specifically, the elasticity of substitution across capital goods is not equal to one – so our analysis requires the computation of a multi-industry growth model in transition.[3] We focus on a generalization of a balanced growth path – an equilibrium where the initial condition for the capital stock is chosen so as to satisfy the Euler equation at date zero – which we refer to as an *Euler growth path*. However, our results hold even off the Euler growth path.

Acemoglu and Guerrieri (2008, hereafter AG) assume that both productivity growth rates and capital shares vary across industries. We assume that capital intensity is the same for all industries in our model (as do NP). This allows us to focus on the productivity mechanism in our paper. At the same time, we do not find clear evidence that differences in capital shares are related to the "stages." By contrast, we do find evidence that countries systematically shift resources between industries and sectors with different rates of TFP growth, as predicted by the model.

Also related is Ilyina and Samaniego (2012, henceforth IS). The IS model predicts that R&D-driven TFP growth is the driving force behind structural change, and they conjecture but do not explore the possibility of a U-shaped specialization pattern *over time*, not in relation to GDP per person. Indeed, the literature has not developed an explicit, quantifiable theoretical model that attempts to account for the stages of diversification in IW.

Duarte and Restuccia (2010) examine the impact of productivity differences across countries in agriculture and services on aggregate productivity. Our experiment is different: we assume that the rate of productivity growth in a given industry is constant across countries, and focus on accounting for economic structure rather than aggregate productivity. Our model is also much more disaggregated, allowing us to provide a more detailed picture of industrial structure along the development path at different levels of aggregation. The assumption that the industry TFP growth rate does not vary across countries is driven by the nature of the experiment, as it allows us to talk of high- and low-TFP growth industries, and it is also consistent with the finding in Rodrik (2012) that there is unconditional convergence in labor productivity across countries among disaggregated manufacturing industries.

[3]This is something that to our knowledge has not been done before in an infinite-horizon multi-industry model with capital accumulation, and our methodology may be of independent interest. Rogerson (2008), Duarte and Restuccia (2010) and others compute transition dynamics in growth models with many industries: however, their models do not have capital so there are no intertemporal decisions.

Still, we perform a variety of robustness checks to examine the importance of this assumption, finding that the results are robust to significant variation in TFP growth rates across countries.

The rest of the paper is organized as follows. Section 2 describes the link between industry productivity differences and industrial diversification in a simple heuristic framework. Section 3 presents a general equilibrium model economy with many industries and characterizes the equilibrium. Section 4 calibrates the model economy with a focus on manufacturing, and reports the results concerning the evolution of industrial structure in the model economy within manufacturing. Section 5 calibrates the economy for several sectors and reports results concerning sector level specialization. Section 6 discusses extensions and possibilities for future work.

2 Diversification and TFP Growth differences

We begin by describing the stylized facts of how industrial structure evolves along the development path, and by explaining how productivity differences across industries might account for these facts using a simple model. IW deliver results at the sector level, and also at the industry level within manufacturing. For heuristic purposes in this section we focus on the latter.

2.1 Economic structure along the development path

IW use a nonparametric methodology to investigate the relationship between sectorial diversification and income. Manufacturing industry data are drawn from the INDSTAT3 database distributed by UNIDO, whereas sector-level data are provided by the ILO, and data on aggregate income per capita are from the Penn World Tables. The industry (or sector) share is defined as the share of manufacturing employment.

IW use the Gini coefficient of industry shares $GINI_{c,t}$ to measure the degree of industrial concentration in any country c at date t: the more equal the industry shares (i.e. the lower the Gini), the more diversified the economic structure.[4] Then, they apply a procedure related to robust locally weighted scatterplot smoothing (lowess) to uncover the link between income per capita $GDP_{c,t}$ and specialization. Specifically, they regress the Gini coefficients of industrial specialization on income per capita with country fixed effects, using rolling

[4]IW use a number of other measures of diversification for robustness, as do we.

income windows.

$$GINI_{c,t} = \hat{\alpha}_c(x) + \hat{\beta}(x) GDP_{c,t} + \varepsilon_{c,t},\ GDP_{c,t} \in [x - \Delta/2, x + \Delta/2]. \tag{1}$$

The income interval Δ is fixed in each regression ($\$5,000$ in 1985 dollars) and the midpoint x of the interval gradually moves away from the previous income range (the increment across regressions is $\$25$). Then, they plot the fitted Gini coefficients estimated at the midpoint of the income interval in each regression. They find a U-shaped relationship between Gini coefficients and income levels. Their U-shaped relationship is robust across sectors (ILO data) and within manufacturing (UNIDO data). Figure 1 reproduces their main results.

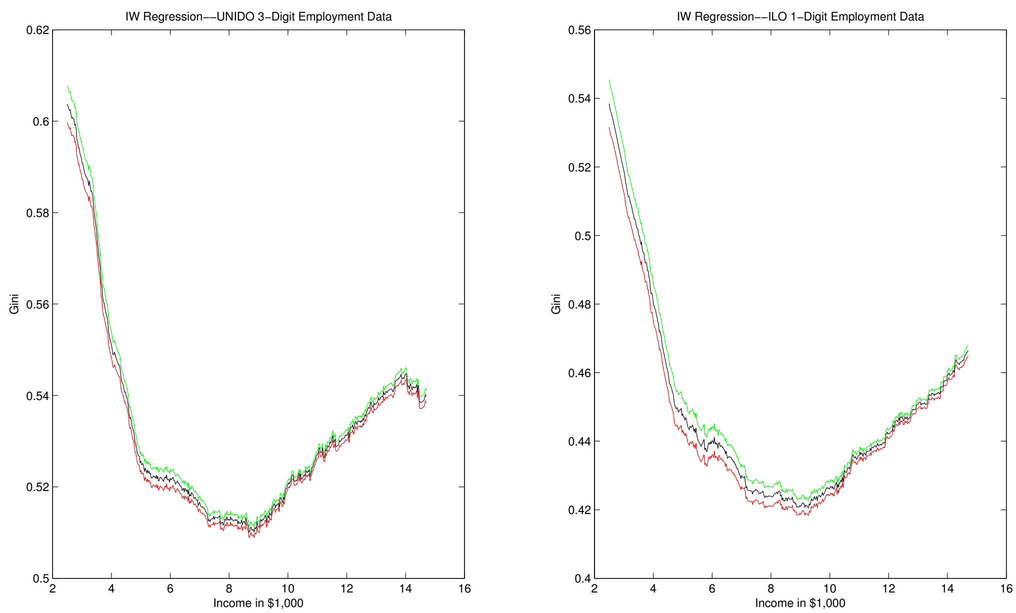

Figure 1. IW results. Each point in the Figure is the fitted value $\hat{\alpha}_c(x) + \hat{\beta}(x)x$ from equation (1), where x is GDP per capita. Confidence bands represent two standard errors of the coefficient $\hat{\beta}(x)$. The left panel is industry concentration within manufacturing estimated using INDSTAT3 data provided by UNIDO. The right panel is sectorial concentration across the entire economy estimated using ILO data.

2.2 Productivity and economic structure

To illustrate the main mechanisms in our model, consider the following simple setup. Suppose there are N competitive industries, with production functions of the form:

$$y_{it} = A_{it} K_{it}^{\alpha} n_{it}^{1-\alpha} \tag{2}$$

where $A_{it} = A_{i0}g_i^t$. The growth factor g_i may vary across industries, but capital shares α are assumed to be constant to highlight the productivity mechanism. Producers solve the problem

$$\max_{k_{it},n_{it}} \{p_{it}y_{it} - w_t n_{it} - r_t K_{it}\} \tag{3}$$

subject to (2), where p_{it} is the price of good i, w_t is the wage and r_t is the rental rate of capital. For now, the series for w_t and r_t may be arbitrary.

Assume these goods are consumed and that preferences are CES, so that, if $\mathbf{y}_t = \{y_{1t}, ..., y_{Nt}\}$, then

$$u(\mathbf{y}_t) = \left[\sum_{i=1}^{N} \xi_i \times y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \sum_{i=1}^{N} \xi_i = 1 \tag{4}$$

where ε is the elasticity of substitution among goods.

Let v_{it} be value added in industry i, so $v_{it} = p_{it}y_{it}$ where p_{it} is the price of good i. Then define the growth factor of value added G_{it} as:

$$G_{it} = v_{i,t+1}/v_{it}.$$

On the demand side, the consumer's first order conditions imply $\frac{p_{it}}{p_{jt}} = \left(\frac{y_{j,t}}{y_{i,t}}\right)^{\frac{1}{\varepsilon_s}} \frac{\xi_i}{\xi_j}$, so that

$$\frac{G_{it}}{G_{jt}} = \left[\frac{\frac{p_{i,t+1}}{p_{it}}}{\frac{p_{j,t+1}}{p_{jt}}}\right]^{1-\varepsilon}. \tag{5}$$

On the supply side, the optimal capital labor ratio is a constant across industries, so that $\frac{A_{it}}{A_{jt}} = \frac{p_{jt}}{p_{it}}$.[5] Thus, for any industries i and j, $\left(\frac{p_{i,t+1}}{p_{it}}\right) \div \left(\frac{p_{j,t+1}}{p_{jt}}\right) = \left(\frac{g_i}{g_j}\right)^{-1}$. In equilibrium (5) becomes:[6]

$$\frac{G_{it}}{G_{jt}} = \left[\frac{g_i}{g_j}\right]^{\varepsilon-1}. \tag{6}$$

Let $s_{i,t}$ be the share of manufacturing of industry i at date t. Given shares $s_{i,t}$ for one year t, we can compute shares for the next year $t+1$ by multiplying $s_{i,t}$ by $g_i^{\varepsilon-1}$ and

[5] To see this, the conditions can be written $p_{it}\alpha y_{it}/K_{it} = r_{it}$ and $p_{it}(1-\alpha)y_{it}/n_{it} = w_{it}$. Dividing one condition by the other we get that $\frac{1-\alpha}{\alpha}\left(\frac{K_{it}}{n_{it}}\right) = \frac{w_t}{r_t}$. Then, dividing any of these conditions for industry i by that for j yields the result.

[6] Notice that, while we defined $G_{it} = v_{i,t+1}/v_{it}$, (6) would also hold if $G_{it} = n_{i,t+1}/n_{it}$. to see this, remember the household's first order conditions imply that $\frac{p_{it}}{p_{jt}} = \left(\frac{y_{j,t}}{y_{i,t}}\right)^{\frac{1}{\varepsilon_s}} \frac{\xi_{s,i}}{\xi_{s,j}}$. Plugging in the production functions and recalling that capital labor ratios are constant across industries yields $\frac{p_{it}}{p_{jt}} = \left(\frac{A_{jt}n_{jt}}{A_{it}n_{it}^{1-\alpha}}\right)^{\frac{1}{\varepsilon_s}} \frac{\xi_{s,i}}{\xi_{s,j}}$. Rearranging, we have that $\frac{n_{it}}{n_{jt}} = \frac{p_{it}y_{it}}{p_{jt}y_{jt}}$.

repeating this procedure to get predicted shares for as many years as desired[7]. Thus, given initial conditions, a value of ε, and productivity growth factors g_i, we can compute model-generated industry shares of manufacturing, and subject the resulting industry structure to the same nonparametric methodology as in IW to study whether productivity differences might be able to generate a U-shaped specialization pattern.

This might occur if "initial" industry composition is skewed towards low-tech industries. For example, suppose that $N = 2$ and that "specialization" is measured using the Gini coefficient. If s_{jt} is the share of industry j, then the Gini coefficient equals $0.5 - \min\{s_{1t}, 1 - s_{1t}\}$.[8] Now suppose that $g_1 < g_2$. Then, if $\varepsilon > 1$, for a sufficiently low initial share of industry 1 the economy will start off specialized in industry 1 whereas in all periods thereafter the share of 2 will increase and that of 1 will decrease. Thus, the minimum of the two (s_{2t}) will rise until it reaches 0.5 and the Gini coefficient has dropped to 0. After this, the minimum of the two becomes s_{1t} and, as its share continues to decrease, the Gini coefficient rises again. Thus, for a time, specialization decreases, until s_{1t} drops below half – after which specialization will begin increasing again. Alternatively, if $\varepsilon < 1$, for sufficiently low initial productivity in 2 the economy will start off specialized in industry 2, whereas in all periods thereafter the share of 1 will increase, and the same dynamics obtain.

We now examine whether the heuristic model presented above can generate a U-shaped specialization pattern for the 28 manufacturing industries examined in IW. We use the initial industry shares in 1963 from the UNIDO employment data, and simulate a time series of future industry shares until 1992 using equation (6). We then include the same country-time pairs as IW, so that we have a model-generated unbalanced panel that is of the same dimensions as that in the IW database. We simulate industry shares for the 28 manufacturing industries in the ISIC revision 2 industry classification used by the UNIDO INDSTAT3 database, from 1964 until 1992 given the initial share in 1963 drawn from the UNIDO employment data. To perform this simulation we adopt the value $\varepsilon = 3.75$, which is estimated in Ilyina and Samaniego (2012) by observing that the logarithm of (6) indicates that regressing value-added growth rates (or employment growth rates) on TFP growth rates yields a coefficient equal to $\varepsilon - 1$.[9] Finally, TFP growth data are computed using the

[7] Literally, this procedure would yield shares that do not add to one. To be precise, let $z_{i,t+1} = g_i^{\varepsilon-1} s_{i,t}$,Then $s_{i,t+1} = \frac{z_{i,t+1}}{\sum_{n=1}^{N} z_{n,t+1}}$.

[8] To see this, note that the Lorenz curve of industry composition when $N = 2$ is a line joining $(0, 0)$ to $(0.5, \min\{s_1, s_2\})$ and another line joining $(0.5, \min\{s_1, s_2\})$ to $(1, 1)$. The Gini coefficient is defined as the integral of the area above this line.

