DOMESTIC EXCHANGE RATE DETERMINATION IN EARLY RENAISSANCE FLORENCE

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Abstract: We explore the strategic dynamics of the price setting protocol used by the money-changers guild to set the daily domestic exchange rate between gold and silver coins in early Renaissance Florence. We frame our analysis as a repeat, non-constant sum game of indeterminate length characterized by voluntary participation and asymmetric information. A dynamically stable Nash equilibrium obtains and guild profits maximized if only a few informed players participate, suggesting that the Florentine protocol was an effective price setting mechanism. It is not surprising, therefore, that the protocol was in force for at least 100 years and that some of its characteristics are echoed in modern financial markets.

JEL Classification: G10, G21, N23, P12

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1. Introduction

Throughout his writings on political and moral philosophy, Binmore (1994, 1998 and 2005) argues that people of a common culture coordinate their actions to find a Nash equilibrium and they create institutional norms and devices to stabilize and enhance this coordination. Grief (2006) supplements this argument by suggesting that the presence of Nash equilibria results in self-reinforcing behavior, thereby causing institutions to perpetuate themselves. He also maintains that institutions evolve and that the trajectory of their evolution is influenced by elements inherited from the past because they reflect what is understood, expected, and appropriate. We explore these notions in the context of trading protocols that are used to ensure orderly markets for financial assets.

Designing trading protocols is a challenging task, with determining the price at which to trade when a trading session opens (or reopens after a halt in trading) being particularly vexing. The objective is to arrive at the true price of an asset, which is the price that clears the market or, equivalently, the price that maximizes trading volume. Nowadays the prevalent market structures are order driven, quote driven or a mix of the two. To begin the trading session, most order-driven markets use some type of oral or written order matching auction that links buyers and sellers, with a uniform pricing rule being common. Quote-driven markets, often referred to as dealer markets, provide prices at which dealers are willing to buy or sell based on their assessment of market conditions as well as their inventory position.

Price discovery protocols are not unique to 21st century markets. A notable example existed in Florence (Italy) during the early Renaissance. Underpinning the city's economy from the second half of the 13th century into the 16th century was a bimetallic monetary standard based on the gold florin and several petty coins containing varying amounts of silver. Unlike modern bimetallic systems, there was no government mandated fixed exchange rate between the gold and silver coins. Instead, there were two moneys of account and the exchange rate between these coins fluctuated according to market conditions and was determined daily by small local banks (*banchi a minuti*) and the local subsidiary of large banks with international correspondents and branches (*banchi grossi*).[1] Both types of local banks were required to be members of the *Arte del Cambio*, the money-changer's guild that regulated local banking activities. This environment provides a rich setting for our analysis of economic arrangements that have endured for a significant amount of time.[2]

Our purpose is to analyze the price setting protocol used to determine the exchange rate between the gold florin and the denaro, quattrino and grosso, which were denominations of silver coins that were standard multiples of each other. At the end of each trading day, the exchange rate was set by the *Arte del Cambio* based on the input of its members. Specifically, guild members submitted individual estimates of the next day's exchange rate to the guild manager, who then set the price equal to the arithmetic average of these estimates. This rate was then used throughout the next trading day for all transactions. We conjecture that this protocol, which we refer to as the 'florin fix', was designed to arrive at the effective bid and ask exchange rates that maximized guild profits. The portion of profits that accrued to an individual member was then determined by his market share, which in turn may have depended on factors such as the money-changer's place of business, customer relationships, and other attributes related to non-price competition.[3] Using Vickery's (1961) taxonomy, the guild acted as an "exclusive marketing agency" that attempted to determine the equilibrium exchange rate by aggregating the supply and demand information provided by its members.

[1] Milan and Venice also had a similar monetary arrangement in which the domestic exchange rate was determined by market forces. Lane and Mueller (1985, pp. 54 – 55), however, observe that sometimes the Milanese government set their domestic exchange rate for a period of time. They footnote Bernocchi (1978), which is our source describing the Florentine protocol, but they make no mention of the procedure used by Milan or Venice.

[2] McCloskey (1976) cogently points out that history provides plethora of varied economic facts and circumstances to test propositions, which may lead to better economic theory and policy, and, hence, better economists.

[3] Our conjecture recognizes MacIntyre's (1966, p. 130) position that prior to the Reformation (commonly thought to have begun on October 31, 1517 when Martin Luther nailed his 95 theses to the door of the All Saints' Church) the relevant social unit was the community and not the individual. The preeminence of the community has its roots in ancient Greece. For example, Aristotle believed that contributing to the common good was essential if the individual was to lead a good life. Thus, it is natural to assume that the guild's objective was to maximize the collective well being of its members, especially after recalling that, during the Renaissance, Florence was one of the centers of Greek scholarship in Europe.

Our analysis rests on the notion that the Florentine price-fixing protocol can be viewed essentially as a repeat, non-constant sum game of indeterminate length in which players (i.e., individual guild members) have asymmetric information regarding the true exchange rate. In addition, since the daily submission of estimates was not mandatory, individual guild members strategically choose whether or not to participate in the price-setting process. We find that a profit-maximizing Nash equilibrium is obtained when informed members choose to submit their estimates of the true exchange rate to the guild manager and uninformed money-changers choose not to submit their estimates (i.e., uninformed members choose not to participate). Moreover, we show that this Nash equilibrium is an evolutionarily stable outcome, suggesting that the Florentine protocol was indeed self-perpetuating in the spirit of Binmore (1994, 1998 and 2005) and Greif (2006). We then argue that if their information content and sources are diverse, only a few informed money-changers need to participate to obtain an accurate estimate. Finally, we identify elements of the florin fix that are echoed in the pricing protocols of many modern financial markets, which we submit supports the notion that institutions evolve in a purposeful manner.

2. Money and Banking in Renaissance Florence

Florence declared itself an independent republic in 1115 and by the 14th century it was a major cultural and banking center. This prominence was brought about by the entrepreneurship successes of the Florentine business families, many of which were connected by marriage and business interests (Padgett and Ansell, 1993), and their willingness to be patrons of the arts and look after the extensive financial interests of the Roman Catholic Church. These families were skilled, sophisticated merchants and as Padgett (2010) documents, socially mobile. According to Staley (1906, p. 179), as a result of their systematic financial training, these merchant bankers were ingenious and resourceful users of capital.[4]

[4] Commercial mathematics was taught in "reckoning" schools under the vigilance of a "reckoning master," and Florence was the home to more than a dozen of these schools (Swetz, 1987). According to Van Egmond (1976, p. 229) their curricula typically covered the topics found in modern secondary education. Instruction in the Florentine monetary system and its relationships to its foreign counterparts was also covered. Acquired skills included not only straightforward arithmetic and algebra but also more arcane (by modern standards) operations

They created companies with extensive networks of branches and agents throughout Europe as well as east to the Levant and south to the Maghreb. For instance in 1311, the Bardi Company consisted of nine partners and had branches in France, England, Ireland, Tuscany, Lombardy and Germany as well as Cyprus and Rhodes (Staley, 1906, p. 182).[5] Florence's independence lasted until 1532, at which time Pope Clement VII conquered the city state and declared it a principality.

