Quiet Bubbles*

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Abstract

Classic speculative bubbles are loud – price is high and so are price volatility and share turnover. The credit bubble of 2003-2007 is quiet – price is high but price volatility and share turnover are low. We develop a model, based on investor disagreement and short-sales constraints, that explains why credit bubbles are quieter than equity ones. Since debt up-side payoffs are bounded, debt is less sensitive to disagreement about asset value than equity and hence has a smaller resale option and lower price volatility and turnover. While optimism makes both debt and equity bubbles larger, it makes debt mispricings quiet but leaves the loudness of equity mispricings unchanged. Our theory suggests a taxonomy of bubbles.

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1. Introduction

Many commentators point to a bubble in credit markets from 2003 to 2007, particularly in the AAA and AA tranches of the subprime mortgage collateralized default obligations (CDOs), as the culprit behind the Great Financial Crisis of 2008 to 2009. Indeed, there is compelling evidence that these securities were severely mis-priced relative to their risk-adjusted fundamental values (see Coval, Jurek, and Stafford (2009)). However, stylized facts, which we gather below, suggest that the credit bubble lacked many of the features that characterize classic episodes. These episodes, such as the Internet Bubble, are typically loud – characterized by high price, high price volatility, and high trading volume or share turnover as investors purchase in anticipation of capital gains.[1] In contrast, the credit bubble is quiet – characterized by high price but low price volatility and low share turnover. In this paper, we attempt to understand equity and credit bubbles within a single framework. We show why credit bubbles are quieter and hence fundamentally different than equity ones. Our analysis suggests a taxonomy of bubbles based on the nature of the asset being traded.

Our theory builds on the investor disagreement and short-sales constraints framework, which has been used to generate loud equity bubbles. In these models, disagreement and binding short-sales constraints lead to over-pricing in a static setting as pessimists sit out of the market (Miller (1977) and Chen, Hong, and Stein (2002)). In a dynamic setting, investors value the potential to re-sell at a higher price to someone with a higher valuation due to binding short-sales constraints (Harrison and Kreps (1978), Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006)).[2] This framework generates a bubble or overpricing in which the asset's price is above fundamental value. A distinguishing feature

[1]See Hong and Stein (2007) for a review of the evidence regarding classic bubbles and Ofek and Richardson (2003) for a focus on the dot-com bubble.

[2]See Hong and Stein (2007) for a more extensive review of the disagreement approach to the modeling of bubbles. For other behavioral approaches without this strong turnover implication, see De Long et al. (1990) and Abreu and Brunnermeier (2003). For an agency approach which generates investor optimism, see Allen and Gorton (1993), Allen and Gale (2000) and more recently Vayanos and Gromb (2010). For the possibility of rational bubbles, see Blanchard and Watson (1983), Tirole (1985), Santos and Woodford (1997), and Allen, Morris, and Postlewaite (1993).

of this model is that the resale option is also associated with high price volatility and high share turnover.

Within this framework, we consider the pricing of a debt security. Investors disagree over the underlying asset value.[3] Whereas equity payoffs are linear in the investor beliefs regarding underlying asset value, debt up-side payoffs are capped at some constant and hence are non-linear (concave) in the investor beliefs about fundamental. We make the standard assumption regarding short-sales constraints. There is compelling evidence that such constraints are at least as binding in debt markets as in equity ones.[4] The pricing of debt contracts in this disagreement and short-sales constraints setting is new.

We show that the disagreement or resale framework, which is typically thought of as generating loud equity bubbles, also naturally generates quiet credit bubbles. Since debt upside payoffs are bounded in contrast to equity, the valuation of debt is less sensitive to disagreement about underlying asset value than equity and hence has a smaller resale option.[5] As a result, a debt bubble is smaller and quieter than an equity bubble. This is due simply to the bounded upside payoff of debt compared equity. The safer is the debt claim, the more bounded is the upside, the less sensitive its valuation is to disagreement and therefore the lower the resale option and the smaller and quieter is the bubble.

In addition to this resale option, the bubble can also have an optimism component. As investor optimism rises holding fixed fundamental value, both debt and equity bubbles naturally grow in size. But debt bubbles become quieter in the process whereas an equity bubble's loudness is invariant to the degree of investor optimism. When investors become

[3]In the context of the subprime mortgage CDOs, the underlying asset values are real estate prices.

[4]Indeed, short-sales constraints on CDOs were binding until the onset of the financial crisis (see Michael Lewis (2010)). See for more systematic evidence on cost of borrowing in corporate bonds Asquith et al. (2010).

[5]Here we are implicitly comparing a debt claim with an equity claim that does not have limited liability, so that the only difference between the two claims is the upside of the payoff function. If one compares the debt claim with the complementary equity claim (i.e. with limited liability), then our results hold provided that the fundamental of the economy is good enough or that there is enough optimism among investors. Intuitively, when this is the case, the equity claim is mostly linear in the relevant range while the debt claim has upside bounded payoffs in the relevant range. The formal analysis of equity with limited liability is in Section 3.5

more optimistic about the underlying fundamental of the economy, they view debt as being more risk-free with less upside and hence having a smaller resale option. Hence the resale option component becomes a smaller part of the debt bubble and debt mispricing becomes quieter with investor optimism. In contrast, the linearity of the equity payoff implies that an increase in average optimism does not change the sensitivity of equity to disagreements about the underlying asset. As a result, both the price volatility and share turnover of equity bubbles are independent of investors' average optimism.

Alternatively, one can fix investor optimism and consider a comparative static with respect to fundamental value and derive similar predictions. Interestingly, when fundamentals deteriorate, investors perceive the debt claim as riskier – closer to an equity claim – and beliefs start having a stronger influence on the asset valuation. The resale option grows and both price volatility and trading volume increase.[6] It turns out that working through debt pricing with heterogeneous beliefs about asset value yields many interesting implications.[7] Notably, recent work by Greenwood and Hanson (2011) find that there are credit bubbles in corporate bonds, especially in lower rated debt, which is consistent with our analysis here.

Finally, we provide a formal characterization that allows the ranking of two assets in terms of their disagreement sensitivity. An asset will be more disagreement sensitive than another if its derivative with respect to the underlying asset value is pointwise weakly greater with at least a non-zero measure set where this inequality is strict. Intuitively, when the slope of the asset payoff with respect to the fundamental value increases, the differences in opinion matters more for the asset valuation at the interim period. Thus, holding constant investors' beliefs, this means that short-sales constraint are more likely to bind in the future and hence the resale option is larger today. This also implies larger volatility and turnover.

The contribution of our paper is to generalize the disagreement and short-sales constraints

[6] This result stands in contrast to the traditional arguments in Myers and Majluf (1984) and Gorton and Pennacchi (1990) that debt claims are less sensitive to private information – our model is the counterpart to this view in a world without private information but with heterogenous beliefs.

[7] Earlier work on heterogeneous beliefs and bond pricing such as in Xiong and Yan (2010) assume that investors have disagreement about bond prices or interest rates and they do not model the nature of the concavity of debt pay-offs as a function of underlying asset value disagreement.

models of asset price bubbles by extending it to non-linear payoff functions. We show that the concavity of debt-payoffs and their lower sensitivity to belief differences generate many interesting new insights on asset price bubbles. In this context, we explicitly neglect the role of leverage and assume that agents have access to an efficient borrowing technology. Our taxonomy of bubbles does not depend on the determinants of leverage. An earlier literature has examined how to endogenize leverage when investors trade equity claims–so the credit bubble in their setting is a bubble in lending.[8] In contrast, our paper focuses on bubbles in the trading of actual credits.

Our paper proceeds as follows. We present some stylized facts on the recent credit bubble of 2003-2007 in Section 2. The model and main results are discussed in Section 3. We discuss empirical implications of our model in Section 4. We relate our work to studies on the recent financial crisis in Section 5. We conclude in Section 6. In the Appendix, we collect proofs and derive an extension to show that even in a setting with an unbiased average belief, an increase in the dispersion of priors can make bubbles larger and quieter at the same time, provided that trading costs and asset supply are sufficiently small.

2. Stylized Facts About the Credit Bubble of 2003-2007

We begin by providing some stylized facts regarding the recent credit bubble that motivate our theoretical analysis. By all accounts, subprime mortgage CDOs experience little price volatility between 2003 until the onset of the financial crisis in mid-2007. In Figure 1, we plot the prices of the AAA and AA tranches of the subprime CDOs. The ABX price index only starts trading in January of 2007, very close to the start of the crisis. Nonetheless,

[8]Geanakoplos (2010) shows that leverage effects in this disagreement setting can persist in general equilibrium though disagreement mutes the amount of lending on the part of pessimists to optimists. Simsek (2010) shows that the amount of lending is further mediated by the nature of the disagreement–disagreement about good states leads to more equilibrium leverage while disagreement about bad states leads to less.

one can see that, in the months between January 2007 until mid-2007, the AAA and AA series are marked by high prices and low price volatility. Price volatility only jumps at the beginning of the crisis in mid-2007. This stands in contrast to the behavior of dot-com stock prices–the price volatility of some internet stocks during 1996-2000 (the period *before* the collapse of the internet bubble) exceed 100% per annum, more than three times the typical level of stocks.

Another way to see the quietness is to look at the prices of the credit default swaps for the financial companies that had exposure to these subprime mortgage CDOs. This is shown in Figure 2. The price of insurance for the default of these companies as reflected in the spreads of these credit default swaps is extremely low and not very volatile during the years before the crisis. One million dollars of insurance against default cost a buyer only a few thousand dollars of premium each year. This price jumps at the start of the crisis, at about the same time as when price volatility increases for the AAA and AA tranches of the subprime mortgage CDOs.

The low price volatility coincided with little share turnover or re-selling of the subprime mortgage CDOs before the crisis. Since CDOs are traded over-the-counter, exact numbers on turnover are hard to come by. But anecdotal evidence suggests extremely low trading volume in this market particularly in light of the large amounts of issuance of these securities. Issuance totals around $100 billion dollars per quarter during the few years before the crisis but most of these credits are held by buyers for the interest that they generate. To try to capture this low trading volume associated with the credit bubble, in Figure 3, we plot the average monthly share turnover for financial stocks. Turnover for finance firms is low and only jumps at the onset of the crisis as does turnover of subprime mortgage CDOs according to anecdotal accounts (see, e.g., Michael Lewis (2010)). This stands in contrast to the explosive growth in turnover that coincided with the internet boom in Figure 4–obtained from Hong and Stein (2007). As shown in this figure, the turnover of internet stocks and run-up in valuations dwarfs those of the rest of the market.

At the same time, there is a large amount of evidence that the subprime CDO bubble was derived from the same ingredients of disagreement and short-sales constraints that played a crucial role in the Internet Bubble. First, fixed income traders routinely refer to the credit bubble of 2003-2007 as their dot-com bubble due to disagreement over the path of real estate prices, which serve as the fundamental values underlying the CDOs. Optimists told stories of how home prices had been too depressed historically and would keep rising (see Lereah (2005)) or rationalized the risk of the CDOs by pointing to the fact that national home prices had never fallen in US history. Indeed, optimism over a lack of correlation among regional home prices seem to have played a role in the demise of the credit rating agencies' models (Coval, Jurek, and Stafford (2009)).