[9] To see this, consider that (6) is equivalent to $\log G_i = \alpha + (\varepsilon - 1)\log g_i + \epsilon_i$ where $\alpha = \log G_j - \log g_j$

NBER manufacturing productivity database. Note that the NBER industry classification is 4-digit SIC. We use Domar weights to convert NBER SIC industry TFP growth data into ISIC revision2 data (see Table 5 in the Appendix for values). The value of g_i is the industry average over time. GDP per-capita data are drawn directly from the data for each country-year combination.[10]

For the sector level results, we calibrate the basic model independently. We take the manufacturing sector value of g_i to equal the average value in the NBER productivity database. Then we calibrate g_i for the other sectors using their inverse price growth rates relative to the price of manufacturing, using price data drawn from the US Bureau of Economic Analysis.

2.3 Basic Model: Results

We regress Gini coefficients generated from our TFP growth simulation on income per capita for countries and periods, following the IW methodology. Our results display a similar U-shaped relation between sector concentration and income levels: see Figure 2. In addition, in the case of manufacturing, the turning point is roughly $\$9,000$, as found by IW, something that lends weight to the empirical relevance of the productivity mechanism. Across broad sectors, the turning point is a bit lower, around $\$6,000$.

for some arbitrary industry j and ϵ_i is any unmodeled noise in the relationship. IS estimate this coefficient using the industry TFP and value-added data reported in Jorgenson et al (2007).

[10] It is worth mentioning that the correlation between the NBER productivity values and US industry employment growth in the UNIDO database is 0.46**.

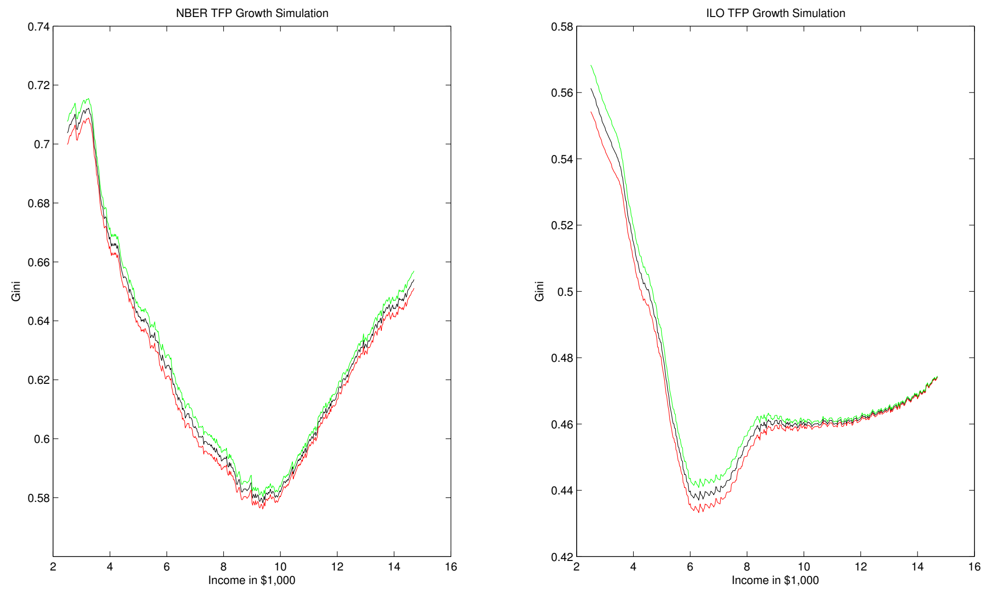

Figure 2. Industry structure along the development path in the simple model. The left panel is the relationship between income and specialization within manufacturing reported in IW. The right panel is the same relationship in the pseudo-data generated using equation (6).

As a robustness check, we use an alternative way of measuring TFP growth rates for manufacturing. Using the UNIDO dataset, we compute the TFP growth rates for the 28 UNIDO manufacturing industries in the United States using the following equation:[11]

$$\ln(TFP_{it}) = \ln(Y_{it}) - (1-\alpha)\ln(L_{it}) - \alpha \ln(K_{it}) \tag{7}$$

where Y_{it} is the production index, L_{it} is the total amount of labor and K_{it} is capital used in industry i at time t. See the Appendix for details.

Also we compute industry price growth rates as a robustness check, so that equation (5) rather than (6) dictates industry dynamics. The price index is computed using value added

[11] It is an important part of the experiment that industry TFP growth rates be the same across countries: all that varies are initial conditions. When we used this procedure to measure industry TFP growth rates in different countries we found that the estimated values in some countries were sometimes absurdly high. We interpret this as indicating that the input data in those countries are likely mismeasured. This implies that we cannot reliably estimate country-specific industry growth rates using the UNIDO data: however, this does not affect the usefulness of the reported initial conditions, which do not depend on input data.

divided by the production index from the UNIDO dataset.[12] Both TFP and price growth rates (in Table 6, see Appendix) are averages over the period $1963-1992$ and assumed to be the same for all countries. TFP growth rates computed this way are highly correlated with those derived from the NBER data, with a correlation coefficient of 0.6 (significant at the 5 percent level). The TFP growth and price growth series based on UNIDO data are highly negatively correlated with a coefficient of -0.9 (significant at the 5 percent level). All of this is encouraging as to the robustness of the productivity measures.

We simulate industry shares following equation (5) for UNIDO price growth and (6) for TFP growth and apply nonparametric methodology to model simulated Gini coefficients on income. Again, we obtain a U-shape in both cases, see Figure 3.

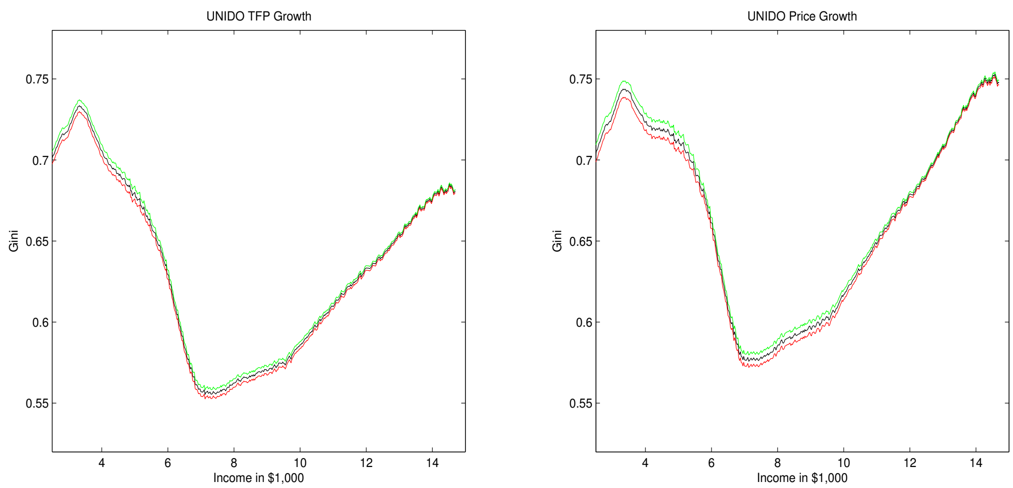

Figure 3. IW nonparametric regression using simulated industry concentration measures based on equation (6) using UNIDO TFP growth rates, and based on equation (5) using UNIDO price growth rates.

There are several reasons why the above results might not extend to the "full" growth model. First, manufacturing can be separated into capital goods and non-capital goods, which serve different purposes and which may hence have different elasticities of substitution. Second, the share of capital goods within manufacturing will be determined by agents' investment behavior, whereas the share of non-capital goods will be determined by their consumption behavior. Third, capital also includes structures, which are built by the construction sector but which is not part of manufacturing. Fourth, the basic model does not

[12] Recall that value added $v_{it} = p_{it}y_{it}$. The assumption is that growth in the UNIDO industrial production index proxies for growth in y_{it}.

generate a series for income per capita: we simply took the data values as given. The general equilibrium model addresses all of these issues to see whether productivity differences can account for the observed evolution of economic structure along the development path within an integrated theoretical framework.

3 Model Economy

We now develop a general equilibrium multi-industry growth model to test whether the mechanisms described above can generate stages of diversification at the industry or sector levels.

3.1 Preferences and Technology

Time is discrete and there is a $[0,1]$ continuum of agents. In the baseline economy, there are S sectors, each of which produces an aggregate of I industries. Let I_s be the set of industries that supplies sector s. We focus on the case in which each industry supplies only one sector, so that $I_s \cap I_{s'} = \varnothing$, $\forall s \neq s'$. Note that this is without loss of generality, as one could have two industries identical in all ways that are distinguished by the fact that they provide a given good to two different sectors.

We assume that sectors $s \in \{1, ..., S-1\}$ produce consumption goods. Only one sector, S, produces capital goods.[13] Now for each sector $s \in \{1, ...S\}$, the production function has the CES form:

$$y_{st} = \left[\sum_{i \in I_s} \xi_i \times u_{s,i,t}^{\frac{\varepsilon_s - 1}{\varepsilon_s}} \right]^{\frac{\varepsilon_s}{\varepsilon_s - 1}}, \; \sum_{i \in I_s} \xi_i = 1, \qquad s = 1, ..., S \tag{8}$$

where u_{sit} is use of good i by sector s, ξ_i is the weight on good i, and ε_s is the elasticity of substitution among goods within sector s.

Agents consume a CES aggregate c_t of the output of the different consumption sectors:

$$c_t = \left[\sum_{s=1}^{S-1} \zeta_s y_{st}^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}.$$

Finally, agents have isoelastic preferences over c_t and discount the future using a factor $\beta < 1$, so that:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta} - 1}{1 - \theta}. \tag{9}$$

[13]We abstract from intermediate goods. As we discuss later, the results are robust to allowing for intermediates.

They are endowed with one unit of labor every period which they supply inelastically, and start period zero with capital K_0. Let q_{st} be the price of the sector aggregate s, with r_t as the interest rate and w_t as the wage. Agents choose expenditure on each good so as to maximize (9) subject to the budget constraint

$$\sum_{s=1}^{S} q_{st} y_{st} \leq \sum_{s=1}^{S} \sum_{i \in I_s} r_t K_{it} + \sum_{s=1}^{S} \sum_{i \in I_s} w_t n_{it} \tag{10}$$

and the capital accumulation equation

$$K_{t+1} = y_{St} + (1 - \delta) K_t. \tag{11}$$

On the supply side, each industry features a Cobb-Douglas production function:

$$y_{it} = A_{it} K_{it}^{\alpha} n_{it}^{1-\alpha}, \;\; A_{it} = A_{i0} g_i^t \tag{12}$$

where $g_i = A_{i,t+1}/A_{it}$ is the TFP growth factor of industry i and A_{i0} is given. Producers maximize profits

$$\max_{n_{it}, K_{it}} \{p_{it} y_{it} - w_t n_{it} - r_t K_{it}\} \tag{13}$$

subject to (12), where p_{it} is the output price of industry i at time t. Capital and labor are freely mobile across sectors.

3.2 Equilibrium

The producers' first order conditions imply that the capital labor ratio is constant across industries, which implies that $A_{it} p_{it} = A_{jt} p_{jt}$. Thus, as in related models, goods that experience rapid productivity growth display a decline in their relative price. This result, combined with the consumer's first order conditions implies that the ratio of value added $p_{it} y_{it}$ in any two industries in the same sector s depends on preference parameters and the productivity terms.

$$\frac{p_{it} y_{it}}{p_{jt} y_{jt}} = \left(\frac{\xi_{s,i}}{\xi_{s,j}} \right)^{\varepsilon_s} \left(\frac{A_{it}}{A_{jt}} \right)^{\varepsilon_s - 1} = \frac{n_{it}}{n_{jt}} \qquad \forall s \tag{14}$$

Notice that the same relationship holds for the ratio of employment – just as with the basic model – except that it only holds comparing industries that are in the same sector.

Define the growth factor of employment (or value added) in industry i as

$$G_{it} \equiv \frac{n_{i,t+1}}{n_{i,t}} = \frac{p_{i,t+1} y_{i,t+1}}{p_{it} y_{it}}. \tag{15}$$

Then, the expression G_{it}/G_{jt} then denotes the growth of employment (or value added) in industry i relative to industry j. Using (14) we have that

$$\frac{G_{it}}{G_{j,t}} = \left(\frac{g_i}{g_j}\right)^{\varepsilon_s - 1} \qquad \forall s. \tag{16}$$

Consequently, within sectors, structural change depends on relative TFP growth factors $\frac{g_i}{g_j}$ and on the elasticity of substitution ε_s. For comparing industries *across* sectors requires characterizing shifts in expenditure across sectors, as well as investment behavior.

