Mortgage Innovation and the Foreclosure Boom

Dean Corbae
University of Texas at Austin

Erwan Quintin *
Federal Reserve Bank of Dallas

December 19, 2008

Abstract

We present a simple quantitative model where agents with different characteristics select from a set of possible mortgage contracts and choose whether to subsequently default on their payments given realizations of idiosyncratic income and housing price shocks. Two broad types of mortgage contracts are considered: fixed rate mortgages (FRM) which require a downpayment and mortgages which have variable payments that tend to be backloaded. One such example is an interest only mortgage (IOM) which requires no downpayment and payments of interest only for a given number of model periods. We assume a competitive mortgage market where each contract must earn zero expected profits given the characteristics of households that select into it and the possibility of default. We first calibrate a benchmark model with FRMs only to data prior to 2000. In the benchmark equilibrium, people with low income and low assets cannot afford to buy houses, and interest rates on mortgages are decreasing in initial income and assets. We then introduce IOM mortgages which attract equilibrium selection by the low income, low asset, high-default risk part of the economy. The introduction of IOMs causes foreclosure rates to rise markedly because they induce participation by the "subprime" segment of the market, and because households who choose IOMs build little home equity early in the contract.

Very preliminary, comments welcome.

*E-mail: corbae@eco.utexas.edu, erwan.quintin@dal.frb.org. We especially wish to thank Daphne Chen who has provided outstanding research assistance. The views expressed in this paper are not necessarily those of the Federal Reserve Bank of Dallas or the Federal Reserve System.

1 Introduction

Since 2003 there has been a significant rise in nontraditional (non-FRM) mortgages as well as a significant rise in residential loan delinquency rates in the United States (see figure 1.) Many of the innovations in mortgages were designed to substantially reduce the initial obligations of borrowers. These innovations made it possible for more households to obtain the financing necessary to purchase a house and in other papers (e.g. Chambers, et. al. (2007)) have been associated with the rise in homeownership.

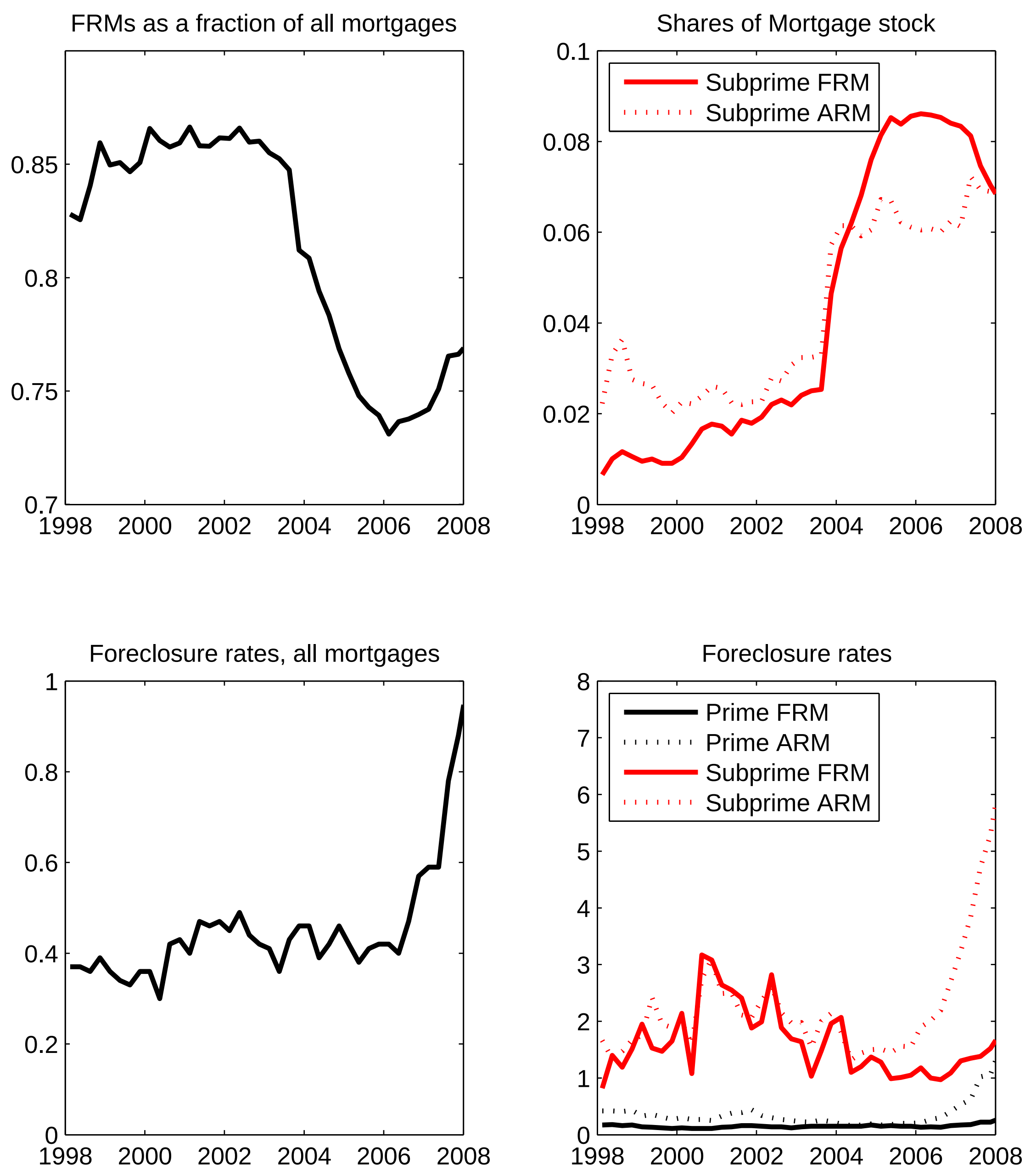
Our objective is to quantify the importance of mortgage innovation for the recent flare-up in foreclosure rates. Specifically, we answer the following questions. First, how much of the rise in foreclosures can be attributable solely to innovation in mortgage contracts? How much does mortgage innovation magnify the effect of downturns in house values on default rates? What is the welfare gain associated with mortgage innovation? To answer these questions, we first describe a model that mimics salient features of the US housing market up to the turn of the century. We consider an economy where households value both consumption and housing services and move stochastically through several stages of life. For simplicity, agents who are young are constrained to obtain housing services from the rental market and split their remaining income between consumption and the accumulation of liquid assets. Given the idiosyncratic income shocks, despite the fact that households begin life ex-ante identical in our model, there is an endogenous distribution of assets among the set of people who turn middle aged.

When agents become middle-aged, they are given the option to purchase a fixed quantity of housing capital (*a house*). We assume they must finance the house purchase via a mortgage drawn from a set of contracts with properties like those available in the United States. Until the contract matures, agents can choose to terminate the contract in any given period. If termination occurs, the house is sold and agents receive any proceeds in excess of the outstanding loan principal.[1] Two events trigger the choice to terminate: an adverse income shock which makes it impossible for agents to meet their current payment or a shock to the value of a house which makes the agent's home net equity negative.

Early terminations are costly for financial intermediaries – which issue all mortgages – because we assume that foreclosure carries transaction costs and because terminations occur in many cases where the value of the house falls below the remaining principal. As a result, intermediaries demand higher yields from agents whose asset and income position make default more likely. In fact, intermediaries do not issue loans to some agents because their default risk is too high or because the agents are too poor to make a downpayment. In particular, our model is consistent with the fact that agents at lower asset and income positions are less likely to become home-owners, face more expensive borrowing terms, and are more likely to default on their loan obligations. In particular, we allow contracts to depend on both the

[1] Here we are assuming the default law is consistent with antideficiency (as in California for example) where the defaulting household is not responsible for the deficit between the proceeds from the sale of the property and the outstanding loan balance.

Figure 1: Recent trends in US housing



Sources: Haver analytics, National Delinquency Survey (Mortgage Bankers Association), and Statistical Abstract of the United States.

household's income and asset position at the time they take out the mortgage and price these contracts under the assumption of competitive markets.