Optimism also seemed to have played a big role in institutional exposures to these structured credits. Michael Lewis (2010) points to AIG-FG – the financial markets group of AIG – insuring 50 billion dollars worth of subprime AAA tranches between 2002 and 2005 at extremely low prices. During these years, the issuance in the market was still relatively low, on the order of 100 billion dollars a year. Why did AIG-FP take on such a large position? Because Joe Cassano, the head of AIG-FP, did not think home prices could fall – i.e. our optimism assumption. Michael Lewis (2010) writes, "Confronted with the new fact that his company was effectively long $50 billion in triple-B rated subprime mortgage bonds, masquerading as triple A-rated diversified pools of consumer loans – Cassano at first sought to rationalize it. He clearly thought that any money he received for selling default insurance on highly rated bonds was free money. For the bonds to default, he now said, U.S. home prices had to fall and Joe Cassano didn't believe house prices could ever fall everywhere in the country at once. After all, Moody's and S&P had rated this stuff triple-A!" Indeed, AIG FP continued to keep their insurance contracts even after 2005.

Finally, we provide evidence that short-sales constraints were tightly binding until around 2006 when synthetic mezzanine ABS CDOs allowed hedge funds to short subprime CDOs, thereby leading to the implosion of the credit bubble. In Figure 5, we plot issuance of investment grade synthetic mezzanine ABS CDOs which is how hedge funds such as John Paulson's finally were able to short the subprime mortgage CDOs. Notice that there was very little shorting in this market until the end as issuances of this type of CDO are not sizable until 2006. In other words, short-sales constraints were binding tightly until around 2007, consistent with the premise of our model. The collapse coincided with a large supply of these securities in 2007, similar to what happened during the dot-com period. This collapse effect is already modeled in Hong, Scheinkman, and Xiong (2006) using disagreement and short-sales constraints. We are interested in seeing whether the properties of credit bubbles (as described in Figures 1-4) arise within the same disagreement and short-sales constraint framework. It is to this analysis that we now turn.

3. Model

3.1. Set-up

Our model has three dates $t = 0$, 1, and 2. There are two assets in the economy. A risk-free asset offers a risk-free rate each period. A risky debt contract with a face value of D has the following payoff at time 2 given by:

$$\tilde{m}_2 = \min\left(D, \tilde{G}_2\right), \tag{1}$$

where

$$\tilde{G}_2 = G + \tilde{\epsilon}_2 \tag{2}$$

and G is a known constant and $\tilde{\epsilon}_2$ is a random variable drawn from a standard normal distribution $\Phi(\cdot)$. We think of $\tilde{G}_t$ as the underlying asset value which determines the payoff of the risky debt or the fundamental of this economy. There is an initial supply Q of this risky asset.

There are two groups of agents in the economy: group A and group B with a fraction

1/2 each in the population. Both groups share the same belief at date 0 about the value of the fundamental. More specifically, both types of agents believe at $t = 0$ that the underlying asset process is:

$$\tilde{V}_2 = G + b + \tilde{\epsilon}_2, \tag{3}$$

where b is the agents' optimism bias. When $b = 0$, investor expectations are equal to the actual mean of the fundamental G and there is no aggregate bias. The larger is b, the greater the investor optimism.

At $t = 1$, agents' beliefs change stochastically: agents in group A believe the asset process is in fact

$$\tilde{V}_2 = G + b + \eta^A + \epsilon_2 \tag{4}$$

while agents in group B believe it is:

$$\tilde{V}_2 = G + b + \eta^B + \tilde{\epsilon}_2, \tag{5}$$

where η^A and η^B are drawn from a normal standard distribution with mean 0 and standard deviation 1. These revisions of beliefs are the main shocks that determine the price of the asset, its volatility and turnover at $t = 1$.

The expected payoff of an agent with belief $G + b + \eta$ regarding the mean of $\tilde{V}_2$ for this standard debt claim is given by:

$$\pi(\eta) = E[\tilde{V}_2|\eta] = \int_{-\infty}^{D-G-b-\eta} (G + b + \eta + \tilde{\epsilon}_2)\phi(\tilde{\epsilon}_2)d\tilde{\epsilon}_2 + D\left(1 - \Phi\left(D - G - b - \eta\right)\right). \tag{6}$$

If the fundamental shock $\tilde{\epsilon}$ is sufficiently low such that the value of the asset underlying the credit is below its face value ($G + b + \eta + \tilde{\epsilon} < D$), then the firm defaults on its contract and investors become residual claimant (they receive $G + b + \eta + \tilde{\epsilon}$). If the fundamental shock $\tilde{\epsilon}$ is sufficiently good such that the value of the fundamental is above the face value of debt ($G + b + \eta + \tilde{\epsilon} \geq D$), then investors are entitled to a fixed payment D. Our analysis below applies more generally to any (weakly) concave expected payoff function, which would include equity as well standard debt claims. Note also that the unlimited liability assumption – the fact that debtholders may receive negative payoff – is not necessary for most of our analysis, but it allows us to compare our results with the rest of the literature.

Agents are risk-neutral and can borrow from a perfectly competitive credit market.[9] The discount rate is normalized to 0 without loss of generality. Finally, the last ingredient of this model is that our risk-neutral investors face quadratic trading costs given by:

$$c(\Delta n_t) = \frac{(n_t - n_{t-1})^2}{2\gamma}, \tag{7}$$

where n_t is the shares held by an agent at time t. The parameter γ captures the severity of the trading costs – the higher is γ the lower the trading costs. These trading costs allow us to obtain a well-defined equilibrium in this risk-neutral setting. Note that $n_{-1} = 0$ for all agents, i.e. agents are not endowed with any risky asset. Investors are also short-sales constrained. This set-up is similar to the CARA-Gaussian platform in Hong, Scheinkman, and Xiong (2006) except that we consider non-linear payoff functions over disagreement about underlying asset value.

[9]This can be viewed as the limiting case of the following model with borrowing constraints. Agents are endowed with zero liquid wealth but large illiquid wealth W (which becomes liquid and is perfectly pledgeable at date 2). Credit markets are imperfectly competitive so that banks charge a positive interest rate, which we call $\frac{1}{\mu} - 1$, so that μ is the inverse of the gross rate charged by banks. μ is increasing with the efficiency of the credit market. We consider here the case where $\mu = 1$. The derivation of the model with $\mu < 1$ is available from the authors upon request. Results are qualitatively similar.

3.2. Date-1 equilibrium

Let P_1 be the price of the asset at $t = 1$. At $t = 1$, consider an investor with belief $G + b + \eta$ and date-0 holding n_0. Her optimization problem is given by:

$$J(n_0, \eta, P_1) = \left| \begin{array}{l} \max\limits_{n_1} \left\{ n_1 \pi(\eta) - \left((n_1 - n_0) P_1 + \dfrac{(n_1 - n_0)^2}{2\gamma} \right) \right\} \\ n_1 \geq 0 \end{array} \right. \tag{8}$$

where the constraint is the short-sales constraint.

Call $n_1^\star(\eta)$ the solution to the previous program. If $n_1^\star(\eta) - n_0$ is positive, an agent borrows $(n_1^\star(\eta) - n_0)P_1 + \frac{(n_1^\star(\eta) - n_0)^2}{2\gamma}$ to buy additional shares $n_1^\star(\eta) - n_0$. If $n_1^\star(\eta) - n_0$ is negative, the agent makes some profit on the sales but still has to pay the trading cost on the shares sold $(n_0 - n_1^\star(\eta))$. This is because the trading cost is symmetric (buying and selling costs are similar) and only affects the number of shares one purchases or sells, and not the entire position (i.e. $n_1 - n_0$ vs. n_1). In equation 8, $J(n_0, \eta, P_1)$ is the value function of an agent with belief $G + b + \eta$, initial holding n_0 and facing a price P_1. Clearly, $J(n_0, \eta, P_1)$ is driven in part by the possibility of the re-sale of the asset bought at $t = 0$ at a price P_1.

Our first theorem simply describes the date-1 equilibrium. At date 1, three cases arise, depending on the relative beliefs of agents in group A and B. If agents in group A are much more optimistic than agents in group B ($\pi(\eta^A) - \pi(\eta^B) > \frac{2Q}{\gamma}$), then the short-sales constraints binds for agents in group B. Only agents A are long and the price reflects the asset valuation of agents A ($\pi(\eta^A)$) minus a discount that arises from the effective supply of agents B who are re-selling their date-0 holdings to agents A.

Symmetrically, if agents in group B are much more optimistic than agents in group A ($\pi(\eta^B) - \pi(\eta^A) > \frac{2Q}{\gamma}$), then the short-sales constraints binds for agents in group A. Only agents B are long and the price reflects the valuation of agents B for the asset ($\pi(\eta^B)$) minus a discount that arises from the effective supply of agents A who are re-selling their date-0 holdings to agents B.

Finally, the last case arises when the beliefs of both groups are close (i.e $|\pi(\eta^A)-\pi(\eta^B)| < \frac{2Q}{\gamma}$). In this case, both agents are long at date 1 and the date-1 equilibrium price is simply an average of both groups' beliefs ($\frac{\pi(\eta^A)+\pi(\eta^B)}{2}$).

Theorem 1. ***Date-1 equilibrium.***

At date 1, three cases arise.

1. *If $\pi(\eta^A) - \pi(\eta^B) > \frac{2Q}{\gamma}$, only agents in group A are long (i.e. the short-sales constraint is binding). The date-1 price is then:*

$$P_1 = \pi(\eta^A) - \frac{Q}{\gamma}.$$

2. *If $\pi(\eta^B) - \pi(\eta^A) > \frac{2Q}{\gamma}$, only agents in group B are long (i.e. the short-sales constraint is binding). The date-1 price is then:*

$$P_1 = \pi(\eta^B) - \frac{Q}{\gamma}.$$

3. *If $|\pi(\eta^A) - \pi(\eta^B)| \leq \frac{2Q}{\gamma}$, both agents are long. The date-1 price is then:*

$$P_1 = \frac{\pi(\eta^A) + \pi(\eta^A)}{2}.$$

Proof. Let (η^A, η^B) be the agents' beliefs at date 1. Agents in group i are solving the following problem:

$$\left| \begin{array}{l} \max\limits_{n_1} \left\{ n_1 \pi(\eta^i) - \left((n_1 - n_0) P_1 + \frac{(n_1 - n_0)^2}{2\gamma} \right) \right\} \\ n_1 \geq 0 \end{array} \right.$$

Consider first the case where both agents are long. Then, the date-0 holdings are given by the F.O.C. of the unconstrained problem and yield

$$n_1^A = n_0 + \gamma \left(\pi(\eta^A) - P_1 \right) \quad \text{and} \quad n_1^B = n_0 + \gamma \left(\pi(\eta^B) - P_1 \right)$$

The date-1 market-clearing condition ($n_1^A + n_1^B = 2Q$) combined with the date-0 market-clearing condition ($n_0^A + n_0^B = 2Q$) gives:

$$P_1 = \frac{\pi(\eta^A) + \pi(\eta^B)}{2},$$

and

$$n_1^A - n_0^A = \gamma \frac{\pi(\eta^A) - \pi(\eta^B)}{2} \quad \text{and} \quad n_1^B - n_0^B = \gamma \frac{\pi(\eta^B) - \pi(\eta^A)}{2}.$$