3.3 Sectorial and Aggregate Growth

Notice that in equilibrium we can aggregate the industries in a given sector into a sectorial production function. To see this, define q_{st} as the price index for final goods in sector s, so that

$$q_{st} y_{st} = \sum_{i \in I_S} p_{it} A_{it} k_t^{\alpha} n_{it}$$

where k_t is the equilibrium capital-labor ratio, which is common across industries. Define input use in sector s as $K_{st} = \sum_{i \in I_s} K_{it}$ and $n_{st} = \sum_{i \in I_s} n_{it}$. Then, define a sectorial production function:

$$y_{st} = A_{st} K_{st}^{\alpha} n_{st}^{1-\alpha}, \; A_{st} = A_{s0} \bar{g}_s^t \tag{17}$$

The problem of the sector firm and the industry firms can be combined as

$$\max_{n_{it}} q_{st} \left[\sum_{i \in I_s} \xi_i \times \left(A_{it} k_t^{\alpha} n_{it}\right)^{\frac{\varepsilon_s - 1}{\varepsilon_s}} \right]^{\frac{\varepsilon_s}{\varepsilon_s - 1}} - r_t k_t \sum n_{it} - w_t \sum n_{it} \tag{18}$$

The first order conditions imply that:

$$\frac{n_j}{n_i} = \left(\frac{\xi_j}{\xi_i}\right)^{\varepsilon} \left(\frac{A_i}{A_j}\right)^{1-\varepsilon} \tag{19}$$

We also have that $\sum_i n_i = n_s$ by definition, so we can use (19) write n_i in terms of n_s. Substituting this back into the problem (18), we have

$$\max_{n_{it}} q_{st} A_{st} k_t^{\alpha} n_s - rkn_{st} - wn_{st}$$

where

$$A_{st} = \left[\sum_{i \in I} \xi_{s,i}^{\varepsilon_s} \times A_{it}^{\varepsilon_s - 1} \right]^{\frac{1}{\varepsilon_s - 1}} = \left[\sum_{i \in I} \xi_{s,i}^{\varepsilon_s} \times A_{i0}^{\varepsilon_s - 1} g_i^{t(\varepsilon_s - 1)} \right]^{\frac{1}{\varepsilon_s - 1}} \tag{20}$$

and

$$\bar{g}_s = \prod_{i \in I_s} g_i^{x_{it}/X_{st}} \tag{21}$$

where

$$x_{it} = \xi_{s,i}^{\varepsilon_s} A_{it}^{\varepsilon_s - 1},\ X_{st} = \sum_{i \in I_s} x_{it}.$$

Since the total production of consumption sectors $c_t = \left[\sum_{s=1}^{S-1} \zeta_s y_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$, we can also aggregate all the consumption goods production sectors. Then we have that

$$c_t = A_{c_t} K_{ct}^{\alpha} n_{ct}^{1-\alpha},\ A_{c_t} = \left[\sum_{s=1}^{S-1} \zeta_s^{\varepsilon} \times A_{st}^{\varepsilon - 1}\right]^{\frac{1}{\varepsilon-1}} \tag{22}$$

As a result, the aggregate behavior of the model economy with many sectors is the same as that of a 2-sector economy that produces c_t using technology (22) and produces capital goods using technology (17). In the consumption goods sector, firms maximize

$$\max_{K_{c_t}, n_{c_t}} \left\{p_{c_t} A_{c_t} K_{c_t}^{\alpha} n_{c_t}^{1-\alpha} - r_t K_{c_t} - w_t n_{c_t}\right\}$$

$$\text{where} \qquad A_{c_t} = \left[\sum_{s=1}^{S-1} \zeta_s^{\varepsilon} \times A_{st}^{\varepsilon-1}\right]^{\frac{1}{\varepsilon-1}}$$

whereas in the capital goods sector:

$$\max_{K_{h_t}, n_{h_t}} \left\{p_{h_t} A_{h_t} K_{h_t}^{\alpha} n_{h_t}^{1-\alpha} - r_t K_{h_t} - w_t n_{h_t}\right\}$$

$$\text{where} \qquad A_{h_t} = \left[\sum_{i \in I_S} \xi_i^{\varepsilon_S} \times A_{it}^{\varepsilon_S - 1}\right]^{\frac{1}{\varepsilon_S - 1}}$$

Consumers choose consumption c_t and investment h_t to solve:

$$\max_{c_t, h_t} \left\{\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta} - 1}{1-\theta}\right\} \tag{23}$$

$$s.t. \qquad p_{c_t} c_t + p_{h_t} h_t \leq r_t K_t + w_t \tag{24}$$

$$K_{t+1} = K_t (1 - \delta) + h_t \tag{25}$$

$$K_0 \text{ given.} \tag{26}$$

In equilibrium, capital and labor markets must clear at all dates, so

$$c_t = A_{c_t} K_{c_t}^{\alpha} n_{c_t}^{1-\alpha} \tag{27}$$

$$h_t = A_{h_t} K_{h_t}^{\alpha} n_{h_t}^{1-\alpha}$$

$$K_t = K_{h_t} + K_{c_t} \tag{28}$$

$$n_{c_t} + n_{h_t} = 1 \tag{29}$$

It will be convenient to set $p_{h_t} = 1 \forall t$, so that consumption goods prices p_{c_t} are expressed relative price to the price of capital goods.

Solving the 2-sector problem and using the equilibrium conditions, we obtain expressions for labor shares in the capital goods sector n_{ht} and the consumption goods' sector $n_{ct} = 1 - n_{ht}$ along an unbalanced growth path[14]. These turn out to be functions only of the productivity growth rates g_i, parameters, and of the equilibrium growth rate of aggregate consumption $g_{c_t} = \frac{p_{c,t+1}c_{t+1}}{p_{ct}c_t}$ which is endogenous. This will be true at all dates except possibly date zero, where n_{ht} is determined by the initial condition K_0.

Define real GDP as $y_t = h_t + p_{ct}c_t$. Notice it is measured in units of capital.

Proposition 1 *Equilibrium exists and is unique. In equilibrium, the growth factors of total capital K,capital per capita k, and total output y depend on the growth factors of TFP in the consumption and capital sectors and on the growth factor of consumption sector (as well as parameters):*

$$g_{k_t} = \frac{k_{t+1}}{k_t} = \frac{K_{t+1}}{K_t} = \overline{g}_{A_{ht}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}} \right)^{\frac{1}{1-\alpha}} \tag{30}$$

and

$$g_{yt} = \frac{y_{t+1}}{y_t} = \overline{g}_{A_{ht}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}} \right)^{\frac{\alpha}{1-\alpha}} \tag{31}$$

where GDP is defined as $y_t = h_t + p_{ct}c_t$ and the equilibrium interest rate is $r_t = \frac{\left(\frac{\overline{g}_{A_{ht-1}}}{\overline{g}_{A_{ct-1}}}\right)^{1-\theta} g_{c_{t-1}}^{\theta}}{\beta} - 1 + \delta$ for $t > 0$. At date zero, r_0 is determined by market clearing given K_0.

Proposition 2 *The model economy converges to a balanced growth path where in each sector*

$$\lim_{t\to\infty} A_{st} = A_{jt} \text{ where } j = \begin{cases} \arg\max_{i\in I_s} \{A_i\} & \text{if } \varepsilon_s > 1 \\ \arg\min_{i\in I_s} \{A_i\} & \text{if } \varepsilon_s < 1 \end{cases},$$

and

$$\lim_{t\to\infty} A_{ct} = A_{jt} \text{ where } j = \begin{cases} \arg\max_{s<S} \{A_s\} & \text{if } \varepsilon > 1 \\ \arg\min_{s<S} \{A_s\} & \text{if } \varepsilon < 1 \end{cases}.$$

[14] We do not focus on the balanced growth path (BGP), because BGP results understate the impact of productivity differences on stages of diversification. One reason is that we ruled out SoD among capital producing industries when assuming Cobb-Douglas production (which leads to constant growth rate and is required for the existence of BGP) in the capital sector. Moreover, from our estimations of the elasticity of substitution for both capital and non-capital manufacturing sectors, we find the elasticities of substitution in both sectors are not statistically different from each other, and the elasticity of substitution of capital sector is statistically different from one. So we abandon C-D production function in capital sector. Instead, we use CES function for capital sector and focus on an unbalanced growth path in this paper.

Recalling that the only endogenous variable that affects r_t for $t > 0$ is g_{ct},[15] Proposition 1 implies that we can compute the equilibrium for th multi-industry model economy in transition, provided we can derive the series for g_{ct}. The economy with many consumption goods sectors will asymptotically converge to an economy with one consumption sector which has either the highest or lowest TFP growth rate depending on the elasticity of substitution. The same occurs within the capital goods sector. As a result, the expression r_t converges to some constant r and, although in general the model does not possess a balanced growth path (see Ngai and Pissarides (2007)), it converges to one. This suggests that the equilibrium may be computed by finding a sufficiently good approximation to the series for g_{ct}. In the limit, since by assumption $\varepsilon_s \neq 1$ for all $s \leq S$, one industry will end up dominating each sector. However, we wish to study the behavior of the model economy in transition, where sectors are relatively diversified.

4 UNIDO Calibration: Manufacturing

In the remainder of the paper we will focus on a particular type of equilibrium. Observe that the capital stock will be set to satisfy the Euler equation (30) at all dates except date zero. In other words, the investment share of the model economy will in general be smooth over time, except between dates zero and one. The model will be calibrated to the available data and, since the initial year in which data for a given country become available has no economic content, it is difficult to justify why the first year we have data for (generally 1963) happens to be the only date when the intertemporal optimization (30) is not satisfied. For this reason, we focus on an equilibrium where this does not occur.

Definition 3 *An Euler Growth Path (EGP) is an equilibrium and an initial condition K_0 such that equation* (30) *holds at date zero.*

The Euler growth path is a generalization of a balanced growth path which may exist in models that do not exhibit balanced growth. For our benchmark results, we calibrate the model to match an Euler growth path by matching the composition of manufacturing but not necessarily its *size*. Details are in the Appendix. Nonetheless, it is important to underline that our results concerning the structure of the economy turn out not to hinge on whether we focus on an Euler growth path: results on the equilibrium calibrated to match the initial conditions in the data are almost indistinguishable.

[15] In general, at $t = 0$, the value of r_0 is determined by market clearing and the value of K_0.

Table 1: Sectors and Industries in the model economy

	Industries			
Sector	Services, etc	Agriculture	UNIDO	Construction
Services, etc	X	-	-	-
Agriculture	-	X	-	-
Capital Manufacturing	-	-	X	X
Non-Cap Manufacturing	-	-	X	-

Calibrating the model economy requires a choice of industries, and values of the following parameters and variables.

1. Technological parameters α, δ.

2. Preference coefficients $\xi_{s,i}, \zeta_i, \beta$.

3. Elasticities of substitution ε_s for $s \leq S$, and ε, the elasticity across consumption sectors

4. The intertemporal elasticity parameter θ.

5. Productivity growth values g_i.

6. Productivity initial conditions $A_{i0.}$

We provide two selections of industries. In this section, we calibrate the model so as to focus on the "stages" in manufacturing in the UNIDO data. In the following section, we disaggregate the non-manufacturing sector further to focus on the "stages" across broad sectors in the ILO data. We calibrate the model twice because the data required for the ILO sector calibration are available for fewer countries. We refer to them as the UNIDO or manufacturing calibration and the ILO or sector calibration. Details are provided below. The simulation requires computing transition dynamics in a model without a balanced growth path, and the procedure is described in the Appendix for the interested reader.

For the UNIDO calibration, we group all industries into four sectors: Agriculture, Services, Capital and Non-capital manufacturing. Agriculture, services and non-capital manufacturing sectors produce consumption goods, and the capital sector only produces capital goods. Industries include agriculture, services, the 28 UNIDO manufacturing industries, and

Table 2: Capital good-producing manufacturing industries

Industry	ISIC code
Wood products	331
Furniture, except metal	332
Fabricated metal products	381
Machinery, except electrical	382
Machinery, electric	383
Transport equipment	384
Prof. & sci. equip.	385
Other manufactured prod.	390

construction (see Table 1). Thus, the agriculture and services industries only contain one industry. The UNIDO industries serve either the capital or the non-capital manufacturing sectors. We assigned an industry to the capital sector if the US NIPA tables count it in their "fixed asset" tables (see Table 2). Construction serves capital sector too. The initial shares of agriculture, services, manufacturing and construction sectors out of GDP are derived from World Development Indicators data (WDI).[16]

1. We assume that $\delta = 0.06$ as in Greenwood et al (1997): this is a standard values in models in which the productivity of the investment technology exceeds that in the consumption sector. We use a standard value for the capital share, $\alpha = 0.3$.

2. To calibrate the utility weights $\xi_{s,i}$, it should be noted that in a sense these weights are arbitrary, as they depend on the exact unit of measurement for good i.[17] Thus, without loss of generality, we set $\xi_{s,i} = \frac{1}{I_s}$, where I_s is the number of industries in sector s. The same applies to ζ_i, so $\zeta_i = \frac{1}{S-1}$. We set $\beta = 0.95$, a standard value.

3. For each sector, equation (16) is equivalent to $\log G_i = \alpha + (\varepsilon - 1) \log g_i + \epsilon_i$ where $\alpha = \log G_j - \log g_j$ for some arbitrary industry j and ϵ_i is any unmodeled noise in

[16] For countries with missing data, we use predicted values computed by regressing sector shares on income, income squared and UNIDO industry shares in the manufacturing sector for all countries and years in our sample.

[17] For example, if I measure apples and get $\xi_{s,apples} = 2$ (and $A_{apples,0} = 3$), I could choose to measure apples in units of "half an apple" and then $\xi_{s,apples} = 1$ (and $A_{apples,0} = 1.5$).

the relationship. We regress U.S. value added growth rates on TFP growth rates[18] for capital and non-capital manufacturing goods respectively, finding that they were not statistically significantly different: $\varepsilon_{noncapmanuf} = \varepsilon_{capital}$. Pooling the data, we estimate that $\varepsilon_{noncapmanuf} = \varepsilon_{capital} = 3.73$. Across sectors, we use the value $\varepsilon = 0.3$, which is the estimate in Ngai and Pissarides (2004). We will examine other values of $\varepsilon \in [0.1, 0.9]$ and find results to be visually indistinguishable, as the value of ε has negligible impact on what occurs *within* the manufacturing sector.

4. The preference parameter θ is calibrated so that in the long run the investment share of GDP converges to 12 percent, which is roughly the share in the US: investment shares in transition turn out not to be very different. This implies that $\theta = 3$: typical values used in calibration fall in the range $\theta \in [1, 5]$,[19] so it is encouraging that our value falls in the middle.