Since high early payments are prohibitively costly for asset and income poor agents, there is a natural role to play in our economy for mortgage innovation in the form of contracts that do not front-load payments. We find that in an economy calibrated to match key aspects of the US housing market prior to 2003 where FRMs are the predominant form of contracts, the introduction of a mortgage that approximates the features of interest-only mortgages (IOM) has drastic effects on both home-ownership rates and default rates. In particular, asset and income poor households (those who could be interpreted as subprime) endogenously select into IOMs. Again, since we allow contracts to depend on the household's income and asset position at the time they take out the mortgage, the selection issue means that these contracts bear higher interest rates than FRM contracts. Since IOM mortgages back-load payments this implies that the future IOM payments over a shorter horizon will be higher than the payments of "longer" FRM mortgages thereby making it more likely they will be subject to both involuntary default (arising from an empty budget set) and voluntary default (arising from the decision to walk away from a mortgage on a house with negative equity).

These findings have a number of implications for how one should interpret current events. Mortgage innovation serves an important purpose and, in a model that abstracts from the possibility of spillovers from housing finance to the financial system at large, can raise welfare by expanding the range of choices for a number of households, particularly agents at the bottom of the asset and income distributions. The nature of these innovations, however, does make an increase in default rates unavoidable since agents are much slower to accumulate home equity.

We provide an experiment to show that a correlated rise in the probability of house value shocks causes a large increase in default rates in an economy with mortgages that back-load payments. The resemblance with current events is not purely fortuitous, in our view, and makes models such as ours useful tools for designing safeguards against the collapse of financial systems in modern economies.

Our paper is closely related to several studies of the recent evolution of the US housing market and mortgage choice.[2] First is the paper by Chambers et al. (2007) which, in a model without the possibility of default, argues that the development of mortgages with gradually increasing payments has had a positive impact on participation in the housing market. Garriga et al. (2008) quantify the impact of aggregate house price shocks on default rates where there is cross-subsidization of mortgages **within** but not **across** mortgage types

[2] There are other papers which are a bit less closely related. Campbell and Cocco (2003) study the microeconomic determinants of mortgage choice but do so in a model where all agents are home-owners by assumption, and focus their attention on the choice between adjustable rate mortgages and standard FRMs with no option for default. Rios-Rull and Sanchez-Marcos (2008) develop a model of housing choice where agents can choose to move to bigger houses over time, a feature from which we abstract to keep computations manageable. A different strand of the housing literature (see e.g. Gervais, 2002, and Jeske and Krueger, 2005) studies the macroeconomic effects of various institutional features of the mortgage industry, again where there is no possibility of default.

(e.g. FRM or IOM). A key difference between our paper and theirs is that we consider a menu of different terms on contracts **both** within **and** across mortgage types.[3] This enables us to build a model that is consistent with the heterogeneity of foreclosure rates and mortgage terms across wealth and income categories, which we view as basic ingredients of a satisfactory model of the US housing market. Along this dimension our paper is more closely related to Guler (2008) where intermediaries offer a menu of FRMs at different possible downpayment rates. He then studies the impact of an innovation to the screening technology on default rates.

2 The environment

We study an economic environment where time is discrete and infinite. Each period a mass one of agents is born. Over time, agents move stochastically through four stages of life: young (Y), middle-aged (M), old (O) and dead. All agents are born young. At the beginning of each period, young agents become middle-aged with probability ρ^M, middle-age households become old with probability ρ^O, while old agents die with probability ρ^D. We assume that population size is at its unique invariant value, and that the fraction of agents of each type obeys a law of large numbers.

Each period, as long as they are young or middle-aged, agents earn positive income denominated in terms of the unique consumption good. All agents begin life at the lower bound of income support $\{y_L, y_M, y_H\}$ where $0 < y_L < y_M < y_H$. Income then evolves stochastically according to a transition matrix π which we assume ergodic. Agents begin life at a value $y \in \{y_L, y_M, y_H\}$ drawn from the unique invariant distribution associated with π. When old, agents earn a fixed, certain amount of y^O of income.

Until they become old, agents can save in one-period bonds that earn rate $1 + r_t \geq 0$ at date t with certainty. When old, the agents can buy annuities that pay rate $\frac{1+r_t}{1-\rho^D}$ in the following period provided they are alive, and pay nothing otherwise.

Agents value both consumption and housing services. They order non-negative processes $\{c_t, s_t\}_{t=0}^{+\infty}$ according to:

$$E_0 \sum_{t=0}^{+\infty} \beta^t U(c_t, s_t)$$

where U satisfies standard assumptions. They can rent quantity $\frac{\bar{h}}{\eta} > 0$ of housing services at price R_t at date t. We assume that $\eta \geq 1$. In the period when agents move from youth to middle-age, agents can choose instead to purchase quantity $\bar{h}$ of housing capital for price q_t, an asset which we refer to as a house that delivers quantity $\bar{h}\theta$ of housing every period with $\theta \geq 1$.

[3] Effectively, Garriga et al. (2008) apply the equilibrium concept in Athreya (2002) while we apply the equilibrium concept in Chatterjee et al. (2007).

Every period, a fraction $\lambda > 0$ of house-owners see the quantity of capital they own fall permanently to $\frac{\bar{h}}{\eta} > 0$. Like apartments, these houses generate quantity $\frac{\bar{h}}{\eta} > 0$ of housing services. We will interpret this shock as an idiosyncratic house price shock. Assuming that following this shock the house size becomes that of rental units and yields the same quantity of services as rental units simplifies the analysis by keeping the size of the housing choice set small, namely $\{\frac{\bar{h}}{\eta}, \bar{h}\}$. Since depreciated houses provide no advantage over rental units, no agent who becomes middle-aged would strictly prefer to purchase a depreciated house, and all agents who experience a house shock are at least as well off selling their house and becoming renters as they would be if they keep their house.[4]

Owners of a house of size $h \in \{\frac{\bar{h}}{\eta}, \bar{h}\}$ bear maintenance costs δh in all periods where $\delta > 0$. Maintenance costs are denominated in terms of the consumption good. We assume that once agents sell or foreclose their house, they are constrained to relying on the rental market for the remainder of their life. We also assume that in the period in which agents become old, they must sell their house immediately and become renters for the remainder of their life.

A financial intermediary holds household savings. The intermediary can store savings at exogenously given return $1 + r_t$ at date t. It can also transform each unit of consumption good saved into quantity $A > 0$ of housing capital, and each unit of housing capital back into quantity $\frac{1}{A}$ of the consumption good. Housing capital can be rented at rate R_t at date t. The intermediary incurs maintenance cost δ on each unit of housing capital rented, measured in terms of the consumption good. At date t, each unit of consumption good rented thus earns net return $A(R_t - \delta)$. The intermediary can also sell housing capital as houses to eligible households.

We assume that households that can purchase a house at a given date are constrained to finance this purchase with one of two possible types of mortgage contracts. The first contract (which we design to mimic the basic features of standard fixed-rate mortgage, or FRM) requires a downpayment of size $(1-\alpha)\bar{h}q$ where $\alpha \in (0, 1)$ and stipulates a yield $r^{FRM}(a_0, y_0)$ that depends on the household wealth and income characteristics (a_0, y_0) at the time of origination of the loan. Given the yield $r^{FRM}(a_0, y_0)$ the constant payments $m^{FRM}(a_0, y_0)$ and principal balance schedule $\{b_\tau^{FRM}(a_0, y_0)\}_{\tau=0}^{T}$ can be computed using standard calculations, where T is the maturity of the loan.

Specifically, suppressing the initial characteristics for notational simplicity,

$$m^{FRM} = \frac{r^{FRM}}{1 - (1 + r^{FRM})^{-T}}(1 - \alpha)\bar{h}q$$

and, for all $\tau \in \{0, T - 1\}$,

$$b_{\tau+1}^{FRM} = b_\tau^{FRM}(1 + r^{FRM}) - m^{FRM},$$

where $b_0^{FRM} = (1 - \alpha)\bar{h}q$ so that $b_T^{FRM} = 0$.

[4] Arbitrage implies that the present value of renting housing services each period is the same as purchasing a depreciated house. Selling the depreciated house, however, can relax an agent's liquidity constraint.

The second contract (interest-only mortgage, or IOM) stipulates an interest rate $r^{IOM}(a_0, y_0)$, no down-payment, constant payments $m^{IOM}(a_0, y_0) = \bar{h}qr^{IOM}(a_0, y_0)$ that do not reduce the principal for the first $k < T$ periods, and fixed-payments for the following $T - k$ periods with a standard FRM-like balance schedule $\{b_\tau^{IOM}(a_0, y_0)\}_{\tau=k}^{T}$. Notice that households accumulate no house equity for k periods under the second contract.