This can be an equilibrium provided that these date-1 holdings are indeed positive:

$$\frac{2n_0^A}{\gamma} > \pi(\eta^B) - \pi(\eta^A) \quad \text{and} \quad \frac{2n_0^B}{\gamma} > \pi(\eta^A) - \pi(\eta^B).$$

If this last condition is not verified, two cases may happen. Either agents in group B are short-sales constrained ($n_1^B = 0$). In this case, the date-1 market clearing condition imposes that:

$$P_1 = \pi(\eta^A) - \frac{Q}{\gamma}.$$

This can be an equilibrium if and only if group B agents' F.O.C. leads to a strictly negative holding or

$$\pi(\eta^A) - \pi(\eta^B) > \frac{2n_0^B}{\gamma}.$$

Or the agents in group A are short-sales constrained ($n_1^A = 0$). In this case, the date-1 market clearing condition imposes that

$$P_1 = \pi(\eta^B) - \frac{Q}{\gamma}.$$

This can be an equilibrium if and only if group B agents' F.O.C. leads to a strictly negative holding or

$$\pi(\eta^B) - \pi(\eta^A) > \frac{2n_0^A}{\gamma}$$

□

3.3. Date-0 equilibrium

We now turn to the equilibrium structure at date 0. Let P_0 be the price of the asset at $t = 0$. Then at $t = 0$, agents of group $i \in \{A, B\}$ have the following optimization program:

$$\left| \begin{array}{l} \max_{n_0} \left\{ -\left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right) + \mathbb{E}_\eta [J(n_0, \eta, P_1)] \right\} \\ n_0 \geq 0 \end{array} \right. \tag{9}$$

where the constraint is the short-sales constraint and the expectation is taken over the belief shocks (η^A, η^B).

The next theorem describes the date-0 equilibrium. In this symmetric setting, it is particularly simple. Both groups of agents are long and hold initial supply Q. The date-0 demand is driven by the anticipation of the date-1 equilibrium. When agents consider a large belief shock, they anticipate they will end up short-sales constrained. In this case, holding n_0 shares at date 0 allows the agents to receive $n_0 P_1$ at date 1 minus the trading cost associated with the reselling of the date-0 holding or $\frac{n_0^2}{2\gamma}$. Or agents consider a small belief shock, and thus anticipate to be long at date 1, i.e. that they will not become too pessimistic relative to the other group. In this case, it is easily shown that their utility from holding n_0 shares at date 0 is proportional to the expected payoff from the asset conditional on the date 1 belief $\pi(\eta^i)$.

Theorem 2. ***Date-0 equilibrium.***

At date-0, each group owns Q shares. The date-0 price is given by:

$$P_0 = \int_{-\infty}^{\infty} \left[\left(\pi(y) - \frac{2Q}{\gamma} \right) \Phi \left(\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}] \right) + \int_{\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}]}^{\infty} \pi(x)\phi(x)dx \right] \phi(y)dy - \frac{Q}{\gamma} \tag{10}$$

Proof. At date 0, group A's program can be written as:

$$\max_{n_0} \left\{ \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]} \left(n_0 \left(\pi(y) - \frac{Q}{\gamma} \right) - \frac{n_0^2}{2\gamma} \right) \phi(x) dx \right.\right.$$

$$\left.\left. + \int_{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]}^{\infty} \left(n_1^\star(x)\pi(x) + (n_0 - n_1^\star(x))P_1(x,y) - \frac{(n_1^\star(x)-n_0)^2}{2\gamma} \right) \phi(x) dx \right] \phi(y) dy - \left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right) \right\}$$

Let $G + b + x$ be the date-1 belief of group A agents and $G + b + y$ be the date-1 belief of group B agents. The first integral corresponds to the case where group A agents are short-sales constrained. This happens when $\pi(x) < \pi(y) - \frac{2Q}{\gamma} \Leftrightarrow x < \pi^{-1}\left(\pi(y) - \frac{2Q}{\gamma}\right)$. In this case, group A agents re-sell their date-0 holdings for a price $P_1 = \pi(y) - \frac{Q}{\gamma}$ and pay the trading cost $\frac{n_0^2}{2\gamma}$. The second integral corresponds to the case where group A agents are not short-sell constrained and their date-1 holding is given by the interior solution to the F.O.C., $n_1^\star(x)$. The corresponding payoff is the expected payoff from the date-1 holding with date-1 belief, i.e. $n_1^\star(x)\pi(x)$ plus the potential gains (resp. cost) of selling (resp. buying) some shares $((n_0 - n_1^\star(x))P_1(x,y))$ minus the trading costs $(\frac{(n_1^\star(x)-n_0)^2}{2\gamma})$ of adjusting the date-1 holding.

Note that the bounds defining the two integrals depend on the aggregate holding of group A, but group A agents have no impact individually on this aggregate holding n_0^A. Thus, they maximize only over n_0 in the previous expression and take n_0^A as given. Similarly, agents consider $P_1(x,y)$ as given (i.e. they do not take into account the dependence of P_1 on the aggregate holdings n_0^A and n_0^B).

To derive the F.O.C. of group A agents' program, use the envelope theorem to derive the second integral w.r.t. n_0. For this integral, the envelope theorem applies as $n_1^\star(x)$ is determined according to the date-1 interior F.O.C.. We thus have:

$$\frac{\partial \left(n_1^\star(x)\pi(x) + (n_0 - n_1^\star)P_1(x,y) - \frac{(n_1^\star - n_0)^2}{2\gamma} \right)}{\partial n_0} = P_1(x,y) + \frac{n_1^\star(x) - n_0}{\gamma} = \pi(x).$$

Thus, the overall F.O.C. writes:

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]} \left(\pi(y) - \frac{Q}{\gamma} - \frac{n_0}{\gamma} \right) \phi(x) dx + \int_{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\gamma}]}^{\infty} \pi(x)\phi(x) dx \right] \phi(y) dy - \left(P_0 + \frac{n_0}{\gamma} \right) = 0$$

The model is symmetric. Hence, it has to be that $n_0^A = n_0^B = Q$. Substituting in the previous F.O.C. gives the date-0 equilibrium price:

$$P_0 = \int_{-\infty}^{\infty} \left[\left(\pi(y) - \frac{2Q}{\gamma} \right) \Phi \left(\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}] \right) + \int_{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]}^{\infty} \pi(x)\phi(x) dx \right] \phi(y) dy - \frac{Q}{\gamma}$$

$\square$

3.4. Comparative Statics

Now that we have solved for the dynamic equilibrium of this model, we are interested in how mispricing, price volatility and share turnover depend on the following parameters: the structure of the credit claim (D), the bias of the agents' prior (b) and the fundamental of the economy (G). We will relate the predictions derived from these comparative statics to the stylized facts gathered in Section 2.

To be more specific, we first define the bubble or mispricing, which we take to be P_0, the equilibrium price, minus $\bar{P}_0$, the price of the asset in the absence of short-sales constraints *and* with no aggregate bias ($b = 0$). This benchmark or unconstrained price can be written as:[10]

$$\bar{P}_0 = \int_{-\infty}^{\infty} \pi(\eta - b)\phi(\eta)d\eta - \frac{Q}{\gamma}. \tag{11}$$

Now define $\hat{P}_0$ as the date-0 price when there are no short-sales constraint but the aggregate bias is b. This price is given by

$$\hat{P}_0 = \int_{-\infty}^{\infty} \pi(\eta)\phi(\eta)d\eta - \frac{Q}{\gamma}. \tag{12}$$

The date-0 price can then be decomposed in the following way:

$$P_0 = \hat{P}_0 + \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\gamma}]} \left(\pi(y) - \pi(x) - \frac{2Q}{\gamma} \right) \phi(x)dx \right) \phi(y)dy. \tag{13}$$

[10]First, if there is no bias b, then the belief of an agent with belief shock η will be $G + \eta$. Thus, this agent will expect a payoff $\pi(\eta - b)$. Moreover, when there is no short-sales constraint, the formula for the price is similar to equation 10, except that the short-sales constraint region shrinks to 0.

Then we can decompose the bubble into the following two terms:

$$\text{bubble} = \underbrace{\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]}\left(\pi(y)-\pi(x)-\frac{2Q}{\gamma}\right)\phi(x)dx\right)\phi(y)dy}_{\text{resale option}} + \underbrace{\hat{P}_0-\bar{P}_0}_{\text{optimism}} \tag{14}$$

In this simple model, the bubble emerges from two sources: (1) there is a resale option due to binding short-sales constraints in the future and (2) agents are optimistic about the asset payoff and thus drive its price up.

The second quantity we are interested in is expected share turnover. It is simply defined as the expectation of the number of shares exchanged at date 1. Formally:

$$\mathbb{T} = \mathbb{E}_{\{\eta^A,\eta^B\}}\left[|n_1^A - n_0^A|\right] \tag{15}$$

Share turnover can be expressed in our setting as:

$$\mathbb{T} = \int_{-\infty}^{\infty}\left(\underbrace{Q(\Phi(\underline{x}(y)) + (1-\Phi(\bar{x}(y)))}_{\text{A,B short-sales constrained}} + \underbrace{\int_{\underline{x}(y)}^{\bar{x}(y)}\mu\gamma\frac{|\pi(y)-\pi(x)|}{2}d\Phi(x)}_{\text{no short-sales constraint}}\right)d\Phi(y),$$

where $\pi(\underline{x}(y)) = \pi(y) - \frac{2Q}{\gamma}$ and $\pi(\bar(x)(y)) = \pi(y) + \frac{2Q}{\gamma}$. Intuitively, if y is the interim belief shock of agents in group A, then when agents in group B have an interim belief shock x below $\underline{x}(y)$ (resp. above $\bar{x}(y)$) agents in group B (resp. agents in group A) are short-sales constrained. Conditional on one group of agent being short-sales constrained, share turnover is maximum and equal to Q. When neither group is short-sales constrained, turnover is just proportional to the difference in valuation between the optimistic and the pessimistic group.

The third object is price volatility between $t = 0$ and $t = 1$. Price volatility is defined simply by:

$$\sigma_P = \text{Var}_{\{\eta^A,\eta^B\}}\left[P_1(\eta^A,\eta^B)\right] \tag{16}$$

The following proposition shows how these three quantities depend on D.

Proposition 1. *An increase in D (the riskiness of debt) leads to an increase in (1) mispricing (2) share turnover and (3) price volatility.*

Proof. See Appendix. □

Proposition 1 offers a rationale for why debt bubbles are smaller and quieter than equity ones. The main intuition is that because the credit payoff is bounded by D, it is insensitive to beliefs on the distribution of payoffs above D. Thus, when D is low, there is very little scope for disagreement – the credit is almost risk-free and its expected payoff is close to its face value, and is in particular almost independent of the belief about the fundamental value. Short-sales constraint are thus not likely to bind (as short-sales constraints at date 1 arise from large differences in belief about the expected payoff). As a result, the resale option is low (i.e. the asset will most likely trade at its "fair" value at date 1) and mispricing is low. This, in turns, leads to low expected turnover as turnover is maximized when the agents' short-sales constraint binds.