5. Productivity growth values g_i are drawn from the NBER productivity database, as described in Section 2. We use the average value over the period 1963-1992. See Table 5. To calibrate the growth factors of the consumption goods sectors, first we use equation (20) to compute TFP growth in the capital sector (excluding construction) over the period 1963-1992, and get the average value $\widetilde{g}_h = 1.0241$. According to NIPA, the relative price of construction has risen at a rate of 0.0109 each year relative to other capital. This means the growth factor of construction sector $g_{construction} = \tilde{g}_h / e^{0.0109} = 1.0130$. For the services sector, the relative price of services has risen at a rate of 0.0103 each year relative to other capital. This means the growth factor of services sector $g_{services}$ is then $\tilde{g}_h / e^{0.0103} = 1.0136$. For agriculture, we have that the relative price of agriculture has dropped at a rate of 0.004 each year relative to other capital. So the growth factor of the agricultural sector is $g_{agriculture} = \tilde{g}_h / e^{-0.004} = 1.0282$.

6. For the initial productivities of the capital and consumption sectors, we initially set $A_{capital,0} = 1$ and $A_{consumption,0} = 1$. The former is a normalization, and the latter is without loss of generality because the size of the non-investment sectors is independent of the level of $A_{consumption,0} = 1$.[20] Then, using (14) and (20), for the capital sector

[18] We use data from Jorgenson et al (2007): although they are a little more disaggregated, we want a value estimated at roughly the same level of aggregation as the UNIDO data. The UNIDO data themselves are too few so we were unable to obtain a good estimate from them directly.

[19] Growth models tend to use $\theta = 1$, whereas asset pricing studies tend to use larger values. For an example with $\theta = 5$, see for example Jermann (1998).

[20] Proof available upon request.

Table 3: Calibrated Parameters: Baseline Model

$g_{construction}$	$g_{services}$	$g_{agriculture}$	$\varepsilon_{capital}$	$\varepsilon_{consumption}$
1.0130	1.0136	1.0282	3.73	0.3

$\varepsilon_{noncapmanuf}$	θ	δ	α	β
3.73	3	0.06	0.3	0.95

industries $i \in I_S$, we set initial TFP to equal $A_{i0} = \left[\frac{n_{i0}}{\sum \xi_i^{\varepsilon_S} n_{i0}}\right]^{\frac{1}{\varepsilon_S - 1}}$, thus matching the initial share of capital industries in each country. For the consumption sectors, set A_{s0} (where $s \in \{$services, agriculture and non-capital manufacturing$\}$) so as to match the initial share of that sector in each country: $A_{s0} = \left[\frac{n_{s0}}{\sum \zeta^{\varepsilon} n_{s0}}\right]^{\frac{1}{\varepsilon - 1}}$. Finally, for industry productivity in non-capital manufacturing, we have again that $A_{i0} = \left[\frac{n_{i0} A_{s0}}{\sum \xi_i^{\varepsilon_s} n_i}\right]^{\frac{1}{\varepsilon_s - 1}}$. Industry shares are drawn from UNIDO and sector shares are based on the WDI.[21] Finally, we multiply A_{i0} in all industries and sectors by a country-specific constant so that the country GDP per head relative to US GDP per head in the initial year is the same as in the data.

4.1 Simulation

For each country we use initial conditions from 1963[22] as starting points, and simulate the share of GDP n_{it} of any industry or sector along unbalanced growth path.[23]

[21] As mentioned, an adjustment to industry shares is required due to our focus on an EGP: see the Appendix for details.

[22] For some countries initial data in 1963 are not available: then we use the earliest available year.

[23] In our derivations, the model measures GDP in terms of capital goods (remember we normalize capital goods price to 1 and consumption goods prices are expressed as relative to capital goods price). In the data, however, GDP is measured in terms of consumption goods, see Greenwood et al (1997). Since in our model, $A_{h_t} = p_{c_t} A_{c_t}$, we can express the GDP growth factor measured in units of *consumption* using the formula $\widetilde{g}_{yt} = g_{yt} \frac{g_{Ah}}{g_{Ac}}$. This is the notion of GDP we use in the graphs below. The model simulated using GDP defined in terms of capital goods yields very similar results. An issue here is that the values of A_{h_0} and A_{c_0} are arbitrary in the calibration, but not when we wish to express cross-country GDP in common units. We handle this by assuming that the real GDP data are measured in units of consumption and are internationally comparable, and then use the model to compute growth rates (which do not depend on GDP

Figure 4 shows the estimated curve of Gini using industry shares simulated in our baseline model. We can see that our baseline model is able to capture the U-shape of stages of diversification very well. Again, the turning point is around $9,000, as found by IW. Thus, the results derived using the simple model are robust to allowing the composition and size of the capital sector to evolve independently of the non-capital manufacturing sector, and to allowing the model to generate the GDP series as well as just industry structure. It is notable that, in the full growth model, the re-specialization after the turning point is slower than the initial specialization, just as in IW.[24]

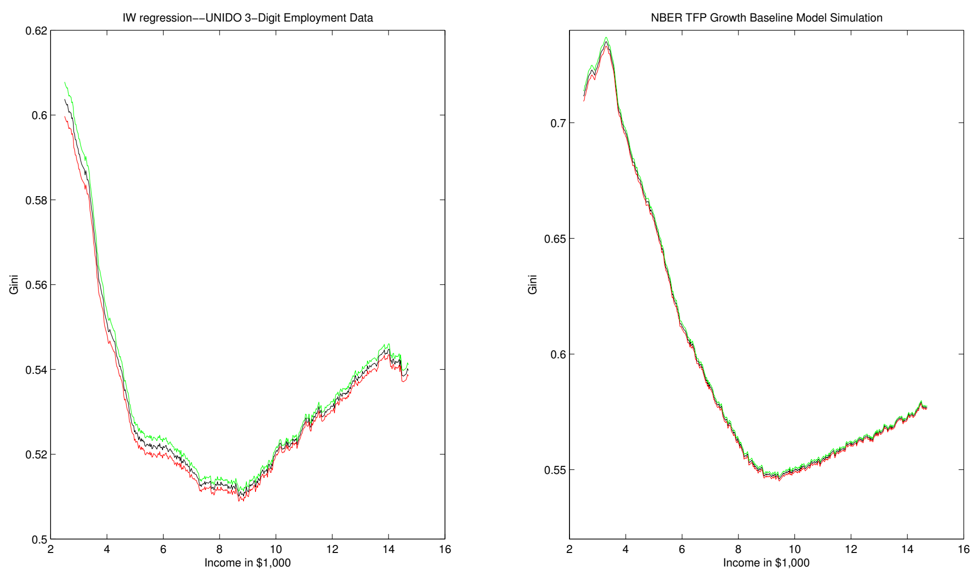

Figure 4. Industry structure along the development path in the full model. The left panel is the relationship between income and specialization within manufacturing reported in IW. The right panel is the same relationship in the pseudo-data generated from the model economy. The range of income is the same as that reported in IW.

For robustness, we also repeat our calibration using the TFP measures in table 6 which were derived from the UNIDO data instead of the NBER data. The simulated Gini displays similar U-shape as our baseline model, and the turning point appears between $8,000 and $10,000. See Appendix.

levels) extrapolating from initial GDP in the data.

[24] If we extend the simulated curve from $15,000 to $20,000, the rising right hand side of the curve continues to increase linearly.

In the model economy, the concept of capital includes any goods that are durable. Some items that are not classified in the fixed asset tables may be thought of as durable goods. As a result we redefined capital in broader terms to include things that could be durable but are not classified as such (e.g. pottery, iron products, and so on). This raises the number of capital goods to 15, plus construction. The calibrated parameters using the alternative classification of capital industry is listed in Table 7 in the Appendix. The simulated industry shares display similar stages of diversification to our baseline model (see Appendix).

5 ILO Calibration: Broad sectors

IW report how economic structure evolves along the development path for 9 broad sectors, drawing on data from the ILO. In this section, we will show that TFP growth differences in our model can explain economic structure through the economy at sector level. We recalibrate the model economy so that $S = 10$, where the capital and non-capital manufacturing sectors are the same as in the UNIDO calibration, whereas the other 8 sectors correspond to the non-manufacturing sectors in the ILO 1-digit data. Thus, whereas before we had capital, non-capital manufacturing, agriculture and services, we know disaggregate services into several new sectors. See Table 4 for the list. Within manufacturing, we still use 28 UNIDO industries, of which 8 produce capital goods as the UNIDO calibration (Table 2). The definition of the capital goods sector is the same as the UNIDO calibration, i.e. 8 UNIDO industries and construction. All other sectors in Table 4 produce consumption goods. For sectorial TFP growth factors, we use the growth in the price of the output of each sector relative to the price of capital (excluding construction), as before (see Table 4).[25] The initial sector shares are taken from the ILO dataset.[26] In our baseline results, we again assume that $\varepsilon = 0.3$. However, we studied values of $\varepsilon \in [0.1, 0.9]$, with generally similar results, as shown below.

Figure 5 shows the estimated link between income and sectoral concentration using the 9 ILO sector shares simulated in our model. It is clear that economic structure at sector level still displays a U-shape. Again, the turning point is around \$9,000 as in the data.

[25] We take the average growth rate of sectoral relative prices over the period 1963-1992. One exception is the mining and quarrying sector, which displays very high variance in the relative price. We take the median value for this sector.

[26] Again, an adjustment is required due to the focus on an EGP. See Appendix.

Table 4: ILO Sectors

	ILO 1-Digit Classification (9 sectors)	Growth Factor g
1	Agriculture, Hunting, Forestry and Fishing	1.0282
2	Mining and Quarrying	1.0196
3	Manufacturing	-
4	Electricity, Gas and Water	1.0042
5	Construction	1.0130
6	Wholesale and Retail Trade and Restaurants and Hotels	1.0150
7	Transport, Storage and Communication	1.0157
8	Financing, Insurance, Real Estate and Business Services	0.9978
9	Community, Social and Personal Services	0.9902

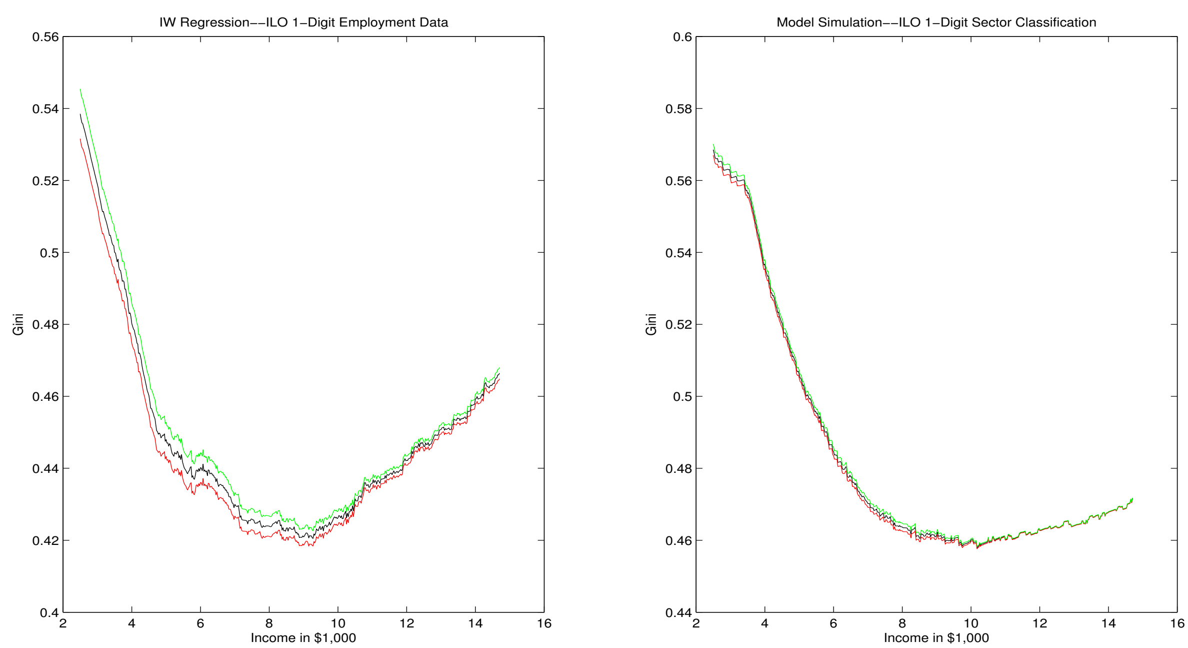

Figure 5. Economic Structure along the Development path: results for the entire economy, using the ILO 1-Digit Sector Classification.

Within the manufacturing sector (for both capital and non-capital manufacturing industries), resources will shift towards high-TFP growth industries, as calibrated elasticities of substitution are above one. However across *sectors*, because the elasticity of substitution across the consumption sectors is *less* than unity, the model predicts that the economy will shift resources towards the *slowest* TFP growth sector. In our calibration, the TFP growth rate community, social and personal services sector is the lowest. So, if an economy starts out being specialized in other sectors, e.g., agriculture, then eventually its resources will re-allocated towards this section of the service sector. During this process of structural change, the economy displays a U-shaped pattern of stages of diversification. Thus we can see that persistent TFP growth differences are able to account for stages of diversification not only within manufacturing sector but also across sectors.[27] Again, we repeated the results using different values of ε, ranging from 0.1 to 0.9. When $\varepsilon = 0.1$, the process of re-specialization was a bit more more rapid, more closely resembling the original IW results in Figure 5. By contrast, when $\varepsilon = 0.9$, the process of re-specialization is more slow. The initial diversification, however, is similar regardless of the value. See Appendix.

Notice that a strong but testable prediction of the model is that, within manufacturing, high-TFP industries should begin to dominate, whereas across sectors the opposite should be the case. We will check this prediction later.

A lot of attention has been devoted to explaining changes in the share of agriculture and services along the development path – see Ngai and Pissarides (2004, 2007), Rogerson (2008) and Duarte and Restuccia (2010), among others. Although not designed to do so, the model matches extremely well the observed changes in the link between agricultural shares and service shares and income. See Figure 6.[28]

[27] We also run the IW regression on the GDP share of agriculture and services in the UNIDO manufacturing calibration model. The shape of the curves are similar to those in the data. See Appendix for the figures.