Florence's economy was organized around 21 guilds of varying importance and individuals often held multiple memberships. By statute the members of some guilds transacted their business and kept their accounts in florins, while members of other guilds reckoned in petty coins. Typically, wholesale prices were stated in gold and retail prices in silver. International transactions, however, were generally conducted in gold, and, according to Cippola (1956), the florin and the Venetian ducat were the preferred currency for international transactions. Although wages were usually paid in silver regardless of the type of business, Spufford (1988, p. 335) points out that ordinary people used both gold and silver coins. Thus, money-changing was not only an integral part of business but also of every day life.

The florin was first minted in 1252 from 24 carat gold bullion and weighed approximately 3.5 grams. During its reign as coin of the realm, which ended in 1523, there was no significant change in its metal content or design. It was the first pure gold coin to appear in significant quantities in Western Europe after the Dark Ages ended in the 7th century, and its name was derived from the Italian *fiorino* (flower) in reference to the figure of a lily on the obverse. The original petty coins were the denaro and the quattrino, with four denari equaling one quattrino by government mandate. Each coin consisted of an alloy of silver (less than 50%) and copper (together called billion). Because of gradual debasement over time (on average 0.8% annually from 1252 to 1500 (Spufford, 1988, p. 291)), the denaro fell into disuse (although it remained a unit of account for record keeping purposes) and a new coin, the grosso, was

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such as vigesimal and duodecimal divisions (Van Egmond, 1976, p. 128). During the time period in which we are concerned, reckoning schools were not associated with Renaissance universities.

[5] Padgett (2010) points out that the Florentines invented many of the modern practices in use today, including the partnership system, limited liability, double-entry bookkeeping, and the use of the current account; in doing so, the city-state became the birthplace of financial capitalism.

minted, which contained significantly more silver than the quattrino. The number of coins in circulation was not regulated by the Florentine government, but city leaders influenced the amount indirectly by deciding the silver content of the petty coins. Thus, if the monetary value of the small change was higher than its commodity value, bullion was minted into coin at the city's mint. If the value was lower, coins were melted into bullion.[6]

Much like modern day exchanges and other similar trading venues, the *Arte del Cambio* was a self-regulating body. According to Staley (1906, p. 173 – 179) the earliest records supporting its existence are dated circa 1200, although a complete set of guild statutes dated before 1299 is not available. Like the other Florentine guilds, the *Arte del Cambio* was more than a business organization; it was a social entity as well and its members were concerned with the well being of their community. Admission to the guild required passing a rigorous exam that included assessing the character of the aspiring member and paying a substantial (relative to capital at risk) entry fee. Thus, membership in the guild certified that the money-changer was honest, financially sound, and committed to obeying the guild's rules. The penalties for disobedience could be very harsh. For instance, according to Rule 70, a money-changer found guilty of being untruthful concerning his monetary dealings could be subjected to the "rack or other corrective instruments" at the guild's headquarters. In addition to being routinely audited, once a year each money-changer was given a performance review by the guild. Inadequate performance, which included unethical behavior, resulted in suspension.

It is worth noting that money-changers entered and exited the market on a regular basis and the life spans of their individual businesses varied greatly. For example, the bank founded by Giovanni di Bicci de' Medici, which became one of Florence's most prominent institutions, survived for almost a century until 1494 under the stewardship of his descendants. In contrast, the banking partnership of Francesco di Marco Datini (a.k.a. da Prato) and Bartolomeo Cambroni & Partners lasted only

[6] According to Sargent and Velde's (2002, p. 37) model, for the florin and denaro or its companion coins to circulate freely after accounting for costs of minting, the exchange rate between the two coins could not be greater (less) than the exchange rate value that aligned the melting (minting) point for the florin with the minting (melting) point of the denaro.

approximately three years, from 1398 to 1401. According to de Roover (1963, p. 16), 57 banks were in operation in Florence in the middle of the 14th century, and this number increased to 71 by the century's end. By the middle of the 15th century, however, the number of banks had been halved and remained relatively stable over the next 50 years. Goldthwaite (1983) suggests that the banking industry was highly decentralized. McLean and Padgett (1997) provide some empirical evidence supporting the notion that banks were decentralized but note that they were more concentrated than most Florentine industries.

Banks that engaged in money-changing ranged in size from a one-person operation to one having several partners served by a clerical staff of a half dozen or more. These money-changers were typically open for business Monday through Saturday. Exceptions to this schedule were made for holidays, inclement weather and epidemics so that on average trading occurred four to five days per week, although there were periods when there was no trading for many days. Money-changers exchanged monies in four different market locations for the convenience of their customers. They sat behind a bench (*banco*) of sorts that was usually elevated and covered by a green cloth on which were placed stacks of coins and an account book. The money-changers acted as dealers and were always ready to physically exchange coins, make transfer entries in their account books, and accept and record deposits. Transactions that involved more that one money-changer were periodically netted out by the money-changers involved. Not only did money-changers exchange Florentine coins for one another, but they also exchanged florins and foreign gold coins.[7] They were familiar with the quality of the coins and were able to tell whether their physical condition was worse than that associated with normal wear and tear. They could also detect counterfeit coins and coins that had been tampered with, for example, by filing the edges or shaking them in a bag with other coins to remove some of their metal. Detailed written records were kept and archived in locked strongboxes along with coins held in reserve, and in the case of a dispute these records were used by the guild manager to assist in its resolution. These activities have led most economic historians (e.g., Usher,

[7] It was common for money-changers to exchange the florin with the ducat (Venice), pound sterling (London), livre tournois (Paris), besan (Rhodes and Tunis) and carlin (Naples).

1944; de Roover, 1948; Lane and Mueller, 1985) to maintain that money-changing was the precursor to modern deposit banking.

