The Optimality of Interbank Liquidity Insurance*

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Abstract

This paper studies banks' incentives to engage in liquidity cross-insurance. In contrast to previous literature we view interbank insurance as the outcome of *bilateral* (and non-exclusive) contracting between pairs of banks and ask whether this outcome is socially efficient. Using a simple model of interbank insurance we find that this is indeed the case when insurance takes place through pure transfers. This is even though liquidity support among banks sometimes breaks down, as observed in the crisis of 2007-2008. However, when insurance is provided against some form of repayment (such as is the case, for example, with credit lines), banks have a tendency to insure each other less than the socially efficient amount. We show that efficiency can be restored by introducing seniority clauses for interbank claims or through subsidies that resemble government interbank lending guarantees. Interestingly, even considering generic externalities among banks we find that there cannot be situations where banks insure more than is efficient. Such insurance, however, may arise if banks receive regulatory subsidies (explicit or implicit) in case they fail *jointly*.

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1 Introduction

The liquidity problems experienced in the banking system during the crisis of 2007 and 2008 have raised concerns about whether the insurance banks provide to one another is efficient. This is because during this crisis some banks were clearly suffering from liquidity shortages, while others seemed to have liquidity surpluses.[1] It thus appears that liquidity insurance among banks had broken down, which in turn induced many countries to guarantee interbank claims in one form or another. However, observing a breakdown of lending does not necessarily imply that there are any inefficiencies in interbank insurance that could create a role for government intervention. It may well be that such a breakdown is part of an efficient outcome. In order to address the question of efficiency one needs to explicitly consider banks' private incentives to engage in insurance, and to analyze whether they lead to a socially desirable outcome. Only if this is not the case, there is a potential role for regulation.

This paper provides such an analysis. Its main idea is as follows. Previous literature has mostly focused on analyzing the optimal form of interbank insurance. Thus (implicitly) a situation is considered where all banks meet centrally and decide jointly about the interbank insurance structure to implement. The question we ask then is whether this insurance is also an equilibrium outcome. The reason why it may not be is that it is not feasible for all banks to decide jointly on their insurance arrangements and, furthermore, commit to them. Instead, interbank insurance is typically implemented bilaterally (for example, bank A may decide to grant a credit line to bank B etc.).[2] Such insurance arrangements are also non-exclusive in nature, that is, when bank A insures bank B against certain shocks, it cannot prevent (or force) bank B to also engage in bilateral insurance with other banks. As a result, even if each pair of banks in the financial system chooses their bilaterally optimal insurance arrangement, there is no guarantee that this produces the efficient outcome for the financial system as a whole. Efficiency may in particular break down when contracting between two banks poses an externality on other banks.

[1] Acharya and Merrouche [3] and Heider et al [17], for example, provide evidence of liquidity hoarding by banks active in the interbank market during the recent crisis.

[2] Interbank lending, in particular, is almost exclusively carried out bilaterally, either directly (such as in the case of OTC markets) or through intermediated contact (brokered lending), or indirectly as in the case of electronic exchanges.

We develop a simple model of interbank insurance to analyze these issues. In our model there are three banks. One of the banks is randomly hit by an idiosyncratic liquidity shock of variable size. Banks can decide ex-ante to (bilaterally) insure each other against this shock.[3] Insurance, however, comes at a cost. This is because when the insurance is provided to the bank hit by the shock, the liquidity holdings at the insuring banks are diminished. This puts them at risk since they may be later hit by another shock, which then may cause their failure.

Providing insurance thus gives rise to a trade-off. It may save the bank which has received the idiosyncratic shock, but may also result in the failure of other banks when an additional shock hits. We show that the optimal form of insurance is to insure against small (idiosyncratic) shocks, but not to insure against large shocks. More precisely, optimal insurance stipulates that banks fully insure each other up to a certain level of idiosyncratic liquidity shortages, but do not provide any insurance beyond this level. In other words, interbank insurance optimally breaks down when banks are hit by large shocks. We also show that the marginal shortage to insure against is determined by the likelihood of additional shocks hitting banks in the future.[4]

We then turn to the analysis of banks' bilateral incentives to form insurance. We first consider a version of the model where insurance can take place in the form of pure transfers (i.e., the bank which has the liquidity shortage does not have to repay an insurance transfer it may receive). In this version there are no interactions among the banks beyond the insurance agreements (such interactions may arise, for example, due to spillovers from bank failures). Using the concept of pairwise-stability (Jackson and Wolinsky [18]), we

[3]While we focus on ex-ante insurance, our main ideas also apply to ex-post liquidity smoothing. The relevance of ex-ante insurance arrangements, however, is indicated by evidence that documents that relationship lending plays an important role in providing liquidity to banks (see, for example, Furfine [15], King [21] and Cocco et al. [12]), a role which is also retained in times of crisis (see Furfine [16]).

[4]While in our model the cost of insurance comes from the risk of experiencing an *aggregate liquidity* shock in the future, the nature of this shock is not crucial. What is important is that providing insurance to other banks in a crisis makes a bank more vulnerable, which would be, for instance, also the case for solvency shocks. In the case of aggregate liquidity shortages it may be argued that those could be remedied by the central bank. While this is certainly true in a world without imperfections, the 2007/2008 financial crisis has shown that even in the presence of large liquidity injections, banks may still encounter liquidity issues. Thus banks arguably always face (ex-ante) the risk of encountering liquidity problems in the future, hence making a provision of insurance to other banks costly.

show that in this case equilibrium insurance is efficient, that is banks' bilateral incentives to insure each other produce the efficient outcome. The reason for this result is as follows. Efficiency will obtain when there are no externalities among banks, that is when the bilateral insurance decision between two banks (say, bank A and B) does not affect the third bank (bank C). Consider first a situation where bank C does not provide insurance payments to either bank A or bank B. Clearly, in this case insurance between A and B does not affect bank C since we have ruled out any spillovers from bank failures beyond the insurance itself. Consider next the situation where bank C provides insurance to bank A and/or B. When bank A and bank B decide not to insure each other, they may fail if they are hit by the idiosyncratic shock because C's insurance payment may then be insufficient to avoid failure. However, this again does not affect bank C (bank C has to provide the insurance payment in any case). For these reasons the bilateral insurance choice between the three banks can be efficient.[5]

We next consider a setting where insurance payments have to be provided against a repayment at a later stage (in our model this is in order to solve a moral hazard problem at banks). Such insurance arrangements can be interpreted as credit lines which banks grant to one another. We show that banks then have an incentive to "underinsure", that is, to insure each other against fewer shocks than is optimal from a second-best perspective (the equilibrium is hence constrained inefficient). The reason for this is that now a bank may suffer from the failure of a bank it has provided insurance to since it may then not receive the repayment. This gives rise to a positive externality from bilateral insurance. When bank A and bank B insure each other, they each reduce their likelihood of failing in response to the idiosyncratic shock. This also benefits bank C because it then becomes more likely that the latter will be able to recover any insurance payments it has made to bank A or bank B. Bank A and B will, however, ignore this effect. As a result, "underinsurance" (relative to the second best) among banks may arise in equilibrium. This analysis hence suggests that the breakdown of liquidity support among banks could indeed be the result of inadequate insurance arrangements, in particular since interbank insurance in practice typically requires some form of repayment.

We also analyze whether there may conceivably be situations in which banks insure

[5] Nevertheless, it turns out that there can be inefficiencies due to banks coordinating on a dominated equilibrium. For our welfare analysis, however, we rule out such equilibria.

more than the efficient amount. Aside from coordination problems, such situations may arise if two banks decide to insure each other even though this is not jointly efficient for all three banks. To analyze this possibility we consider generic externalities that arise because a joint failure of two banks causes the failure of the third bank, for example, because of spillovers of informational nature or through asset prices. In the presence of such externalities, if two banks insure each other, there will be a negative effect on the third bank. This is because, as already explained, insurance comes at the cost of a higher likelihood of joint failure.[6] One may conjecture that banks may hence insure excessively as a result. However, we show that this cannot be the case. In fact, the only situation in which overinsurance can occur in our framework is when regulators provide a subsidy to *jointly* failing banks. For example, when the regulator adopts a too-many-to-fail policy banks may be bailed out in a joint failure. This increases their incentives to insure themselves since the cost of insurance (which comes in the form of a higher likelihood of joint failure) is reduced. As a result, banks may overinsure in equilibrium.

Our analysis yields interesting implications for how to judge apparent breakdowns in interbank insurance. In principle, as we have shown, a breakdown can be efficient as banks may optimally withhold liquidity for fear of future shocks. However, we have also shown that banks may have a tendency to underinsure each other. This is because they do not fully internalize that saving each other has an effect on other banks in the financial system. From this perspective, the breakdown of interbank insurance during 2007-2008 may have been inefficient, suggesting that there is a role for financial regulation in encouraging banks to provide more support to each other.

We consider two policies that can restore efficiency, while retaining the decentralized nature of interbank insurance. One is a seniority clause in interbank claims. This policy achieves efficiency by removing the externality from bank failures in our model.[7] Another

[6] While we consider here interbank externalities, banking failures may also have effects outside the banking system. Kahn and Santos [20] consider a setting in which the joint failure of banks causes a failure of the payment system and incurs costs for producers in the economy. Similar to excessive insurance in our paper, they show that this can cause the banking system to connect 'too much'.