[28] The model does not produce a hump shape in the share manufacturing, which some authors have focused on (e.g. see Buera and Kaboski (2012)). Pooling lots of countries and using the IW method, the data do seem to display a hump but it is weak compared to the dramatic rise of services and fall in agriculture, which the model does reproduce. Duarte and Restuccia (2010) show that a model with our basic mechanisms plus non-homothetic preferences can reproduce the hump.

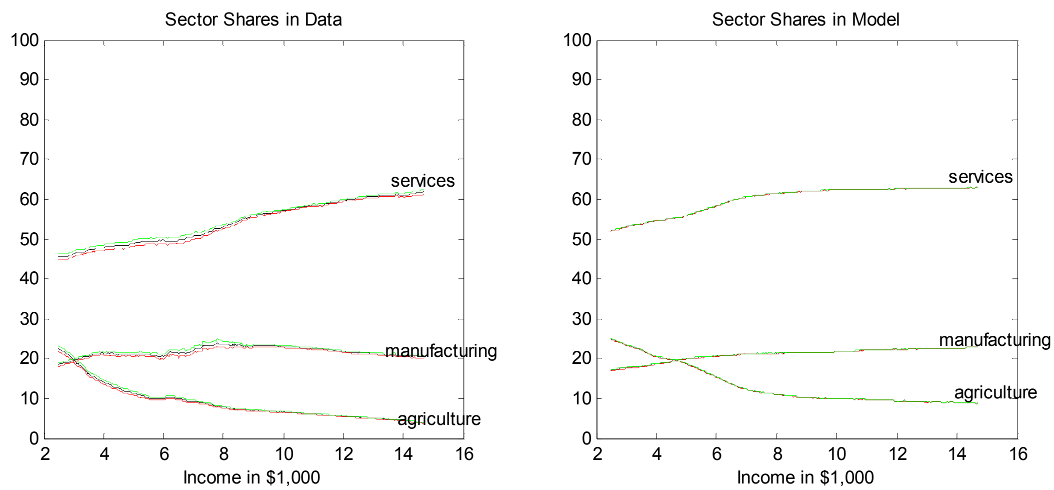

Figure 6. IW regressions for sector shares in data and baseline model

6 Discussion and extensions

In this section we provide some evidence in support of the mechanisms in the paper, and discuss how the results might extend to modifications of the model.

6.1 Productivity and structural change

As noted, the calibrated economy has two strong predictions:

1. within manufacturing, as countries develop they shift resources towards high-TFP growth industries;

2. conversely, across broad sectors, as countries develop they shift resources towards high-TFP growth industries.

To test whether the data support these predictions, we compute a time series for the weighted average of the measure of industry TFP growth rates oin manufacturing for each country and at each date. First, we take the NBER productivity numbers g_i, and normalize them so that the mean measure is zero and the standard deviation is one. Then, for each country at each date, we compute the weighted average TFP growth measure, where the weights are the value added shares of each industry in total manufacturing, computed using UNIDO data. Then, we apply the nonparametric method in IW to this measure, examining its relationship to real GDP per capita. TFP growth rates are assumed constant in each industry across time and across countries, so any patterns are solely due to patterns of specialization among industries with different averege rates of TFP growth.

Figures 7 shows the estimated curve (nonparametric) of manufacturing sector weighted average TFP growth rate and R&D intensity, respectively. There is a mostly positive relationship with income, indicating that, behind the "stages of diversification", economic structure shifts towards manufacturing industries with rapid TFP growth, as predicted by the model. In the ILO data, by contrast, there is a negative relationship, consistent with the assumption that $\varepsilon < 1$ and as predicted by the model. These results strongly support the assumption that TFP growth differences can be a driving force behind structure change along the development path, and it is striking once more that the opposite is happening within manufacturing and across broad sectors.

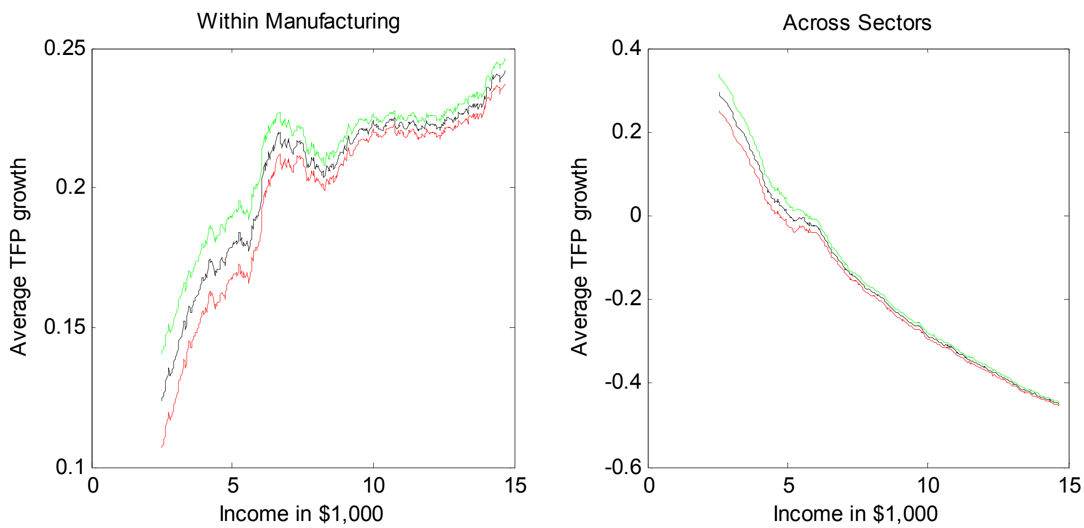

Figure 7 – Trends in average TFP growthwithin manufacturing and across sectors along the development path.

6.2 Other factors of structural change

In the paper we have focused on productivity differences as the mechanism that drives structural change. However, there are other theories of long-run structural change that imply a shift in resources towards particular industries in the long run. As long as countries begin specialized in industries other than those that dominate in the long run, those models too might display stages of diversification.

At least four general equilibrium frameworks have recently been developed to think about long-term structural change:

1. Ngai and Pissarides (2007, NP) emphasize persistent productivity differences across industries, as we do.

2. Ilyina and Samaniego (2012, IS) emphasize productivity differences driven by differences in desired R&D intensity. This theory is not at odds with that of NP, but digs deeper as to the underlying causes of TFP growth differences.

3. Acemoglu and Guerrieri (2008, AG) consider both productivity differences and differences in capital shares. Specifically they predict that TFP growth rates divided by labor shares determine which industries tend to dominate in the long run.

4. Buera et al (2011) argue that structural change is affected by industry differences in firm size, with poorer countries less able to finance large-scale technologies.

To see whether structural change appears related to any of the factors of structural change other than TFP growth rates (R&D intensity, labor intensity, firm size), we repeat the experiment illustrated in Figure 7 and compute series for the weighted average of each of these measures (R&D intensity, etc.) for each country over time. Industry R&D intensity and labor intensity measurements are 3-decade averages of the measures RND and LAB drawn from Ilyina and Samaniego (2011). The industry firm size is the average number of employees per establishment in the US over the period 1963-1992, as reported by UNIDO in INDSTAT3. Again, each measure is normalized so that the mean measure is zero and the standard deviation is one. Then, as before, weighted averages are computed for each country-year, where the weights are value added shares of each industry in total manufacturing, computed using UNIDO data. Finally, we apply the same nonparametric method to these four measures, examining their relationship to real GDP per capita.

Figure 8 shows the estimated curve (nonparametric) for each of these measures. Average R&D displays a positive relationship with income within manufacturing, indicating that, behind the "stages of diversification", economic structure shifts towards industries with rapid TFP growth, and industries with high R&D intensity. Figure 8 also shows the same for the broad sectors in the ILO data:[29] in this case the trend is downwards, consistent with the assumption that $\varepsilon < 1$. These results support the assumption that TFP growth

[29] For most measures, unfortunately we do not have data for sectors outside of manufacturing.

differences can be a driving force behind structure change along the development path, and that TFP growth is related to R&D intensity.

Regarding the other measures, AG argue that differences in labor shares could also be a driving force behind structural change. We also report the estimated curve (nonparametric) of the weighted average labor intensity in the manufacturing sector. We can see that labor intensity shows a hump-shaped relationship with income. In particular, labor intensity declines beyond the income level of $10,000. Thus, structural change seems more closely linked to productivity differences than to differences in labor shares.[30] This justifies our focus on a model with productivity differences, abstracting from differences in labor shares. As for firm size, average firm size declines along the development path, which contradicts the idea that countries are more able to overcome large optimal scales of production as they develop.[31]

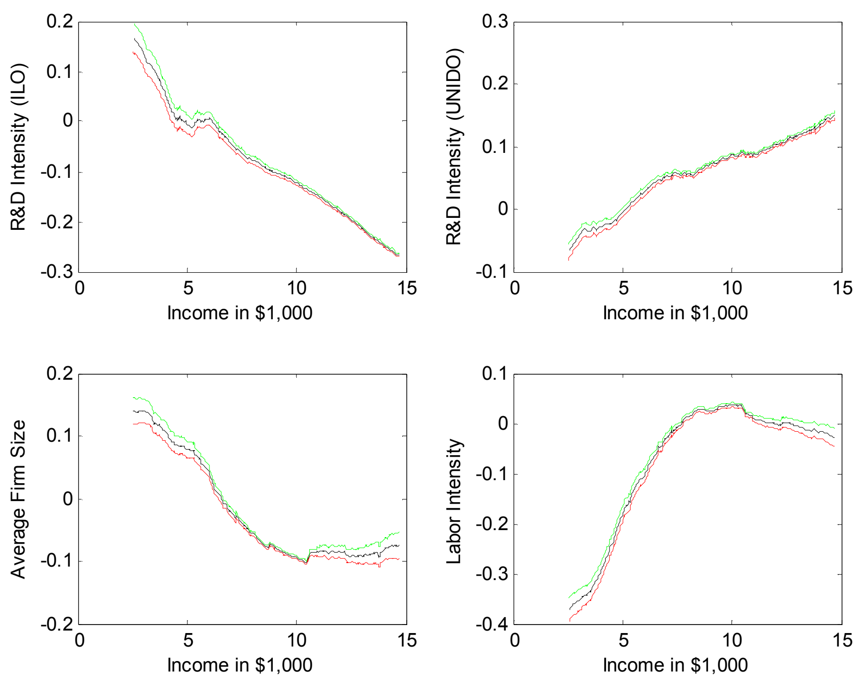

Figure 8 – Trends in average firm size, R&D intensity and labor intensity within manufacturing along the development path.

[30] More specifically, AG argue that the relevant variable is the productivity growth rate divided by the labor share. The weighted average of this variable across manufacturing industries is a hump shape with an upturn towards the right tail.

[31] This result, however, is consistent with high TFP growth industries having a relatively small firm size, as found by Mitchell (2002).

6.3 Intermediate goods

We have abstracted from the existence of intermediate goods. There is a question as to whether results might change if we allowed for intermediates. For one thing, when there are intermediate goods, Ngai and Samaniego (2009) point out that TFP growth measures computed using gross output data (as is the case for the NBER numbers) understate TFP growth in a value added model.

Consider that the IO matrix is largely diagonal: industries tend to use intermediates produced within the industry. Let us abstract from the off-diagonal elements. In this case, the production function is

$$y_{it} = A_{it} \left(K_{it}^{\alpha} n_{it}^{1-\alpha} \right)^{1-\beta} x^{\beta} \tag{32}$$

where x_{it} are intermediate goods and β is the intermediate goods share. Producers solve the problem

$$\max_{k_{it}, n_{it}, x_{it}} \left\{ p_{it} y_{it} - w_t n_{it} - r_t K_{it} - p_{it} x_{it} \right\}. \tag{33}$$

Solving for optimal use of x_{it}, It is easy to show that this is equivalent to solving

$$\max_{k_{it}, n_{it}} \left\{ p_{it} \tilde{y}_{it} - w_t n_{it} - r_t K_{it} \right\}. \tag{34}$$

where $\tilde{y}_{it} = y_{it} - x_{it}$ is value added in terms of good i, with the value-added production function

$$\tilde{y}_{it} = p_{it} \tilde{A}_{it} K_{it}^{\alpha} n_{it}^{1-\alpha} \tag{35}$$

$$\tilde{A}_{it} = A_{it}^{\frac{1}{1-\beta}} \left[\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right].$$

While (35) is of the same form as (12), note that the growth factor of $\tilde{A}_{it}$ is equal to $g_i^{\frac{1}{1-\beta}} > g_i$.

At the same time, this does not matter for the results. Recall that what affects rates of structural change in the model is the combination of g_i and ε. If we regress the log value added growth on the log real value-added productivity factor $g_i^{\frac{1}{1-\beta}}$, we would obtain a different value of epsilon. Recall that (6) is equivalent to $\log G_i = \alpha + (\varepsilon - 1) \log g_i + \epsilon_i$ where $\alpha = \log G_j - \log g_j$ for some arbitrary industry j and ϵ_i is any unmodeled noise. If we use g_i instead of $g_i^{\frac{1}{1-\beta}}$, we would have an estimated elasticity $\tilde{\varepsilon}$ where $\tilde{\varepsilon} - 1 = (\varepsilon - 1)(1 - \beta)$ – a lower value, since $\beta < 1$. However, structural change within sectors would be driven by the following relationship

$$\frac{G_{it}}{G_{jt}} = \left[\frac{g_i^{\frac{1}{1-\beta}}}{g_j^{\frac{1}{1-\beta}}} \right]^{\tilde{\varepsilon}-1} = \left[\frac{g_i}{g_j} \right]^{\frac{\tilde{\varepsilon}-1}{1-\beta}} = \left[\frac{g_i}{g_j} \right]^{\varepsilon-1}, \tag{36}$$

which is exactly equivalent quantitatively to patterns of structural change in our model without intermediates. Thus, significant off-diagonal elements in the input-output tables would be required to change our quantitative results.