In other words,

$$m_\tau^{IOM} = \begin{cases} \bar{h}qr^{IOM} & if \quad \tau < k \\ \frac{r^{IOM}}{1-(1+r^{IOM})^{-(T-k)}}\bar{h}q & if \quad \tau \geq k \end{cases}$$

and, for all $\tau \in \{0, T - 1\}$,

$$b_{\tau+1}^{IOM} = b_\tau^{IOM}(1 + r^{IOM}) - m_\tau^{IOM},$$

where $b_0^{IOM} = \bar{h}q$. Notice that for $\tau < k$, $b_{\tau+1}^{IOM} = b_0^{IOM}$ so that the principal is not paid down for k periods and hence the mortgage payments are backloaded and $b_T^{IOM} = 0$.

Mortgages are issued by the financial intermediary. The intermediary incurs service costs which we model as a premium $\phi > 0$ on the opportunity cost of funds loaned to the agent for housing purposes.

The agent can terminate the contract at the beginning of any period, an event which can occur for several possible reasons. First, because of the possibility of house value shocks, terminations occur when the outstanding principal exceeds the house value. We will think of that event as a foreclosure. In that event, the intermediary loses fraction $\kappa > 0$ of the principal payment it collects. Termination also occurs when the agent's income falls short of the stipulated payment for the period. In that case, the house is sold immediately, the agent's house equity, if any, augments his liquid asset position, and the agent is constrained to become a renter. Agent may also choose to sell their house even when they can meet the payment and have positive equity, for instance because they are borrowing constrained in the current period. Finally, termination occurs by assumption when agents become old.

The timing in each period is as follows (see appendix for flow chart.) At the beginning of the period, agents discover whether or not they have aged, and receive a perfectly informative signal about their income draw for the period. Middle-aged agents who own homes also observe the realization of their devaluation shock at the beginning of the period, hence the market value of their home. These agents then decide whether to remain home-owners, or, instead, to become renters in which case their house is sold. Agents who just became middle-aged also make their home-buying and mortgage choice decisions at the beginning of the period, after all uncertainty for the period is resolved. At the end of the period, agents receive their income, mortgage payments are made, and consumption takes place.

3 Equilibrium

We will only study equilibria in which all prices are constant.For simplicity, we now drop all time subscripts.

3.1 Agent's problem

We will state the household problem recursively. In general, the household value functions will be written as $V_{age}(\omega)$ where $\omega \in \Omega_{age}$ is the state facing an agent of $age \in \{Y, M, O\}$.

3.1.1 Old agents

For old agents, the state space is $\Omega_O = I\!R^+$ with typical element $\omega \equiv a \geq 0$. The value function (that is, the expected present value of future utility) for an old agent with assets $a \in I\!R^+$ solves

$$V_O(a) = \max_{a' \geq 0} \left\{ U\left(c, \frac{\bar{h}}{\eta}\right) + \beta(1-\rho^D)V_O(a') \right\}$$

s.t.

$$c = a\frac{(1+r)}{1-\rho^D} + y^O - \frac{\bar{h}}{\eta}R - a' \geq 0$$

3.1.2 Mid-aged agents

For mid-aged agents, the state space is $\Omega_M = I\!R^+ \times \{y_L, y_M, y_H\} \times \{0,1\} \times I\!N \times \{0,1\} \times \{FRM, IOM\} \times I\!R^+$ with typical element $\omega = (a, y, H, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$. Here, $H = 1$ denotes whether the agent begins the period as a house owner, while $H = 0$ if they are renters. We write $\tau \in \{0, 1, \ldots\}$ for the number of periods the agent has been mid-aged, hence the age of their mortgage when they have one, and $\sigma = 1$ if the agent's house has devalued. The last two arguments, $(\zeta, r^\zeta(a_0, y_0)) \in \{FRM, IOM\} \times I\!R^+$, are the agent's mortgage type and yield. The long notation for r is meant to emphasize that the mortgage terms are a function of the agent's wealth-income position when they become mid-aged. In order to define the agent's' value function $V_M : \Omega_M \mapsto I\!R$, , it is useful to consider different cases.

Case 1: $\tau \geq 1$

- If the agent enters the period as a renter (i.e. $H = 0$):

$$\begin{aligned} V_M(a, y, 0, \cdot) &= \max_{c,a'} U\left(c, \frac{\bar{h}}{\eta}\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a', y', 0, \cdot) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y + a(1+r) - R\frac{\bar{h}}{\eta} \end{aligned}$$

- If the agent is a home-owner (i.e. $H = 1$) but it is not budget feasible for her to make her mortgage payment $m(\tau;\zeta,r^{\zeta})$, or:

$$y + a(1+r) - m(\tau;\zeta,r^{\zeta}) - \delta h(\sigma) < 0 \tag{3.1}$$

where $h(\sigma) \equiv (1-\sigma)\overline{h} + \sigma\frac{\bar{h}}{\eta}$,then the value function solves

$$\begin{aligned} V_M(a,y,1,\tau,\sigma;\zeta,r^{\zeta}(a_0,y_0)) &= \max_{c,a'} U\left(c,\frac{\bar{h}}{\eta}\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a',y',0,\cdot) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y + a(1+r) + \max\left\{(1-\kappa)qh(\sigma) - b(\tau;\zeta,r^{\zeta}),0\right\} - R\frac{\bar{h}}{\eta} \end{aligned}$$

- If it is budget feasible for a homeowner (i.e. $H = 1$) to make her mortgage payment (that is, (3.1) does not hold) but it has negative equity in his house, that is:

$$qh(\sigma) - b(\tau;\zeta,r^{\zeta}) < 0, \tag{3.2}$$

then, if the household chooses to rent in the next period (so that $H' = 0$), define the value function by

$$\begin{aligned} V_M^{H'=0}(a,y,1,\tau,\sigma;\zeta,r^{\zeta}(a_0,y_0)) &= \max_{c,a'} U\left(c,\frac{\bar{h}}{\eta}\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a',y',0,\cdot) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y + a(1+r) - R\frac{\bar{h}}{\eta}. \end{aligned}$$

- If it is budget feasible for a homeowner (i.e. $H = 1$) to make her mortgage payment and she has nonnegative equity in her house (that is, (3.2) does not hold), then if the household chooses to sell her house ($H' = 0$), define the value function by

$$\begin{aligned} V_M^{H'=0}(a,y,1,\tau,\sigma;\zeta,r^{\zeta}(a_0,y_0)) &= \max_{c,a'} U\left(c,\frac{\bar{h}}{\eta}\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a',y',0,\cdot) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y + a(1+r) + qh(\sigma) - b(\tau;\zeta,r^{\zeta}) - R\frac{\bar{h}}{\eta}. \end{aligned}$$

- If the agent is in the same state but chooses to keep her house ($H' = 1$), define the value function by

$$\begin{aligned} V_M^{H'=1}(a,y,1,\tau,\sigma;\zeta,r^{\zeta}(a_0,y_0)) &= \max_{c,a'} U\left(c, h(\sigma)\left[\sigma + (1-\sigma)\theta\right]\right) \\ &+ \beta E_{(y',\sigma')|(y,\sigma)}\left[\begin{array}{c} (1-\rho_O)V_M(a',y',1,\tau+1,\sigma';\cdot) \\ +\rho_O V_O(a' + \max\left\{qh(\sigma) - b(\tau+1;\zeta,r^{\zeta}),0\right\}) \end{array}\right] \\ \text{s.t. } c + a' &= y + a(1+r) - m(\tau;\zeta,r^{\zeta}) - \delta h(\sigma). \end{aligned}$$

where $m(\tau;\zeta,r^{\zeta}) = 0$ for $\tau \geq T$. Notice that we are making the assumption that the age shock occurs before the price shock since we use $h(\sigma)$ instead of $h(\sigma')$.