At the end of each trading day, the new official denaro-florin exchange rate and the trading commission for the next trading day were established and announced in the four markets located in the city. These rates were used by all money-changers, who acted as a single economic unit, for all transactions throughout the next trading day. Relying on an archival document dated May 21, 1492, which reports a discussion held by the Guardians of the Republican Mint, Bernocchi (1974) indicates that the official rate was determined in the following way. First, the money-changers individually and simultaneously submitted to the guild manager their best estimates of the next day's "true" price of the florin in terms of denari. The guild manager then set the price equal to the arithmetic average of the submitted estimates, which was referred to as the gross exchange rate. Importantly, it was not mandatory that a money-changer submit a price. Thus, the number of estimates included in the calculation of the average could range from one guild member to all of them. A money-changer could submit only one estimate, thereby eliminating any learning during the fixing process. Nevertheless, this restriction most likely did not preclude the money-changers from interacting with each other prior to submission. Bernocchi (1974) asserts that this protocol had been in effect since at least the 14th century.

The gross exchange rate was recorded by the guild and stored in the State Archives of Florence (*Archivo di Stato di Firenze*). Also recorded was the net exchange rate, which is the gross rate less the commission. In modern parlance, the net rate is the bid rate. The gross rate is the midpoint between the bid and ask rates, with the latter being the gross rate plus the commission. The difference between the bid and ask rates is the bid-ask spread and is equal to twice the commission. Thus, from the money-changers perspective buying florins at the bid rate and selling them at the ask rate is the cash flow equivalent to buying and selling at the midpoint rate and charging a commission.

Bernocchi (1974) is unclear as to how the commission or net rate is determined. However, the way that the gross and net rates are recorded in the State Archives suggests that the money changers, who were skilled in manipulating exchange rates, likely thought in terms of rates whether they be bid, ask or their midpoint so that the commission was the residual in their calculation. In any case, a money-changer who submitted an estimate undoubtedly had a commission value in mind. This commission had to reflect not only profits but costs attributed to adverse selection (e.g., the cost of exchanging money with a counterparty more informed than the collective level of informedness exhibited by the guild members), processing orders, and maintaining an inventory of coins.[8] In our analysis below it does not matter whether the money-changer supplied this information to the guild as an estimate of a commission or net rate. To be consistent with the modern literature we develop our analysis in terms of bid and ask prices.

3. An Analysis of the Florin Fix

We model the denaro-florin protocol as a repeat, non-constant sum game of indeterminate length. Recall, guild members simultaneously submitted individual estimates of the next day's exchange rate to the guild manager, who then set the price equal to the arithmetic average of these estimates. Moreover, since the submission of estimates was not mandatory, individual guild members could strategically choose whether or not to participate. Thus, it is possible for the number of active participants in the price-fixing process to change at every stage the game is played. The choice to actively participate is independent of past or anticipated future decisions, but it may be influenced by what the money-changer learns about market conditions, including past movements of the exchange rate and what strategies that he believes are being pursued by other guild members. We assume that each

[8] Bernocchi (1974) compiled daily quote and commission data from 1389 to 1432. These data were originally recorded by various members of the *Arte del Cambio*. A review of the two time series indicated that the florin-denaro quotes varied over time, usually in multiples of 1/5 of a denaro, as did the commission, which commonly was about two denari, which is about 0.2% of the trade's value. This percentage is similar to that found in today's financial markets. For example, relative half-spreads of U.S. common stocks and treasuries are typically 0.25% and 0.05%, respectively. Foreign exchange relative half-spreads range from 0.002% for wholesale quotes to 1.6% for retail transactions. In addition, Booth and Gurun (2008) show that there is significant, irregularly spaced volatility clustering in daily gross exchange rates and that this volatility is positively related to the magnitude of the commission charged. They also find that the adverse selection component of the half-spread is, on average, 31.7% while the portion attributed to order processing and inventory costs is 37.3%. The remaining portion is associated with persistence, i.e., the phenomenon of a day with more buyer (seller) than seller (buyer) initiated transactions being followed by a similar day. These percentages are similar to those found in modern markets.

money-changer is subjectively rational, knows the rules of the florin fix, is able to form beliefs about the economic environment and the strategies of the other money-changers, and, based on the preceding, can determine what he believes to be his optimal strategy.[9] Because money-changers make their choices (nearly) simultaneously, the game is one of imperfect information, although it is possible some sort of explicit or tacit cooperation may occur.

The objective of the individual money-changer is to select a strategy (i.e., whether or not to participate by submitting an estimate) so as to maximize expected short- and long-run guild profits, given the strategies selected by the other guild members.[10] In the short run, the number of money-changers is fixed and the focus is on daily profits. However, perturbations such as the entry and exit of money-changers may cause temporary deviations from equilibrium. Over time, as a result of social mechanisms such as imitation and learning, money-changers' behavior evolves so that "fitter" strategies (i.e., those with higher payoffs) eventually dominate in the long run. Our task, then, is to ascertain whether the price-fixing game yields a static Nash equilibrium in which guild profits are maximized and, if so, whether this outcome is a dynamically stable long-run equilibrium.

3.1. The Set-Up and Single (Static) Game Solution

Consider the following stylized game structure. There are two assets, one that is the riskless numeraire (the florin), and another that has a stochastic liquidation value representing the exchange rate (the price in denaro of one florin). There are two money-changer populations in the guild that differ in terms of their information regarding the true (intrinsic) value of the exchange rate. The money changers that know the true value of the exchange rate are informed *(I)*, while those that have a noisy signal of the true price are uninformed (*U*). Both money-changer types know whether they themselves are informed

[9] Under these conditions Kalai and Lehrer (1993) point out that learning is possible. In the context of financial markets, Seru, Shumway and Stoffman (2010) demonstrate that some individuals can consistently achieve higher returns than others. Moreover, they empirically show that investors learn from their trading experience and find that some become better with experience while others stop trading once they realize that their ability is poor.

[10] As previously discussed, maximization of guild profits translates into optimal profits for individual members, since guild profits were allocated among individual members via non-price mechanisms.

and how informed they are relative to each other. Finally, there are two types of money-changing customers: liquidity customers, who trade denari and florin for the purpose of transactions, and speculators, whose demand is driven by information.[11] Liquidity volume *L* on any trading day depends on stochastic, exogenous market conditions and is distributed with mean $\overline{L}$ and finite variance. Speculative volume is positively driven by mispricing $|\varepsilon|$ on the part the guild, where $\varepsilon \sim N(0,\sigma^2)$ is the pricing error. Thus, all else equal, the larger the (absolute) discrepancy sensed by speculators between the guild price and the true price, the more they will move to exploit the mispricing by appropriately buying or selling florins. Speculative volume is also negatively related to the size of the commission *C*.