Similarly, volatility will be low as prices will be less extreme (intuitively, the date-1 price will be representative of the average of the two groups beliefs rather than of the maximum of the two groups' beliefs). This mechanism builds on the analysis in Hong, Scheinkman, and Xiong (2006) which relies on risk averse investors and a positive supply of the security so that there are regions in which both groups of investors are long. As D increases, agents' belief matters more for their valuation of the credit, both because of the recovery value conditional on default and because of the default threshold. In the extreme, when D grows to infinity, the credit becomes like an equity, beliefs become relevant for the entire payoff distribution of the asset and the scope for disagreement is maximum. This leads to more binding short-sales constraint at date 1, and hence more volatility and expected turnover.

A simple remark emerges from this analysis. In this pure resale option model, mispricing and turnover/volatility go hand in hand – an increase in mispricing is associated with more price volatility and more turnover. There is, however, one mechanism that leads to a decoupling of prices and turnover volatility. When the aggregate bias increases (i.e. b increases), mispricing increases *and* turnover/volatility decrease. Increasing the aggregate bias b decreases the scope for disagreement and thus reduces the resale option, volume and volatility. However, because G is held fixed, the fundamental price of the asset has not changed. Thus, even though the speculative component of the price decreases as b increases, the price overall gets further away from its fundamental value and thus mispricing increases. In this sense, an increase in aggregate optimism leads to a quieter credit bubble but interestingly does not lead to a quieter equity bubble.

Proposition 2. *Assume that $D < \infty$. An increase in aggregate optimism (i.e. b) leads to (1) higher mispricing (2) lower price turnover, and (3) lower volatility.*
If the asset is an equity ($D = \infty$), an increase in aggregate optimism leads to higher mispricing but leaves price turnover and volatility unaffected.

Proof. We first look at mispricing. Note that $\bar{P}_0$ is independent of b. Thus:

$$\frac{\partial \text{mispricing}}{\partial b} = \frac{\partial P_0}{\partial b} = \int_{-\infty}^{\infty} \left(\Phi\left(\underline{x}(y)\right) \Phi(D - G - b - y) + \int_{\underline{x}(y)}^{\infty} \Phi(D - G - b - x)\phi(x)dx \right) \phi(y)dy > 0$$

Formally, the derivative of turnover and price volatility w.r.t. b is equal to the derivative of turnover and price volatility w.r.t. G. Thus, thanks to the proof of proposition 3 below:

$$\frac{\partial \mathbb{T}}{\partial b} < 0 \text{ and } \frac{\partial \mathcal{V}}{\partial b} < 0$$

□

In our model, provided that the payoff function is strictly concave (or equivalently that $D < \infty$), an increase in average optimism makes the bubble bigger and quieter at the same time. This can be contrasted with the case of a straight equity claim, where both volatility and turnover would be left unaffected by variations in the average optimism – even in the case of binding short-sales constraint. This is because differences in opinion about an asset with a linear payoff are invariant to a translation in initial beliefs. Thus while an increase in optimism would obviously inflate the price of an equity, it would not change its price volatility nor its turnover.

Propositions 1-2 explain why the recent credit bubble is quieter than the Internet bubble. These two results rationalize Figures 1-4, which presented stylized facts that the recent credit bubble was quiet in contrast to classic speculative episodes such as the dot-com bubble. The reason is that an increase in optimism makes both credit and equity bubbles big. But it makes credit bubbles quiet while leaving the loudness of equity bubbles unchanged.

In Proposition 2, we held fixed G and considered how a change in b influence properties of the credit bubble. In the next proposition, we hold fix b and consider the comparative static with respect to G.

Proposition 3. *A decrease in G (the riskiness of debt) leads to an increase in (1) mispricing (2) share turnover, and (3) price volatility.*

Proof. See Appendix. □

When fundamentals deteriorate, the credit claim becomes riskier and hence disagreement becomes more important for its valuation. This increase in disagreement sensitivity leads to an increase in the resale option (the speculative component of the date-0 price increases as short-sales constraints are more likely to bind at date 1) and hence higher mispricing. This triggers an increase in both price volatility and turnover as argued above and the bubble stops being quiet. This time-series behavior can be seen in Figure 1 above as price volatility increases in the year preceding the crisis and the collapse of these markets. Notice that the price volatility of even the highest rated AAA tranches begin to exhibit significant price movements in the year preceding the financial crisis.

Our model thus leads to very different predictions than standard model of adverse selections (e.g., see the discussion by Holmstrom (2008)). In these models, a deterioration in the fundamental of the economy destroys the information-insensitiveness of the credit, which reinforces adverse selection and potentially leads to a market breakdown. Thus a worsening of the economy leads to lower trading activity. In our model, however, when the economy worsens, agents realize that disagreement matters for the pricing of the credit in future periods – which drives up the resale option and subsequently increases volatility and trading volume.

3.5. Equity with Limited Liability

Our analysis so far has implicitly compared a credit claim (finite D) with an unlevered equity claim ($D = \infty$) on the same underlying asset. It is fairly direct to extend our results to the case where we compare the levered equity claim that complements the credit claim in the asset value space. Formally, we define the expected payoff function for the levered equity claim under fundamental G, aggregate optimism b, belief shock η and principal on the debt claim D as:

$$\pi^E(\eta) = \int_{D-G-b-\eta}^{\infty} (G + b + \eta + \epsilon - D)\, d\Phi(\epsilon)$$

The expected payoff of the debt claim is similar to our previous analysis and is easily defined by: $\pi^D(\eta) = G + b + \eta - \pi^E(\eta)$.

The following proposition compares the loudness of these two tranches:

Proposition 4. *There exists $\bar{G}$ such that if the fundamental is high enough ($G \geq \bar{G}$), the equity tranche has greater mispricing, share turnover and volatility than the debt tranche. Similarly, provided that aggregate optimism is high enough ($b \geq \bar{b}$) or that the principal on the loan is small enough $D \leq \bar{D}$, the equity tranche has greater mispricing, share turnover and volatility than the debt tranche.*

Proof. See Appendix. □

Intuitively, when G increases, the debt tranche becomes less disagreement sensitive while the equity tranche becomes more disagreement sensitive. As a consequence, the relative mispricing of equity vs. debt increases, as well as their relative turnover and volatility. One purpose of this proposition is to show that provided that b is large, i.e. provided that the bubble is large enough, our results that credit bubbles are quieter than equity bubbles is robust to the consideration of shareholders' limited liability. One can thus interpret the main results of this paper – the relative quietness of credit and equity bubbles – as the comparison between two "bubbly" states of nature, i.e. two states of nature where b is large enough, and the ranking in this environment of the relative quietness of a credit bubble (the CDO bubble of 2005-2007) and an equity bubble (the dot-com bubble of the 90's).

3.6. Characterizing Disagreement Sensitivity

In this section, we move away from the simple debt/equity dichotomy we have emphasized up to now. Our objective is to provide a characterization of the payoffs of various assets that allows us to rank them according to their disagreement sensitivity. We consider the following problem. Take two derivatives on the same underlying fundamental, with payoff function $\pi_1()$ and $\pi_2()$. To get rid of level effects, we make the assumption that π_1 and π_2 have the same expected fundamental value, i.e.:[11]

$$\int_0^\infty \pi_1(x)\phi(x)dx = \int_0^\infty \pi_2(x)\phi(x)dx$$

The next proposition proposes a sufficient condition under which π_1 will lead to a larger and louder bubble than π_2 for any distribution of the belief shocks (η^A, η^B):

Proposition 5. *Assume that for all $x \in \mathbb{R}$, $\pi_1'(x) \geq \pi_2'(x)$ and that the inequality holds strictly on a non-empty set. Then asset 1 has a strictly larger date-0 price, a strictly larger expected turnover and a strictly larger expected volatility than asset 2.*

Proof. Consider the function $\Delta(x) = \pi_1(x) - \pi_2(x)$. Thanks to our assumption on π_1 and π_2, Δ is increasing over $\mathbb{R}$. We thus have:

$$\forall\ y \geq x, \quad \Delta(y) \geq \Delta(x) \Leftrightarrow \pi_1(y) - \pi_1(x) - \frac{Q}{2\gamma} \geq \pi_2(y) - \pi_2(x) - \frac{Q}{2\gamma}$$

[11] Note that in our debt setup, an increase in D was increasing both the disagreement sensitivity of the debt *and* its expected payoff. In this sense, the exercise we consider in this section is more precise in that we only consider variations in the slope of the payoff function for a constant expected value.

Moreover:

$$
\begin{aligned}
& \pi_1^{-1}(\pi_1(y) - \tfrac{2Q}{\gamma}) \geq \pi_2^{-1}(\pi_2(y) - \tfrac{2Q}{\gamma}) \\
\Leftrightarrow \quad & \pi_1(y) - \pi_1\left[\pi_2^{-1}(\pi_2(y) - \tfrac{2Q}{\gamma})\right] \geq \tfrac{2Q}{\gamma}
\end{aligned}
$$

But because π_2 is increasing, we know that $y \geq \pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma})$. Moreover, because Δ is increasing, we know that: $\Delta(y) \geq \Delta(\pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma}))$. This implies:

$$
\pi_1(y) - \pi_2(y) \geq \pi_1\left[\pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma})\right] - \pi_2(y) + \frac{2Q}{\gamma} \Leftrightarrow \pi_1(y) - \pi_1\left[\pi_2^{-1}(\pi_2(y) - \frac{2Q}{\gamma})\right] \geq \frac{2Q}{\gamma}
$$

Thus, this proves that $P_0(\pi_1) \geq P_0(\pi_2)$. This is because (1) short sales constraint are binding more often under π_1 than π_2 and (2) when short-sales constraints are binding, the difference between the actual price and the no-short sales constraint price (which is proportional to the difference in beliefs between the two groups) is larger under π_1 than under π_2. Note that the inequality will be strict as soon as the derivatives of π_1 is strictly greater than the derivative of π_2 on a non-empty set of $\mathbb{R}$.

Similarly, it is direct to show that turnover and volatility will be greater under π_1 than under π_2. For instance, short-sales constraints bind more often with π_1 and turnover is then maximum and equal to Q. Moreover, when short-sales constraint do not bind, turnover is proportional to the difference in belief between the optimistic and the pessimistic group and we know that this difference will be larger under π_1 than under π_2. □

Intuitively, the payoff function with the largest slope will be such that differences in beliefs lead to larger differences in valuation for the asset. It will thus have the largest probability that short-sales constraints are binding and hence will have the largest price, expected turnover and expected volatility. Note that because of the constant expected value assumption, the condition in proposition 5 is similar to a single crossing condition.[12]

Finally, note that to derive a necessary condition to rank the two assets, one needs to make assumptions on the p.d.f. of the beliefs shocks (η_1, η_2). In particular, if the condition has to hold for any distribution of the belief shocks, then the condition in proposition 5 is

[12] We have $\lim_{x\to-\infty} \Delta(x) \leq 0$. If this was not the case, then $\Delta(x)$ would be strictly positive for all x and the two assets could not have the same expected value. Similarly, $\lim_{x\to-\infty} \Delta(x) \geq 0$.

also a necessary condition.