We let an indicator function for involuntary default be denoted $D^I(\omega) = 1$ in the event $H = 1$ and (3.1) where $\omega = (a, y, 1, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$, zero otherwise. Similarly, we let an indicator function for voluntary default be denoted $D^V(\omega) = 1$ in the event that $H = 1$, (3.1)) does not hold but (3.2) holds, and

$$V_M^{H'=0}(\omega) > V_M^{H'=1}(\omega)$$

where $\omega = (a, y, 1, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$,zero otherwise. Finally, we let an indicator function for a house sale be denoted $S(a, y, 1, \tau, \sigma; \zeta, r^\zeta(a_0, y_0)) = 1$ in the event that $H = 1$, (3.1) and (3.2)do not hold, but:

$$V_M^{H'=0}(\omega) > V_M^{H'=1}(\omega)$$

where $\omega = (a, y, 1, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$, zero otherwise. With these definitions:

$$V_M(\omega) = (1 - S(\omega)) \cdot V_M^{H'=1}(\omega)) + S(\omega) \cdot V_M^{H'=0}(\omega)$$

where $\omega = (a, y, 1, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$.

Case 2: $\tau = 0$ (The agent just became mid-aged)

- In the agent continues to rent, her value function is:

$$\begin{aligned} V_M^{H'=0}(a_0, y_0, 0, 0, \cdot) &= \max_{c,a'} U\left(c, \frac{\bar{h}}{\eta}\right) + \beta E_{y'|y_0}\left[(1-\rho_O)V_M(a', y', 0, 1, \cdot) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y_0 + a_0(1+r) - R\frac{\bar{h}}{\eta} \end{aligned}$$

- If a mortgage contract is budget feasible (i.e. $y_0 + \left(a_0 - \alpha q\overline{h} \cdot 1_{\{\zeta=FRM\}}\right)(1+r) - m(0; \zeta, r^\zeta) - \delta\overline{h} \geq 0$ for $\zeta \in \{FRM, IOM\}$ **and** $a_0 - \alpha q\overline{h} \cdot 1_{\{\zeta=FRM\}} \geq 0$), then the value function of an agent choosing contract ζ is given by:

$$\begin{aligned} V_M^\zeta(a_0, y_0, 0, 0, 0; \zeta, r^\zeta(a_0, y_0)) &= \max_{c\geq 0, a'\geq 0} U\left(c, \overline{h}\theta\right) \\ &+ \beta E_{(y',\sigma')|(y,0)}\left[\begin{array}{c} (1-\rho_O)V_M^\zeta(a', y', 1, \tau+1, \sigma'; \zeta, r^\zeta(a_0, y_0)) \\ +\rho_O V_O(a' + \max\left\{q\overline{h} - b(1; \zeta, r^\zeta(a_0, y_0)), 0\right\}) \end{array}\right] \\ \text{s.t. } c + a' &= y_0 + \left(a_0 - \alpha q\overline{h} \cdot 1_{\{\zeta=FRM\}}\right)(1+r) - m(0; \zeta, r^\zeta) - \delta\overline{h} \\ \text{and } a_0 &- \alpha q\overline{h} \cdot 1_{\{\zeta=FRM\}} \geq 0. \end{aligned}$$

A newly mid-aged agent will continues to rent (that is, $H'(a, y, 0, 0, 0; \zeta, r^\zeta(a_0, y_0)) = 0$) if:

1. there exists no contract $(\zeta, r^\zeta(a_0, y_0))$ that is budget feasible;

2. for any budget feasible contract $(\zeta, r^\zeta(a_0, y_0))$, we have

$$V_M^{H'=0}(a_0, y_0, 0, 0, \cdot) > V_M^\zeta(a, y, 0, 0, 0; \zeta, r^\zeta(a_0, y_0))$$

for any $(\zeta, r^\zeta(a_0, y_0))$ such that $y_0 + \left(a_0 - \alpha q\overline{h} \cdot 1_{\{\zeta=FRM\}}\right)(1+r) - m(0; \zeta, r^\zeta) - \delta\overline{h} \geq 0$.

The agent chooses a feasible contract $(\zeta, r^\zeta(a_0, y_0))$, that is $\Xi(a, y, 0, 0, 0; \zeta, r^\zeta(a_0, y_0)) = \zeta$ and $H'(a, y, 0, 0, 0; \zeta, r^\zeta(a_0, y_0)) = 1$, if

$$V_M^\zeta(a, y, 0, 0, 0; \zeta, r^\zeta(a_0, y_0)) \geq \max\{V_M^{H'=0}(a_0, y_0, 0, 0, \cdot), V_M^\zeta(a, y, 0, 0, 0; \backslash\zeta, r^{\backslash\zeta}(a_0, y_0))\}$$

where $\backslash\zeta$ is defined to be the other type of mortgage. With these definitions:

$$V_M(\omega) = (1 - H'(\omega))V_M^{H'=0}(\omega) + H'(\omega)\left[1_{\{\Xi(\omega)=FRM\}}V_M^{FRM}(\omega) + 1_{\{\Xi(\omega)=IOM\}}V_M^{IOM}(\omega)\right].$$

3.1.3 Young agents

For young agents, the state space is $\Omega_Y = I\!R^+ \times \{y_L, y_M, y_H\}$ with typical element $\omega = (a, y)$. The value function $V_Y : \Omega_Y\Omega_Y \mapsto I\!R$ for a young agent with assets a and income y solves

$$\begin{aligned} V_Y(a, y) &= \max_{a'\geq 0}\left\{U\left(c, \frac{\bar{h}}{\eta}\right) + \beta E_{y'|y}\left[(1-\rho_M)V_Y(a', y') + \rho_M V_M(a', y', 0, 0, \cdot)\right]\right\} \\ \text{s.t. } c + a' &= y + a(1+r) - R\frac{\bar{h}}{\eta}. \end{aligned}$$

3.2 Intermediary's problem

All possible uses of loanable funds must earn the same return for the intermediary. In other words, the following arbitrage conditions must hold at all dates:

$$r = A(R - \delta) = Aq(1+r) - 1.$$

This pins down all prices as a function of returns to storage. Note that the value of the house is multiplied by $(1+r)$ because, by convention, the purchase of the house takes place at the beginning of period t.

The arbitrage condition implies as usual that the market value of each unit of houses is the present discounted value of all future rents that unit would earn on the spot market. Indeed:

$$\begin{aligned} \sum_{t=1}^{+\infty}\frac{R-\delta}{(1+r)^t} &= \sum_{t=1}^{+\infty}\frac{r}{A(1+r)^t} \\ &= \frac{r}{A(1+r}\sum_{t=0}^{+\infty}\frac{1}{(1+r)^t} \\ &= \frac{1}{A} \\ &= q. \end{aligned}$$

Arbitrage also requires that for all mortgage issued at a given date, the expected return on the mortgage net of expected foreclosure costs cover the opportunity cost of funds, which by assumption is the returns to storage plus the servicing premium ϕ.

To make this precise, denote the value to the intermediary of a mortgage contract held by a mid-aged agent in state $\omega \in \Omega_M$ by $W(\omega)$. Again, we need to consider several cases.

- If the agent a homeowner whose mortgage is not paid off, so that $\omega = (a, y, 1, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$ with $\tau \in (0, T-1]$, then:

$$\begin{aligned} W(\omega) &= \left(D^I(\omega) + D^V(\omega)\right) \max\{(1-\kappa)qh(\sigma), b(\tau; \zeta, r^\zeta)\} + S(\omega)b(\tau; \zeta, r^\zeta) \\ &\quad + \left(1 - D^I(\omega) - D^V(\omega) - S(\omega)\right) \left(\frac{m(\tau; \zeta, r^\zeta)}{1+r+\phi} + E_{\omega'|\omega} \left[\frac{W(a', y', 1, \tau+1, \sigma'; \cdot)}{1+r+\phi} \right] \right) \end{aligned}$$

- If the household just became mid-aged and her budget set is not empty so that $\omega = (a_0, y_0, 0, 0, 0; \zeta, r^\zeta(a_0, y_0))$ and $y_0 + a_0(1+r) - m(0; \zeta, r^\zeta) - \delta\overline{h} \geq 0$

$$W(\omega) = \frac{m(0; \zeta, r^\zeta)}{1+r+\phi} + E_{\omega'|\omega} \left[\frac{W(\omega')}{1+r+\phi} \right]$$

- In all other cases, $W(\omega) = 0$.[5]

Then, the expected present discounted value of a loan contract $(\zeta, r^\zeta(a_0, y_0))$ offered to a household that just turned mid-age with characteristics (a_0, y_0) is denoted $W(\omega_0)$ with $\omega_0 = (a_0, y_0, 0, 0, 0; \zeta, r^\zeta(a_0, y_0))$. The zero profit condition on a loan contract $(\zeta, r^\zeta(a_0, y_0))$ $W(\omega_0)$ with $\omega_0 = (a_0, y_0, 0, 0, 0; \zeta, r^\zeta(a_0, y_0))$. can now be written as

$$W(a_0, y_0, 0, 0, 0; \zeta, r^\zeta(a_0, y_0)) - (1 - \alpha^\zeta)q\overline{h} = 0. \tag{3.3}$$

3.3 Distribution of agent states

The agent's problem yields optimal policies functions for a given set of prices. In turn, these policy functions imply in the usual way transition probability functions across possible agent states. We will study equilibria in which the distribution of agent states is invariant under those probability functions. This section makes this notion precise.