[7] Interbank lending was typically secured before the early 1990s (King [21]). In the mid-90s regulatory reforms in the U.S. encouraged unsecured lending with the view of facilitating interbank monitoring. As secured lending is tantamount to seniority in our setting, our analysis suggests that this policy change may have had negative side effects by increasing externalities from bank failures and hence resulting in less efficient interbank insurance. Consistent with this, King [21] reports that interbank lending was reduced

policy is subsidies for interbank insurance, which are paid to the insuring bank in case the insured bank fails. Such subsidies are essentially guarantees of interbank claims. Interestingly, we find that optimal guarantees have to be complete (and not partial as moral hazard considerations would suggest). This provides justification for the interbank guarantee schemes introduced by many countries during the crisis of 2007-2008, which are for the most part complete guarantees.

The remainder of this paper is organized as follows. In the next section we discuss related literature. Section 3 presents the model. In Section 4 we analyze interbank insurance in the case of pure transfers. In Section 5 we consider insurance that is provided against repayment and analyze policy interventions that can restore efficiency. Section 6 studies the impact of spillovers from joint bank failures on the efficiency of interbank insurance. Section 7 contains the conclusions.

2 Related Work

The analysis of interbank insurance has received widespread attention in the literature. In the seminal paper of Bhattacharya and Gale [7] banks under-invest in liquidity reserves in the presence of moral hazard and adverse selection problems. In Rochet and Tirole [24] interbank markets act as a device that banks use to monitor each other. In Freixas, Parigi and Rochet [14] interbank markets operate in a given spacial economy and a solvency shock can cause a gridlock in the system.[8] In Acharya, Gromb and Yorulmazer [2] inefficiencies in interbank lending arise due to monopoly power. Banks with liquidity surplus may rationally not provide liquidity to needy banks in the hope that the latter will fail, enabling them to purchase their assets at fire-sale prices. Diamond and Rajan [13] show how the fear of future fire sales in the banking sector can induce banks with excess liquidity to hold onto cash. This effect can work to worsen fire-sales and also cause a credit freeze. In Allen, Carletti and Gale [5] inefficiencies are due to interest rates fluctuating too much in response to shocks, precluding efficient risk sharing. Heider, Hoerova and Holthausen [17] present a model where interbank lending may breakdown in times of crisis because increased

following this policy change.

[8] While in Freixas, Parigi and Rochet inefficiencies arise because banks coordinate on an inefficient equilibrium, in our paper inefficiencies arise even absent coordination problems, yielding different policy implications.

counterparty risk intensifies adverse selection problems. In contrast to these papers, our paper considers lending breakdowns due to fear of future shocks[9] and inefficiencies are due to the bilateral nature of interbank insurance.

Several papers have also studied the optimal *structure* of interbank connections (e.g., Allen and Gale [4] and Brusco and Castiglionesi [10]). These papers analyze interbank insurance under the (implicit) assumption that banks meet centrally and commit to a certain interbank structure. We depart from this approach by assuming that insurance contracts are formed bilaterally. Relatedly, the network formation approach to financial systems (see Leitner [22], Castiglionesi and Navarro [11], and Babus [6]) analyzes the endogenous formation of interbank connections. The main interest of these papers is to study how financial institutions form linkages despite (or because of) the possibility of contagion. While these papers are interested in the structure of connections (i.e., which banks connect to each other), in our paper the amount of insurance provided by each connection (that is, the intensity of each link) is endogenous. Another key difference is that we are not interested in the form interbank insurance takes per se, but in whether equilibrium insurance is efficient.

Our approach has its roots in the literature on non-exclusive contracts, initiated by Pauly [23] (more recent contributions include Bizer and DeMarzo [9], Bisin and Guaitoli [8], and Kahn and Mookherjee [19]). In these papers the relationship between borrowers and lenders is analyzed when borrowers can sign contracts with different lenders that are in competition with each other. The borrower can choose (either sequentially or simultaneously) more than one contract since exclusive clauses are not feasible or not enforceable. Our paper is, to our knowledge, the first to apply this notion of non-exclusivity to interbank insurance.

3 The Model

The economy consists of three banks, denoted by A, B and C. There are four dates. At date 0 each bank invests in a long term asset with return R. The assets are subject to both aggregate and idiosyncratic uncertainty. They mature at the final date (date 3) with

[9] Our explanation of a lending breakdown is consistent with evidence provided in Acharya and Merrouche [3], suggesting that tensions in U.K. money markets arose primarily due to precautionary liquidity hoarding and not due to other mechanisms such as counterparty risk.

an overall return of R.

At date 1 and 2 the assets may be hit by liquidity shocks. These shocks may be either positive (=liquidity surplus) or negative (=liquidity deficit). A liquidity surplus may arise because parts of the asset mature early. A liquidity deficit may reflect unexpected liquidity needs from an asset. If in any period a bank's asset faces a liquidity need and the bank cannot provide the required liquidity injection, the asset cannot be continued. The return on the asset is then zero. However, any previously injected liquidity can still be recouped in this case.[10] Liquidity deficits are *pure* liquidity shocks, that is they are offset by corresponding changes in the final date payoffs and hence do not change the overall return on an asset.[11] There is also a storage technology which allows liquidity to be moved from one period to the next.

The date 1 liquidity shock is idiosyncratic. Nature selects at date 1 with equal probability one of the assets. This asset then has a liquidity need of $l \in [0, 2]$, while the other two assets generate a liquidity surplus of 1. The size of the shock is known at date 0 (alternatively, one may assume that the size of the liquidity shock can be contracted upon). At date 2 an aggregate liquidity need hits all banks with probability π^A ($\pi^A \in (0, 1)$). The size of the aggregate shock, denoted l^A, is assumed to be uniformly distributed on $[0, 1]$.

Before the idiosyncratic shock hits at date 1, banks can engage in bilateral contracts to insure each other against the idiosyncratic shock.[12] An insurance contract specifies a transfer $t \geq 0$ between two banks if one of them is hit by the idiosyncratic shock. The insurance contract may also specify a repayment $r \geq 0$, taking place at the final date. A (symmetric) insurance contract between two banks i and j is thus a pair $(t_{i,j}, r_{i,j})$. Note that the contract cannot be conditional on any insurance contracts with the third bank. That is, bilateral insurance between A and B, for example, cannot be conditional on the insurance A or B have agreed upon with C. This is the imperfection which is the focus of our analysis.

The model can be summarized as follows. At date 0 banks each invest in an asset

[10] We thus model liquidity deficits as resources the project needs in the intermediate period to continue, but which are not project-specific (for example, an extra machine or an extra equipment has to be bought/hired).

[11] That is, if an asset, for example, has a deficit of l at date 1 and the bank injects this amount of liquidity into the project, its final date return will be $R + l$ whenever the project does not fail.

[12] We do not consider insurance against the aggregate shock. Footnote 13 contains a discussion of this, suggesting that our analysis also applies when the aggregate shock is contractible.

and form bilateral insurance contracts $(t_{i,j}, r_{i,j})$. At date 1 the bank whose asset has the liquidity need l (the "deficit bank") may receive transfers from the other banks (the "surplus banks"). For example, if A is the deficit bank it collects $t_{AB}+t_{AC}$ from B and C. If $t_{AB}+t_{AC} \geq l$ the bank can meet the liquidity injection and its asset can be continued. If $t_{AB}+t_{AC} < l$ the liquidity injection cannot be provided and the asset is worthless. The bank then "fails", and its return consists only of any transfer payments $(t_{AB}+t_{AC})$ it has received. Any excess liquidity at surviving banks is stored for the next period.

At date 2, the aggregate shock may hit. If it does not hit, nothing happens and all remaining assets can be continued. If it hits, the remaining banks have new liquidity needs l^A. The assets of the banks that have a liquidity of less than l^A fail and return only any previously injected liquidity. The other assets can be continued. Again, any excess liquidity is stored. At date 3 all remaining assets mature and return R net of any previous liquidity deficit or surplus.

The timing of the model is summarized in Figure 1 (the date 1 effort choice referred to in brackets at will be discussed in Section 5).

[Figure 1]

4 Insurance Through Pure Transfers

We first analyze the case where there are no interactions between banks beyond their insurance payments at date 1. For this we constrain the repayment r to be zero, thus the insurance payments take the form of pure transfers.

We first solve for the (socially) efficient insurance contract. Maximizing welfare requires us here to maximize the sum of the expected returns of all banks. Recall that the liquidity deficits only affect the timing of an asset's returns but not its overall return. Therefore, the return on an asset depends only on whether it survives or not (in the latter case R is lost). Since transfers between banks cancel out in the aggregate, maximizing welfare thus simply requires minimizing the expected number of asset liquidations, that is minimizing the expected number of banking failures.

Proposition 1 *An efficient contract $t^* = t_{AB} = t_{AC} = t_{BC}$ is given by*

$$t^* = \begin{cases} \frac{l}{2} \text{ if } l \leq \frac{1}{\pi^A} - 1 \\ 0 \text{ if } l > \frac{1}{\pi^A} - 1. \end{cases} \tag{1}$$

Proof. Due to the symmetry of the setup we can focus, without loss of generality, on the case where bank A is the deficit bank (that is the bank whose asset has a liquidity deficit at date 1). For a given liquidity deficit l there can be in principle two different situations: either bank A is insured against this deficit ($t_{AB} + t_{AC} \geq l$) or not ($t_{AB} + t_{AC} < l$). Note that insurance is always possible since $l \leq 2$ and hence never exceeds the combined liquidity holdings of the surplus banks (which are 2 units).