4. Discussions of The Financial Crisis of 2008

Our theory complements recent work on the financial crisis. An important narrative is that a wave of money from China needed a safe place to park and in light of the lack of sovereign debt, Wall Street created AAA securities as a new parking place for this money. This search for safe assets let to inflated prices through a variety of channels. Caballero and Krishnamurthy (2009) provides a theory of how search for safe assets led to a shrinking of the risk premium. Gennaioli, Shleifer, and Vishny (2010) focuses on the neglected risks from this demand for safety. Another narrative is that of agency and risk-shifting as in Allen and Gale (2000), which can also deliver asset price bubbles and would seem vindicated by the bailouts of the big banks. Gorton, Holmstom, and Dang (2010) and Holmstom (Forthcoming) provide a theory of why these securitized debt products are a good substitute for Treasuries since they are information insensitive except when the economy is hit by a negative shock and they become more sensitive to private information, resulting in a loss of liquidity and trade.

Our mechanism shares the most with the last approach in terms of our emphasis on the insensitivity of debt to underlying disagreement about asset values. Focusing on disagreement instead of asymmetric information leads to an opposite prediction compared to theirs, in which there is little trading in good times and more trading in bad times in CDOs. Our prediction is borne out in anecdotal accounts of the rise in speculative trade of CDOs after 2006-2007 but before the financial crisis of 2008. Our approach cannot capture the financial crisis and the ensuing freeze in trade.

Our theory also has a number of implications that are distinct from this earlier work. First, it offers a unified approach to explain both "classic" bubbles such as the dot-com and the recent so-called credit bubble.[13] Second, it offers a story for how a financial crisis of this

[13] Our paper focuses on mispricing of the mortgages as opposed to the homes since evidence from Mian and Sufi (2009) indicate that the run-up in real estate prices was due to the cheap subprime home loans and that

size could emerge despite our recent experience with bubbles. Anecdotal accounts of the crisis invariably point to how the crisis and the credit bubble caught everyone by surprise. Indeed, this is an especially large conundrum when one considers that sophisticated finance companies such as Goldman Sachs were able to survive and indeed thrive through many prior speculative episodes, including the dot-com boom and bust. So why did companies like Goldman Sachs get caught this time?

Our unified theory offers a rationale for why smart investment banks and regulators failed to see the bubble on time. The low price volatility made these instruments appear safer in contrast to the high price volatility of dot-com stocks which made traders and regulator more aware of their dangers. Interestingly, our theory also predicts that riskier tranches of CDOs trade more like equity and hence have more price volatility and turnover, which is also consistent with the evidence. These securities were also less responsible for the failure of the investment banks which were mostly caught with the higher rated AAA tranches of the subprime mortgage CDOs. Other theories for the crisis based on agency or financial innovation cannot explain why banks did not get caught during the dot-com bubble since they had the same agency problems and innovative products to price.

Finally, our analysis also has implications for thinking about the Dodd-Frank financial reform package. This legislation establishes, among many things, an Office of Financial Research and a Financial Stability Oversight Committee that are meant to detect the next bubble. The premise of the Office of Financial Research is that by forcing finance firms to disclose their positions, regulators will be in a better position to detect the next speculative episode. Our model suggests that the quietness of the credit bubble would have made it difficult to detect regardless. Hence, regulators should distinguish between loud versus quiet bubbles as the signs are very different.

households took out refinancing from the equity of their homes for consumption as opposed to the purchase of additional homes, suggesting that there was limited speculation among households in the physical homes themselves. Khandani, Lo, and Merton (2010) make a similar point that the financial crisis was due to the refinancing of home loans as opposed to the speculative aspects described in the media.

5. Conclusion

In this paper, we attempt to understand the dynamics of equity and credit bubbles within a unified framework built on investor disagreement and short-sales constraints. Our analysis is motivated by the observation that the classic speculative episodes such as the dot-com bubble usually come with high price, high price volatility and high turnover, while the recent credit bubble appears much quieter. We show that credit bubbles are quieter than equity ones because the up-side concavity of debt payoffs means debt instruments (especially higher rated ones) are less disagreement-sensitive than lower rated credit or equity. As a consequence, optimism which increases the size of credit and equity bubbles makes credit bubbles quiet but leaves the loudness of equity bubbles unchanged. We offer a first attempt at a taxonomy of bubbles that distinguishes between loud equity bubbles and quiet credit bubbles. Future work elaborating on this taxonomy and providing other historical evidence would be very valuable.

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A. Appendix

A.1. Proof of Proposition 1 and 3

As shown in the text, mispricing can be written as:

$$\text{mispricing} = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]}\left(\pi(y)-\pi(x)-\frac{2Q}{\gamma}\right)\phi(x)dx\right)\phi(y)dy+\int_{-\infty}^{\infty}\left(\pi(y)-\pi(y-b)\right)\phi(y)dy$$

Note that $x < \pi^{-1}\left[\pi(y)-\frac{2Q}{\gamma}\right] \Rightarrow x < y$. Moreover, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} = \Phi(D-G-b-x)-\Phi(D-G-b-y)$. Thus, for all $x < \pi^{-1}\left[\pi(y)-\frac{2Q}{\gamma}\right]$, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} > 0$. Similarly, as $b > 0$, $\frac{\partial(\pi(y)-\pi(y-b))}{\partial D} = \Phi(D-G-y)-\Phi(D-G-b-y) > 0$. Thus, the derivative of mispricing w.r.t. D is strictly positive:

$$\frac{\partial(\text{mispricing})}{\partial D} = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]}\underbrace{\frac{\partial(\pi(y)-\pi(x))}{\partial D}}_{>0}\phi(x)dx\right)\phi(y)dy+\int_{-\infty}^{\infty}\underbrace{\frac{\partial(\pi(y)-\pi(y-b))}{\partial D}}_{>0}\phi(y)dy \tag{17}$$

Thus, as D increases, both the resale option and the mispricing due to aggregate optimism increases, so that overall mispricing increases.

We now turn to expected turnover. To save on notations, call $\bar{x}(y)$ the unique real number such that: $\pi(\bar{x}(y)) = \pi(y)+\frac{2Q}{\gamma}$. Similarly, call $\underline{x}(y)$ the unique real number such that: $\pi(\underline{x}(y)) = \pi(y)-\frac{2Q}{\gamma}$. Obviously, $\underline{x}(y) < y < \bar{x}(y)$. Expected turnover is:

$$\mathbb{T} = \int_{-\infty}^{\infty}\left(\underbrace{\int_{-\infty}^{\underline{x}(y)}Q\phi(x)dx}_{\text{A short-sales constrained}}+\underbrace{\int_{\underline{x}(y)}^{\bar{x}(y)}\gamma\frac{|\pi(y)-\pi(x)|}{2}\phi(x)dx}_{\text{no short-sales constraint}}+\underbrace{\int_{\bar{x}(y)}^{\infty}Q\phi(x)dx}_{\text{B short-sales constrained}}\right)\phi(y)dy$$

We can take the derivative of the previous expression w.r.t. D. Note that the derivative of the bounds in the various integrals cancel out, so that:

$$\frac{\partial\mathbb{T}}{\partial D} = \int_{-\infty}^{\infty}\left(\int_{\underline{x}(y)}^{\bar{x}(y)}\frac{\gamma}{2}\frac{\partial\,|\pi(y)-\pi(x)|}{\partial D}\phi(x)dx\right)\phi(y)dy \tag{18}$$

If $y \geq x$, $\frac{\partial|\pi(y)-\pi(x)|}{\partial D} = |\Phi(D-G-b-x)-\Phi(D-G-b-y)| > 0$. Thus turnover is strictly increasing with D.

We now turn to variance. Formally, note: $P_1(\tilde{x},\tilde{y},D)$ the date-1 price when one agent has belief shock $\tilde{x}$, the other $\tilde{y}$ and the face value of debt is D.

$$P_1(\tilde{x},\tilde{y},D) = \begin{cases} \pi(\tilde{x}) - \dfrac{Q}{\gamma} & \text{if } \pi(\tilde{x}) \geq \pi(\tilde{y}) + \dfrac{2Q}{\gamma} \\ \dfrac{\pi(\tilde{x})+\pi(\tilde{y})}{2} & \text{if } |\pi(\tilde{x}) - \pi(\tilde{y})| \leq \dfrac{2Q}{\gamma} \\ \pi(\tilde{y}) - \dfrac{Q}{\gamma} & \text{if } \pi(\tilde{y}) \geq \pi(\tilde{x}) + \dfrac{2Q}{\gamma} \end{cases}$$

We have:

$$\mathbb{E}_{x,y}[(P_1(\tilde{x},\tilde{y})^2] = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\underline{x}(y)} \left(\pi(y) - \frac{Q}{\gamma}\right)^2 \phi(x)dx + \int_{\underline{x}(y)}^{\bar{x}(y)} \left(\frac{\pi(x)+\pi(y)}{2}\right)^2 \phi(x)dx + \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma}\right)^2 \phi(x)dx \right) \phi(y)dy \tag{19}$$

We can take the derivative of the previous expression w.r.t. D. Call $K = D - G - b$:

$$\begin{aligned} &\tfrac{1}{2}\tfrac{\partial \mathbb{E}[(P_1(\tilde{x},\tilde{y})^2]}{\partial D} \\ = \quad & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\underline{x}(y)} (1 - \Phi(K-y)) \left(\pi(y) - \frac{Q}{\gamma}\right) \phi(x)dx + \int_{\underline{x}(y)}^{\bar{x}(y)} \left(1 - \frac{\Phi(K-x)+\Phi(K-y)}{2}\right) \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \\ & + \int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} (1 - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma}\right) \phi(x)dx \right] \phi(y)dy \end{aligned}$$

Note first that:

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\underline{x}(y)} (1 - \Phi(K-y)) \left(\pi(y) - \frac{Q}{\gamma}\right) \phi(x)dx \right] \phi(y)dy = \int_{-\infty}^{\infty} (1 - \Phi(K-y)) \left[\int_{-\infty}^{\underline{x}(y)} \left(\pi(y) - \frac{Q}{\gamma}\right) \phi(x)dx \right] \phi(y)dy$$

Now the second term in equation 19 can be decomposed into:

$$\begin{aligned} & \int_{-\infty}^{\infty} \left[\int_{\underline{x}(y)}^{\bar{x}(y)} \left(1 - \frac{\Phi(K-x)+\Phi(K-y)}{2}\right) \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \\ = \quad & \int_{-\infty}^{\infty} \left[\int_{\underline{x}(y)}^{\bar{x}(y)} (1 - \Phi(K-y)) \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \\ & \quad + \underbrace{\int_{-\infty}^{\infty} \left[\int_{\underline{x}(y)}^{\bar{x}(y)} \frac{\Phi(K-y) - \Phi(K-x)}{2} \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy}_{=0} \\ = \quad & \int_{-\infty}^{\infty} (1 - \Phi(K-y)) \left[\int_{\underline{x}(y)}^{\bar{x}(y)} \left(\frac{\pi(x)+\pi(y)}{2}\right) \phi(x)dx \right] \phi(y)dy \end{aligned}$$

Thus, eventually:

$$\frac{1}{2}\frac{\partial \mathbb{E}[(P_1(\tilde{x},\tilde{y})^2]}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\gamma}\right)\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}\right)\phi(x)dx\right]\phi(y)dy$$

Call $m=\mathbb{E}_{x,y}[P_1(\tilde{x},\tilde{y},D)]$. The derivative of m^2 w.r.t. to D is simply:

$$\frac{1}{2}\frac{\partial\left(\mathbb{E}[P(x,y)]\right)^2}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}m\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}m\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\,m\phi(x)dx\right]\phi(y)dy$$

and $\mathcal{V}=Var(P_1(\tilde{x},\tilde{y},D))=\mathbb{E}_{x,y}[(P_1(\tilde{x},\tilde{y})^2]-m^2$. We have:

$$\frac{1}{2}\frac{\partial\mathcal{V}}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\gamma}-m\right)\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}-m\right)\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}-m\right)\phi(x)dx\right]\phi(y)dy$$
$$= \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\gamma}-m\right)d\Phi(x)+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}-m\right)d\Phi(x)+\int_{\bar{x}(y)}^{\infty}\left(\pi(x)-\frac{Q}{\gamma}-m\right)d\Phi(x)\right]d\Phi(y)$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(\Phi(K-y)-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}-m\right)\phi(x)dx\right]\phi(y)dy$$

First note that $(1-\Phi(K-y))$ is an increasing function of y, as well as $\mathbb{E}[P(x,y)-m|y]$. Thus, these two random variables have a positive covariance and because $\mathbb{E}_y[\mathbb{E}_x[P(x,y)-m|y]]=0$, this implies that $\mathbb{E}_y[(1-\Phi(K-y))\times\mathbb{E}_x[P(x,y)-m|y]]\geq 0$, i.e. the first term in the previous equation is positive.