Before entering the price-fixing game, *U* observes noisy signals of the ask (*Ask*) and bid (*Bid*) prices: $Ask = Ask^{*} + \varepsilon$ and $Bid = Bid^{*} + \varepsilon$, where *Ask** and *Bid** are the true (intrinsic) ask and bid prices, respectively (which are known by *I*). At the start of the price-fixing game each money-changer type decides whether to pursue an active strategy (*A*) by submitting an exchange rate estimate to the guild manager, or a passive one (*P*), in which case the money-changer chooses the strategy of non-participation. Thus, the action space is given by $\{A, P\}$. When an *I*-type money-changer chooses strategy *A*, he will submit the true prices *Ask** and *Bid**, whereas if a *U*-type agent chooses strategy *A*, he will submit the noisy estimates *Ask* and *Bid*. The set of possible strategy pairs for (*I*, *U*) is thus given by *S* = {(*A*, *P*), (*A*, *A*), (*P*, *A*), (*P*, *P*)}, which forms a 2x2 game matrix with each element corresponding to a different payoff to the guild. These payoffs, or more precisely, expected guild profits, are analyzed below.

At the end of the trading day, the gross profit of the guild as a whole is given by:

$$(1) \qquad \Pi = CL + (2C - u|\varepsilon|)\frac{\gamma u|\varepsilon|}{C}\ell_{2C-u|\varepsilon|<0},$$

where $C = (Ask^{*} - Bid^{*})/2 = (Ask - Bid)/2$ is the denari commission (or half-spread) earned on each florin bought or sold, $u \in [0,1]$ is the proportion of those active in the price fixing that are uninformed,

[11] Our specification differs from those present in typical microstructure models; see, e.g., Biais, Glosten and Spatt (2005) for a recent survey. In these models, the dealer is information neutral and his clients are informed or uninformed.

and $\gamma > 0$ is a parameter that governs the effect of mispricing on speculative volume.[12] The term *CL* represents guild profits generated from liquidity customers. The term $\gamma u|\varepsilon|/C$ represents speculative volume, which increases with the degree of mispricing $|\varepsilon|$, which in turn is exacerbated by the proportion u of participants in the price setting process that are uninformed. In addition, speculative volume decreases with C due to limits-to-arbitrage. The term $\ell_{2C-u|\varepsilon|<0}$ is an indicator variable that equals 1 if $2C-u|\varepsilon|<0$ and 0 otherwise, where $2C-u|\varepsilon|$ represents the net profit (loss) per transaction from dealing with informed speculators.[13]

Speculative volume disappears when the guild price contains no mispricing (u = 0) or speculation against the guild is unprofitable ($2C > u|\varepsilon|$), in which case guild profits are generated solely from liquidity transactions (*CL*). This suggests that the guild can protect itself from speculative volume by setting a commission sufficiently large so as to discourage speculation on the part of informed customers. It is likely, however, that such protection comes at the cost of liquidity volume, since higher commissions may cause a decline in liquidity demand in the long run. Given the central role of the *Arte del Cambio* in the Florentine monetary system and economy, it is likely that the primary function of the guild was to provide transaction liquidity, and so the commission would have accordingly been set with the objective of maximizing liquidity volume rather than minimizing speculative volume.

Since $|\varepsilon|$ follows a half-normal distribution, we have $E(|\varepsilon|) = \sigma\sqrt{2/\pi}$, so that expected guild profits are given by:

$$(2) \qquad E(\Pi) = C\bar{L} + \gamma u\sigma\left(2\sqrt{\frac{2}{\pi}} - \frac{u\sigma}{C}\right).$$

[12] More precisely, u equals the number of uninformed players choosing strategy *A* divided by the total number of players (both informed and uninformed) choosing strategy *A*.

[13] We use the approximation that money changers earn a total commission of 2*C* on speculative transactions. This is because speculators must engage in a round-trip transaction in order to exploit any mispricing. However, since the 'unwinding' transaction takes place at a future date, the commission is unknown. We thus assume that money changers "book" the future revenue based on today's commission, *C*.

Consistent with current microstructure literature, we have $2C < u\sigma\sqrt{2/\pi}$, indicating that money changers lose money, on average, when dealing with informed customers (Madhavan, 2000). Because $\partial E(\Pi)/\partial u < 0$ and continuous in *u*, expected guild profits decrease monotonically with *u*. Intuitively, the larger the proportion of those involved in setting the guild price that are uninformed, the larger the degree of mispricing and therefore the more vulnerable the guild is to informed speculators, causing expected profits to decline. Conversely, expected guild profits are larger the smaller the proportion of uninformed money changers participating in the price-fixing process, with expected guild profits maximized when *u* = 0.

We next link the payoff space to the strategy space. Since each strategy pair in *S* maps into a unique value or range of values of *u*, it becomes possible to calculate and rank payoffs. Specifically, when informed (uninformed) money changers are active (passive), *u* takes on the value of $u(A,P) = 0$, so that there are no uninformed agents participating in the price fixing. This corresponds to expected guild profits of $\Pi_1 = C\overline{L}$. When both informed and uninformed agents are active in the price fixing, we have $u = u(A,A) \in (0,1)$, so that the proportion of uninformed now takes on a range of values depending on the actual number of uninformed participants relative to the total number of participants. Thus, expected guild profits also take on a range of values, which is given by $\Pi_2 = C\overline{L} + \gamma u\sigma(2\sqrt{2/\pi} - u\sigma/C)$, with $0 < u < 1$, i.e., Π_2 represents a continuous vector containing any and all values in the open interval $(C\overline{L} + \gamma\sigma(2\sqrt{2/\pi} - \sigma/C), C\overline{L})$. When informed (uninformed) money-changers are passive (active), $u = u(P,A) = 1$, so that all participants in the price-fixing are uninformed, corresponding to expected guild profits of $\Pi_3 = C\overline{L} + \gamma\sigma(2\sqrt{2/\pi} - \sigma/C)$. Finally, when both players adopt a passive strategy, i.e., (*P*, *P*), no market exists, resulting in expected profits of zero so that $\Pi_4 = 0$. Since expected guild profits decline monotonically with *u*, we have $\Pi_1 > \Pi_2 > \Pi_3 > \Pi_4$.

Table 1 contains the normal form representation of the above strategy and payoff spaces, and we see that the strategy pair (*A*, *P*), in which only the informed actively participate in setting the price, is the unique Nash equilibrium of the price-fixing game. This is because *A* is *I*'s best response given that *U* plays *P*, and *P* is *U*'s best response given that *I* plays *A*. Since the strategy pair (*A*, *P*) involves each player playing their best response against the other player's best response, it is by definition a Nash equilibrium. At this point, no player has an incentive to deviate by playing a different strategy.