Consider first the case of no insurance ($t_{AB} + t_{AC} < l$). If bank A is not fully insured against the liquidity deficit, it fails at date 1. Bank B and C survive date 1 and store their remaining liquidity (net of any transfers to A) of $1 - t_{AB}$ and $1 - t_{AC}$, respectively. At date 2 they will thus survive the aggregate shock if the latter does not exceed $1 - t_{AB}$ and $1 - t_{AC}$, respectively. Given that the aggregate liquidity shock is uniformly distributed on $[0, 1]$, the likelihood that it exceeds the liquidity holdings of these banks is t_{AB} and t_{AC}, respectively. Recalling that the aggregate shock hits with probability π^A, the expected number of bank failures at date 2 is hence $\pi^A(t_{AB} + t_{AC})$. It follows that it is optimal to set transfers equal to zero ($t^*_{AB} = t^*_{AC} = t^* = 0$). The total number of banking failures is hence 1 (the deficit bank is certain to fail, while the surplus banks survive).

Consider next the case of insurance ($t_{AB} + t_{AC} \geq l$). The deficit bank is now insured against the idiosyncratic shock. Hence no bank fails at date 1 and any excess liquidity is stored. If at date 2 the aggregate shock (l^A) does not hit, no bank will fail at all. If it hits, bank A fails if $l^A > t_{AB} + t_{AC} - l$, while bank B and C fail if $l^A > 1 - t_{AB}$ and $l^A > 1 - t_{AC}$, respectively. The total expected number of banks failing when the aggregate shock arrives is hence $(1 - (t_{AB} + t_{AC} - l)) + (1 - (1 - t_{AB})) + (1 - (1 - t_{AC}))$, which simplifies to

$$1 + l \tag{2}$$

and is independent of t_{AB} and t_{AC}.[13] Thus, without loss of generality we can set for the optimal contract $t^*_{AB} = t^*_{AC} = \frac{l}{2}$ $(=: t^*)$, that is a deficit bank gets just enough transfers

[13] Since the expected losses are independent of the distribution of the liquidity holdings, contracts which are conditional on the *arrival* of the aggregate state cannot improve upon the allocation. However, if in addition also the *size* of this shock were contractible, the analysis becomes more complicated as losses may then be partly avoided (for example, if at date 2 two banks each have small shortfalls, one bank still

to survive at date 1 and the surplus banks contribute equally to its insurance. Given a probability of the aggregate shock arriving of π^A, the total expected number of banking failures is then

$$\pi^A(1+l). \tag{3}$$

From comparing this to the expected number of banking failures when there is no insurance ($=1$), it follows that it is optimal to insure the deficit bank iff $1 \geq \pi^A(1+l)$, or, rearranging, iff $l \leq \frac{1}{\pi^A} - 1$. ■

Proposition 1 shows that it is optimal to insure against small liquidity deficits ($l \leq \frac{1}{\pi^A} - 1$) but not against large ones ($l > \frac{1}{\pi^A} - 1$). What is the intuition for this result? Providing insurance creates a trade-off here. On the one hand, sufficient transfers can always save the deficit bank at date 1. On the other hand, transfers reduce the available liquidity at surplus banks. This may cause their failure when they get into troubles as well due to the aggregate shock.[14] If the liquidity deficit is small, small transfers are sufficient to save the deficit banks. Then only a large aggregate shock can cause the failure of the surplus banks. It is hence less likely that they fail as a result of insuring the deficit bank. Conversely, if the liquidity deficit is large, large transfers are required to save the deficit bank. Already relatively small aggregate shocks then cause the failure of the surplus banks. Thus, it is (socially) more costly to insure the deficit bank against larger liquidity shocks.

Moreover, Proposition 1 also shows that the condition for insurance being optimal becomes tighter when the likelihood of the aggregate shock π^A increases. This is because it is then more likely that the surplus banks themselves have (future) liquidity needs, which makes it (socially) more costly to insure the deficit bank.

Note that, while we focus here on (ex-ante) insurance among banks, an alternative way to deal with liquidity shocks is for a deficit bank to borrow at date 1 against the date 3 returns on its asset. In some situations this can indeed replace interbank insurance but in many situations it cannot. The reason is that when the liquidity deficit is significant, the required repayment for the bank becomes large. This is, among others, because the repayment also has to compensate the lenders for their higher likelihood of failure.

could be saved by transferring liquidity from the other bank to it). At any rate, as banks cannot remedy aggregate shortages the latter will always induce bank failures (if large enough) and hence date 1 interbank insurance remains subject to the same trade-off regardless of the contractibility of the aggregate shock.

[14] There is hence "contagion" in the sense that a liquidity problem at one bank can cause the failure of other banks.

Such repayments may not be feasible because they may exceed the date 3 return of the deficit bank. We analyze the case for borrowing and lending (as an alternative to ex-ante insurance) in the Appendix.

4.1 Equilibrium Insurance

We now consider banks' individual incentives to form insurance. For this we study whether banks have an incentive to deviate from the efficient contract. In other words, we ask whether there exists an equilibrium that is efficient. Since banks can form insurance only bilaterally, it is a priori unclear whether an equilibrium can also be efficient for the banking system as a whole. In particular, bilateral insurance between two banks may impose externalities on the third bank, which may preclude an efficient outcome.

We first define an equilibrium based on the notion of pairwise-stability (Jackson and Wolinsky [18]).[15] For this denote with t_{ij} ($\in [0,1]$) the bilateral transfer level between bank i and j ($i, j \in \{A, B, C\}$, $i \neq j$) and the expected pay-off for bank i ($i \in \{A, B, C\}$) for a given a set of transfers (t_{AB}, t_{AC}, t_{BC}) by $U_i(t_{AB}, t_{AC}, t_{BC})$.

Definition 1 *A set of transfers $\{t^*_{AB}, t^*_{AC}, t^*_{BC}\}$ forms a pairwise-stable equilibrium if*

(i) for all i, j ($i, j \in \{A, B, C\}$ and $i \neq j$) with $t^*_{i,j} > 0$ we have:

$$U_i(t^*_{ij}, t^*_{-ij}) \geq U_i(0, t^*_{-ij}) \text{ and } U_j(t^*_{ij}, t^*_{-ij}) \geq U_j(0, t^*_{-ij}),$$

(ii) for all i, j ($i, j \in \{A, B, C\}$ and $i \neq j$) we have

$$\text{for all } t_{ij} \neq t^*_{ij} \ (\in [0,1]) \text{: if } U_i(t^*_{ij}, t^*_{-ij}) < U_i(t_{ij}, t^*_{-ij}) \text{ then } U_j(t^*_{ij}, t^*_{-ij}) > U_j(t_{ij}, t^*_{-ij}),$$

Condition (i) states that in equilibrium it should not be possible for any bank to profit from severing a link with another bank, that is, no bank should be better off if it unilaterally changes from insurance with another bank ($t > 0$) to no insurance ($t = 0$). Condition (ii), in addition, requires that there is no transfer level that leads a pareto-improvement for any pair of banks, that is, if there is another transfer that makes one bank strictly better off, it should make the other strictly worse off.

Note that we do not model the procedure through which the transfers among banks are formed: the notion of pairwise stability is not dependent on any particular formation

[15] Note that the standard Nash-equilibrium is not an apropriate concept here as actions (bilateral insurance) are chosen jointly by pairs of banks.

process. Pairwise stability is a relatively weak concept and modeling explicitly the formation process might lead to a more restrictive definition of the equilibrium (for a further discussion of these points, see Section 5 in Jackson and Wolinsky [18]).

For symmetric transfers ($t^*_{AB} = t^*_{AC} = t^*_{BC}$) it is easy to check whether a transfer scheme forms an equilibrium. Due to the symmetry of the payoffs, the payoffs for each bank in a given pair are the same. Their interests when choosing the transfer scheme are hence completely aligned. Thus, it suffices to verify that for one of the banks there is no other transfer level that makes the bank better off (or, alternatively, there is no transfer level that increases the sum of banks' expected payoffs).

Proposition 2 *A symmetric transfer scheme ($t^*_{AB} = t^*_{AC} = t^*_{BC}$) forms an equilibrium iff for all i, j ($i, j \in \{A, B, C\}$ with $i \neq j$) and for all $t_{ij} \neq t^*_{ij}$ ($\in [0, 1]$): $U_i(t^*_{ij}, t^*_{-ij}) \geq U_i(t_{ij}, t^*_{-ij})$*

Proof. From the symmetry of the payoffs we have that for $t^*_{ik} = t^*_{jk}$: $U_i(t_{ij}, t^*_{-ij}) = U_j(t_{ij}, t^*_{-ij})$. This implies that when a pair of banks deviates from a symmetric scheme, their expected payoff changes are the same. The proposition then follows by noting that in Definition 1 condition i) is then equivalent to $U_i(t^*_{ij}, t^*_{-ij}) \geq U_i(0, t^*_{-ij})$, and likewise, condition ii) is equivalent to $U_i(t^*_{ij}, t^*_{-ij}) \geq U_i(t_{ij}, t^*_{-ij})$. ■

Figure 2 depicts the payoffs for bank A when the interbank insurance is provided with pure transfers (payoffs for bank B and bank C are equivalent). The payoffs are shown for given transfers t_{AB}, t_{AC}, t_{AC} and given liquidity shock l. With probability 1/3 bank A is the deficit bank at date 1 with liquidity need l. If the transfers it receives from the other banks are less than the liquidity need (i.e., $t_{AB} + t_{AC} < l$), bank A fails and its overall return is $t_{AB} + t_{AC}$ as the return on project is zero. Otherwise, if $t_{AB} + t_{AC} \geq l$ bank A survives at date 1. The aggregate shock arrives with probability π^A at date 2. If the aggregate liquidity shock is greater than the remaining liquidity (i.e., $l^A > t_{AB} + t_{AC} - l$), an event that occurs with probability $1 - (t_{AB} + t_{AC} - l)$, bank A fails and the overall return consists of return on liquidated project l plus remaining liquidity $t_{AB} + t_{AC} - l$, thus $t_{AB} + t_{AC}$ in total. Otherwise, if $l^A \leq t_{AB} + t_{AC} - l$ (which occurs with probability $t_{AB} + t_{AC} - l$), bank A survives and overall payoff consists of return on matured project $R + l + l^A$, plus remaining liquidity $t_{AB} + t_{AC} - l - l^A$, hence $R + t_{AB} + t_{AC}$ in total. With probability $1 - \pi^A$ aggregate shock does not arrive and bank A survives and overall return consists of return on matured project $R + l$, plus remaining liquidity $t_{AB} + t_{AC} - l$, hence $R + t_{AB} + t_{AC}$ in total.