[5]Specifically, this is the case when:

1. the agent just turned mid-aged and her budget set is empty so that $\omega = (a_0, y_0, 0, 0, 0; \zeta, r^\zeta(a_0, y_0))$ and $y_0 + a_0(1+r) - m(0; \zeta, r^\zeta) - \delta\overline{h} < 0$
2. the agent is a renter $\omega = (a, y, 0, \tau, \sigma; \zeta, r^\zeta(a_0, y_0))$ with $\tau > 0$,
3. the agent has been mid-aged for more than T periods.

In our environment, the transition matrix across ages is given by:

$$\begin{bmatrix} (1-\rho_M) & \rho_M & 0 \\ 0 & (1-\rho_O) & \rho_O \\ \rho_D & 0 & 1-\rho_D \end{bmatrix}$$

since the old are immediately replaced by newly born young people. Let (n_Y, n_M, n_O) be the corresponding invariant distribution of ages. The invariant mass of agents born each period is then given by

$$\mu_0 \equiv n_O \rho_D.$$

With this notation in hand, we can define invariant distributions over possible states at each demographic stage.

3.3.1 The young

The invariant distribution μ_Y on Ω_Y solves, for all $y \in \{y_L, y_M, y_H\}$ and $A \subset I\!R^+$:

$$\mu_Y(A, y) = \mu_0 1_{\{0 \in A, y = y_L\}} + (1-\rho_M) \int_{\omega \in \Omega_Y} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) d\mu^Y(\omega)$$

where $a'_Y : \Omega_Y \mapsto I\!R^+$ is the optimal saving policy for young agents, and, abusing notation somewhat, $\Pi(y|\omega)$ is the likelihood of income draw $y \in \{y_L, y_M, y_H\}$ in the next period given current state $\omega \in \Omega_Y$.

3.3.2 The mid-aged

For mid-aged, recall that the state space is $\Omega_M = I\!R^+ \times \{y_L, y_M, y_H\} \times \{0,1\} \times I\!N \times \{0,1\} \times \{FRM, IOM\} \times I\!R^+$ with typical element $\omega = (a, y, H, \tau, \sigma; \zeta, r)$. The invariant distribution μ_M on Ω_M solves, for all $y \in \{y_L, y_M, y_H\}$, $A \subset I\!R^+$ and $(H, \tau, \sigma; \zeta, r) \in \{0,1\} \times I\!N \times \{0,1\} \times \{FRM, IOM\} \times I\!R^+$:

$$\begin{aligned} \mu^M((A, y, H, \tau, \sigma; \zeta, r)) &= \rho_M \int_{\Omega_Y} 1_{\{(H,\tau,\sigma)=(0,0,0)\}} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) d\mu_Y(\omega) \\ &+ (1-\rho_0) \int_{\Omega_M} 1_{\{(H'(\omega)=H, \tau(\omega)=\tau-1, a'_M(\omega) \in A\}} \Pi(y|\omega) P(\sigma|\omega) d\mu_M(\omega) \\ &\times \left\{ 1_{\{\tau(\omega)=0, \Xi(\omega)=\zeta, r^\zeta(a(\omega), y(\omega))=r\}} + 1_{\{\tau(\omega)>0, \zeta=\zeta(\omega), r=r(\omega)\}} \right\} \end{aligned}$$

where $a'_M : \Omega_M \mapsto I\!R^+$ is the optimal saving policy for mid-aged agents, $\tau(\omega)$ extracts the contract age argument of ω, $\zeta(\omega)$ extracts the contract type argument of ω, $r^\zeta(a, y)$ is the intermediary's pricing policies for mortgages of type $\zeta \in \{FRM, IOM\}$ for agents whose initial wealth-income position is (a, y), and $r(\omega)$ is the contract rate argument of ω.

The first term corresponds to agents who go from being young to being mid-aged, while the second integral corresponds to agents who were mid-aged in the previous period and do not get old. The indicator functions reflect the fact that agents make their mortgage choice in first period but cannot revisit that choice in subsequent periods.

3.3.3 The old

The invariant distribution μ^O on $\Omega_O \equiv I\!R^+$ solves, for all $A \subset I\!R^+$:

$$\mu^0(A) = (1-\rho_D)\int_{\Omega_O} 1_{\{a'_O(\omega)\in A\}} d\mu^O(\omega) + \rho_O \int_{\Omega_M} 1_{\{a'_M(s)+\max\{H'(\omega)[qh(\omega)-b(\tau+1,\zeta,r)],0\}\in A\}} d\mu^M(\omega)$$

where, for $\omega \in \Omega_M$,

$$h(\omega) = \sigma\frac{\bar{h}}{\eta} + (1-\sigma)\bar{h}$$

while $b(\tau+1,\zeta,r)$ is the principal balance on a mortgage of type ζ with yield r after $\tau+1$ periods.

3.4 Steady state equilibrium

Equipped with this notation, we may now define an equilibrium. A steady-state equilibrium is a pair $r^{FRM} : I\!R^+ \times \{y_L, y_M, y_H\} \mapsto I\!R^+$ and $r^{IOM} : I\!R^+ \times \{y_L, y_M, y_H\} \mapsto I\!R^+$ of mortgage pricing functions, agent value functions $V_{age} : \Omega_{age} \mapsto I\!R$ for $age \in \{Y, M, O\}$, saving policy functions $a'_{age} : \Omega_{age} \mapsto I\!R^+$, a mortgage choice policy function $\Xi : \Omega_Y \mapsto \{FRM, IOM\}$, a housing policy function $H' : \Omega_Y \mapsto \{0,1\}$, involuntary and voluntary default policy functions $D^I, D^V : \Omega_Y \mapsto \{0,1\}$, and distributions μ_{age} of agent states on Ω_{age} such that:

1. Agent policies are optimal given pricing functions.

2. The intermediary expects to make zero profit on all mortgages. In other words, condition (3.3) holds for all $(a, y) \in I\!R^+ \times \{y_L, y_M, y_H\}$ and for $\zeta \in \{FRM, IOM\}$.

3. The distribution of states is invariant given pricing functions and agent policies.

The next section simulates this economy under various calibrations. We will be particularly interested in the fraction of agents who choose to terminate their mortgages early. As we have discussed, this may occur for voluntary or involuntary reasons.

4 Parameterization

The benchmark version of the model only makes standard FRM contracts available to agents. We choose parameters so that this benchmark model matches the relevant features of the US housing market before 2000. We will then introduce the option to finance house purchases with an IOM instead of an FRM, and ask whether the model can replicate the pick-up in ownership and foreclosure rates that has taken place in the US since 2000.

We will think of a model period as representing 1 year. Then, we set demographic parameters to $(\rho^M, \rho^0, \rho^D) = (0.06, 0.03, 0.06)$ so that, on average, agents are young for 16 years, middle-aged for 33 years, and retired for 16 years.

The labor income process takes values in $\{(y_L, y_M, y_H)\} = \{(0.2823, 1, 3.5428)$, while $y_O = 0.25$. The income transition probability matrix for young and middle-aged agents, which is defined over $\{y_L, y_M, y_H\} \times \{y_L, y_M, y_H\}$, is given by:

$$\begin{bmatrix} 0.9980 & 0.0020 & 0 \\ 0.0007 & 0.9986 & 0.0007 \\ 0 & 0.0020 & 0.9980 \end{bmatrix}$$

This implies a variance of log income among active agents of 0.4 and an autocorrelation of 0.95 which is consistent with the evidence on US labor earnings discussed for instance by Krueger and Perri (2005).