3.2. The Repeat (Dynamic) Game Solution

The static game above results in a Nash equilibrium in which only the informed participate in setting the guild price and daily guild profits are maximized. Given the repeated strategic nature of the game, we now examine long-run equilibrium outcomes by asking the following: if perturbations to the system cause short-run deviations from equilibrium, does the pricing protocol allow for the eventual convergence to steady-states that correspond to the optimal Nash outcome? To answer this question, we turn to evolutionary game theory.

First, let (p, q) be the state that specifies the current distribution of strategies employed in each population. At any point in time, p (q) is the proportion of the population of informed (uninformed) money-changers that chooses strategy *P*. Conversely, $1 - p$ ($1 - q$) is the proportion of informed (uninformed) money-changers that chooses strategy *A* (see Table 1). Next, it is necessary to specify the dynamics by which the population states evolve. A hallmark of evolutionary games is the notion that actions or strategies that are more fit (i.e., those that have higher payoffs), given the current distribution of behaviors, tend over time to displace ones that are less fit (Friedman, 1991). In evolutionary biology, this notion of 'survival of the fittest' is captured by so-called replicator dynamics, which are enforced by the genetic mechanism of natural selection. In economic contexts, replicator dynamics do not have such a compelling motivation, but evolutionary economic theorists suggest that that the social mechanisms of learning and imitation are sufficient to deliver similar patterns (sometimes called Malthusian dynamics). For instance, according to Malaith (1998, p. 1348), "[i]t is important to note that successful behavior becomes more prevalent not just because market forces select against unsuccessful behavior, but also because agents imitate successful behavior."

Formally, let $\alpha \in (0,1)$ be the share of guild profits accumulating to the population of informed money changers *I*, with the remaining share $1-\alpha$ going to the population of uninformed money-changers *U*, where market share α is determined by non-price competition. Further, let F_i^a be the payoff to agents from population $i \in \{I, U\}$ from playing strategy $a \in \{A, P\}$. Then, for agents in population *I*, the payoffs from playing strategies *P* and *A* are given by $F_I^P = (1-q)\alpha\Pi_3$ and $F_I^A = q\alpha\Pi_1 + (1-q)\alpha\Pi_2$, respectively. In addition, the average payoff for population *I* is given by $\overline{F}_I = pF_I^P + (1-p)F_I^A$. Similarly, for population *U*, we have $F_U^P = (1-p)(1-\alpha)\Pi_1$ and $F_U^A = p(1-\alpha)\Pi_3 + (1-p)(1-\alpha)\Pi_2$, with average payoff $\overline{F}_U = qF_U^P + (1-q)F_U^A$. To capture the notion that strategies with higher payoffs increase relative to strategies with lower payoffs over time, we make the standard Malthusian assumption that the growth rate of a strategy is proportional to its relative payoff, or $\dot{p}/p = F_I^P - \overline{F}_I$ and $\dot{q}/q = F_U^P - \overline{F}_U$, where $\dot{p} = dp/dt$ and $\dot{q} = dq/dt$ are derivatives with respect to time. Rearranging, the replicator dynamics of the price-fixing game are then given by the following system of ordinary differential equations:

(3) $\dot{p} = p(1-p)\left[(1-q)(\Pi_3 - \Pi_2) - q\Pi_1\right]\alpha$, and

(4) $\dot{q} = q(1-q)\left[(1-p)(\Pi_1 - \Pi_2) - p\Pi_3\right](1-\alpha)$.

Equations (3) and (4), along with initial conditions, uniquely determine a path for the state (*p*, *q*), thereby permitting us to analyze how the behavior of the different money-changer populations evolves over time.

In particular, we are interested in asymptotically stable, fixed-point equilibria, or evolutionary equilibria.[14] Inspection of the system of differential equations (3) and (4) reveals four fixed points (i.e., steady-states) corresponding to the four corners of the 2-simplex, (*p*, *q*) = {(0,0), (1,0), (0,1), (1,1)}, and a fixed-point at (*p*, *q*) = $\left(\frac{\Pi_1-\Pi_2}{\Pi_1-\Pi_2+\Pi_3}, \frac{\Pi_3-\Pi_2}{\Pi_1-\Pi_2+\Pi_3}\right)$. However, since this latter fixed-point involves a negative

[14] Friedman (1998) contains a formal definition: A state $s \in S$ is a fixed point of the dynamic *F* if *F*(*s*) = 0 and is locally asymptotically stable if every open neighborhood $N \subset S$ of *s* has the property that every path starting sufficiently close to *s* remains in *N* and converges asymptotically to *s*. Such asymptotically stable fixed points are referred to as evolutionary equilibria.

value of q, we restrict our attention to the corner solutions on the unit square. To determine whether fixed-points are asymptotically stable, we calculate the eigenvalues of the system's corresponding Jacobian matrix (see Appendix A).

Note, the Jacobian evaluated at the fixed point (0,1) is given by:

$$J(0,1) = \begin{pmatrix} -\alpha\Pi_1 & 0 \\ 0 & -(1-\alpha)(\Pi_1 - \Pi_2) \end{pmatrix},$$

so that $tr[J(0,1)] < 0$ and $Det[J(0,1)] > 0$. These inequalities imply that the eigenvalues of $J(0,1)$ are both real and negative, making the fixed-point (0,1) an asymptotically stable equilibrium. All other fixed points are unstable. Since this point corresponds to the strategy pair (A, P), we see that the Nash equilibrium of the stage game, in which only informed money-changers actively participate in the price setting process (while uninformed money-changers are passive), is a dynamically stable outcome. That is, the Nash equilibrium of the price-fixing game is also an evolutionary equilibrium.

The preceding analysis, however, only establishes local stability (see fn. 14). To make a stronger statement about equilibrium play, we must also consider where the replicator dynamic will lead when set into motion from a much wider range of initial conditions. Thus, we must also examine the game's global convergence properties. From inspection of Table 1, it is clear that the price-fixing game is dominance solvable (i.e., iteratively removing strategies leaves only one strategy for each money-changer population), and that the dominance solution corresponds to the game's unique Nash equilibrium. Sandholm (2010, p. 260) shows that for dominance solvable games, all interior solution trajectories of any imitative dynamic (including the replicator dynamic) converge to the dominance solution (Theorem 7.4.5).

To illustrate, Figure 1 shows the phase portrait corresponding to the evolutionary price-fixing game. Every trajectory in the interior of the 2-simplex converges to the unique Nash equilibrium (0,1). In addition, all starting points along the left-hand (p = 0) and upper (q = 1) boundaries (excluding corner steady states) also converge to this point.[15] Thus, for any initial distribution in which a non-zero fraction of informed players are active ($p < 1$) and a non-zero fraction of uninformed players are passive ($q > 0$), over time only informed money-changers will actively participate in price setting in equilibrium.