With probability 2/3 bank A does not get the liquidity shock and it has a liquidity surplus of 1 at date 1. Bank B is deficit bank with probability 1/2 so bank A has to transfer t_{AB} to bank B. The aggregate shock arrives with probability π^A. If $l^A > 1 - t_{AB}$ (which occurs with probability t_{AB}), bank A fails and payoff is $1 - t_{AB}$. Otherwise, if $l^A \leq 1 - t_{AB}$ (which occurs with probability $1 - t_{AB}$), bank A survives and its payoff is $R + 1 - t_{AB}$. With probability $1 - \pi^A$ the aggregate shock does not arrive, then bank A survives and its payoff is $R + 1 - t_{AB}$. Finally, with probability $\frac{1}{2}$ bank C is deficit bank and we have a similar situation of the previous case when bank B is hit by the liquidity shock.

[Figure 2]

The next proposition establishes that the efficient outcome can be decentralized as an equilibrium.

Proposition 3 *The efficient outcome t^* also constitutes an equilibrium.*

Proof. We have to show that under the efficient transfer scheme, no pair of banks can benefit by changing to another level of transfers. For this we have to distinguish between the case where the efficient outcome is insurance ($l \leq \frac{1}{\pi^A} - 1$) and where it is not ($l \geq \frac{1}{\pi^A} - 1$). We start with the second case.

1. Insurance is inefficient ($l \geq \frac{1}{\pi^A} - 1$). The optimal contract is then $t^* = 0$. In order to analyze banks' incentives to deviate from the efficient outcome, we consider without loss of generality bank A and B's incentives to change their bilateral insurance contract t_{AB} from zero to an amount larger than zero. In doing so both banks take as given their other insurance agreements, that is between A and C and B and C: $t_{AC} = t_{BC} = 0$. A deviation will only affect bank A and B if either of them is the deficit bank at date 1. This is because the bilateral insurance between A and B is not invoked if bank C is the deficit bank. Without loss of generality presume again that bank A is the deficit bank. If bank A and B decide to remain uninsured (that is, if they leave t_{AB} at 0), bank A fails at date 1 and its expected return is zero. Bank B always survives since it can store one unit of liquidity and thus withstand any aggregate shock. Thus, the combined expected payoff is $R + 1$. Now consider the case where they both choose to insure each other by agreeing on an $t_{AB} \geq l$ (note that this is only feasible if $l \leq 1$ because bank B has only one unit of liquidity). The banks may then fail when the aggregate

shock arrives at date 2. From Figure 2 we have that the expected payoff of bank A is $(1-\pi^A)(R+t_{AB})+\pi^A[(t_{AB}-l)(R+t_{AB})+(1+l-t_{AB})t_{AB}]$, while the expected payoff of bank B is $(1-\pi^A)(R+1-t_{AB})+\pi^A[(1-t_{AB})(R+1-t_{AB})+(1-t_{AB})t_{AB}]$. Summing up the two expected payoff, and rearranging, we get $2R-\pi^A R(1+l)+1$. According to Proposition 2 an equilibrium requires that no bank benefits from unilaterally changing to another transfer lever. Given the symmetry of the payoffs and given $t_{AC}=t_{BC}$, this will be exactly the case when changing to another transfer level does not increase the joint expected payoffs of the two banks. Comparing the expected payoff under deviation and no deviation, we see that the former are only greater if $2R-\pi^A R(1+l)+1>R+1$, which implies $l<\frac{1}{\pi^A}-1$. The previous condition contradicts the assumed (social) optimality of no insurance ($l \geq 1/\pi^A-1$ from above).

2. Insurance is efficient ($l<\frac{1}{\pi^A}-1$). The efficient contract now stipulates to save the deficit bank, with each of the surplus banks contributing $t_{AB}=t_{AC}=\frac{l}{2}$. Consider again the incentives of bank A and B to deviate, and presume that bank A is the deficit bank. If both banks do not deviate, bank A is saved at date 1 but bank A and B may fail when the aggregate shock arrives at date 2. From Figure 2 we have that the expected payoff for bank A is $(1-\pi^A)(R+l)+\pi^A l$, while the expected payoff for bank B is $(1-\pi^A)(R+1-l/2)+\pi^A[l/2(1-l/2)+(1-l/2)(R+1-l/2)]$. Summing up the two expected payoff, and rearranging, we get $2R-\pi^A R(1+l/2)+l/2+1$ (observe that we now have the term $\frac{l}{2}$ rather than l as under 1.; this is because in the case above B had to insure bank A alone, making insurance more costly). If both banks deviate from insurance, there is one bank which fails with certainty (bank A). However, bank A in this case gets $l/2$ from bank C. Bank B always survives and it expects to get $R+1$. Thus, the two banks will benefit from a deviation if and only if $2R-\pi^A R(1+l/2)+l/2+1<R+1+l/2$, which implies $l>\frac{2(1-\pi^A)}{\pi^A}$. The latter condition contradicts the condition that insurance is optimal ($l<\frac{1}{\pi^A}-1$). There are hence no parameter values for which bank A and B would like to deviate. ■

What is the reason why banks do not deviate from the efficient outcome? Loosely speaking, the bilateral insurance choice between two banks will be efficient if it does not pose any externalities to the third bank. In the case of no insurance the third bank does not interact at all with the first two banks and hence there cannot be any externality. In the case of insurance the third bank does interact with the other banks because it has to make transfers to the deficit bank. But these transfers only depend on the idiosyncratic liquidity shock and are not affected by the bilateral insurance between the other two banks. Hence, there is again no externality and the equilibrium can be efficient.

Notice, however, that a potential inefficiency may arise because banks may coordinate on an inefficient equilibrium. Recall from the proof of Proposition 3 that the condition for banks not finding it optimal to deviate from an insurance outcome is $l < 2(\frac{1}{\pi^A} - 1)$, while the condition for the optimality of insurance is $l < \frac{1}{\pi^A} - 1$. Thus there are parameter constellations for l $(2(\frac{1}{\pi^A} - 1) > l > \frac{1}{\pi^A} - 1)$ where insurance is inefficient but banks would not deviate from an insurance outcome. Intuitively, this is because when the third bank already provides insurance, lower insurance payments are required by the other two banks in order to save a deficit bank. As a result, these two banks perceive lower costs of insuring.

5 Credit Lines

So far interbank insurance took place in the form of pure transfers. There was hence no interaction between banks beyond the risk sharing at date 1. In this section we consider insurance against repayments. For this we introduce moral hazard at the bank level, which makes pure transfers undesirable. In fact, a certain repayment on the insurance transfers is now needed in order to (at least partially) force a bank to internalize the costs of ending up in a liquidity deficit. Such insurance contracts with repayments are essentially credit lines granted by banks to each other. The repayment adds another interaction among banks which, as we will see, provides a potential source of inefficiency. We want to stress that even though moral hazard is a natural way to induce repayment, it is not the cause of the inefficiency. Repayments are the ultimate source of inefficiency. Any alternative reason that induces repayments would generate similar results.

We now assume that at date 0, after insurance contracts have been signed, each bank can decide whether or not to exert effort. If each bank undertakes effort, the date 1 (idiosyncratic) liquidity shock arrives only with probability π^I $(\pi^I \in (0, 1))$. When it arrives it hits with equal probability one of the banks, as before. A bank's probability of receiving the shock is thus $\frac{\pi^I}{3}$. If one bank does not exert effort, the liquidity shock arrives with probability 1. It then hits this bank with probability $1 - \frac{2}{3}\pi^I$ $(> \frac{\pi^I}{3})$ and the other two banks with probability $\frac{\pi^I}{3}$ each. Thus, if a bank does not exert effort it increases its risk of receiving the liquidity shock by $1-\pi^I$, while the risk for all other banks stays the same. If more than one bank does not exert effort, we assume for simplicity that each of the assets without effort become worthless.[16] Effort is assumed to incur private (non-monetary) costs of e ($e>0$) per bank. The timing of actions can be seen from the previous Figure 1, now including the effort choice at date 1.