We let $r = 0.04$ and choose the maintenance cost (δ) to match the estimated gross rate of depreciation of housing capital, which is 2.5% annually according to Haring et al., 2007.

We then normalize the initial house size $\bar{h}$ and fix A (hence q, given r and δ) so that the ratio of house price $\bar{h}q$ to income among home buyers is roughly 2.5, a number consistent with pre-2000 evidence available from the American Housing Survey (AHS).

As for preferences, we initially specify, for all $(c, h) > (0, 0)$,

$$U(c, s) = \log c + \log s,$$

although we will also study the impact of various degrees of substitutability between consumption and housing services. We select θ so that the overall home-ownership rate is $\frac{2}{3}$ among middle-aged agents, which is consistent with the pre-2000 evidence available from the Census Bureau.

We select β so that the average non-housing assets to income ratio is about 2.25. This is the average ratio of non-housing assets to income among households whose head age is between 35 and 70 in the 1998 Survey of Consumer Finance (SCF).

The stipulations of FRM contracts are set to mimic the features of common standard fixed-rate mortgages in the US. The down-payment ratio α is 20% while the maturity T is 15 years. The IOM we introduce in our main experiment has $k = 3$ and $T = 10$ so that agents make no payment for 3 years, and make fixed payments for the remaining 7 contract periods unless the contract is terminated before maturity.

We set κ and η jointly to match estimates of the loss incidence rate for lenders on foreclosed properties and estimates of the price discount on these properties relative to other, similar properties. The loss severity rate is the present value all losses on a given loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are caused both by transaction and time costs associated with the foreclosing process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a data set of a dataset of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. They also estimate that foreclosed properties sell a discount relative to their appraised value that ranges from 10% among properties with appraisal values over $180,000 to 45% among properties with appraisal values near $20,000. Other studies of foreclosure discounts (see Pennington-Cross, 2004, for a review) typically find discount rates near one quarter, with some exceptions. Based on these studies we select κ and η so that 1) houses that sell following a default in our model (whether voluntary or involuntary) sell at 25% discount on average relative to other houses, and 2) in the event of default and on average,

$$\frac{\min\{(1-\kappa)qh, d)}{d} = 0.5$$

where d is the outstanding principal at the time of default and qh is the house value. In other words, on average, the intermediary recovers 50% of the outstanding principal it is owed on defaulted loans.

We assume that agents experience a house value shock with probability $\lambda = 0.03$. This yields a default rate of 1.25% which is near the average foreclosure rate among all mortgages in during the 1990s in the Mortgage Bankers Association's National Delinquency survey.

Finally, we select the mortgage service premium (ϕ) so that the average yield on FRM contracts in the benchmark economy is roughly 7.5%. This was the average contract rate on conventional, fixed rate mortgages between 1990 and 2000 according to Federal Housing Finance Board data.

Table 1 summarizes our benchmark parameterization.

Table 1: **Benchmark parameters**

Parameter	Description	Value	Target	Value
(ρ_M, ρ_0, ρ_D)	Demographics	$(0.06, 0.03, 0.06)$	Average length of each stage	$(16, 33, 16)$
$\{y_L, y_M, y_H\}$	Income process	$\{0.2823, 1, 3.5428\}$	Perri and Krueger, 2005	$\rho(y) = 0.95$, $Var(\log y) = 0.4$
r	Storage returns	0.04	Risk-free rate	
δ	Maintenance rate	2.5%	Residential depreciation (Harding et al., 2007)	
$\bar{h}$	House size	1	Normalization	
A	Housing technology TFP	0.2	Average LTY at loan origination (AHS)	2.5
κ	Foreclosing costs	0.4	Loss-incidence estimates (Hayre and Saraf, 2008)	50%
λ	Housing shock probability	0.03	Foreclosure rates (MBA)	1.25%
η	House value shock	1.67	Foreclosure discount (Hayre and Saraf, 2008)	25%
θ	Owner-occupied premium	2.5	Home-ownership rates (Census Bureau)	67%
β	Discount rate	0.96	Average ex-housing asset-to-income ratio (1998 SCF)	2.25
α	Downpayment on FRMs	0.2	AHS	
ϕ	Mortgage service cost	0.02	Average FRM yields (FHFB)	7.5%

Table 2: Rent-or-own decision, Benchmark economy

	Rent	Own
y_L	$0.0000 \leq a_0 < 1.6565$	$1.6565 \leq a_0$
y_M	$0.0000 \leq a_0 < 1.0000$	$1.0000 \leq a_0$
y_H	$0.0000 \leq a_0 < 1.0000$	$1.0000 \leq a_0$

Table 3: Steady state statistics

	Data	Bench.	IOM (1)	FRM (2)	IOM (2)
Home-ownership rate	66.00	60.12	63.02	55.83	58.48
Default rate	1.2-4.8	1.06	1.18	1.43	1.58
Average LTY	2.5	2.26	2.41	2.15	2.28
Average LTV	0.85	0.80	0.83	0.80	0.83
Average Asset/income	2.23	2.69	2.64	3.03	2.98
Recovery rate	0.50	54.02	51.07	53.27	50.48
Foreclosure discount	75.00	74.51	73.97	76.28	75.95

5 Results

The benchmark economy delivers a number of intuitively appealing predictions. The top panel of figure 2 shows the FRM rates agents at various asset-income positions can obtain from the intermediary when they become middle-aged. The figure shows, first, that agents whose income and assets are low do not get a mortgage in equilibrium. This occurs for several reasons. First, asset and income poor agents cannot meet the down-payment requirement and/or mortgage payments. Second, these agents are more likely to default, hence receive less favorable borrowing terms.

In some cases in fact, there is no yield such that the intermediary would expects to break even on the mortgage, even when the agents have the means to finance the initial down-payment. Among agents who do receive a mortgage offer, yields fall both with assets and income. Given the monotonicity of rates and mortgage availability in asset and income, ownership rates are also monotonic in assets and income, as table 2 shows. Overall (see table 3), home ownership-rates are near 60%.

Once IOMs are introduced (an experiment we call IOM(1)), agents face two potential payment schedules. Since prices do not change by assumption, the FRM payment schedule as a function of agent's initial asset-income position is unchanged. The new menu of IOM yields is shown in the bottom panel of figure 2. Several facts are immediately apparent. First, IOM

Figure 2: Equilibrium yield schedules, IOM(1)

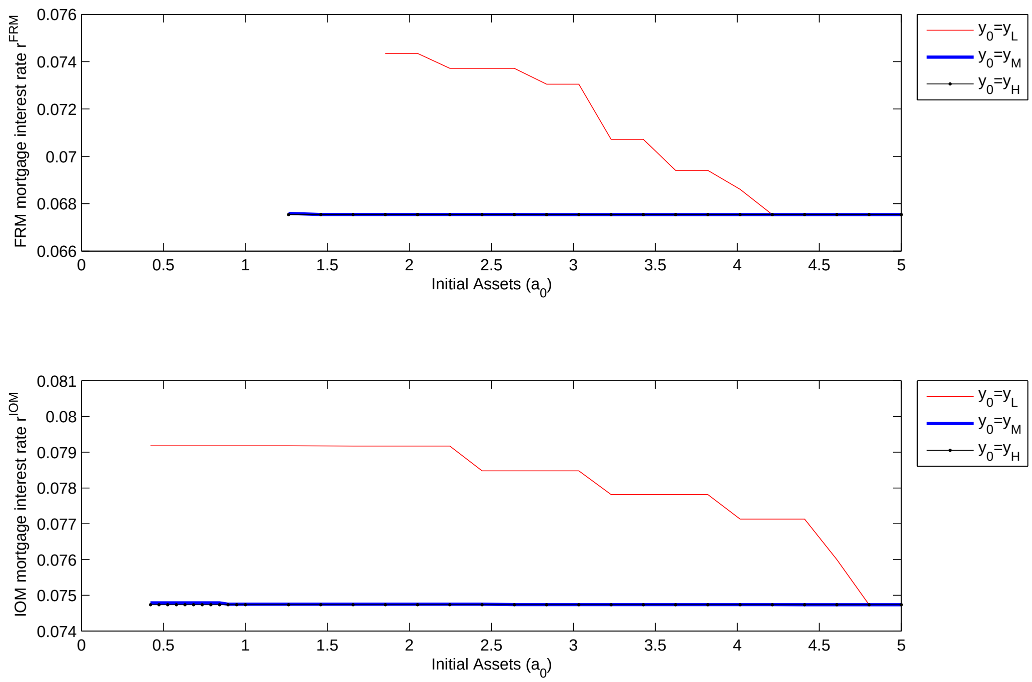


Table 4: Rent-or-own decision, IOM(1)