While dominance solvability guarantees global convergence to the unique Nash equilibrium from all interior starting points, and convergence along the left-hand and upper boundaries is easily verified (see fn. 15), it is important to note that initial conditions lying on the opposite boundaries do not converge to (0,1). Specifically, Figure 1 shows that for the right-hand ($p = 1$) and lower ($q = 0$) boundaries, in which all informed money-changers are passive and all uninformed money-changers are active, respectively, the game converges instead to the unstable steady states (1,0) and (0,0), respectively. This result, however, is not a severe limitation because, as mentioned above, we are primarily interested in perturbations to the system (e.g., from the entry and exit of money-changers) rather than particular starting points, per se. Since these steady states are unstable, any shock resulting in $p < 1$ and $q > 0$ (e.g., the entry of an informed, active money-changer) will draw the system away from these unstable fixed points and towards the asymptotically stable Nash equilibrium.

3.3. Additional Considerations

In this section we extend our analysis to address the following questions. Who were the informed money-changers? How much did it cost to become informed and what is the role of these costs in our analysis? Was it possible for money-changers individually or jointly to be sufficiently informed so that on average the quoted exchange rate was the true exchange rate? Did the money-changers always exchange at the official guild rate? And finally, did the Florin fix result in money-changing being a profitable business?

[15] Along the left-hand boundary, where $p = 0$ and $0 < q < 1$, the replicator dynamic is given by $\dot{p} = 0$ and $\dot{q} = q(1-q)(\Pi_1 - \Pi_2)(1-\alpha) > 0$. Thus, all starting points on the left-hand boundary converge to (0,1). Similarly, along the upper boundary, where $0 < p < 1$ and $q = 1$, $\dot{p} = p(1-p)(\Pi_3 - \Pi_2)\alpha < 0$ and $\dot{q} = 0$, again resulting in convergence to (0,1). Note that under the replicator dynamic, strategies that are initially unused remain unused.

Undoubtedly, some money-changers were more informed than others. One can only speculate, however, as to who the more informed and less informed money-changers were and how they came to be so. For instance, the informed money-changer may have been a representative of the local subsidiary of a *banco grosso* having strong international networks that served as a conduit for news on business conditions and exchange rates. These local subsidiaries may have acquired potentially superior information concerning foreign factors that affected supply and demand of gold and silver bullion and coins from their parent unit.[16] Possible factors include pressures in the foreign money markets (such as Venice, Bruges and Barcelona), large one-way remittances (often papal), and the opening and closing of mines. Because these networks were needed to conduct both non-banking and international banking businesses, the marginal cost of obtaining news relevant to the money-changers was likely very low or nil. In contrast the less informed money-changer may have been a *banco a minuto* that did not have an external news source. Nevertheless, both categories of money-changers were undoubtedly cognizant of local money market conditions, including liquidity needed for routine transactions, for the money-changers often employed a network of local brokers (*sensali*) whose function was to bring business to their employer.

Thus, it is doubtful that the difference in marginal costs incurred by the informed and uninformed money-changers that were necessary to conduct their business was large enough to affect our game results. If there were substantial costs incurred in being informed, in our model the higher-cost informed money-changers would change their role and become uninformed until there were no cost differences

[16] According to de Roover (1968, p.49) *banchi grossi* focused on making profits from betting on fluctuations in international exchange rates and actively engaged in arbitrage. They accomplished this by the issuing bills of exchange payable at one location and then at maturity reversing the transaction. The maturity date depended on the issuance date and the usance. The usance is a standard time that it took to travel between the two locations and was published in merchant manuals such as that provided by da Uzzano (1442). For example, the usance between Florence and Bruges was 60 days, and between Bruges and Barcelona it was 30 days. Thus the time to payment was determined by the amount of time that it took a courier to travel between locations and present the bill for acceptance. Often there was written commentary from the individuals at the issuing and receiving cities concerning the state of the local economy and other items of interests accompanying the bill. Thus there was a steady flow of information through the business network of a *banco grosso*, especially if it employed its own couriers. Although by modern standards the information was certainly dated and most likely stale, it was still news to the Florentines.

between the informed and uninformed money-changers or until there was only one money-changer left. This raises the phenomenon of free-riding by the uninformed money-changers. However, this does not appear to be a serious concern for the *Arte del Cambio* since, as mentioned above, they permitted non-guild members to engage in money changing.

Our game assumes that informed money-changers are able to estimate, on average, the true exchange rate. We cannot directly address this point, but we note that the guild's protocol is equivalent to a consensus forecast where individual forecasts are equally weighted. Supporting a body of theoretical work, Ashton and Ashton (1985) analyze consensus subjective forecasts and show that precision increases at a decreasing rate as individual forecasts are added to the consensus computation.[17] They empirically show that only a few independent forecasts are necessary to achieve most of the precision possible. They also show that simple averaging may not necessarily be the best way to combine the forecasts but improvements using other methods are small, which is a well documented phenomenon (e.g., Clemen, 1989; Capistrán and Timmerman, 2009). Thus, statistical theory and empirical evidence supports the notion that the Florentine protocol had the potential to provide relatively accurate estimates, especially since our analysis indicates that, in equilibrium, less informed money-changers will not participate in the price formation process. It also appears that it is not necessary for the less informed money-changers to incur the cost of becoming more informed unless they can insert new information into the process.

The Florentines were an enterprising lot and undoubtedly there were coins exchanged at a rate other than the official guild rate.[18] While ordinary citizens may have engaged in such gray trading,

[17] Makridakis and Winkler (1983) and many others show that, when time series techniques are used to forecast, consensus forecasts tend to be more accurate than single forecasts. Of course these techniques were not available to the money-changers as the requisite mathematics for their development did not begin to be developed until the middle of the 17th century. However, it is not unreasonable to believe that the money-changers plotted daily exchange rates and were involved in searching for patterns and extrapolating, much like today's financial chartists. Moreover, for our purposes, if we rely on the Friedman and Savage (1948) "as if" contention, all that is necessary is for some of the money-changers to be able make accurate forecasts regardless of their approach.

[18] It was not uncommon for a person who was not in the *Arte del Cambio* to engage in money changing. The guild permitted this. The non-sanctioned money-changers could have a table in the marketplace but they could not cover the table with a green cloth nor could they have a chair on which to sit.

cheating by a guild member was perhaps very unlikely. Recall that the penalties imposed by the guild for disobeying guild rules were very harsh and the loss of individual reputation had severe social consequences. Becker (1976) and many others point out that economic success in Florence was based on public trust. He supports this assertion by putting the incidence of business crimes (e.g., falsification of records, counterfeiting, and forgery) in perspective, noting that it was much smaller in Florence in the late 1300s than in modern Tuscany by factors ranging from one-fourth to one-half.