We first write the condition that exerting effort is productive, which requires that a bank would also undertake effort in the absence of any insurance. A bank fails in the absence of insurance whenever it receives the idiosyncratic liquidity shock. The probability of this is $\frac{\pi^I}{3}$ if effort is undertaken and $1-\frac{2}{3}\pi^I$ if not. Given effort costs e, effort is hence exerted when $\frac{1}{3}\pi^I R+e \leq (1-\frac{2}{3}\pi^I)R$. Rearranging this gives

$$(1-\pi^I)R \geq e. \tag{4}$$

Quite intuitively, this condition states that the expected reduction in liquidation costs induced by exerting effort, $(1-\pi^I)R$, has to at least offset the effort costs e. We assume this condition in the following to be fulfilled.

The efficient contract, as defined in Proposition 1, remains unchanged. There is only the added requirement that repayments have to be chosen such that effort is induced.

Lemma 1 *When insurance takes the form of credit lines, a minimum repayment of*

$$\overline{r}=\frac{1}{1-\pi^A}\left(\frac{e}{1-\pi^I}-\pi^A R+l\right) \tag{5}$$

is needed in order to induce effort.

Proof. We derive the required repayment r which induces effort under insurance for the optimal contract ($t^*=\frac{l}{2}$). For this we analyze how a bank's payoff changes when it decides to exercise effort. If it does so, it has to incur the effort costs e but it also becomes less likely to receive the idiosyncratic shock. Receiving this shock is now costly for two reasons. First, because it may lead to the failure of the bank, and second, because the bank has to make a repayment when it survives. More specifically, when the bank receives this liquidity shock it fails with probability π^A (inducing costs of R) but survives with probability $1-\pi^A$ (inducing costs r due to repayment). However, receiving the liquidity shock also has a

[16] This assumption serves to avoid a no-effort Pareto-inferior equilibrium.

benefit because the bank gets a total transfer of l from the other banks. Given that effort reduces the probability of receiving the liquidity shock by $1-\pi^I$, effort is hence exerted iff

$$(1-\pi^I)(\pi^A R + (1-\pi^A)r - l) \geq e. \tag{6}$$

Rearranging for r gives $r \geq \frac{1}{1-\pi^A}(\frac{e}{1-\pi^I} - \pi^A R + l)$. From this we can define with $\overline{r}$ the critical repayment that just induces effort, which yields equation (5). This critical repayment is larger than zero, for example, for sufficiently large effort costs e. Note that $\overline{r}$ refers to the total repayment a bank has to make (that is, the sum of the repayments to the two other banks). ■

Figure 2 reports the payoffs for bank A when interbank insurance is provided with repayment (again, payoffs for bank B and bank C are identical). Payoff are shown for given transfers t_{AB}, t_{AC}, t_{AC} and given liquidity shock l and repayment r. Bank A is the deficit bank at date 1 with probability $\pi^I/3$ if it exercise effort. Otherwise, it will experience a liquidity deficit with probability $1-(2/3)\pi^I$. Similarly to Figure 2, if $t_{AB}+t_{AC} < l$ then bank A fails and its overall payoff is $t_{AB}+t_{AC}$ as return on project is zero. Otherwise, if $t_{AB}+t_{AC} \geq l$ bank A survives at date 1. The aggregate shock arrives at date 2 with probability π^A. If $l^A > t_{AB}+t_{AC}-l$, which occurs with probability $1-(t_{AB}+t_{AC}-l)$, the payoff is $t_{AB}+t_{AC}$. Otherwise, if $l^A \leq t_{AB}+t_{AC}-l$, which occurs with probability $t_{AB}+t_{AC}-l$, bank A survives and overall payoff is $R+t_{AB}+t_{AC}-r$ since insurance is now provided with repayment. If the aggregate shock does not arrive, bank A survives and overall payoff is $R+t_{AB}+t_{AC}-r$.

Bank A is not hit by the liquidity shock and instead has a liquidity surplus of 1 with probability $(2/3)\pi^I$. If bank B is the deficit bank (an event that occurs with probability $1/2$), bank A has to transfer t_{AB} to bank B. If bank B does not survive the liquidity shock at date 1 (which occurs whenever $t_{AB}+t_{AC} < l$) then bank A does not receive any repayment. The aggregate shock arrives with probability π^A, and if $l^A > 1-t_{AB}$ (which occurs with probability t_{AB}), bank A fails and its payoff is $1-t_{AB}$. Otherwise, if $l^A \leq 1-t_{AB}$ (which occurs with probability $1-t_{AB}$) or the aggregate shock does nor arrive, bank A survives and its payoff is $R+1-t_{AB}$. If bank B survives the liquidity shock at date 1 (i.e., if $t_{AB}+t_{AC} \geq l$) then the repayment is possible. If the aggregate shock arrives, then bank A can fail with probability t_{AB} and its payoff is $1-t_{AB}$ if bank B does not survive the aggregate shock (which occurs with probability $1+l-t_{AC}-t_{AB}$), or the payoff is $1-t_{AB}+r$ if bank B survives the aggregate shock (which occurs with probability $t_{AC} + t_{AB} - l$). With probability $1 - t_{AB}$ bank A survives the aggregate shock and its payoff is $R + 1 - t_{AB}$ if bank B does not survives the aggregate shock, otherwise, if bank B survives the aggregate shock, bank A gets $R + 1 - t_{AB} + r$. If the aggregate shock does not arrive then bank A payoff is $R + 1 - t_{AB} + r$. If bank C is deficit bank, we get similar payoff when bank B is the deficit bank. Finally, bank A gets $R + 1$ when no bank is affected by liquidity shock, an event that occurs with probability $1 - \pi^I$ when bank A exercises effort (otherwise, the liquidity shock arrives with certainty).

[Figure 3]

Proposition 4 shows next that the equilibrium may now display underinsurance:

Proposition 4 *When insurance is provided with repayments, banks may deviate from an efficient insurance outcome (equilibrium underinsurance).*

Proof. We show that when insurance is optimal there are parameter values for which banks deviate from the insurance outcome. For this suppose that $l < \frac{1}{\pi^A} - 1$, that is insurance is optimal (from Proposition 1). We consider the incentives for banks to deviate from an efficient insurance outcome with $t^* = \frac{l}{2}$ and a total repayment r that induces effort. As we will see later the scope for an inefficient deviation is minimized when repayments are low. We thus set the total repayment to its minimum possible value $r = \overline{r}$. Furthermore we assume that repayments are equally split among banks: $r_{AB} = r_{AC} = r_{BC} = \overline{r}/2$. The deviation we consider is one where banks A and B deviate from the insurance outcome by no longer insuring A through transfers from B (that is $t_{AB} = 0$ if A gets the liquidity shock). Note that any asymmetry in the transfers between A and B does not affect their joint pay-off and hence not their deviation incentives. We focus on the situation where bank A is the deficit bank (otherwise the deviation does not matter).

The two bank's expected payoffs if they do not deviate are as follows (see Figure 3). The probability of bank A being the deficit bank is $\frac{\pi^I}{3}$. With probability $1 - \pi^A$ the aggregate shock does not arrive subsequently and both banks survive. Their joint costs are then $\frac{\overline{r}}{2}$ because this is the repayment bank A has to make to bank C at date 3. With probability $\pi^A(1 - \frac{l}{2})$ the aggregate shock arrives but is not so high that bank B fails (recall that its liquidity holdings at date 2 are $1 - \frac{l}{2}$). The costs are then R because of the failure of bank A. With probability $\pi^A \frac{l}{2}$ the aggregate shock arrives and is high enough to make bank B

fail as well. The joint costs are then $2R$. In addition to the above effects, whenever bank A is the deficit bank, the two bank's payoffs are enhanced by a transfer from bank C of $t_{AC} = \frac{l}{2}$. Finally, there are also the effort costs for both banks: $2e$. The total costs for bank A and bank B are thus

$$\frac{\pi^I}{3}\left((1-\pi^A)\frac{\overline{r}}{2} + \pi^A(1-\frac{l}{2})R + \pi^A\frac{l}{2}\cdot 2R - \frac{l}{2}\right) + 2e. \tag{7}$$

If the banks decide to deviate as described above, their costs are as follows. Since now the critical repayment is no longer reached, bank A does not undertake effort. The probability of it getting the liquidity deficit is then $1-\frac{2}{3}\pi^I$. When bank A receives the liquidity shock it now always fails, while bank B survives. The costs from this are $R-\frac{l}{2}$ in this case (the costs are reduced by $\frac{l}{2}$ through the transfer A receives from C). Since effort costs are e (effort for bank B), the total costs for banks A and B are

$$(1-\frac{2}{3}\pi^I)(R-\frac{l}{2}) + e. \tag{8}$$

Comparing both costs we can see that a deviation takes place iff

$$\frac{1}{3}\pi^I\left((1-\pi^A)\frac{\overline{r}}{2} + \pi^A(1-\frac{l}{2})R + \pi^A\frac{l}{2}\cdot 2R - \frac{l}{2}\right) + 2e > (1-\frac{2}{3}\pi^I)(R-\frac{l}{2}) + e. \tag{9}$$

Note that this condition is more easily fulfilled when $\overline{r}$ is higher. The reason is that when there is deviation from the insurance outcome, the deficit bank fails more often and there are hence less occasions where it has to do the repayment.