A similar intuition regards the possibility of adjustment costs in the reallocation of capital cross industries. Given other parameters, capital adjustment costs could slow the reallocation of resources across industries. However in the presence of adjustment costs the value of ε_s required to match the link between industry growth and TFP growth in the data would be larger. Thus, results would not be affected.

6.4 Country-Specific Productivity Growth

There are many country factors the literature has related to growth which are not featured in the model (see for example Barro (1991) or Sala-i-Martin (1997)). Thus, we would not expect the model to match per capita GDP growth rates around the world.[32] Still, one might ask whether modifying the model so as to match country GDP growth rates might affect the results regarding economic structure. To check, we add country-specific productivity growth factor that affects all industries, and calibrate it to match average GDP growth rates in each country over the sample period. This factor could be interpreted as capturing policies that affect technological diffusion, trends in policy, or any of the factors commonly included in growth regressions. When we do this to the baseline model (the manufacturing calibration), we find that results are almost identical, just that the curve appears slightly stretched to the right (see Figure 9). From all the experiments discussed in our paper, we can conclude that the U-shape generated in our TFP growth differences driven model is robust.

[32] In general we do not find a robust statistically significant correlation between model GDP growth rates and those in the data, even though in the manufacturing calibration the correlation is generally positive. Still, it is worth noting that the mean annual growth rate in the model is 1.9%, compared to 2.0% in the data.

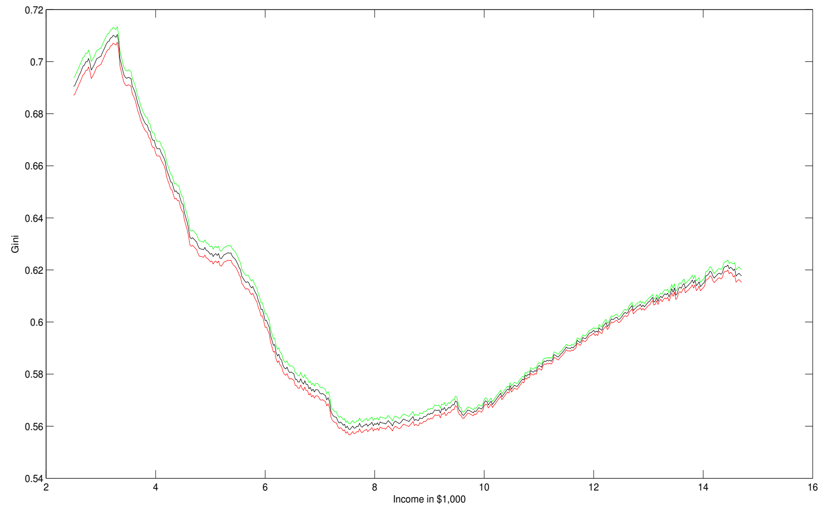

Figure 9 – Industry structure along the development path.

Account for Country Specific Productivity Growth

The above exercise assumed that g_i is constant across countries. It is interesting to examine whether this assumption is key. To see this, we draw on the finding of Rodrik (2012) that there is unconditional convergence in productivity across manufacturing industries.[33] This would suggest that countries have g_{ict} productivity values where $g_{ict} \to g_i$ over time.

Suppose that that

$$g_{ict} = g_c g_i f\left(x_{c,t}\right)$$

where $x_{c,t} = Y_{c,t}/Y_{US,t}$, the relative GDP gap between country c and the United States. In the basic model, it should be clear that industry shares of manufacturing s_{ict} follow

$$\frac{s_{ict}}{s_{jct}} = \left(\frac{g_c g_i f\left(x_{c,t}\right)}{g_c g_j f\left(x_{c,t}\right)}\right)^{\varepsilon-1} = \left(\frac{g_i}{g_j}\right)^{\varepsilon-1}.$$

Thus the only way in which convergence could affect the results is if the convergence function f is different across industries. For example, suppose that

$$g_{ict} = g_c g_i^{x_{c,t}^{\eta}}$$

In this case, if $\eta > 0$, poorer countries have disproportionately more rapid convergence in high-g_i industries, capturing the idea that in high-g_i industries there is greater room for

[33]The finding of Rodrik (2012) relates to labor productivity: however, it is hard to think of reasonable conditions under which labor productivity would converge while TFP does not.

catchup. If $\eta < 0$, then the reverse is the case – catchup is slow in high-g_i industries, as in the model of Ilyina and Samaniego (2012) where financing constraints limit the R&D that would be necessary for poor economies to catch up in high-tech industries. $\eta = 0$ is the case explored in the paper.[34]

We solve the basic model with values $\eta \in \{-1, 1\}$. The results show that, although the exact shape of the curve is sensitive to the value of η, the U-shape in manufacturing is preserved. Still, the results with $\eta = 1$ are better in the sense that when $\eta = -1$ the Gini coefficients are uniformly too high. Thus, the results are robust to allowing for unconditional convergence in productivity growth rates.

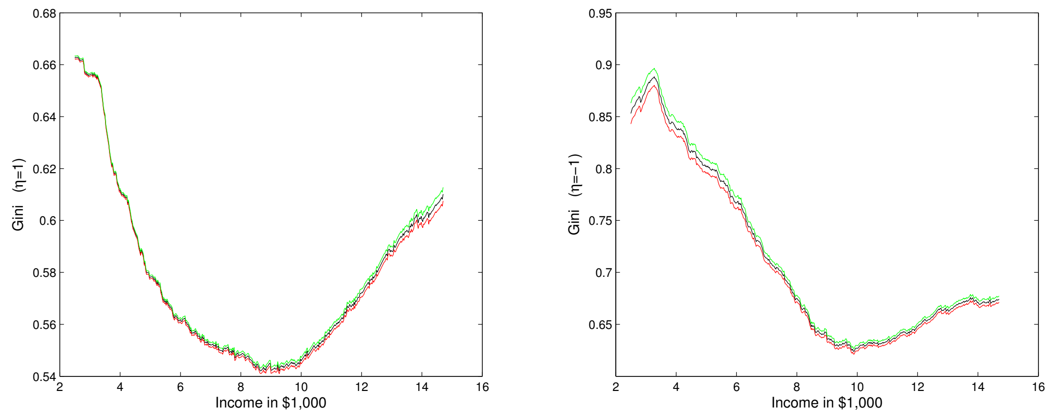

Figure 10 - Industry structure along the development path

Convergence in TFP growth rate across countries.

Another question we ask is, what kind of productivity dynamics are suggested by the model? Using the model, we can map between changes over time in employment or value-added shares in any given country and productivity growth rates by means of equation (6). All that is required is knowing g_{jt} for some benchmark industry j in each country. We derive values of g_{ict} for all countries in this manner, using the assumption that in all countries the productivity growth rate is the same in the industry ISIC 342, Printing and Publishing. We choose this industry because in the NBER data $g_j \simeq 1$, for ISIC 342. We did the same at the sector level, assuming g_i is the same for all c, t for Community, Social and Personal Services.

Then we computed g_{ic}, the average of the time series for each country and industry. We found several things. First, there is a lot of variation in the correlation between g_{ic} and the values calibrated for g_i using NBER data. However, the correlation *between* these correlations

[34]It is worth noting that, in that paper, both effects coexist and in the quantitative results the convergence effect dominates.

and initial GDP is 0.23 for manufacturing and 0.44 across broad sectors, both correlations being significant at the one percent level. This is stronly indicative of the productivity convergence mentioned earlier, at both levels of aggregation. We find also that the median value of g_{ic} – called g_i^{median} – is very highly correlated with the calibrated values of g_i. In manufacturing it is 0.40 and across sectors it is 0.70, both significant at the five percent level. This suggests that the data are consistent with countries converging towards a measure of productivity that is similar to the calibrated values.

Figure 11 displays the link between specialization and development using the basic model, using g_i^{median} instead of the values used earlier. The findings are generally robust. Also Figure 12 displays the weighted average of g_i^{median} (similar to Figure 7): as before, countries appear to shift resources towards high-tech sectors within manufacturing, whereas the opposite takes place across sectors.

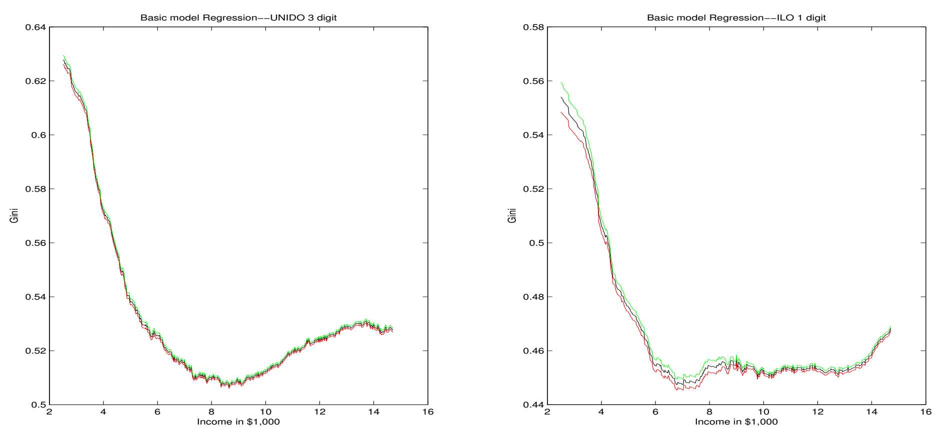

Figure 11 - Basic model using median TFP growth rate.

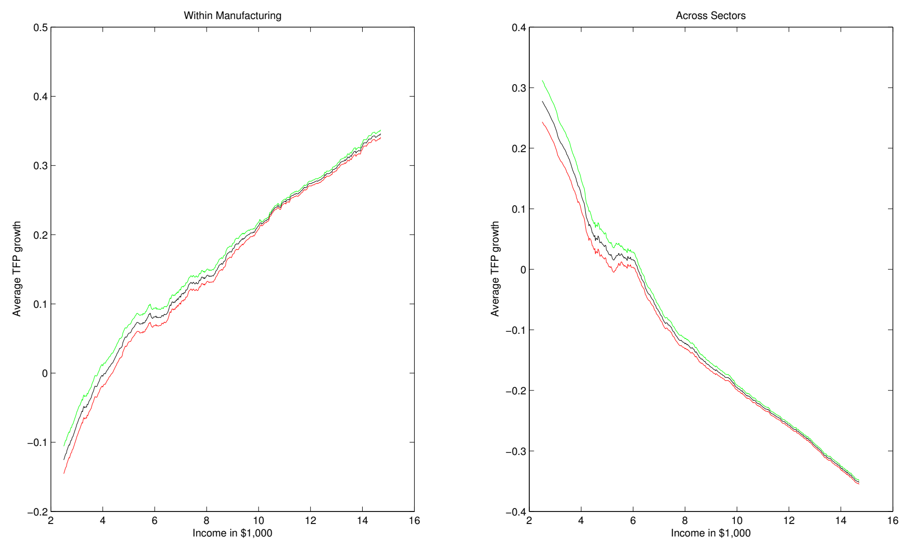

Figure 12- Average TFP growth within Manufacturing

6.5 Concluding Remarks

We develop a multi-sector model in which differential TFP growth rates across industries and sectors lead to structural change along an unbalanced growth path. We find that the model accounts for the pattern of diversification followed by specialization – stages of diversification – that is well-known in the literature. The results are robust to a variety of extensions and modifications, and hold both across sectors and within manufacturing. We do not exclude other possible factors, such as differences in factor shares (as in AG) or international trade (as in IW). However, the paper provides quantitative evidence that productivity differences can account on their own for the dynamics of industrial structure along the development path.

Earlier explanations of the "stages of diversification" have relied on very different factors. IW attribute the U-shaped "stages" to the interaction of aggregate income and openness to trade. IW argue that an interaction of income per capita and openness determines the observed stages of diversification, whereas Koren and Tenreyro (2007) interpret the "stages" in terms of shifts between sectors with differing levels of volatility. However, before concluding that international trade or volatility are important determinants of the evolution of economic structure along the development path, we need to understand how economic structure would evolve in the absence of these factors. This paper shows that even in a closed economy and in a deterministic setting, persistent total factor productivity (TFP) growth differences across industries are sufficient to generate a U-shaped pattern of specialization. It would be interesting in future work to develop a theoretical or empirical model that nests all the various possibilities, and estimate the contribution of different factors to the evolution of economic structure.

The results suggest that a productivity driven theory can account for both income levels and economic structure. This lends further emphasis to the question of the ultimate determinants of productivity growth rates. Ngai and Samaniego (2011) relate productivity growth rates to the technological determinants of R&D intensity and Ilyina and Samaniego (2012) are able to account for country *differences* in industry productivity growth rates based on an interaction of research intensity and institutional frictions, suggesting fruitful avenues for future research.

An implication of the quantitative results is that the composition of capital shifts towards the most high-tech industries, providing an explanation for the observed acceleration of investment-specific technical progress. Krusell et al (2000) relate this to increasing wage inequality, and it would be interesting more broadly to understand the link between economic structure and wage inequality.