Economic historians generally believe that, overall, Florentine banking was a profitable undertaking. Unfortunately, it is difficult to determine exactly how profitable, because although financial transactions were meticulously recorded by guild members, comprehensive accounting records were not kept and some of the entries that were recorded often referred to non-bank activities.[19] Goldthwaite (1985) points out that this was most likely done on purpose to guard against the possibility that usurious activities would be discovered.[20] Nevertheless, Staley (1906, p. 179) points out that the guild was very wealthy. He supports this viewpoint by noting that after a fire destroyed the guild's headquarters in 1304, it erected a new edifice that was more sumptuous than the homes of all the other guilds. The new headquarters is described as having multicolored ceilings and stained glass windows with its walls being

[19] de Roover (1963) reports some highly aggregated profit (Table 8, p. 47) and asset (Table 10. p. 50) information for the Medici Bank for 1397 to 1420. These data suggest that overall the bank earned an annual return on assets in the neighborhood of 20%. This includes returns from non-banking activities, but such profits only accounted for 6% of the total. In addition, using a limited sample of actual transactions from 1437 to 1465 , he (pp. 117 – 120) reports that annualized gross trading profits from round trip dealings in bills of exchange between Venice and London and Venice and Bruges by the Medici Bank ranged from -5.5% to 28.8%, and averaged 14.5%. Using a larger sample size, Booth (2009) estimates that annualized profits from lending using bills of exchange between Barcelona and Bruges and originating in either Barcelona or Bruges averaged around 14%. This lending was often done under the auspices of Florentine banks. Biscaro (1913) provides data that suggest that transactions costs around this time (1436) amounted to approximately 10% of the trading profits.

[20] In the first millennium the received doctrine of the Roman Catholic Church was that a loan was a gratuitous contract and any payment to the lender other than the return of principal (such as interest) was usury, which was a sin. By the 12th century, however, a distinction between usury and interest emerged. Interest became viewed not as an inappropriate gain to the lender but instead as compensation for damages associated with not having access to his funds and only excessive interest charges were considered usurious. It appears that as much as 15% was permissible (Staley, 1906, p. 195). For instance, in the mid 1300s Florence itself paid between 12% and 20% to its creditors. Moreover, Staley (1906, p. 197), citing a banker's ledger, reports a return to capital of nearly 30%. He also reports that a goldsmith paid a 40% annual rate for six-month loan.

covered with embossed and gilt leather hangings and fine oil paintings. In addition, marble statues and fine furniture displaying intricate carvings and inlays were located throughout the building.

Despite these visual trappings of wealth, there is little evidence directly addressing the profitability of money-changing itself. Except for noting that a commission was earned on every trade, the available evidence is largely anecdotal and circumstantial. For instance, Goldthwaite (1985) maintains that the volatility of exchange rates in the early 14th century, coupled with frequent debasements, was ideal for turning quick profits. Moreover, using records from the State Archives, he finds that Bindaccio di Michele de' Cerchi, a man of little or no means, was able to cobble together enough money to start a bank in 1472 and retire wealthy 13 years later while still in his mid-thirties. In addition, de la Roncière (1973, pp. 81 – 86) suggests that the presence of money-changers who were not guild members and yet actively conducted business, is consistent with generous profit possibilities and cites the surviving account book of Lipo di Fede del Sega to bolster his point. Staley (1906, p. 176) supports this notion by noting that the gray market relieved the pressure on the official market and thus was tolerated by the guild.

4. Concluding Remarks

Our analysis suggests that the *Arte del Cambio* devised an exchange rate determination process that was capable of maximizing expected guild profits in the context of a Nash equilibrium. Our conclusion is based on very mild assumptions that the daily volume of liquidity transactions is exogenous while the volume of speculative transactions is influenced by the amount of mispricing on the part of the guild. Moreover, our results are robust to the specification of utility function and risk aversion (i.e., a similar outcome is obtained when assuming that the guild has negative exponential preferences over profits and constant absolute risk aversion).

The Florentine protocol encouraged informed money-changers to submit their best estimate of the next day's true price, while discouraging the participation of less informed or uninformed money-changers. These pricing rules resulted in a Nash equilibrium that was self-reinforcing, because no money-changer had an incentive to deviate from it, and evolutionarily stable, since, through learning and imitation, guild members would eventually re-converge to this equilibrium following shocks to the system. The profits attributed to the individual money-changers were then determined by non-price competition. The honoring of guild regulations on the part of individual members was encouraged through peer pressure, and violators were dealt with harshly (e.g., through social and professional ostracizing and, in the extreme, physical torture). Evidence supporting the notion that money changing was a highly profitable business is a bit sketchy, but this is not unexpected since we must rely on records that are several hundreds of years old.[21]

Although the physical locations where the florin and the petty coins were exchanged can still be found, the denaro-florin market disappeared centuries ago. Nevertheless, certain aspects of the price-setting protocols currently being used in various modern venues are reminiscent of the florin fix. Examples include having a pre-trading period and allowing market participants to transact at a single quoted price. In addition, some markets, such as those for U.S. Treasuries, discourage the participation of uninformed agents in the price-setting process. When auctioning its debt issues, the U.S. Treasury accepts competitive and noncompetitive sealed bids. Those submitting noncompetitive bids receive the amount of the securities that they order but the price that they pay is the lowest price (the highest rate) from the competitive bidders. Notably, auction instructions recommend that those not skilled in bond trading, supposedly the uninformed, use the noncompetitive option (www.treasurydirect.gov). Arguably, the most similar modern protocol is the one used by the London Bullion Market Association (LBMA) to determine the gold forward offer rate (GOFO), which is the rate at which the nine LBMA market-making members are prepared to lend gold to swap for U.S. dollars and which forms the basis of some loan

[21] The paucity of detail, however, does not mean that the results should be discounted for, as Aristotle (350 BCE, Book 1, Chapter 3) wrote, "… it is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits."

agreements.[22] The GOFO is determined daily for five different maturities (1-, 2-, 3-, 6-, and 12-months). Between 10:30 a.m. and 11:00 a.m. GMT market makers submit rates at which they would prefer to transact. At 11:00 a.m. the rate is fixed for each maturity by averaging the rates supplied after the highest and lowest quotes are removed. At least six market-making members must participate for a fixing to occur. This participation rule permits up to three market-making members to recuse themselves from the price-setting process for any reason whatsoever.