In the following we show that there are parameter values for which there are deviations from an (efficient) insurance. For this we show that there is a deviation from insurance for the idiosyncratic shock at which insurance is marginally efficient: $l = \frac{1}{\pi^A} - 1$. Furthermore, we also assume that effort is just worthwhile: $e = (1-\pi^I)R$ (from equation 4). Using this and the equation for $\overline{r}$ to substitute l, e and $\overline{r}$ into equation (9) one obtains after rearranging $R + \frac{1}{\pi^A} > R - \frac{3(1-\pi^I)}{\pi^A\pi^I}$. Since the left-hand side of this inequality is larger than R but the right-hand side is smaller than R, this condition is fulfilled for all feasible π^A, π^I and R. ■

The intuition for why there can now be underinsurance is the following. When two banks move away from insurance, they become more likely to fail. This induces a negative effect on the third bank because in this case the bank is not repaid. This effect is not internalized by the two deviating banks, creating an incentive to insure insufficiently.

5.1 Efficiency Restoring Policies

We now analyze whether there are any policy interventions that can remedy underinsurance. Obviously, if the regulator has full control over interbank insurance he could directly implement the efficient outcome. However, we are looking here for less demanding policy interventions. To this end we first consider a policy where the regulator subsidizes interbank transfers. In particular, we consider a policy where the regulator fully insures any interbank transfers a bank has made to another bank in case the latter fails at $t = 1$. We show that such a policy can induce efficient insurance.

Proposition 5 *Assume that whenever a bank fails at $t = 1$ the regulator pays at $t = 3$ to each bank that has insured the failing bank an amount equal to the insurance transfer. This payment is financed by taxing other banks at $t = 3$. Equilibrium interbank insurance is then efficient.*

Proof. In order to show efficiency we have to show that banks neither have an incentive to deviate from an efficient insurance outcome, nor from an efficient no-insurance outcome. To this end suppose first that insurance is efficient and consider banks' incentives to deviate from an insurance outcome. We again focus on the case where bank A and B want to deviate and where bank A is the deficit bank. If the two banks remain insured, bank A never fails in response to the idiosyncratic shock at $t = 1$ and hence a subsidy never materializes. The total costs for both banks are thus the same as in the absence of a policy intervention and given by equation (7). If the banks deviate by no longer insuring each other, bank A will fail when it is the shock bank. In this case the subsidy will amount to a transfer to bank C equal to the amount bank C insured bank A with ($\frac{l}{2}$), financed through taxation from bank B. The total costs of deviating for A and B are thus increased by (in expected terms) $(1 - \frac{2}{3}\pi^I)\frac{l}{2}$ relative to the situation without subsidies (given by equation (8)). Using equations (7) and (8) we hence obtain the condition for a deviation being worthwhile

$$\frac{1}{3}\pi^I \left((1-\pi^A)\frac{\overline{r}}{2} + \pi^A(1-\frac{l}{2})R + \pi^A\frac{l}{2} \cdot 2R - \frac{l}{2} \right) + 2e > (1 - \frac{2}{3}\pi^I)R + e. \tag{10}$$

Substitution $\overline{r}$ using equation (5) and using the condition that effort is worthwhile (equation (4)) we obtain $l > \frac{1}{\pi^A} - 1$, contradicting the efficiency of insurance (equation (1)). Hence, there are no cases where banks want to deviate from an efficient insurance outcome.

We next consider banks' incentives to deviate from a no-insurance situation. When banks remain uninsured, the subsidies are obviously never invoked and hence the policy intervention does not change bank A and $B's$ total costs. These costs are hence only arising from failures of bank A (bank B always survives) and are (in expected terms) given by the probability of bank A being the shock bank times R:

$$\frac{\pi^I}{3}R. \tag{11}$$

Consider now that banks A and B deviate and insure each other with an amount l. Again, the presence of the subsidy will not affect the (combined) costs as the subsidy is only paid in case where insurance takes place and bank A fails at $t = 1$. The probability of bank A being the deficit bank is $\frac{\pi^I}{3}$. With probability $1 - \pi^A$ the aggregate shock does not arrive and both banks survive. There are no joint costs in this case because no repayment is due to bank C (as it does not provide any insurance). However, banks A and B may fail if the aggregate shock arrives. With probability $\pi^A(1 - l)$ the aggregate shock arrives but is not so high that bank B fails (recall that its liquidity holdings at date 2 are $1 - l$). The costs are then R because of the failure of bank A. With probability $\pi^A l$ the aggregate shock arrives and is high enough to make bank B fail as well. The joint costs are then $2R$. We then have that the total expected costs are

$$\frac{\pi^I}{3}(\pi^A(1 - l)R + \pi^A l \cdot 2R). \tag{12}$$

Banks A and B will deviate from the optimal no-insurance if this reduces the expected costs, that is, iff

$$\frac{\pi^I}{3}R > \frac{\pi^I}{3}(\pi^A(1 - l)R + \pi^A l \cdot 2R). \tag{13}$$

Note that the subsidies do not play a role for the deviation incentives. The above condition implies that $l < 1/\pi^A - 1$, which contradicts the optimality of no insurance (equation (1)). Banks thus do not deviate inefficiently.[17] ■

The intuition why subsidies can improve the allocation is straightforward: since the problem is one of underinsurance, any device that makes insurance more desirable for banks improves upon efficiency. It is worthwhile to note though that the required subsidy

[17] There is one complication though. Bank A and B may try to insure each other *partially* in order to obtain a subsidy from bank C (as the subsidy is only paid when the shock bank fails). However, this does not have any welfare implications since as long as the shock bank is only partially insured the outcomes (in terms of banking failures) are the same as when there is no insurance.

to restore efficiency is full and not partial, as one may have expected from usual moral hazard considerations. Note also that the subsidy in fact resembles government insurance of interbank lending, as it is only invoked when the insuring bank fails. The proposition thus provides a justification for full interbank guarantees, as introduced in many countries during the crisis of 2007/2008. In line with our model, however, such subsidies should only be paid when banks fail in response to idiosyncratic shocks.

One may wonder why we focus on subsidies that are only paid if bank fails in response to the idiosyncratic shock. If the subsidy is paid regardless of the nature of the shock that makes an insured bank fail, the total expected value of the subsidy arising from insurance increases. This implies that the subsidy itself has to be reduced in order to achieve the same incentives as before (and not to cause overinsurance). The degree to which this is the case then depends on the model's parameters as well as the insurance contracts chosen by banks, making it a presumably much more demanding policy intervention.

We consider next a different measure to obtain efficiency, which is to make interbank insurance claims senior. Interbank claims are (implicitly) junior in the current setting as we have presumed that when a bank fails, any agreed repayments on insurance are not met. That is, any other claimants on the bank are implicitly assumed to be senior to the interbank claims. This is plausible to the extent that such claims (e.g., ordinary debt) may have been established before interbank insurance and that existing debt has seniority clauses. Note, however, that the shock bank in principle always has sufficient resources to pay back any insurance transfer. This is because when it fails, it still recoups any liquidity injection it has provided earlier, which will obviously always be at least as large as the combined transfers a bank received.

We consider next the equilibrium impact of a seniority clause for interbank insurance claims, showing that it can obtain efficiency.

Proposition 6 *Assume that if a bank fails at $t = 1$ interbank insurance claims are made senior to any other obligations a bank may have and assume, moreover, that the seniority applies to an amount up to the original insurance transfer (thus not including any agreed repayment that exceeds the transfer). Equilibrium interbank insurance is then efficient.*

Proof. We proceed as in Proposition 5. We first consider the incentives of bank A and B to deviate from an efficient insurance outcome under the presumption that A is the shock bank. If A and B do not deviate, the seniority clause does not apply since A then never fails at $t = 1$. Hence bank A and $B's$ combined expected costs do not change compared to the case of no seniority. In case they deviate by not insuring each other, seniority is invoked as bank A fails. In this case bank A has to repay the transfer $\frac{l}{2}$ to bank C. The total expected costs of bank A and B thus increase by $\frac{l}{2}$, and in expected terms by $(1 - \frac{2}{3}\pi^I)\frac{l}{2}$ relative to the absence of seniority. The costs thus increase by the same amount as in the case of subsidies. It follows with Proposition 5 that the deviation decision cannot be inefficient.

We next consider the two banks' incentives to deviate from an efficient no-insurance outcome. In the case of no deviation, seniority does not play a role since there is then no interbank insurance. If bank A and B deviate by insuring each other, seniority still does not affect their (combined) expected costs. This is because bank C does not provide any insurance, the seniority clause can never lead to any payments from A or B to C. The deviation incentives from no incentives are hence the same as in the absence of seniority, and hence efficient as shown in Proposition 5. ■

The seniority of interbank insurance tackles the root of the inefficiency in our setting. This inefficiency arises because deviating from insurance poses a negative externality by increasing the likelihood of the deficit bank failing, in which case other insuring banks do not get repaid their transfers. Seniority removes this externality, and hence makes interbank insurance efficient.

The obvious question arises why if seniority of interbank claims allows for efficiency, banks do not incorporate such clauses themselves. This would make regulatory intervention obsolete. A first answer is that interbank insurance may be established after debt has been raised and that clauses in this debt may prevent banks from making interbank insurance claims senior. Second, even if banks are free to make senior interbank claims, they do not have an incentive to (bilaterally) switch to seniority. To see this suppose that we are in a situation of mutual insurance and suppose that interbank claims are junior. If two banks decide to bilaterally make senior their claims this will not lower their expected costs as seniority is a zero-sum game between the two involved parties. Hence they have no incentives to switch to senior claims. The choice of seniority thus poses a coordination problem: once claims are junior banks have no incentives to change them to senior. The existence of this coordination problem creates a rationale for regulators to force banks to make senior their claims, even if other obstacles to seniority are absent.

