Assignment, Hierarchies and Upper Tails

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Abstract

This paper examines whether changes in upper tails could explain increases in inequality in recent decades. First, methods are discussed that would determine the relative contributions of educational differentials or variance of outcomes within educational levels to overall inequality. Preliminary analysis suggests that educational differentials could not explain increases in inequality. The paper then examines empirically Pareto upper tails and shows that inequality in upper tails within industries have increased substantially.

1 Introduction

This paper considers whether changes in the upper tail of the distribution of income explain increases in inequality in recent decades. The paper then examines changes in the upper tails of distributions of earnings by industry empirically and analytically in terms of hierarchical structure.

Increases in inequality (in either income or labor earnings) have been widely documented (see reviews by Katz and Autor, 1999; Cunha and Heckman, 2007; Autor, Katz and Kearney, 2006; Lemieux, 2007). Much of the analysis has focused on the increases in skill differentials that occurred over most of the period of increasing inequality (Juhn, Murphy and Pierce, 1993; Bound and Johnson, 1992; Levy and Murnane, 1992; Heckman, Lochner and Taber, 1998). Major explanations for increasing skill differentials have been increasing returns to education, skill biased technological change, international trade, or migration patterns (Acemoglu, 2002; Berman, Bound and Machin, 1998).

However, it has not been established that increases in skill differentials are the major source of the increase in inequality. Other possible sources are the lower tail, the upper tail, and increasing dispersion within constituent groups, which may be related to both lower and upper tails. If inequality in income from all sources is being considered, it would be further necessary to consider the distribution of non-labor sources of income and changes in factor rewards.

There is substantial evidence that the distributions of earnings and income in upper tails have changed dramatically. Piketty and Saez (2003) examine top income and wage shares for 1913 to 1998. The results show increasing income and wage shares from the 1970's to the most recent year, 1998 for the top 10%, 5%, 1%, .5% and .1%. They also conclude that labor earnings are now the largest source of income for top recipients. Capital gains may be biased upward because of regression towards the mean, so the contribution of labor earnings could be even greater. Dew-Becker and Gordon (2005) analyze top income shares to determine who benefited from productivity growth between 1972 and 2001. They find that the share for wage earners from the 95th to 99th percentiles grew 29 percent, from the 99th to 99.9th percentiles grew 75 percent, and from 99.9th to 100th percentiles grew 291 percent. The implication of this evidence is that changes in inequality are more likely to arise from these changes in the upper tail than from skills differentials. Nevertheless, the dominance of the upper tail in the determination of increasing inequality needs to be established empirically and analytically. A subsidiary question is whether changes in the upper tail are separate from changes in the rest of the distribution. For example, both increasing skill differentials and increases in the variance of earnings within groups (e.g., in the form of residual wage variance generated by empirical estimation of earnings equations) will generate greater inequality in the upper tail. Also, skill differentials and residual wage variance may be linked by a common cause (Sattinger, 1980, p. 64). Section II considers methods to resolve these questions.

Next, if upper tails are the major contributor to increasing inequality, it would be desirable to understand the reasons for the changes in upper tails. There has been substantial work in this area, including work based on CEO compensation (Gabaix and Landier, 2006; Terviö 2002; Garicano, 2000; Lemieux, 2007), assignment of workers to tasks in the upper tail, and applications of tournaments and games to compensation of individuals at upper levels of hierarchies. Without ruling out other explanations for changes in upper tails, Sections III and IV consider changes in hierarchies. Section III reviews some of the early literature on the Pareto distribution generated by hierarchical assumptions. Section IV presents empirical work on changes in the upper tails by industry. This work does not provide a complete picture of changes in the upper tails. The conclusions, in Section V, consider what remains to be done.

2 Sources of Changes in Inequality

2.1 Measurement of Inequality

As a preliminary to consideration of what has caused recent increases in inequality, it is necessary to recognize that the answer may depend on which measure of inequality is used. Three common measures are the Coefficient of Variation (the standard deviation divided by the mean), the Variance of the Logarithms of earnings, and the Gini coefficient. These measures vary in their sensitivity to changes in numbers at different places in the distribution. Comparisons between the distributions can be generated by taking an empirical distribution of earnings and then increasing the number at some particular level (e.g., the 18th percentile from the bottom), compensating for the increase by reducing the numbers at all other levels to keep the total constant. Then it is possible to calculate the elasticity of the inequality measure with respect to a change at a particular level (or percentile) of income (Sattinger, 1980, page 133). All measures are positively sensitive to increases in the upper and lower tail, and negatively sensitive to increases in the middle of the distribution, but there are significant differences among the measures. Based on very old calculations that can be updated, the Coefficient of Variation is most sensitive to changes in the upper tail, and the Variance of Logarithms is most sensitive to changes in the lower tail. The sensitivity of the Gini coefficient to changes in the upper and lower tails lies between the sensitivities of the other two measures. The Gini coefficient is more sensitive (negatively) to changes in the middle incomes than the Coefficient of Variation, but less sensitive than the Variance of Logarithms.