	Rent	IOM	FRM
y_L	$0.0000 \leq a_0 < 1.2631$	$1.2631 \leq a_0 < 3.0332$	$3.0332 \leq a_0$
y_M	NA	$0.0000 \leq a_0 < 1.0000$	$1.0000 \leq a_0$
y_H	NA	$0.0000 \leq a_0 < 1.0000$	$1.0000 \leq a_0$

rates exceed FRM rates at all possible asset-income positions since they entail a greater risk of default because home equity is much slower to rise. The likelihood that an agent will find herself with negative equity on her home is higher when she holds an IOM than when she holds an FRM.

Despite the default premium, a significant number of agents do opt for IOM when they become available. This is the case for many agents who are not offered an FRM because they cannot meet the down-payment and/or cannot make the first payment. The presence of IOMs, in other words, enables some agents at the bottom of the asset and income distributions to become home-owners instead of renting.

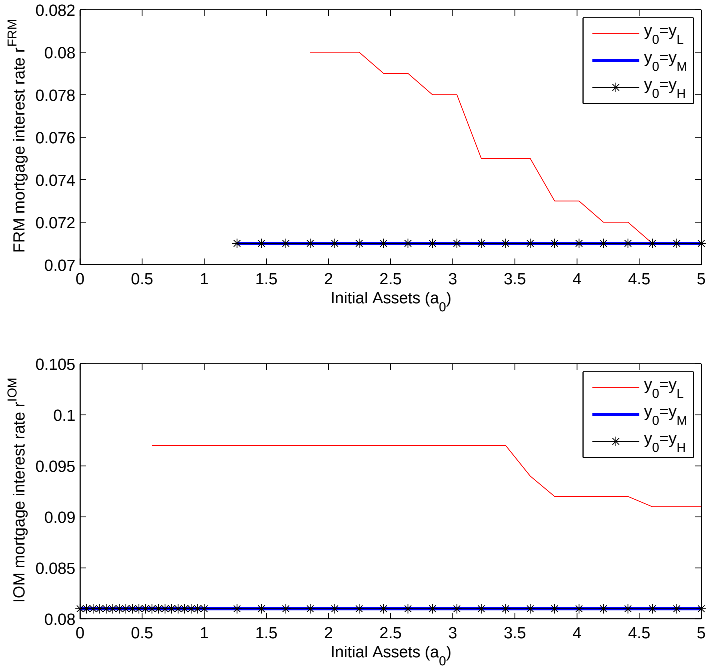
In addition, some agents who were able to obtain a mortgage in the FRM-only economy switch to IOMs to take advantage of the back-loaded payments. This option, once again, is particularly attractive for agents whose current income and assets are low. Asset and/or income rich agents, for their part, tend to opt for lower-yields FRMs since large early payments do not affect their current consumption much. This contract selection pattern is displayed in table 4.

The introduction of IOMs has several consequences on equilibrium statistics (see the second column of table 3.) Home-ownership rates rise as more agents are able to finance the house purchase. Given the back-loaded nature of IOMs, average loan-to-value and loan-to-income ratios rise, as do default rates. These patterns are qualitatively consistent with the recent US evidence displayed in figure 1.

In this experiment, IOMs turn out to be selected by roughly 12% of agents which is a reasonable fraction given the evidence available from the MBA's Mortgage Origination Survey. For these agents, the default risk and the associated premium are too small to offset the front-loaded nature of FRM payments.

Raising the probability of a house devaluation shock (an experiment we call IOM(2)), which causes a drop in average house prices in our model, leads the proportion of IOMs to decline as default premiums become higher. Furthermore, the effect of the devaluation shock is magnified when mortgages with back-loaded payments are available. Figure 3 shows the impact on the yield schedule of a 30% increase in the likelihood of a devaluation shock, while table 5 shows the effect on mortgage selection patterns. Yields rise on both mortgage types at all asset-income positions since the likelihood of default has risen. The associated payments become prohibitive particularly for asset and income poor agents. The asset threshold

Figure 3: Equilibrium yield schedules, IOM(2)



past which low-income agents choose to own rather than rent rises from 0.68 to 0.90. The distribution of agents across asset-income levels is such that this change induces a complete reversal of the mortgage choice distribution. The fraction of agents who opt for IOMs falls from 12 to 11%.

Figure 4 plots the endogenous distribution of assets among agents that just turned middle-aged. In the benchmark experiment, the upper panel shows that, quite intuitively, low-income agents tend to have low assets, and vice-versa. The lower panel shows the change in the distribution when IOMs are introduced, i.e. in experiment IOM(1). Because IOMs do not require a downpayment, the figure makes apparent that agents enter mid-age with less savings.

Figure 5 plots the distribution of contract choice by asset and income level. In the benchmark experiment, the upper panel shows that agents with low income and low assets do not take a mortgage. The lower panel shows that when IOMs are introduced, agents with low income and low assets now select into IOMs.

The impact on steady state statistics is also noticeable (see the last column of table 3.) The home-ownership rate falls down to under 60%. The loan-to-value and loan-to-income ratio also fall as front-loaded mortgages become somewhat more important. The default frequency, however, jumps up by almost 50% despite the lower fraction of IOMs. The direct effect of

Figure 4: Distribution of assets upon entering mid-age

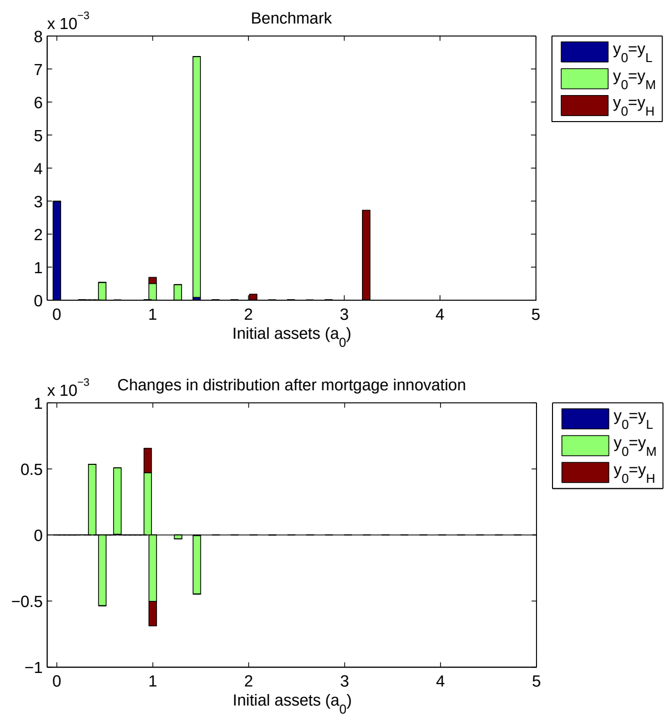


Figure 5: Distribution of contract choice by asset and income level

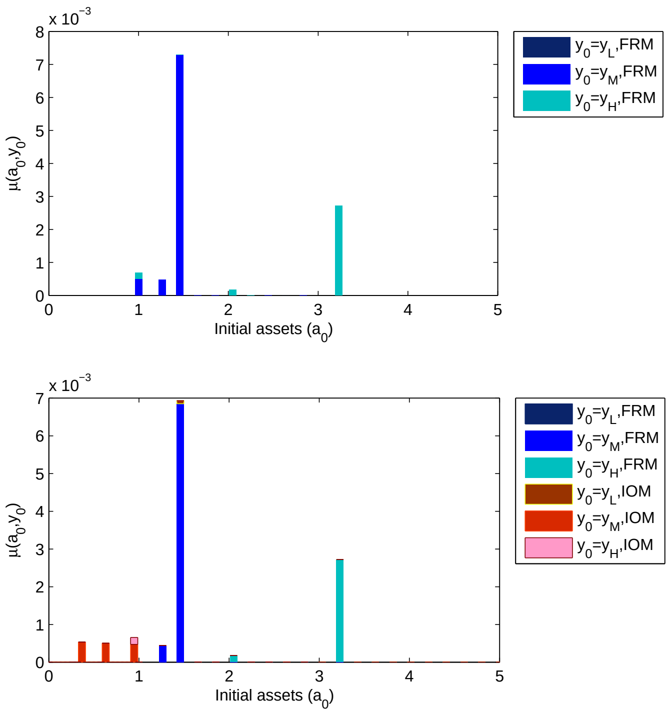


Table 5: Rent-or-own decision, IOM(2)