We argue that the Florentine exchange rate protocol has the hallmarks of being a precursor of many of our modern price determination approaches. Our findings reinforce the belief, eloquently expressed by Silver (1983), that market economies existed and flourished in pre-modern times. Whether this protocol is a Florentine innovation or whether it was a descendant of an earlier market is unknown to us, although given their penchant for inventiveness, it would not be surprising if it originated with the *Arte del Cambio*. Regardless of its origin, the Florentine protocol's longevity (at least a century and perhaps longer) supports the positions of Binmore (1994, 1998 and 2005) and Grief (2006) that communities create Nash-consistent institutions that not only coordinate individual behavior but also perpetuate themselves.

[22] The LBMA also sets the gold bullion price twice a day (10:30 a.m. and 3:00 p.m.) using a Walrasian call process that typically involves several iterations. Five market-making LBMA members, after consulting their order books, agree on a price that clears its market. This price is declared "fixed" by the lead member and the quote (in U.S. dollars) serves as a worldwide benchmark for gold transactions until the next price-fixing session. Likewise the LBMA fixes the silver bullion price once a day at noon by three of the market making members who participate in the gold fixing. The lead responsibility for the gold fixing changes annually but remains the same for the silver fixing.

Appendix A: Local Stability

The Jacobian matrix corresponding to the system of ordinary differential equations specified in (3) and (4) is given by:

$$J(p,q)=\begin{pmatrix}(1-2p)[(1-q)(\Pi_3-\Pi_2)-q\Pi_1]\alpha & -p(1-p)(\Pi_1-\Pi_2+\Pi_3)\alpha \\ -q(1-q)(\Pi_1-\Pi_2+\Pi_3)(1-\alpha) & (1-2q)[(1-p)(\Pi_1-\Pi_2)-p\Pi_3](1-\alpha)\end{pmatrix}.$$

To determine whether a fixed-point is asymptotically stable, it is necessary to compare the signs of the eigenvalues of the Jacobian matrix evaluated at that point. Thus, for each of the four resulting Jacobian matrices (corresponding to the corners of the unit square), we calculate the trace (*tr*) and determinant (*Det*), thereby allowing us to evaluate the signs of the corresponding eigenvalues:

- $J(0,0)=\begin{pmatrix}\alpha(\Pi_3-\Pi_2) & 0 \\ 0 & (1-\alpha)(\Pi_1-\Pi_2)\end{pmatrix}$, so that $tr[J(0,0)]$ is of indeterminate sign and $Det[J(0,0)]<0$. This implies that the eigenvalues of $J(0,0)$ are real and of the opposite sign so that the fixed-point (0,0) is an unstable saddle point.
- $J(0,1)=\begin{pmatrix}-\alpha\Pi_1 & 0 \\ 0 & -(1-\alpha)(\Pi_1-\Pi_2)\end{pmatrix}$, so that $tr[J(0,1)]<0$ and $Det[J(0,1)]>0$. This implies that the eigenvalues of $J(0,1)$ are both real and negative so that the fixed-point (0,1) is a stable sink.
- $J(1,0)=\begin{pmatrix}-\alpha(\Pi_3-\Pi_2) & 0 \\ 0 & -(1-\alpha)\Pi_3\end{pmatrix}$, so that $tr[J(1,0)]$ is of indeterminate sign and $Det[J(1,0)]<0$. This implies that the eigenvalues of $J(1,0)$ are real and of the opposite sign so that the fixed-point (1,0) is an unstable saddle point.
- $J(1,1)=\begin{pmatrix}\alpha\Pi_1 & 0 \\ 0 & (1-\alpha)\Pi_3\end{pmatrix}$, so that $tr[J(1,1)]>0$ and $Det[J(1,1)]>0$. This implies that the eigenvalues of $J(1,1)$ are both real and positive so that the fixed-point (1,1) is an unstable source.

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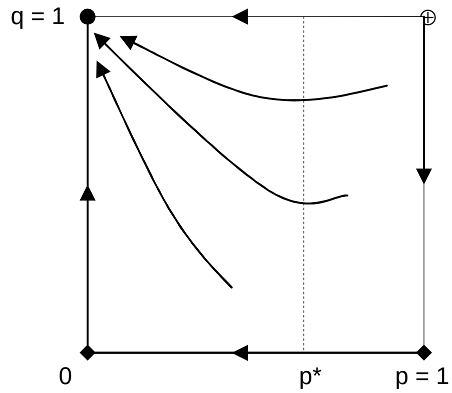
Table 1: Payoff matrix for the florin fix with the proportion of uninformed players *u* and state (*p*,*q*)

			U (Uninformed) $(1-\alpha)$	
			(q)	$(1-q)$
			***P** (Passive)*	***A** (Active)*
	(p)	***P** (Passive)*	(u *does not exist*) Π_4	($u = 1$) Π_3
I (Informed) (α)	$(1-p)$	***A** (Active)*	($u = 0$) Π_1	($0 < u < 1$) Π_2

Note: Since expected guild profits Π decline monotonically with the proportion u of money-changers participating in the florin fix that are uninformed, we have $\Pi_1 > \Pi_2 > \Pi_3 > \Pi_4$. Specifically, $\Pi_1 = C\overline{L}$ ($u = 0$); $\Pi_2 = C\overline{L} + \gamma u\sigma(2\sqrt{2/\pi} - u\sigma/C)$, with $0 < u < 1$ [i.e., Π_2 takes on any and all values in $(C\overline{L} + \gamma\sigma(2\sqrt{2/\pi} - \sigma/C), C\overline{L})$]; $\Pi_3 = C\overline{L} + \gamma\sigma(2\sqrt{2/\pi} - \sigma/C)$ [$u = 1$]; and $\Pi_4 = 0$. The variable p (q) is the proportion of the population of informed (uninformed) money-changers *I* (*U*) that chooses the passive strategy *P*. *I*'s market share is α, with the remaining $1-\alpha$ going to *U*.

Figure 1: Phase portrait for the florin fix with state (*p*,*q*)



Note: The fixed-point (0,1) is an asymptotically stable evolutionary equilibrium corresponding to the unique Nash equilibrium of the price-fixing game. It is globally stable with respect to all interior trajectories and starting points along the boundaries $p = 0$ and $q = 1$. From Eqs. (3) and (4), $\dot{p} < 0$ for all (p, q), $\dot{q} > 0$ for all $p < p^*$, and $\dot{q} < 0$ for all $p > p^*$, where $p^* = (\Pi_1 - \Pi_2)/(\Pi_1 - \Pi_2 + \Pi_3)$. Steady states (0,0) and (1,0) are unstable saddle points, while (1,1) is an unstable source.