Now, consider the function: $x\to\pi(x)-\frac{Q}{\gamma}-m$. It is strictly increasing with x over $[\bar{x},\infty($. Call $x^0=\pi^{-1}(\frac{Q}{\gamma}+m)$. Assume first that $\bar{x}(y)>x^0$ (i.e. $y>\pi^{-1}(m-\frac{Q}{\gamma})$).Then for all $x\in[\bar{x},\infty($:

$$(\Phi(K-y)-\Phi(K-x))\left(\pi(x)-\frac{Q}{\gamma}-m\right)>\left(\Phi(K-y)-\Phi(K-x^0)\right)\left(\pi(x)-\frac{Q}{\gamma}-m\right)$$

Now if $\bar{x} < x^0$, then:

$$\begin{aligned}
\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx &= \int_{\bar{x}(y)}^{x^0} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\quad + \int_{x^0}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\geq (\Phi(K-y) - \Phi(K-x^0)) \int_{\bar{x}(y)}^{x^0} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\quad + (\Phi(K-y) - \Phi(K-x^0)) \int_{x^0}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \\
&\geq (\Phi(K-y) - \Phi(K-x^0)) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx
\end{aligned}$$

Thus, for all $y \in \mathbb{R}$,

$$\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx \geq \left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx$$

This leads to:

$$\begin{aligned}
&\int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx\right] \phi(y)dy \\
\geq \; &\int_{-\infty}^{\infty} \left[(\Phi(K-y) - \Phi(K-x^0)) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx\right] \phi(y)dy
\end{aligned}$$

Now, $\Phi(K-y) - \Phi(K-x^0)$ is a decreasing function of y. $\int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx$ is also a decreasing function of y. Thus, the covariance of these two random variable is positive. But note that:

$$\int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\gamma} - m\right) \phi(x)dx\right] \phi(y)dy = \mathbb{P}\left[\pi(x) \geq \pi(y) + \frac{2Q}{\gamma}\right] \times \left(\mathbb{E}\left[P(x,y) | \pi(x) \geq \pi(y) - \frac{2Q}{\gamma}\right] - \mathbb{E}[P(x,y)]\right)$$

Finally, note that the conditional expectation of prices, conditional on binding short-sales constraints has to be greater than the expected price, m. Thus, this last term is positive and finally the variance of date-1 prices is strictly increasing with D:

$$\frac{\partial Var(P_1(\tilde{x}, \tilde{y}, D))}{\partial D} \geq 0$$

We now turn to the comparative static w.r.t. G. First, note that $\frac{\partial \pi(y)}{\partial G} = \Phi(D - G - b - y) > 0$ and strictly decreasing with y. Now, the derivative of mispricing w.r.t. G is simply

$$\frac{\partial(\text{mispricing})}{\partial G} = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\gamma}]} \underbrace{\frac{\partial(\pi(y)-\pi(x))}{\partial G}}_{<0}\phi(x)dx\right)\phi(y)dy + \int_{-\infty}^{\infty}\underbrace{\frac{\partial(\pi(y)-\pi(y-b))}{\partial G}}_{<0}\phi(y)dy < 0$$

Similarly:

$$\frac{\partial \mathbb{T}}{\partial G} = \int_{-\infty}^{\infty}\left(\int_{\underline{x}(y)}^{\bar{x}(y)} \frac{\gamma}{2}\left|\frac{\partial\left(\pi(y)-\pi(x)\right)}{\partial G}\right|\phi(x)dx\right)\phi(y)dy < 0$$

Finally, note that:

$$\frac{\partial \mathcal{V}}{\partial G} = Cov(\frac{\partial P_1(x,y)}{\partial G}, P_1(x,y)) = Cov(\frac{1-\partial P_1(x,y)}{\partial D}, P_1(x,y)) = -\frac{\partial \mathcal{V}}{\partial D} < 0$$

QED.

A.2. Proof of Proposition 4

Consider first mispricing. The formula for the derivative of mispricing w.r.t. D (equation 17) holds irrespective of the nature of the claim, i.e. whether $\pi = \pi^E$ or $\pi = \pi^D$. We simply remark that π^E, as π^D, is increasing with x (the investor's belief) and $\frac{\partial \pi^E(y)-\pi^E(x)}{\partial D} = \Phi(D-G-b-y) - \Phi(D-G-b-x) = -\frac{\partial \pi^D(y)-\pi^D(x)}{\partial D}$. And similarly: $\frac{\partial \pi^E(y)-\pi^E(y-b)}{\partial D} = \Phi(D-G-b-y) - \Phi(D-G-y) = -\frac{\partial \pi^D(y)-\pi^D(y-b)}{\partial D}$. Thus:

$$\frac{\partial(\text{mispricing on } \pi^E)}{\partial D} = -\frac{\partial(\text{mispricing on } \pi^D)}{\partial D} < 0$$

Thus mispricing of the equity tranche decreases with D. The difference between the mispricing on the equity claim and the mispricing on the debt claim decreases with D as well. When D goes to infinity, there is no mispricing on the equity claim (which is worth 0) so that the difference between the mispricing on the equity claim and the mispricing on the debt claim is strictly negative. Similarly, when D goes to $-\infty$, there is no mispricing on the debt claim (which is worth 0) so that the difference between the mispricing and the equity claim on the mispricing on the debt claim is strictly positive. Thus, there exists a unique $\bar{D}^1 \in \mathbb{R}$ such that for $D \geq \bar{D}^1$, there is a larger mispricing on the equity claim than on the debt claim.

Consider now turnover. The formula for the derivative of turnover w.r.t. D (equation 18) holds irrespective of the nature of the claim, i.e. whether $\pi = \pi^E$ or $\pi = \pi^D$. We simply remark that π^E, as π^D, is increasing with x (the investor's belief) and $\frac{\partial \pi^E(y)-\pi^E(x)}{\partial D} = \Phi(D-G-b-y) - \Phi(D-G-b-x) = -\frac{\partial \pi^D(y)-\pi^D(x)}{\partial D}$.

Thus:

$$\frac{\partial \mathbb{T}(\pi^E)}{\partial D} = -\frac{\partial \mathbb{T}(\pi^D)}{\partial D} < 0$$

Thus, the turnover of the equity tranche decreases with D. The difference between the turnover on the equity claim and the turnover on the debt claim decreases with D as well. When D goes to infinity, there is no turnover on the equity claim (which is worth 0) so that the difference between the turnover on the equity claim and the turnover on the debt claim is strictly negative. Similarly, when D goes to $-\infty$, there is no turnover on the debt claim (which is worth 0) so that the difference between the turnover on the equity claim and the turnover on the debt claim is strictly positive. Thus, there exists a unique $\bar{D}^2 \in \mathbb{R}$ such that for $D \geq \bar{D}^2$, there is a larger mispricing on the equity claim than on the debt claim.

That the variance of $P_1^E(\tilde{x}, \tilde{y})$ is decreasing with D is direct from $\frac{\partial \pi^E}{\partial D}(x) = -\frac{\partial \pi^D}{\partial D}(x)$. Thus, we can apply the same argument as for mispricing and turnover and show the existence of a unique $\bar{D}^3 \in \mathbb{R}$ such that for $D \geq \bar{D}^3$, the variance of the equity tranche is larger than the variance of the debt tranche. To finish the proof, simply define $\bar{D} = \max(\bar{D}^1, \bar{D}^2, \bar{D}^3)$.

The proof for the existence of $\bar{G}$ and $\bar{b}$ follows exactly the same logic.

A.3. Extension: Interim Payoffs and Dispersed Priors

As we showed in the previous section, an increase in aggregate optimism leads to both larger *and* quieter bubbles while leaving unchanged the loudness of equity bubbles. In this section, we highlight another mechanism that makes credit bubbles both larger and quieter while still holding aggregate optimism fixed. In order to do so, we add two additional ingredients to our initial model. First, we introduce heterogenous priors. Group A agents start at date 0 with prior $G + b + \sigma$ and group B agents start with prior $G + b - \sigma$. Second, we introduce an interim payoff $\pi(G + \epsilon_1)$ that agents receive at date-1 from holding the asset at date 0. As a consequence, agents now hold the asset both for the utility they directly derive from it (consumption) and for the perspective of being able to resell it to more optimistic agents in the future (speculation). More precisely, the $t = 1$ interim cash-flow $\pi(G + \epsilon_1)$ occurs before the two groups of agents draw their date-1 beliefs. We also assume that the proceeds from this interim cash flow, as well as the payment of the date-0 and date-1 transaction costs all occur on the terminal date. This assumption is made purely for tractability reason so we do not have to keep track of the interim wealth of the investors.

Our next proposition shows that, provided that dispersion is large enough, an increase in the initial dispersion of belief, σ, leads to an increase in prices *and* simultaneously to a decrease in share turnover an price volatility. Thus, quiet bubbles emerge when there is sufficient heterogeneity among investors about the fundamental.