	Rent	IOM	FRM
y_L	$0.0000 \leq a_0 < 1.6565$	NA	$1.6565 \leq a_0$
y_M	NA	$0.0000 \leq a_0 < 1.0000$	$1.0000 \leq a_0$
y_H	NA	$0.0000 \leq a_0 < 1.0000$	$1.0000 \leq a_0$

the increased frequency of devaluation shocks dominates the mortgage selection effect. At the same time, average house prices fall as more devalued houses are sold.

In fact, the presence of IOMs magnifies the impact of the value shock probability increase. In the economy with FRM only, the same shock only causes default rates to rise by 34% (see the FRM(2) column of table 3.)

Table 6 provides a breakdown of default frequencies by contract type across experiments analogous to the last panel of figure 1. It shows that default rates are much higher for IOM contracts than for FRM contracts. Table 7 shows the contributions of each mortgage type to overall default rates. These two tables show that default frequencies on IOMs are almost double those for FRMs (2% vs. 1% for voluntary defaults).

Furthermore, IOMs account for nearly 25% of the overall default rates even though they only represent 12% of all mortgages. After raising the likelihood of a house value shock in the IOM(2) experiment, IOMs account for 23% of the overall default rates, even though they only represent 11% of all mortgages.

Table 6: Default frequencies by mortgage type

	voluntary	involuntary
Benchmark		
FRM	1.04	0.02
IOM(1)		
FRM	1.04	0.02
IOM	1.90	0.01
IOM(2)		
FRM	1.42	0.01
IOM	2.56	0.01
FRM(2)		
FRM	1.42	0.01

Table 7: Share of overall default rates

	voluntary	involuntary	total
Benchmark			
FRM	1.04	0.02	**1.06**
IOM(1)			
FRM	0.89	0.01	0.90
IOM	0.27	0.00	0.27
Total	1.16	0.01	**1.18**
IOM(2)			
FRM	1.22	0.00	1.22
IOM	0.36	0.00	0.36
Total	1.58	0.00	**1.58**
FRM(2)			
FRM	1.42	0.01	**1.43**

6 The welfare consequences of mortgage innovation

Mortgage innovation induces an increase in default rates hence an increase in the dead-weight loss associated with foreclosure. Yet, in our model, it raises the welfare of all agents by giving them an alternative mortgage choice without affecting any of the other opportunities they face, including the price of houses and the returns to saving.

To calculate consumption equivalent welfare gains, consider agents born with income y_i where $i \in \{L, M, H\}$. Then let k_i be the additional fraction of their lifetime consumption which agents born in the economy with FRMs would need to receive to have the same welfare as agents born with the same income in an economy with both mortgages. Then:

$$\begin{aligned} V^{IOM}(0, y_i) &= E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^{FRM}(1+k_i), s_t^{FRM}) \right] \\ &= E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t^{FRM}) + \ln(1+k_i) + \ln(s_t^{FRM}) \right\} \right] \\ &= V^{FRM}(0, y_i) + \frac{\ln(1+k_i)}{(1-\beta)} \end{aligned}$$

But then

$$\begin{aligned} (1-\beta)\left[V^{IOM}(0, y_i) - V^{FRM}(0, y_i)\right] &= \ln(1+k_i) \\ &\implies 1 + k_i = \exp((1-\beta)\left[V^{IOM}(0, y_i) - V^{FRM}(0, y_i)\right]) \end{aligned}$$

The average consumption equivalent gain is then given by: $\sum_i f_i k_i$ where f_i is the fraction of agents born with income y_i. We calculate that:

$$\begin{aligned} 1 + k_L &= \exp\left((1 - 0.96)\left[(-25.0302) - (-25.0366)\right]\right) = 1.0003 \\ 1 + k_M &= \exp\left((1 - 0.96)\left[(-4.3119) - (-4.6671)\right]\right) = 1.0143 \\ 1 + k_H &= \exp\left((1 - 0.96)\left[9.4550 - 9.4547\right]\right) = 1.0000 \end{aligned}$$

so that $\Rightarrow ce = (0.2056 \cdot 1.0003 + 0.5888 \cdot 1.0143 + 0.2056 \cdot 1.0000) - 1 = 0.0085$, which is near 1%.

Why are gains so low among agents who are born poor? The presence of IOMs raises welfare because agents know that they may eventually take advantage of that option. Because income shocks are very persistent, agents born poor know that they are likely to be poor for a while and to have accumulated little assets by the time the housing option presents itself. They are quite likely, therefore, to remain renters their entire life even when IOMs are offered. Indeed, recall that agents at the very bottom of the asset and income distribution do not take on mortgages in all economies. Likewise, the richer agents know that they will likely find themselves at the top of the income and asset distributions when they become middle-aged, hence are quite likely to opt for FRMs even when IOMs are available. It should be expected, therefore, that gains are particularly large for agents born in the middle-income distribution which, the above calculations show, is exactly what happens.

7 Summary

In our model:

- default frequencies on IOMs are almost double those for FRMs (2% vs. 1% for voluntary defaults);
- introducing IOMs causes default rates to rise by 11%. IOMs account for nearly 25% of the overall default rates;
- raising the likelihood of a house value shock by 30% reduces the frequency of IOMs, but leads to a 50% increase in default rates. IOMs account for 23% of the overall default rate;
- with FRMs only, the same value shock causes default rates to rise by 34%.
- despite the increase in foreclosure rates that results from the introduction of IOMs, welfare rises by 1% in consumption equivalent units.

Bibliography

Athreya, K., "Welfare Implications of the Bankruptcy Reform Act of 1999," *Journal of Monetary Economics*, 49, 2002.

Campbell, J. Y., and Cocco, J. F., "Household Risk Management And Optimal Mortgage Choice," *The Quarterly Journal of Economics*, 118(4), pp 1449-1494, 2003.

Chambers, M. , C. Garriga, and D. Schlagenhauf,"Equilibrium Mortgage Choice and Housing Tenure Decisions with Refinancing," Working Paper, Florida State University, 2007.

Chatterjee, S., D. Corbae, M. Nakajima, and V. Rios-Rull, "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75, pp 1525-1589, 2007.

Conesa, J. C., and Krueger, D., "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, 2(4), pp 757-795, 1999.

Garriga, C., Jeske, K., and D. Schlagenhauf, "House Prices and Foreclosures", Unpublished manuscript, 2008.

Gervais, M., "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 49, pp 1461-1489, 2002.

Guler, B., "Innovations in Information Technology and the Mortgage Market," University of Texas at Austin mimeo, 2008.

Harding J. P., Rosenthal, S. S., and C.F. Sirmans"Depreciation of housing capital, maintenance, and house price inflation: Estimates from a repeat sales model," *Journal of Urban Economics* 61(2), pp 193-217, 2007.

Hayre L. S., and M. Saraf, "A Loss Severity Model for Residential Mortgages", *Journal of Fixed Income*, Fall, pp 5-31, 2008.

Jeske, K. and D. Krueger, "Housing and the Macroeconomy: The Role of Implicit Guarantees for Government-Sponsored Enterprises," Working Paper, Federal Reserve Bank of Atlanta, 2005.

Perri, F. and D. Krueger, "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies*, 73(1), pp. 163-193, 2005.

Ríos-Rull, J.V., and V. Sanchez-Marcos, "An Aggregate Economy with Different House Sizes," *Journal of the European Economic Association*, 6 (2-3), pp 705-714, 2008.