This analysis can be extended to Atkinson's e and to other measures, and needs to be updated. If the updated results demonstrate the same patterns, use of the Coefficient of Variance would generate the greatest effect of upper tails on inequality (among the three measures considered here), while use of the Variance of Logarithms would generate the least effect.

2.2 Decomposition of Inequality Without Selection

As an initial approach to determine the source of changes in inequality analytically, methods developed a half century ago can be applied. In their book on the lognormal distribution, Aitchison and Brown (1957, page 110) consider an aggregate distribution of earnings generated by distributions of earnings in each sector. Suppose the distribution of earnings in each sector is lognormal with variance σ^2, where the variance is the same for each sector. Suppose the arithmetic mean of a sector is given by α, and suppose α varies among sectors. In a sector with arithmetic mean α, the average of the logarithms of means will be $\alpha - \sigma^2/2$. Now suppose the arithmetic mean α is itself lognormally distributed, with parameters μ_0 and σ_0. Then the distribution for the entire population will also be lognormal, with logarithmic mean $\mu_0 - \sigma^2/2$ and variance $\sigma_0^2 + \sigma^2$. This result directly provides an analytic expression for inequality as measured by the Variance of Logarithms, given by $\sigma_0^2 + \sigma^2$. Aggregate inequality is decomposed between differences between sectors and differences within sectors.

Lydall (1968, p. 104) has applied these results to sectors determined by occupation, using U.S. 1959 data. He concluded that the proportion of total variance attributable to between occupation variance is "remarkably small." In terms of the Aitchison and Brown notation, σ_0^2 is small relative to σ^2, so a change in σ_0 contributes much less to an increase in overall inequality than an equally large proportional change in σ.

Consistent with research showing increasing educational returns, the appropriate context in which to apply this lognormal decomposition would be by educational levels. In this context, if the decomposition is applicable, σ_0 would measure the standard deviation of logarithms of earnings by educational level, and σ would measure the standard deviation of logarithms within an educational level. If σ_0 is small relative to σ, an increase in educational differentials would have little effect on overall inequality, but an increase in σ, measuring inequality within educational levels, would have a large effect. An increase in σ_0 would not generate much greater inequality in the upper tail, whereas an increase in σ would. An increase in educational differentials would generate neither much of an increase in overall inequality nor much of an increase in inequality in the upper tail, whereas an increase in within educational level inequality would generate both.

Of course the applicability of this simple decomposition rests on very specific assumptions, which do not hold exactly.

2.3 Decomposition with Selection

The major problem with the Aitchison and Brown methodology is that the sector to which an individual belongs is to some extent determined by self-selection. For example, in the classic Roy model (1951), individuals choose sectors (rabbits or trout) based on income maximization. Any change in the pay for output from a particular sector generates different decisions and movement from one sector to another (as analyzed in Heckman and Sedlacek, 1985). The assumption in the Aitchison and Brown decomposition that no longer holds is that the distribution within a sector stays the same when differences between sectors (reflected in σ_0) increase or decrease. The same principle applies when individuals make human capital decisions on how much education to get (Willis and Rosen, 1979; Heckman, Lochner and Taber, 1998). Figure 1 exhibits the basic diagram showing how selection into the two occupations generates an aggregate distribution of earnings.

Nevertheless, it is possible to apply analytic results from the selection literature to determine the contribution of increasing educational differentials to overall inequality. In the context of the Roy model, when the price of trout increases relative to the price of rabbits, it is possible to determine the overall effects on the distribution of earnings, given the parameters for the means, variances and correlations between individual outputs in the two sectors. Sattinger (1993, page 856) works out the consequences of an increasing skill differential in the Roy model but not the consequences of an increase in the variance of trout performances. Table 1 updates those results to compare the results of an increase in the price of trout relative to rabbits and an increase in the variance of trout performances. These results are preliminary and need to be updated by using relevant earnings data by educational level.