Proposition 6. *Provided the cost of trading are large enough/initial supply is low enough, there is $\bar{\sigma} > 0$ so that for $\sigma \geq \bar{\sigma}$ only group A agents are long at date 0. For $\sigma \geq \bar{\sigma}$, an increase in σ leads to (1) an increase in mispricing and (2) a decrease in trading volume.*

Proof. We now consider the case where group A has prior $G+b+\sigma$ and group B has prior $G+b-\sigma$. Thus, at date 1, beliefs are given by $(G+b+\sigma+\eta^A)$ for group A, with $\eta^A \sim \Phi()$ and $(F-\sigma+\eta^B)$ for group B, with $\eta^B \sim \Phi()$. Agents also receive at date 1 an interim payoff proportional to $\pi()$ from holding the asset at date-0. We first start by solving the date-1 equilibrium. At date 1, three cases arise:

1. Both groups are long. Thus demands are:

$$\begin{cases} n_1^A = n_0^A + \gamma\left(\pi(\sigma+\eta^A) - P_1\right) \\ n_1^B = n_0^B + \gamma\left(\pi(-\sigma+\eta^B) - P_1\right) \end{cases}$$

   The date-1 price in this case is: $P_1 = \frac{1}{2}\left(\pi(\sigma+\epsilon^A) + \pi(-\sigma+\epsilon^B)\right)$. This is an equilibrium if and only if: $\frac{2n_0^A}{\gamma} > \pi(-\sigma+\epsilon^B) - \pi(\sigma+\epsilon^A)$ and $\frac{2n_0^B}{\gamma} > \pi(\sigma+\epsilon^A) - \pi(-\sigma+\epsilon^B)$

2. Only A group is long. The date-1 equilibrium price is then simply: $P_1 = \pi(\sigma+\epsilon^A) - \frac{n_0^B}{\gamma}$

   This is an equilibrium if and only if $\pi(\sigma+\epsilon^A) - \pi(-\sigma+\epsilon^B) > \frac{2n_0^B}{\gamma}$.

3. Only B group is long. The date-1 equilibrium price is then simply: $P_1 = \pi(-\sigma+\epsilon^B) - \frac{n_0^A}{\gamma}$. This is an equilibrium if and only if $\pi(-\sigma+\epsilon^B) - \pi(\sigma+\epsilon^A) > \frac{2n_0^A}{\gamma}$

At date 0, group A program can be written as [14] :

$$\max_{n_0}\left\{n_0\pi(\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}\left(n_0\left(\pi(y-\sigma)-\frac{n_0^A}{\gamma}\right)-\frac{n_0^2}{2\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}^{\infty} n_0\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy - \left(n_0P_0+\frac{n_0^2}{2\gamma}\right)\right\}$$

The F.O.C. of group A's agents program is given by (substituting n_0^A for n_0 in the F.O.C.):

$$0 = \pi(\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}\left(\pi(y-\sigma)-\frac{2n_0^A}{\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\gamma}]-\sigma}^{\infty}\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy - \left(P_0+\frac{n_0^A}{\gamma}\right)$$

[14] In group A agents' program, we note n_0^A group A agents aggregate holding – each agent in group A takes n_0^A as given.

Similarly, at date 0, group B agents' program can be written as:

$$\max_{n_0}\left\{\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}\left(n_0\left(\pi(y+\sigma)-\frac{n_0^B}{\gamma}\right)-\frac{n_0^2}{2\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}^{\infty}n_0\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy\right.$$
$$\left.-\left(n_0P_0+\frac{n_0^2}{2\gamma}\right)\right\}$$

Group B agents' F.O.C.:

$$0=\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}\left(\pi(y+\sigma)-\frac{2n_0^B}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\gamma}]+\sigma}^{\infty}\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy-\left(P_0+\frac{n_0^B}{\gamma}\right)$$

Consider now an equilibrium where only group A is long, i.e. $n_0^A=2Q$ and n_0^B=0. In this case, the date-0 price is given by:

$$P_0=\pi(\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}\left(\pi(y-\sigma)-\frac{4Q}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}^{\infty}\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy-\frac{2Q}{\gamma}$$

This is an equilibrium if and only if:

$$P_0>\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{y+2\sigma}\pi(y+\sigma)\phi(x)dx+\int_{y+2\sigma}^{\infty}\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy$$

We now show that P_0 is increasing with σ (noting $K=D-G-b$):

$$\begin{aligned}
\frac{\partial P_0}{\partial\sigma} &= \pi'(\sigma)+\int_{-\infty}^{\infty}\left[\int_{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}^{\infty}\Phi(K-\sigma-x)\phi(x)dx-\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma}\Phi(K-y+\sigma)\phi(x)dx\right]\phi(y)dy\\
&= \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(\pi^{-1}[\pi(y+\sigma)+\frac{4Q}{\gamma}]+\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\gamma}]-\sigma\right)\Phi(K-y+\sigma)\phi(y)dy\\
&\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(y+2\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(y-2\sigma\right)\Phi(K-y+\sigma)\phi(y)dy\\
&\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(y+2\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(y\right)\Phi(K-y-\sigma)\phi(y+2\sigma)dy\\
&\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\Phi(K-\sigma-y)\left[\phi\left(y+2\sigma\right)\phi(y)-\phi\left(y\right)\phi(y+2\sigma)\right]dy
\end{aligned}$$

Call $\psi(\sigma)=\phi\left(y+2\sigma\right)\phi(y)-\phi\left(y\right)\phi(y+2\sigma)$. $\psi'(\sigma)=2\phi(y+2\sigma)\left(\phi(y)+(y+2\sigma)\phi(y)\right)$ Thus, ψ is increasing if and only if: $2\sigma>-y-\frac{\phi(y)}{\phi(y)}$. Now consider the function $\kappa: y\in\mathbb{R}\to y\phi(y)+\phi(y)$. $\kappa'(y)=\phi(y)>0$. Thus, κ is increasing strictly with y. But $\lim_{y\to-\infty}\kappa'(y)=0$. Thus: $\forall\ y$, $\kappa(y)>0$. Thus, for all $\sigma>0$, $-y-\frac{\phi(y)}{\phi(y)}=-\frac{\kappa(y)}{\phi(y)}<0<2\sigma$ so that ψ is strictly increasing with σ, for all $\sigma>0$ and $y\in\mathbb{R}$. Now $\psi(0)=0$.

Thus, $\psi(y) > 0$ for all $\sigma > 0$. As a consequence:

$$\frac{\partial P_0}{\partial \sigma} \geq \pi'(\sigma) + \int_{-\infty}^{\infty} \Phi(K - \sigma - y)\psi(y)dy > 0$$

Thus, when the equilibrium features only group A long at date 0, the price is strictly increasing with dispersion σ.

We now simply show that in this equilibrium, turnover is strictly increasing. Turnover is $2Q$ when group B only is long at date 1 (i.e. $\pi(-\sigma + \eta^B) > \pi(\sigma + \eta^A) + \frac{4Q}{\gamma}$), it is given by: $\gamma \frac{\pi(y-\sigma)-\pi(x+\sigma)}{2}$ when group B and group A are long at date 1 ((i.e. $\pi(-\sigma + \eta^B) < \pi(\sigma + \eta^A) + \frac{4Q}{\gamma}$ and $\pi(-\sigma + \eta^B) > \pi(\sigma + \eta^A)$ and it is 0 if only group A is long at date 1 (i.e. $\pi(-\sigma + \eta^B) < \pi(\sigma + \eta^A)$). Thus, conditioning over η^B, expected turnover can be written as:

$$\mathbb{T} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\gamma})-\sigma} 2Q\phi(x)dx + \int_{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\gamma})-\sigma}^{y-2\sigma} \gamma \frac{\pi(y-\sigma) - \pi(x+\sigma)}{2} \phi(x)dx \right] \phi(y)dy$$

Again, the derivative of the bounds in the integrals cancel out and the derivative is simply:

$$\frac{\partial \mathbb{T}}{\partial \sigma} = \int_{-\infty}^{\infty} \int_{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\gamma})-\sigma}^{y-2\sigma} -\gamma \frac{\Phi(K - y + \sigma) + \Phi(K - x - \sigma)}{2} \phi(x)dx\phi(y)dy < 0$$

Now consider the equation defining the equilibrium where only group A is long at date 0. This condition is:

$$\delta(\sigma) = P_0(\sigma) - \left(\pi(-\sigma) + \int_{-\infty}^{\infty} \left[\int_{-\infty}^{y+2\sigma} \pi(y+\sigma)\phi(x)dx + \int_{y+2\sigma}^{\infty} \pi(-\sigma + x)\phi(x)dx \right] \phi(y)dy \right) > 0$$

Notice that the derivative of the second term in the parenthesis can be written as:

$$\begin{aligned} & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{y+2\sigma} \pi'(y+\sigma)\phi(x)dx - \int_{y+2\sigma}^{\infty} \pi'(-\sigma + x)\phi(x)dx \right] \phi(y)dy \\ = & \int_{-\infty}^{\infty} \phi(y + 2\sigma)\pi'(y+\sigma)\phi(y)dy - \int_{-\infty}^{\infty} \phi(y - 2\sigma)\pi'(y - \sigma)\phi(y)dy \\ = & \int_{-\infty}^{\infty} \Phi(K - y - \sigma) \left(\phi(y + 2\sigma)\phi(y) - \phi(y)\phi(y + 2\sigma) \right) dy \end{aligned}$$

We thus have:

$$\frac{\partial \delta}{\partial \sigma} \geq (\pi'(\sigma) + \pi'(-\sigma)) > 0$$

Thus, there is $\bar{\sigma} > 0$ such that for $\sigma \geq \bar{\sigma}$, the equilibrium has only group A long at date 0, the price increases with dispersion and turnover decreases with dispersion. QED.

□

The intuition for this result is the following. The condition on trading costs/initial supply allow the optimists to have enough buying power to lead to binding short-sales constraints at date 0. In the benchmark setting, low trading costs/low supply were associated with a louder credit bubble. But it turns out that when there is dispersed priors they can lead to large but quiet mispricings.

To see why, first consider the effect of dispersed priors on mispricing. Mispricing (i.e. the spread between the date-0 price and the no-short-sales constraint/ no bias price) increases with dispersion for two reasons. First, group A agents' valuation for the interim payoff increases. This is the familiar Miller (1977) effect in which the part of price regarding the interim payoff reflects the valuations of the optimists as short-sales constraints bind when disagreement increases. Second, as dispersion increases, so does the valuation of the marginal buyers (or the optimists) at date 1 – which leads to an increase in the resale option and hence of the date-0 price.

As dispersion increases, group A agents – who are more optimistic about the interim payoff than group B agents – own more and more shares until they hold all the supply at date 0 (which happens for $\sigma > \bar{\sigma}$).[15] As σ increases, the probability that group B become the optimistic group at date 1 also becomes smaller. As a consequence, an increase in dispersion leads to an increase in the probability of the states of nature where turnover is zero or equivalently where the group A agents hold all the shares at date 0 and 1. So overall expected turnover decreases.

Price volatility also decreases as σ increases. This is because in these states of nature where only group A agents are long, the expected payoff becomes more concave as a function of their belief shock, so that the price volatility conditional on these states decreases. When σ becomes sufficiently large, group A agents are long most of the time and the asset resembles a risk-free asset and price volatility goes down to zero.

[15] Note that this occurs despite the fact that both types share the same valuation for the date-1 resale option – this is entirely driven by the interim payoff: in a pure resale option setting (i.e. without the interim payoff), all agents would end up long at date 0 as, in the margin, there would be no disagreement about the value of the resale option.

Figure 1: ABX Prices

The figure plots the ABX 7-1 Prices for various credit tranches including AAA, AA, A, BBB, and BBB-.