In the paper we take initial conditions as given for our quantitative experiments. The results suggest that poorer countries tend to begin specialized in industries where TFP growth is low. Although it is beyond the scope of this paper, it is interesting to think about why initial conditions might be biased in this way. One possibility is that there are non-homothetic preferences (see Kongsamut et al (2000)), so that consumption patterns in poor countries are dominated by subsistence considerations that wear off later. If manufacturing industries that produce goods necessary for subsistence (e.g. food products) happen to have slow TFP growth, whereas sectors that are necessary for subsistence (e.g. agriculture) so happen to have rapid TFP growth, then we would observe these initial conditions. This explanation, however, relies on coincidence. Another possibility that does not require non-homothetic preferences involves the transition from a "traditional" technology with low productivity growth to a "modern" technology with more rapid productivity growth. Ngai (2004) shows that small differences across countries in barriers to technology adoption can lead to very large differences in income by delaying the transition from the "traditional" to the "modern" technologies. Initial conditions would be determined by the traditional technology and the date of transition between technologies. The idea that the transition between the "traditional" and "modern" technologies could explain historical economic structure as well as income levels is an interesting topic for future research.

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A Proofs

Proof of decentralized economy. For consumers:

$$\begin{aligned}
& \max_{y_{st}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}-1}{1-\theta} \\
c_t &= \left[\sum_{s=1}^{S-1} \zeta_s y_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
s.t. \qquad \sum_{s=1}^{S-1} q_{st} y_{st} + K_{t+1} &= \sum_{s=1}^{S}\sum_{i\in I_s} r_t K_{it} + \sum_{s=1}^{S}\sum_{i\in I_s} w_t n_{it}
\end{aligned}$$

Capital and labor market clearing conditions are:

$$\begin{aligned}
K_t &= \sum_{s=1}^{S}\sum_{i\in I_s} K_{it} \\
1 &= \sum_{s=1}^{S}\sum_{i\in I_s} n_{it}
\end{aligned}$$

F.O.C w.r.t y_{st} :

$$\frac{q_{st}}{q_{s't}} = \left(\frac{y_{s't}}{y_{st}}\right)^{\frac{1}{\varepsilon}} \frac{\zeta_s}{\zeta_{s'}} \qquad s, s' = 1, ..., S-1 \tag{37}$$

or

$$\frac{y_{st}}{y_{s't}} = \left(\frac{\zeta_s}{\zeta_{s'}} \frac{p_{s't}}{p_{st}}\right)^{\varepsilon} \qquad s, s' = 1, ..., S-1 \tag{38}$$

Final Goods Sector s maximizes profit:

$$\begin{aligned}
& \max_{u_{s,i,t}} q_{st} y_{st} - \sum_{i\in I} p_{it} u_{s,i,t} \\
&= q_{st} \left[\sum_{i\in I} \xi_{s,i} \times y_{i,t}^{\frac{\varepsilon_s-1}{\varepsilon_s}}\right]^{\frac{\varepsilon_s}{\varepsilon_s-1}} - \sum_{i\in I} p_{it} y_{i,t}
\end{aligned}$$

F.O.C w.r.t $y_{i,t}$:

$$q_{st}\left[\sum_{i\in I} \xi_{s,i} \times y_{i,t}^{\frac{\varepsilon_s-1}{\varepsilon_s}}\right] \xi_{s,i} y_{i,t}^{\frac{-1}{\varepsilon_s}} = p_{it}$$

similarly for $y_{j,t}$:

$$q_{st}\left[\sum_{i\in I} \xi_{s,i} \times y_{i,t}^{\frac{\varepsilon_s-1}{\varepsilon_s}}\right] \xi_{s,j} y_{j,t}^{\frac{-1}{\varepsilon_s}} = p_{jt}$$

So we have:

$$\frac{p_{it}}{p_{jt}} = \left(\frac{y_{j,t}}{y_{i,t}}\right)^{\frac{1}{\varepsilon_s}} \frac{\xi_{s,i}}{\xi_{s,j}} \tag{39}$$

or

$$\frac{y_{i,t}}{y_{j,t}} = \left(\frac{\xi_{s,i}}{\xi_{s,j}} \frac{p_{jt}}{p_{it}}\right)^{\varepsilon_s} \tag{40}$$

For industry i in a given sector:

$$\max p_{it} A_{it} K_{it}^{\alpha} n_{it}^{1-\alpha} - r_t K_{it} - w_t n_{it}$$

F.O.C w.r.t K_{it} :

$$p_{it} \alpha A_{it} K_{it}^{\alpha-1} n_{it}^{1-\alpha} = r_t \tag{41}$$

F.O.C w.r.t n_{it} :

$$p_{it}(1-\alpha) A_{it} K_{it}^{\alpha} n_{it}^{-\alpha} = w_t \tag{42}$$

Dividing one F.O.C. by the other we get that

$$\frac{1-\alpha}{\alpha}\left(\frac{K_{it}}{n_{it}}\right) = \frac{w_t}{r_t} \Rightarrow k_t = \frac{w_t}{r_t} \times \frac{\alpha}{1-\alpha} \tag{43}$$

where the capital labor ratio $k_t \equiv K_{it}/n_{it}$ is a constant across industries. Applying this result to (41) implies that

$$\frac{A_{it}}{A_{jt}} = \frac{p_{jt}}{p_{it}} \tag{44}$$

Using (40), (43) and (44) yields

$$\frac{n_{it}}{n_{jt}} = \left(\frac{\xi_{s,i}}{\xi_{s,j}}\right)^{\varepsilon_s} \left(\frac{A_{it}}{A_{jt}}\right)^{\varepsilon_s - 1} \tag{45}$$

which, rearranging (39), implies that $\frac{n_{it}}{n_{jt}} = \frac{p_{it} y_{it}}{p_{jt} y_{jt}}$. Define the industry i growth factor as :

$$G_{it} = \frac{p_{i,t+1} y_{i,t+1}}{p_{it} y_{it}}$$

and the expression G_{it}/G_{jt} then denotes the growth of industry i relative to industry j

$$\begin{aligned}
\frac{G_{it}}{G_{j,t}} &= \frac{\frac{p_{i,t+1} y_{i,t+1}}{p_{it} y_{it}}}{\frac{p_{j,t+1} y_{j,t+1}}{p_{jt} y_{jt}}} = \frac{\frac{p_{it+1}}{p_{jt+1}} \left(\frac{\xi_{s,i}}{\xi_{s,j}} \frac{p_{jt+1}}{p_{it+1}}\right)^{\varepsilon_s}}{\frac{p_{it}}{p_{jt}} \left(\frac{\xi_{s,i}}{\xi_{s,j}} \frac{p_{jt}}{p_{it}}\right)^{\varepsilon_s}} \\
&= \frac{\left(\frac{p_{it+1}}{p_{jt+1}}\right)^{1-\varepsilon_s}}{\left(\frac{p_{it}}{p_{jt}}\right)^{1-\varepsilon_s}} = \frac{\left(\frac{A_{it+1}}{A_{it}}\right)^{\varepsilon_s - 1}}{\left(\frac{A_{jt+1}}{A_{jt}}\right)^{\varepsilon_s - 1}} \\
&= \left(\frac{g_i}{g_j}\right)^{\varepsilon_s - 1}
\end{aligned}$$

■

Proof of Proposition 1. Solving the 2 sector problem and using the equilibrium conditions, we have:

$$A_{h_t} = p_{c_t} A_{c_t} \tag{46}$$

$$\begin{aligned}
r_t &= \frac{\frac{p_{c_t} c_t^{\theta}}{p_{c_{t-1}} c_{t-1}^{\theta}}}{\beta} - 1 + \delta = \frac{\left(\frac{\overline{g}_{A_{ht-1}}}{\overline{g}_{A_{ct-1}}}\right)^{1-\theta} g_{c_{t-1}}^{\theta}}{\beta} - 1 + \delta \text{ IF } \beta \neq 0 && (47)\\
\text{where } g_{ct-1} &\equiv \frac{p_t c_t}{p_{t-1} c_{t-1}} \text{ is the growth factor of aggregate consumption} && (48)\\
\overline{g}_{A_{ct-1}} &= \frac{A_{ct}}{A_{ct-1}}, \overline{g}_{A_{ht-1}} = \frac{A_{ht}}{A_{ht-1}} \text{ are known} && (49)
\end{aligned}$$

Let $\phi_t = \alpha^{-1} r_t^{\frac{-\alpha}{1-\alpha}} - \overline{g}_{A_{ht}}^{\frac{1}{1-\alpha}} r_{t+1}^{\frac{-1}{1-\alpha}} + (1-\delta) r_t^{\frac{-1}{1-\alpha}}$

$$\begin{aligned}
k_t &= \frac{K_{h_t}}{n_{h_t}} = \frac{K_{c_t}}{n_{c_t}} = \left(\frac{\alpha A_{h_t}}{r_t}\right)^{\frac{1}{1-\alpha}}\\
K_t &= k_t
\end{aligned}$$

The growth factor of capital per capita in each sector is:

$$g_{k_t} = \frac{k_{t+1}}{k_t} = \overline{g}_{A_{ht}}^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} \tag{50}$$

Similarly, we get aggregate capital growth factor:

$$g_{K_t} = g_{k_t}$$

Using (46) and (25), we derive capital sector output, i.e., investment:

$$\begin{aligned}
h_t &= \left(\frac{\alpha A_{h_{t+1}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{\alpha A_{h_t}}{r_t}\right)^{\frac{1}{1-\alpha}}\\
&= (\alpha A_{h_t})^{\frac{1}{1-\alpha}} \left[\left(\frac{\overline{g}_{A_{ht}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}\right] && (51)
\end{aligned}$$

and the growth factor of investment h_t is:

$$g_{h_t} = \frac{h_{t+1}}{h_t} = \overline{g}_{A_{ht}}^{\frac{1}{1-\alpha}} \frac{\left(\frac{\overline{g}_{A_{ht+1}}}{r_{t+2}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_{t+1}}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\overline{g}_{A_{ht}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}}$$

so that the labor in capital sector is:

$$n_{ht} = \alpha \left[\frac{1}{r_t}\left(\frac{\overline{g}_{A_{ht}} r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_t}\right] \tag{52}$$

and the growth factor of n_{ht} is:

$$g_{n_{ht}} = \frac{n_{ht+1}}{n_{ht}} = \frac{\frac{1}{r_{t+1}}\left(\frac{\overline{g}_{A_{ht+1}} r_{t+1}}{r_{t+2}}\right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_{t+1}}}{\frac{1}{r_t}\left(\frac{\overline{g}_{A_{ht}} r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_t}} \tag{53}$$

Notice that n_{ht} (and hence $n_{ct} = 1 - n_{ht}$) is independent of the level of technology in c and h as long as the interest rate is too. We can get capital in capital sector:

$$K_{h_t} = \alpha \left[\frac{1}{r_t}\left(\frac{\overline{g}_{A_{ht}} r_t}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - \frac{(1-\delta)}{r_{t-1}}\right]\left(\frac{\alpha A_{h_t}}{r_t}\right)^{\frac{1}{1-\alpha}} \tag{54}$$

Define the aggregate output per capita as $y_t = h_t + p_{ct}c_{ct}$. Since $K_{ct} = K_t - K_{ht}$ and $n_{ct} = 1 - n_{ht}$,

$$\begin{aligned} y_t &= h_t + p_{ct}c_{ct} \\ &= A_{h_t}K_{h_t}^{\alpha}n_{h_t}^{1-\alpha} + p_{c_t}A_{c_t}K_{c_t}^{\alpha}n_{c_t}^{1-\alpha} \\ &= A_{h_t}k_t^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} A_{h_t}^{\frac{1}{1-\alpha}} \end{aligned} \tag{55}$$

and its growth factor is:

$$g_{yt} = \frac{y_{t+1}}{y_t} = \overline{g}_{A_{ht}}^{\frac{1}{1-\alpha}}\left(\frac{r_t}{r_{t+1}}\right)^{\frac{\alpha}{1-\alpha}} \tag{56}$$

Aggregate consumption is:

$$C_t = p_{ct}c_t = y_t - h_t \tag{57}$$

$$\begin{aligned} &= \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} A_{h_t}^{\frac{1}{1-\alpha}} \\ &\quad - (\alpha A_{h_t})^{\frac{1}{1-\alpha}}\left[\left(\frac{\overline{g}_{A_{ht}}}{r_{t+1}}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\left(\frac{1}{r_t}\right)^{\frac{1}{1-\alpha}}\right] \end{aligned} \tag{58}$$

The growth factor of consumption is:

$$g_{ct} = \frac{C_{t+1}}{C_t} = \overline{g}_{Aht}^{\frac{1}{1-\alpha}}\frac{\phi_{t+1}}{\phi_t}. \tag{59}$$

Notice that as $t \to \infty$ the expressions for $\overline{g}_{Aht}$ and $\overline{g}_{Act}$ converge to constants, so the difference equation for g_{ct} converges uniformly to that which characterizes the model of investment-specific technical change in Greenwood, Hercowitz and Krusell (1997). Thus, the result that the transversality condition picks out a single equilibrium solution in that model extends to our case too. ■

Proof of Proposition 2. Corollary of the proof of Proposition 1 and (16). ■

B Measurement of productivity in Manufacturing

We measure productivity using the NBER Manufacturing Productivity Database. The data are more disaggregated that the ISIC3 Classification we need for the UNIDO data, so we aggregate them using Domar weights.

In addition, we use an alternative way of measuring TFP growth rates. Using the UNIDO dataset, we compute the TFP growth rates of 28 UNIDO manufacturing industries of the United States using the following equation:

$$\ln(TFP_{it}) = \ln(Y_{it}) - (1-\alpha)\ln(L_{it}) - \alpha \ln(K_{it}) \tag{60}$$

where Y_{it} is the production index. This requires computing the capital stock at the industry level. The UNIDO dataset provides investment data but not capital stock data K_{it}, so we use a perpetual inventory method

$$K_{it+1} = (1-\delta)K_{it} + I_{it}q_{it} \tag{61}$$

to compute growth rate of capital stock, where I_{it} is investment and q_{it} represents investment-specific technical progress[35]. Then the growth rate of K_{it} is the sum of growth rates of I and q. We set $q_{it} = g_{iq}^{t}$, so that growth rates of q_i vary across industries. We use growth factor g_{iq} from IS. (see table 6) Also, $\delta = 0.06$ and $\alpha = 0.3$. These are standard numbers in the literature.[36] Then, if $\Gamma(x)$ is the log growth rate of x over the time period in the data, note that

$$\ln g_i = \Gamma(Y_i) - (1-\alpha)\Gamma(L_i) - \alpha\Gamma(K_i) \tag{62}$$

We obtain $\Gamma(Y_i)$ and $\Gamma(L_i)$ from UNIDO, and set $\Gamma(K_i) = \Gamma(I_i) + \log g_{iq}$, which is the long run relationship in (61).