Table 1: Effects of Changes on Inequality, Upper Tail

	$p = 1$, $\sigma_2^2 = 1.4$	$p = 1.1$, $\sigma_2^2 = 1.4$	$p = 1$, $\sigma_2^2 = 1.54$
Average Income	145.50	156.58	156.757
Variance of Logs	.888	.929	.952
Upper 1% Share	13.31%	13.60%	16.49%
Upper .1% Share	3.21%	3.28%	3.71%

Considering how the two sectors contribute to the aggregate distribution

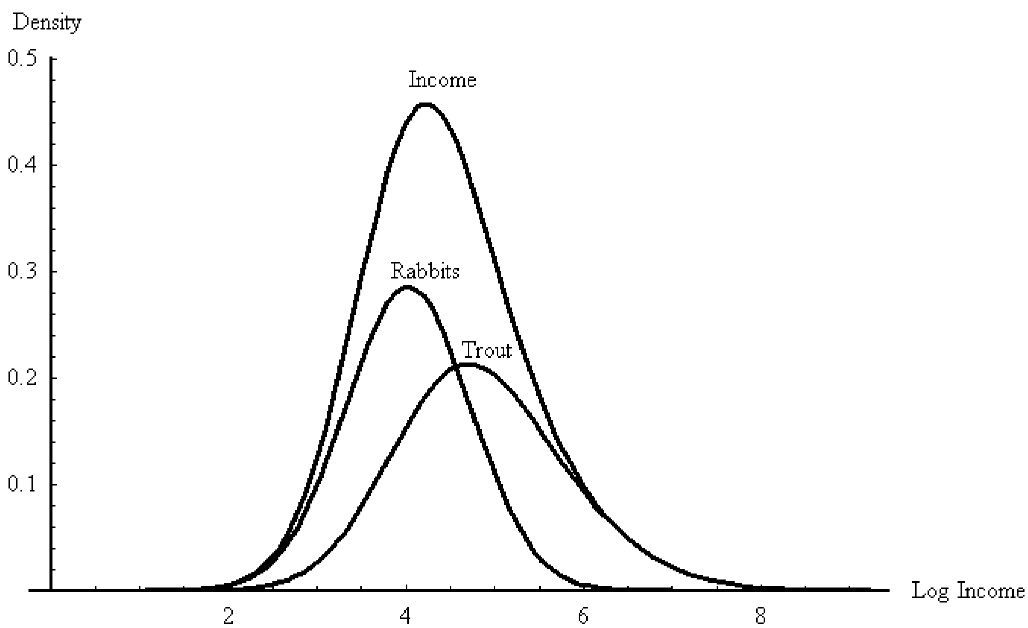

Figure 1: Distribution of Earnings Generated by Selection

of earnings, these results are not surprising. An increase in the differential between the two sectors, reflected in the ten percent increase in the price of trout relative to the price of rabbits, has a relatively small effect on earnings inequality as measured by the Variance of Logarithms (which is less sensitive to the upper tail). The differential affects the distribution by spreading apart the constituent distributions and drawing more of the better rabbit catchers into the trout sector, without changing the shape of the upper tail. Selection effects of an increasing differential include a reduction in average performance of workers continuing to choose to hunt rabbits, and a reduction in their average income. By comparison, the increase in σ_2^2, corresponding to an increase in residual wage variance in the higher-paying sector, has a much greater effect on the Variance of Logarithms and the top shares. Of course, a change in σ_2^2 has a direct effect on the shape of the upper tail, as the source of upper incomes. These results suggest (but do not establish) that skill or educational differentials could not explain the increases in inequality that have taken place.

An important consequence of self-selection is that neither the educational differential (in terms of average earnings by educational level) nor the observed variance of the logarithms of earnings within educational levels can be taken as exogenous explanatory variables that can change independently. An increase in the average return to education (corresponding to an increase in the price of trout in the Roy model) changes both the observed educational differential and the residual variance of earnings in each educational sector, and a change in the underlying variance of returns to education (corresponding to the dispersion of abilities in rabbit or trout catching) also has effects on the observed educational differential. The question of whether the changes in the upper tail have occurred separately from changes in the rest of the distribution or are a part of changes that extend beyond the upper tail, such as the variance of performances in the higher-paying group, is unresolved.

The Roy model can also be extended to incorporate multiple sectors, positive correlations, and unequal variances within sectors by use of order statistics (Sattinger, 1996).

3 The Pareto Distribution for Upper Tails

3.1 Background

The previous section considered increasing dispersion within a sector as a significant cause of both increasing inequality and greater inequality in upper tails (as reflected in greater proportions of income going to the top one percent or the top one-tenth of a percent of all income earners). Other causes that have been suggested in the literature arise from the methods of pay of top executives and changing assignment. This section considers models of incomes at the top of hierarchies as a background to estimation of changes in those hierarchies in the next section.