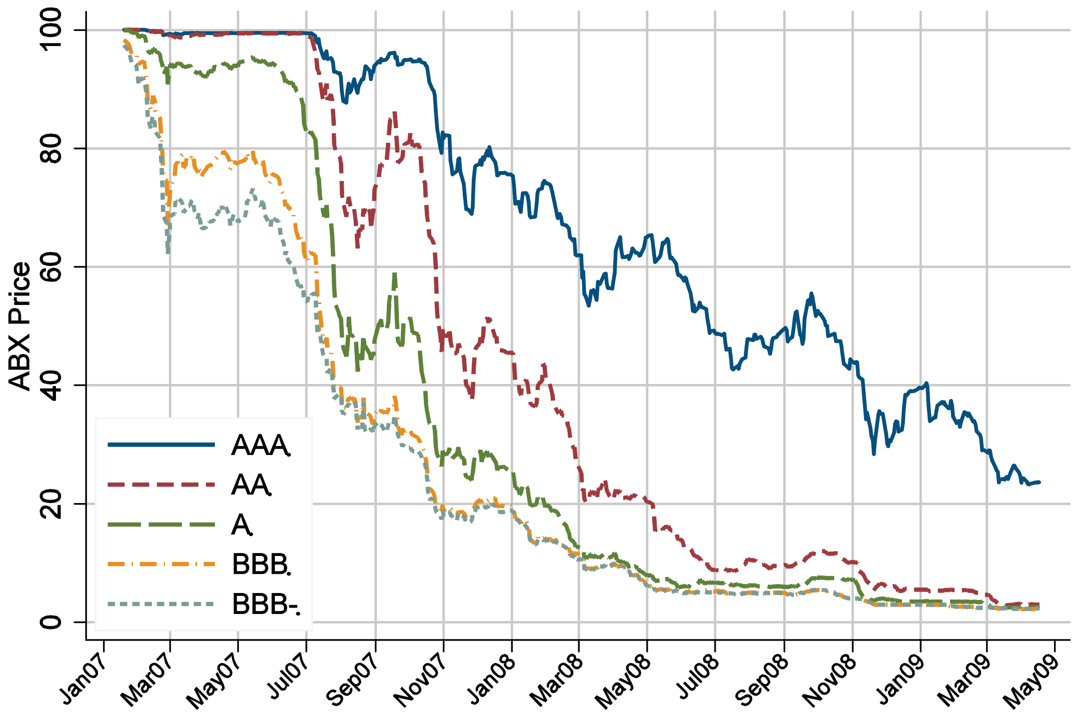

Figure 2: CDS Prices of Basket of Finance Companies

Source: Moodys KMV, FSA Calculations. The figure plots the average CDS prices for a basket of large finance companies between December 2002 and December 2008. Firms included: Ambac, Aviva, BancoSantander, Barclays, Berkshire Hathaway, Bradford & Bingley, Citigroup, Deutsche Bank, Fortis, HBOS, Lehman Brothers, Merrill Lynch, Morgan Stanley, National Australia Bank, Royal Bank of Scotland and UBS.

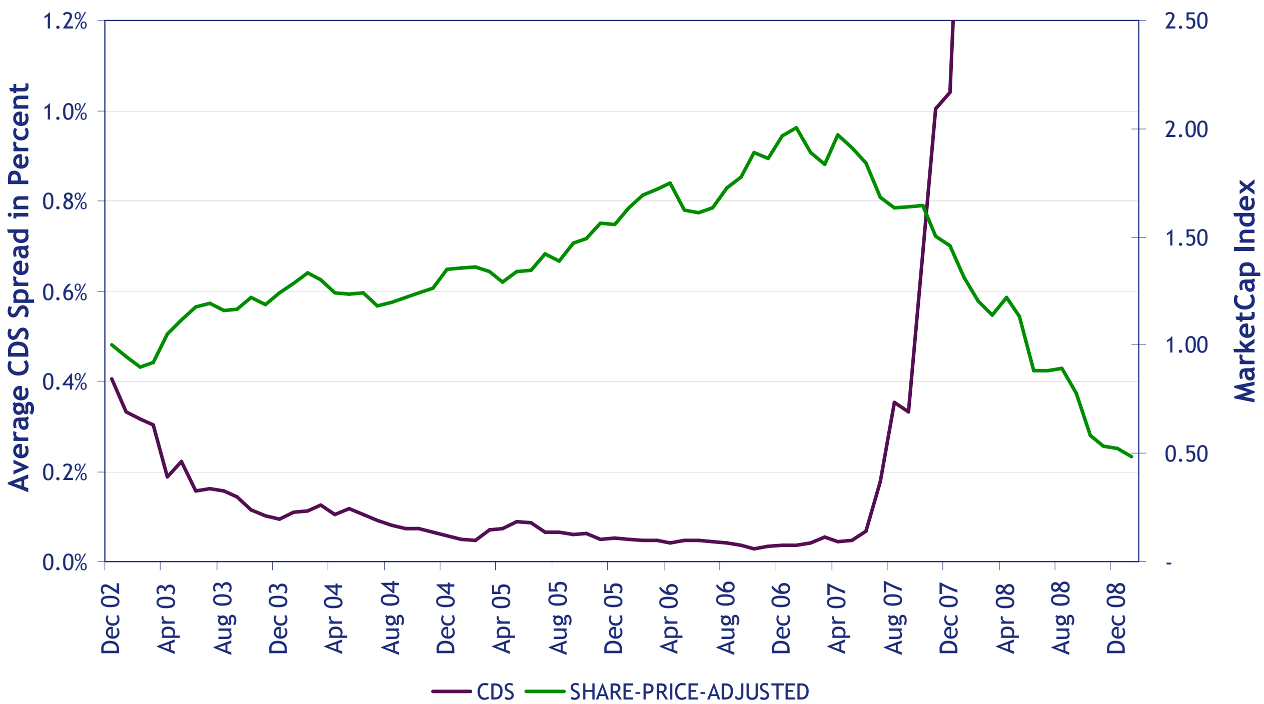

Figure 3: Monthly Share Turnover of Financial Stocks

The figure plots the average monthly share turnover of financial stocks.

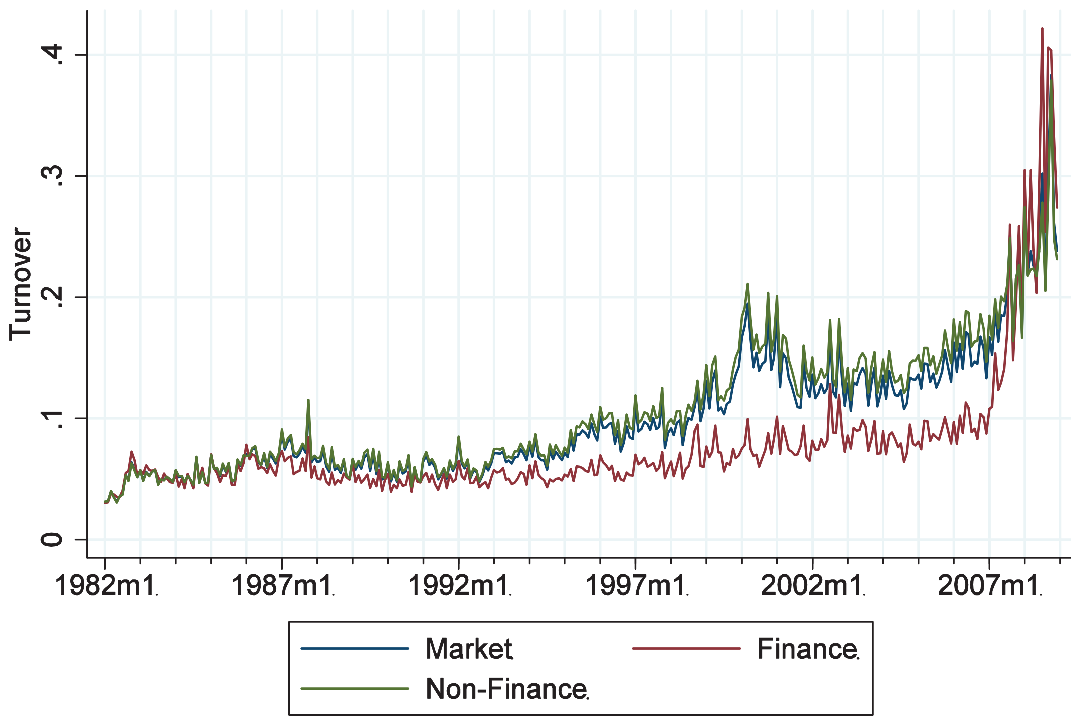

Figure 4: Monthly Share Turnover of Internet Stocks

Source: Hong and Stein (2007). The figure plots the average monthly share turnover of Internet stocks and non-Internet stocks from 1997 to 2002.

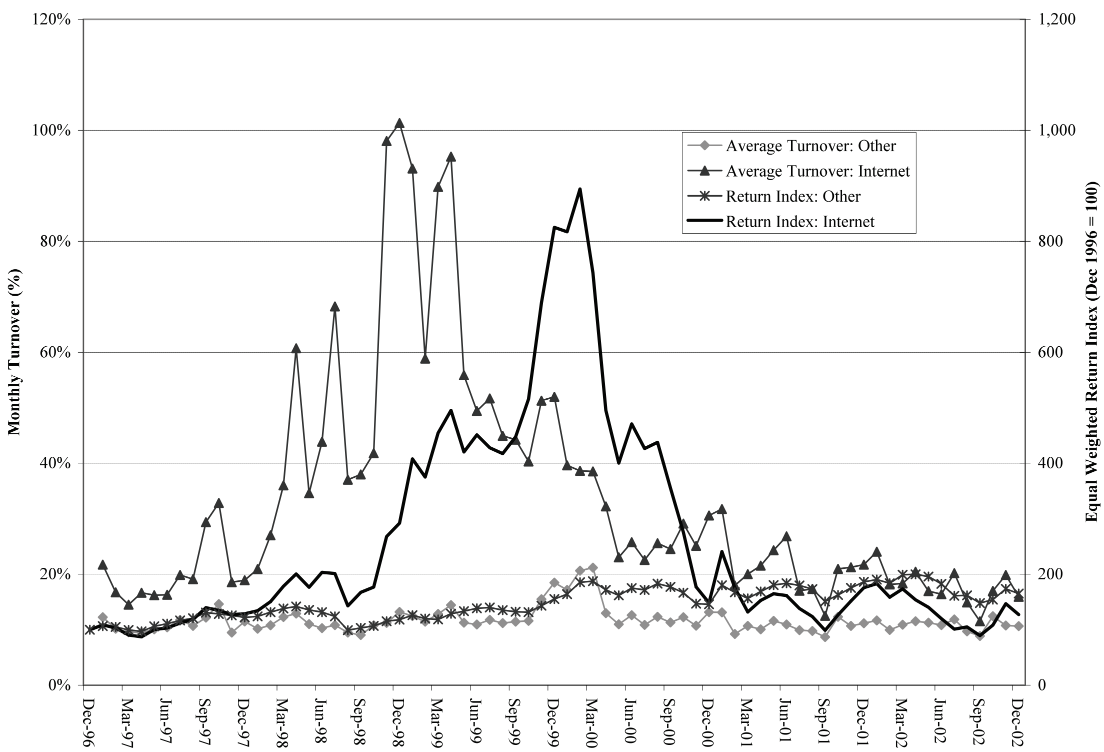

Figure 5: Issuance of IG synthetic CDOs (Monthly)

Source: Citygroup. The figure plots the monthly issuance of investment grade synthetic CDOs.