C Simulation procedure

Simulating the model requires overcoming two distinct problems.

The first concerns matching the model with the data. Notice that the model is essentially a 2 sector model where consumption and investment are made by different sectors. As shown

[35] We need to allow for investment-specific technical progress because the model is one with many industries where productivity growth rates in capital-producing industries may be different from productivity growth elsewhere.

[36] The value of δ is from Greenwood, Hercowitz and Krusell (1997) and is a value typical in models with investment-specific technical change, in other words where $g_q > 1$.

in Greenwood, Hercowitz and Krusell (1997), this is the same as a one-sector model with investment specific technical change. In the one-sector growth model, the equilibrium for any initial conditions is a jump to the stable branch of a saddle path that leads to the long run equilibrium (which in this case is the model where the capital sector has converged to contain only one industry). Thus, for general initial conditions K_0, the share of investment will jump after period 1, so that the structure of the manufacturing sector will change abruptly after period zero (and smoothly thereafter).

We handle this problem in two ways. First, we computed everything without worrying about the jump. Second, we calibrated the model so as to focus on an Euler growth path – which are the results reported in the paper (results were very similar either way).

We did not set the initial value of the capital stock K_0 to match the investment share of GDP in each country. The reason was that, in all other periods after $t = 0$, the investment share will follow the Euler equation. It seems arbitrary to assume that in all countries the Euler equation is satisfied in all years except 1963, or whatever happens to be the year for which data are initially available. As a result, we assume that the Euler equation is also satisfied at date zero. We call this an "Euler growth path" or EGP. To do this requires setting the investment share of GDP at a value that is different from that in the data. At the same time, it is critical that we preserve the composition of manufacturing. Hence, we adopted a recursive strategy. We know from the data the composition of investment in year zero. Given an assumption on the investment to GDP ratio, we can preserve the ratio of capital to manufacturing and find a value for the size of manufacturing that preserves its composition.[37] Then we check whether the assumption on the investment to GDP ratio matches an EGP.[38] If not, then we generate another guess based on the predicted EGP value from the last iteration. We find that 3 loops is sufficient for very tight convergence. Then, the sector shares in the rest of the economy are set so as to preserve their relative values. When we regress data on initial manufacturing shares on the model initial manufacturing shares, we find a coefficient of 1.16 (positive and close to one) and an intercept of -0.026 (close to zero), both significant at the 1% level. We take this to imply that, in general, our procedure does not significantly distort the sector structure of the model economy.

The second computational issue we confront is the fact that we are simulating a model

[37] Other sectors are resized so that, relative to each other, shares of GDP are preserved.

[38] Recall that computing the equilibrium, including the initial share of investment, requires a series for g_c, which in turn depends on sector productivity growth rates. However, sector productivity growth rates depend on the initial composition and size of the economy. This is why an iterative procedure is necessary to find an EGP.

economy that does not have a balanced growth path (although it converges to one). Recall that the aggregate behavior of the model is the same as a one-sector model with investment specific technical change. In the one-sector growth model, any approximation to the saddle path will "shadow" it for a period of time, eventually diverging infinitely from it: see Colucci (2001). As a result, we adopt a procedure to provide this "shadowing" without suffering a significant divergence.

The procedure is to assume limited computational ability among the agents, a procedure we call "rolling windows of consciousness." Specifically, the structure of the model economy can be computed exactly given the investment share of employment. This can be computed exactly given a series for g_{ct}, which is determined by (59) and the transversality condition. This series eventually converges to $g_{ct} = \overline{g}_{Aht}^{\frac{1}{1-\alpha}}$, where $\overline{g}_{Aht}$ is known given initial conditions. We assume that an agent at date t acts as though up to some period $t+T$ difference equation (59) characterizes g_{ct}, whereas after $t+T$ the agent believes that $g_{ct} = \overline{g}_{Aht}^{\frac{1}{1-\alpha}}$. We tried $T = 50$, 90 and 200. For $T = 90$, the error between the realized value of g_{c1} and the value forecast by the agent in period 0 is about 1% of the actual value (because the series for g_{ct} converges uniformly to its long run value, the forecast errors are the highest in the first period). For $T = 200$ these values are indistinguishable to eight decimal places. At the same time, for all values of T, the Gini nonparametric regressions were indistinguishable regardless of the value of T.

This indicates two results. First, this procedure could yield an arbitrarily accurate approximation to the correct aggregate equilibrium dynamics, given a sufficiently large (but finite) value of T. This is distinct from the shadowing property, which provides arbitrarily precise approximations only for a finite period, after which there is increasing divergence. Second, industry dynamics are robust to using values of T such that aggregate dynamics are computed with some degree of imprecision.

Table 5: NBER TFP Growth Rates for the ISIC revision 2 industry classification. Source: NBER productivity database and authors' calculations.

Industry	ISIC code	NBER TFP Growth Rate
Food products	311	0.0101
Beverages	313	0.0303
Tobacco	314	-0.0345
Textiles	321	0.0269
Apparel	322	0.0121
Leather	323	-0.0034
Footwear	324	-0.0035
Wood products	331	0.0113
Furniture, except metal	332	0.0066
Paper and products	341	0.0088
Printing and publishing	342	-0.0022
Industrial chemicals	351	0.0214
Other chemicals	352	0.0135
Petroleum refineries	353	0.0196
Misc. pet. and coal products	354	0.0223
Rubber products	355	0.0142
Plastic products	356	0.0339
Pottery, china, earthenware	361	0.0078
Glass and products	362	0.0051
Other non-metallic mineral prod.	369	0.0120
Iron and steel	371	0.0047
Non-ferrous metals	372	0.0016
Fabricated metal products	381	0.0029
Machinery, except electrical	382	0.0285
Machinery, electric	383	0.0347
Transport equipment	384	0.0160
Prof. & sci. equip.	385	0.0126
Other manufactured prod.	390	0.0089

Table 6: UNIDO TFP and Price Growth Rate

Industry	ISIC code	UNIDO TFP Growth Rate	UNIDO Price Growth Rate
Food products	311	0.0067	0.0424
Beverages	313	0.0244	0.0295
Tobacco	314	-0.0212	0.0941
Textiles	321	0.0090	0.0328
Apparel	322	0.0060	0.0371
Leather	323	-0.0196	0.0570
Footwear	324	0.0034	0.0438
Wood products	331	-0.0014	0.0464
Furniture, except metal	332	0.0064	0.0352
Paper and products	341	0.0019	0.0410
Printing and publishing	342	-0.0062	0.0499
Industrial chemicals	351	0.0225	0.0201
Other chemicals	352	0.0146	0.0322
Petroleum refineries	353	-0.0089	0.0507
Misc. pet. and coal products	354	-0.0168	0.0552
Rubber products	355	0.0287	0.0172
Plastic products	356	0.0321	0.0132
Pottery, china, earthenware	361	-0.0116	0.0428
Glass and products	362	0.0005	0.0358
Other non-metallic mineral prod.	369	0.0040	0.0375
Iron and steel	371	-0.0006	0.0377
Non-ferrous metals	372	-0.0103	0.0472
Fabricated metal products	381	0.0034	0.0392
Machinery, except electrical	382	0.0330	0.0025
Machinery, electric	383	0.0218	0.0167
Transport equipment	384	-0.0118	0.0488
Prof. & sci. equip.	385	-0.0027	0.0425
Other manufactured prod.	390	0.0170	0.0290

D Industry TFP growth data

E Robustness results

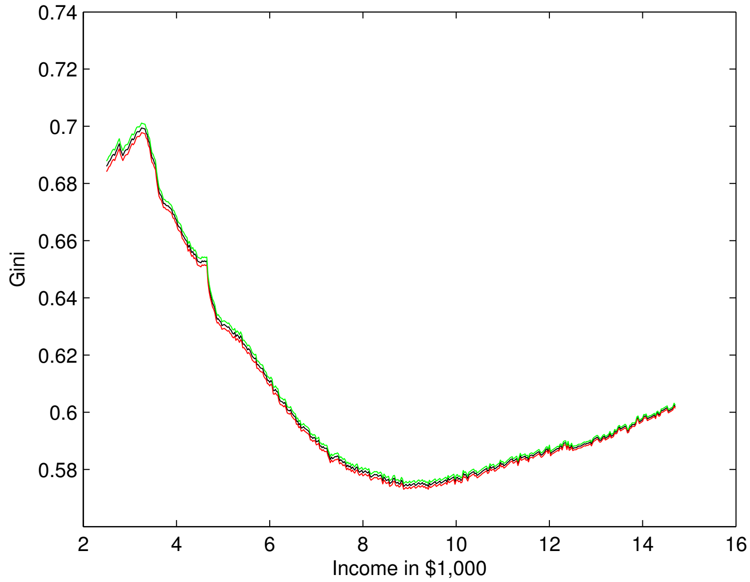

Figure A1. Industry structure along the development path. TFP growth rates are derived from the UNIDO data.

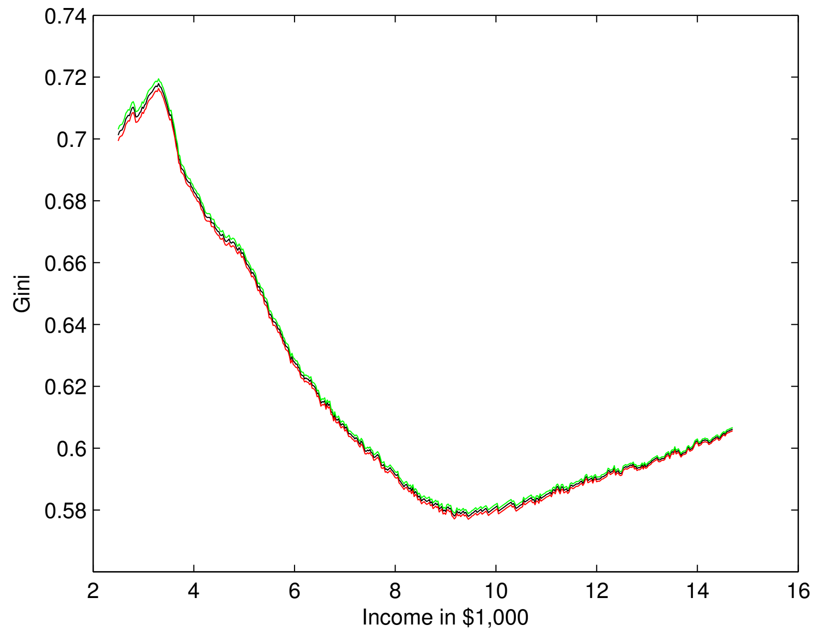

Figure A2. Industry structure along the development path.
Alternative Classification of the Capital Industry.

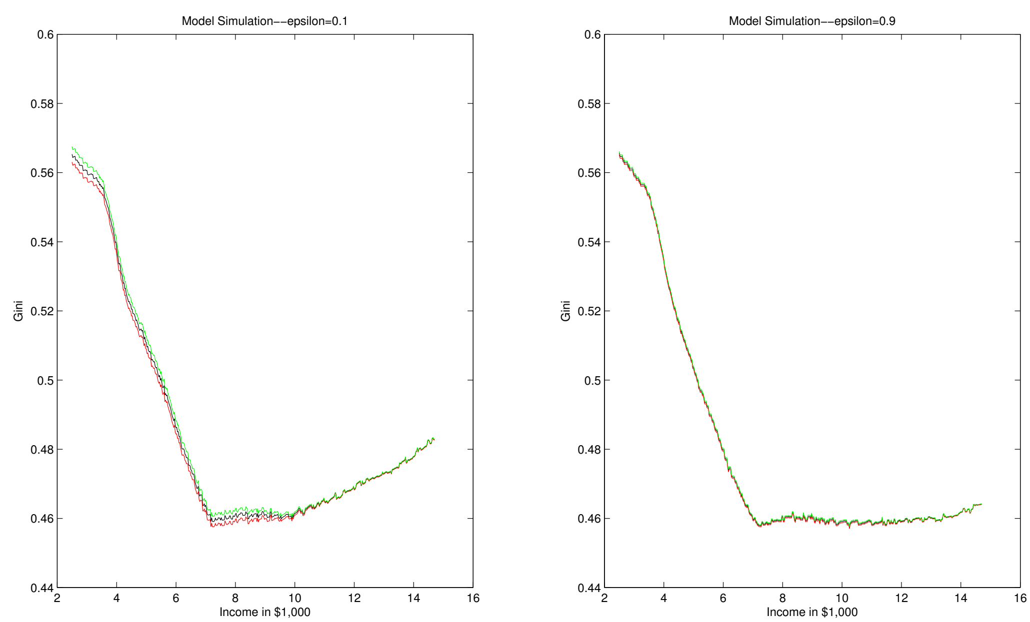

Figure $A3$. Economic Structure along the Development path, results for the entire economy, using the ILO 1-Digit Sector Classification with different values of ε.

Table 7: Capital Industries

Industry	ISIC code
Wood products	331
Furniture, except metal	332
Rubber products	355
Plastic products	356
Pottery, china, earthenware	361
Glass and products	362
Other non-metallic mineral prod.	369
Iron and steel	371
Non-ferrous metals	372
Fabricated metal products	381
Machinery, except electrical	382
Machinery, electric	383
Transport equipment	384
Prof. & sci. equip.	385
Other manufactured prod.	390