The fact that seniority may be a desirable feature has also implications for the evaluation of past changes in U.S. policies towards interbank lending. Interbank lending in the U.S. typically was secured before the early 1990s: "sellers of fed funds to insolvent institutions were often protected from losses... Even those banks that did suffer losses on fed funds typically recovered a relatively high percentage of their principal based on the structure of the payoff hierarchy at the time" (King [21]). In the mid-90s U.S. regulatory reforms encouraged unsecured lending with the view of facilitating interbank monitoring. As secured lending resembles seniority of interbank claims, our analysis suggests that this change had potential negative side-effects. Following the change in regulation, banks reduced interbank lending (see again King [21]). Our model suggests that this may have pushed interbank insurance below efficient levels. The potential market discipline benefits from having unsecured debt may thus have to be weighted against the costs of underinsurance among banks.

One may also ask whether a seniority clause should be preferred to the subsidies. In our model, the inefficiency arises from the fact that a failing bank cannot make the repayment on its interbank obligations. If one believes this indeed to be the main externality, seniority of claims may be the right policy choice as it directly addresses this externality. However, there may be externalities from (single) bank failures beyond the repayment effect. In this case the seniority clause may not be able to correct this externality but subsidies, which generally increase the inclination to insure (and which can be easily extended beyond the amount insured) may be a more appropriate policy device.

Note also that the informational requirements to implement the policy schemes are not excessive. For example, the regulator does not have to observe the size of the idiosyncratic liquidity shock l and who gets it, nor the arrival of the aggregate shock, nor banks' (ex-ante) insurance arrangements. It suffices to observe transfers made at $t = 1$ and whether there are any bank failures at this date. Based on this the subsidies can be arranged or seniority determined. Given central banks' oversight over the interbank market, these requirements may not be overly demanding. Another advantage of our policy interventions is that they do not have to be used in equilibrium. This is because insurance that is insufficient to (initially) save the deficit bank never occurs in equilibrium, and hence neither the subsidy nor the seniority has to be invoked.

6 Overinsurance

We have shown that underinsurance may occur when insurance transfers are provided against a repayment, such as is the case with credit lines. Can there also be overinsurance? That is, is it possible that two banks may decide to deviate from an efficient no-insurance outcome?[18]

Generally, when two banks deviate from a no-insurance outcome by insuring each other they create a trade-off. They reduce the likelihood that a single bank fails (since the deficit bank is now insured) but they increase the likelihood that both banks fail jointly (since now also the surplus bank may fail when the aggregate shock arrives). If either effect poses externalities on the other bank, their deviation decision may become inefficient.

We typically associate the joint failure of banks with external effects. For example, when two banks fail the ensuing liquidation of assets may cause fire-sale prices and thus also lead to the failure of the third bank (e.g., Wagner [25]). Alternatively, there may be negative informational spillovers, causing depositors' runs. Or, the third bank may suffer from a loss of future risk sharing opportunities since there is now no longer a bank with which it can share its idiosyncratic risk.[19] Therefore, a deviation from no insurance may conceivably pose negative externalities that appear to give banks an incentive to insure too much.

However, there cannot be overinsurance as a result of such externalities. To see this, suppose that the joint failure of two banks creates negative costs $K > 0$ for the third bank. For overinsurance to occur we need the two banks to have an incentive to deviate from an efficient no-insurance-outcome. As shown earlier, in the absence of K social and private incentives are completely aligned in this case. That is, two banks only have an incentive to deviate from no insurance if insurance were socially preferable. Since the costs K are external to the two banks, they obviously do not affect their incentives to deviate from a no-insurance outcome. Moreover, the costs K also do not affect the optimality of no insurance relative to a situation where all three banks insure each other. This is because in either case there are no situations where two banks fail and one survives and hence

[18] Note that, as already pointed out earlier, there can be overinsurance in the sense that banks may coordinate on an equilibrium in which there is 'too much' insurance. However, this coordination failure is not our focus here.

[19] We have modelled such an effect in an earlier version (calculations are available on request).

externalities from a joint failure of two banks are absent. The externalities only arise if two banks deviate from a no-insurance outcome to a situation where two banks insure each other; but as just pointed out they only do this if no insurance is in fact not optimal.

The only way in which overinsurance can arise in our setting is if the deviating banks directly gain from their joint failure. This may be, for example, because of a too-many-too-fail policy (e.g., Acharya and Yorulmazer [1]). When several banks fail at the same time, the functioning of the financial system may no longer be guaranteed. The regulator may then decide to bail-out the jointly failing banks, for example by giving them (implicit) subsidies. This increases their incentives to deviate from a no-insurance outcome and may lead to inefficiently high insurance.

7 Conclusion

This paper emphasizes the importance of considering banks' bilateral incentives in the analysis of interbank insurance. While previous literature has analyzed the optimal form of interbank linkages, these linkages may in practice not be implemented. This is because interbank insurance may be formed bilaterally, and not jointly among all banks.

Understanding any potential deviation of the interbank insurance which arises from banks' bilateral contracting from the optimal insurance is of paramount importance for financial regulation. This is because a breakdown of interbank insurance itself does not necessarily indicate inefficiency. Indeed, as we have shown, it may be desirable for banks to not insure each other against certain outcomes. This is because it may be optimal for banks to withhold liquidity for fear of being hit by shocks. Only when the equilibrium deviates from the efficient outcome, is there a role for regulation.

Our analysis has shown, that in principle, such a deviation can go either way. The failure of an insured bank imposes negative externalities on the insuring banks because it can then no longer repay the insurance payment. This creates a tendency for banks to underinsure. That is, there are shocks against which it is optimal to insure, but in equilibrium banks fail to do so. A tendency for underinsurance should be particularly pronounced when there are strong negative externalities of bank failures (which in our model happen to arise from a failure to repay). In such situations there is a role for regulators to encourage banks to cross-insure more. We have shown that such regulation may come in the form of encouraging seniority clauses for interbank claims or in the form of guarantees to interbank insurance.

However, we have also identified a reason why banks may insure “too much”, arising because banks want to exploit the possibility of receiving regulatory subsidies in a systemic crises. The reason is that interbank insurance trades-off a lower probability of individual failures with a higher likelihood of joint failures. Therefore, if banks perceive lower costs from the latter, their incentives to insure increase beyond the efficient level. This effect will be important when public bail-outs of banks are likely. As a result of the experience of the current crisis, the perceived likelihood of such bail-outs has probably increased, and hence this channel may now to a considerable extent offset the underinsurance motive.

Appendix: Borrowing versus Insurance

We show in the following that there are situations where borrowing at date 1 cannot replace (efficient) insurance. The problem with borrowing is that if a deficit bank borrows to withstand a liquidity deficit, this borrowing has to be worthwhile for the other two banks at date 1 (or at least not make them worse off), while insurance only has to be worthwhile from an ex-ante perspective.

Consider a situation where the two surplus banks lend $\frac{l}{2}$ each to save the deficit bank. A surplus bank's pay-off if it does not lend is simply R since it never fails when it does not give away liquidity at date 1. Its pay-off with lending is as follows. With probability $1-\pi^A$ the surplus bank will survive together with the deficit bank. In this case it gets R plus the agreed repayment per bank, denoted $\frac{r_L}{2}$, minus the amount lent out, $\frac{l}{2}$. With probability $\pi^A \frac{l}{2}$ the surplus bank fails and does not receive anything. Finally, with probability $\pi^A(1-\frac{l}{2})$ the surplus bank survives but the deficit bank fails. In this case there is no repayment and the pay-off is $R - \frac{l}{2}$.

The break-even repayment for the surplus banks is determined implicitly by setting equal the payoffs under lending and no lending:

$$R = (1-\pi^A)(R + \frac{r_L}{2} - \frac{l}{2}) + \pi^A \frac{l}{2} \cdot 0 + \pi^A (1 - \frac{l}{2})(R - \frac{l}{2}). \tag{14}$$

From this we can derive the inequality

$$R < (1-\pi^A)(R + \frac{r_L}{2} - \frac{l}{2}) + \pi^A (1 - \frac{l}{2})R. \tag{15}$$

Rearranging for r_L gives

$$r_L > l(\frac{\pi^A R}{1-\pi^A} + 1).$$

The maximum feasible repayment at date 3 is R, thus we must have that $r_L \leq R$. Using this to substitute r_L, and rearranging for l, gives us the condition that determines when lending is feasible

$$l < R\frac{1-\pi^A}{\pi^A(R-1)+1}.$$

Contrast the last expression with the condition for the optimality of saving the deficit bank (Proposition 1), that is $l \leq \frac{1}{\pi^A} - 1$. The condition for the feasibility of lending is more binding when $R\frac{1-\pi^A}{\pi^A(R-1)+1} < \frac{1}{\pi^A} - 1$, or, after rearranging, if $\pi^A \leq 1$. Thus, the lending condition is always more binding. It follows that there are always shocks l for which saving the deficit bank is desirable but which cannot be achieved through lending.

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Figure 1. The Timing of the Model

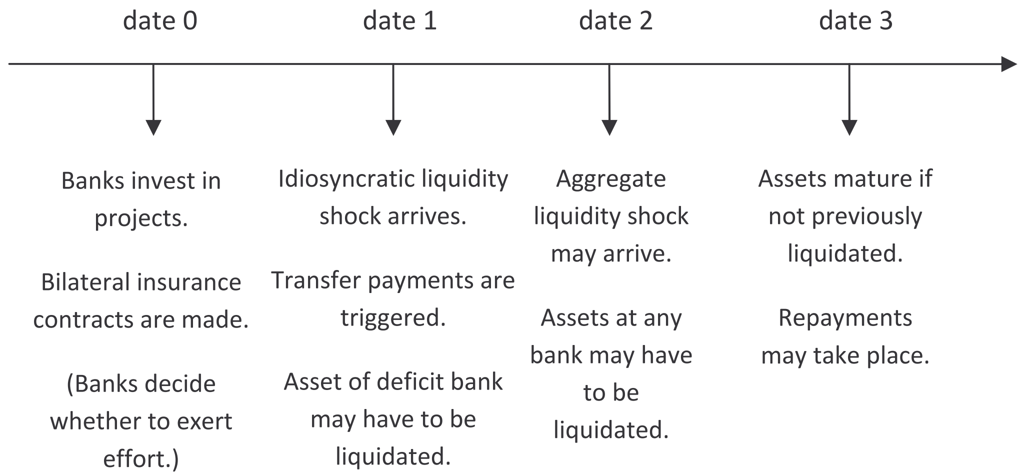
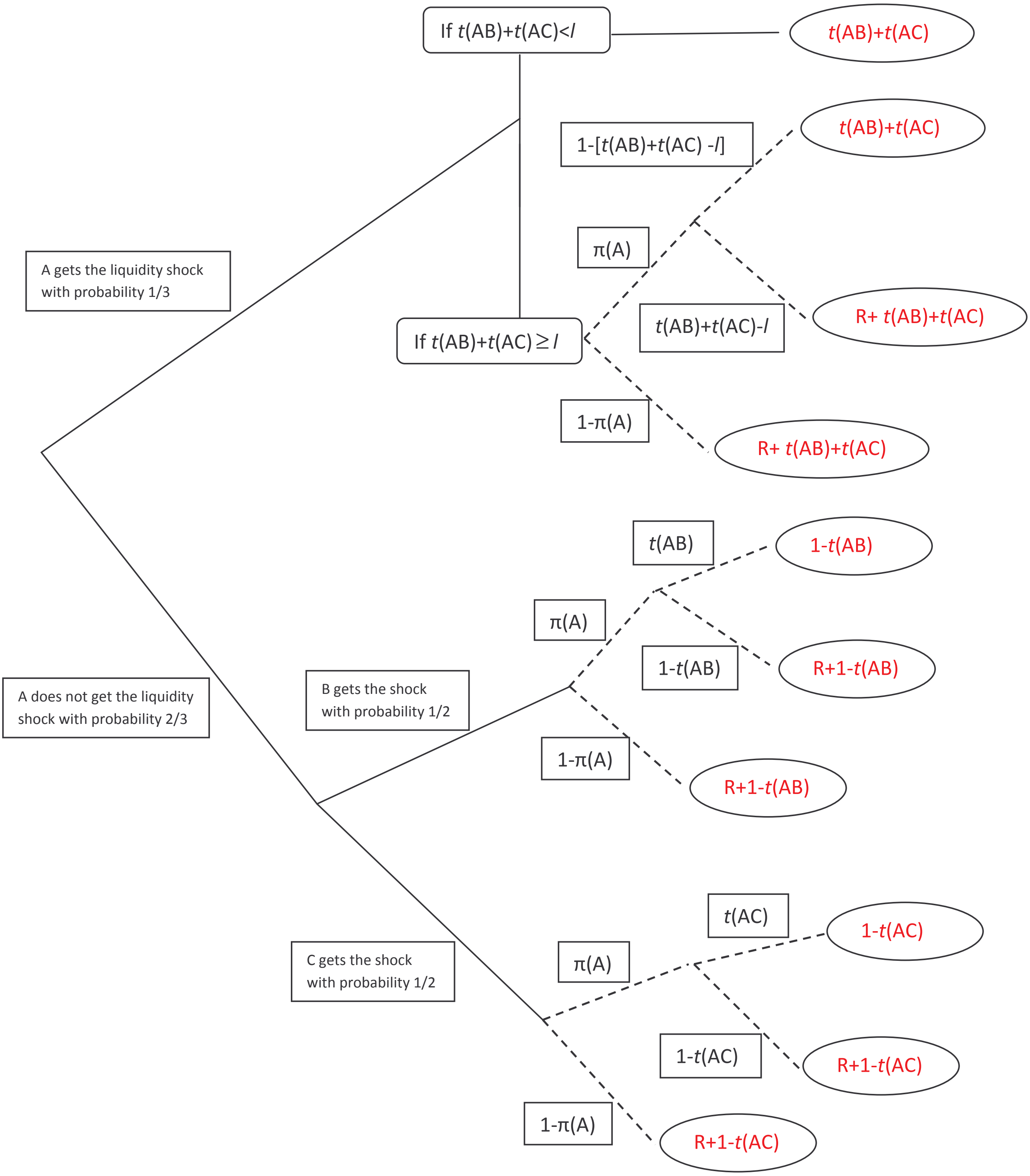
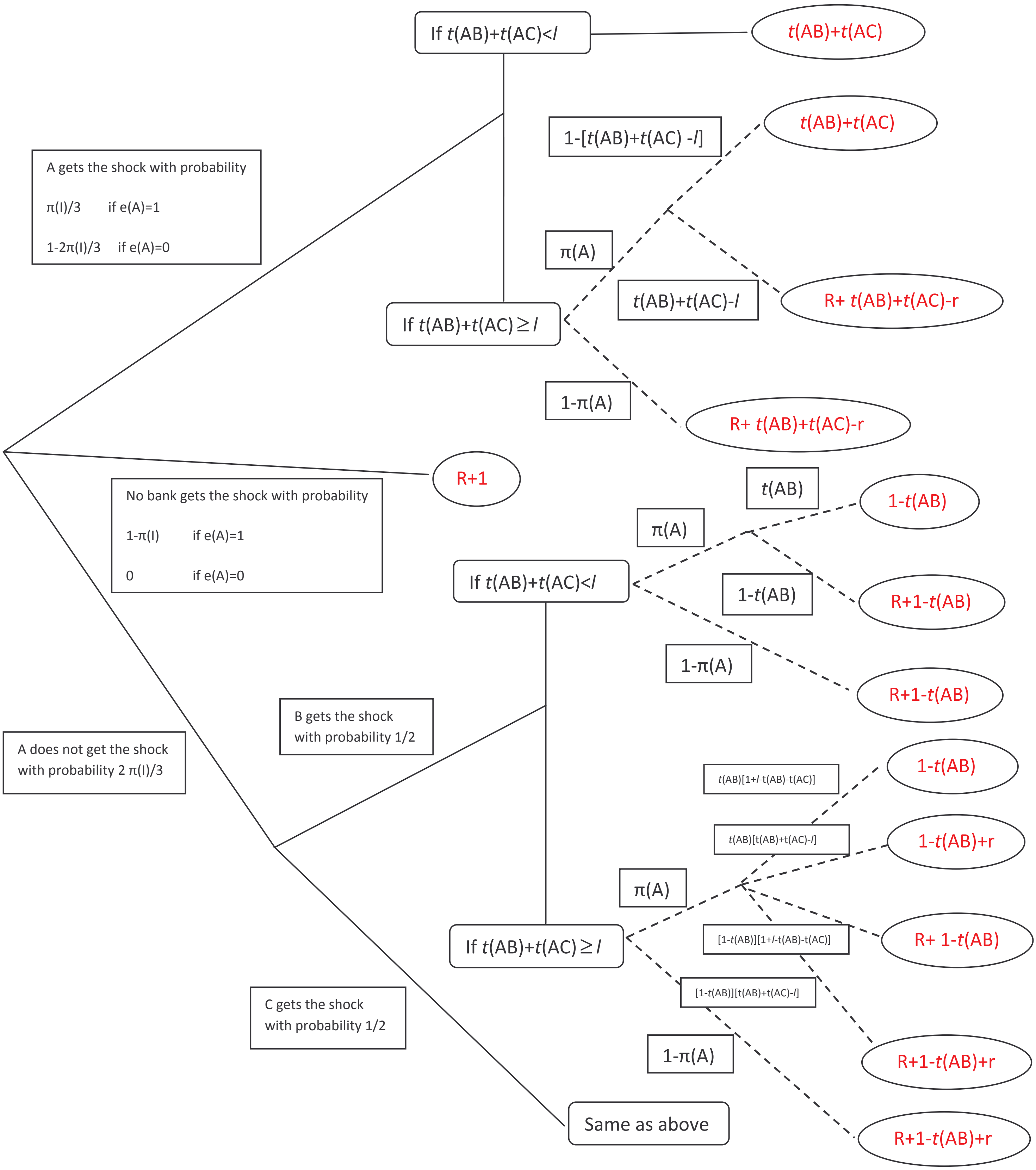


Figure 2. Bank A payoff when insurance is provided with pure transfers.



Bank A payoffs for given transfers t(AB), t(BC), t(AC) and liquidity shock l. Legend: time 1 = ———— ; time 2 = - - - - - ; probabilities = ; payoffs = .

Figure 3. Bank A payoff when insurance is provided with repayment.



Payoffs of bank A for given transfers $t(AB)$, $t(BC)$, $t(AC)$ liquidity shock I and repayment r. Legend: time 1 = ——— ; time 2 = - - - - - ; probabilities = ☐ ; payoffs = ⬭ .