The upper tail of the distribution of earnings has often been charac-

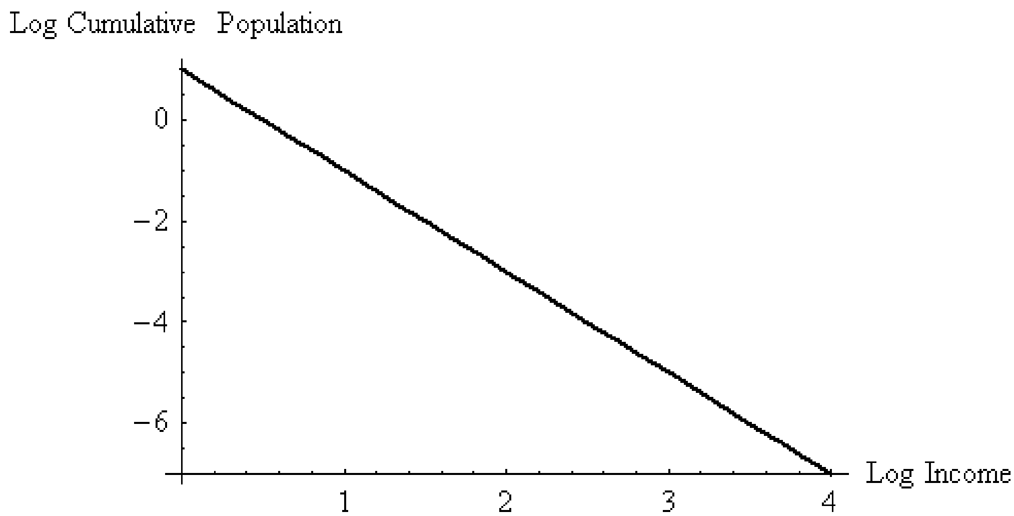

Figure 2: Pareto's Law

terized as having a Pareto distribution. The Pareto distribution, developed by Vilfredo Pareto (1909), is characterized by the following relationship:

$$Log\ N = A - \alpha\ Log\ y \tag{1}$$

where y is income and N is the proportion of the population earning more than y (for historical detail, see John Chipman, 1976). The Pareto distribution was first applied to infant mortality data. Later he applied the distribution to income in an estimate of the demand curve for wheat. In a still later paper, he applied the distribution to incomes in different countries and at different times, and concluded that the exponent α should be between 1.45 and 1.72.

The Pareto distribution is shown in Figure 2. It is a one-tailed distribution that is reflected in a linear relation between *Log N* and *Log y*. The coefficient α, in 1, provides a measure of inequality for this distribution. The slope of the line in Figure 2 is $-\alpha$. The lower the value of α, the more slowly the numbers above each income level decline, and the greater the inequality. On the use of α as a measure of inequality, Pareto (as cited by Chipman 1976, page 117) comments:

> "In general, when the number of persons with incomes less than x decreases relatively to the number of persons with incomes greater than x, I shall say that the inequality of incomes increases. But the

reader is duly warned that by these terms I mean simply to designate this thing and nothing else."

In this quotation, the increase in the number of individuals with income above some level, relative to the number below that level, corresponds to the frequent observation of an increase in the share of income of people at some percentile of the upper tail, for example the share of the top one percent of all income earners. With the Pareto distribution, a decrease in α means that Pareto's statement above would be valid for a range of incomes above some level. Pareto's caution deserves some attention. Consider the statement that a higher proportion of aggregate income going to the top one percent of income recipients indicates greater inequality. This statement requires that other parts of the distribution remain unchanged. Otherwise changes at other levels of income could counter the changes in the share going to the top one percent. The relevance of this rather obvious observation is that it is insufficient to point to increases in upper income shares as a cause of increasing inequality; one should know why upper income shares increased in order to determine what has happened in the lower part of the distribution.

3.2 Changes in Pareto's α Over Time

Pareto's α, regarded as a characteristic arising from hierarchical organizations within industries, varies by industry and can change over time. It is often assumed in textbook developments of industrial organization that production in an industry can expand by additional firms using the exact same technology. It seems reasonable to extend this assumption to suppose that additional firms would have access to the same administrative structure, and would therefore exhibit the same α within an industry. This section estimates changes in α by three digit industry using 1980 and 2000 U.S. Census data (the 5% sample), using procedures applied in Sattinger (1980). An important consequence of the Pareto distribution is that α can be estimated in a straightforward way from cumulative data. Suppose N_1 people earn income greater than or equal to y_1 and N_2 earn income greater than or equal to y_2. Then

$$\frac{Log(N_1/N_2)}{Log(y_1/y_2)} = -\alpha$$

The comparison over time is obtained by using the same income in real terms at two points in time. One of the problems in this application to U.S. Census data is that in 1980 income was top-coded at only \$75,000, meaning that incomes above that level were placed in the same group, i.e. incomes \$75,000 and above. This top-coding can generate an underestimate of the inequality in earnings if an insufficiently high average income for the group is chosen in calculations.

Using industries for which there are sufficient numbers to estimate α for both years, the results show that α declined in 68 out of 74 industries (see

Table 2). The average value of α declined from 2.81 to 2.14, a substantial decrease. This suggests that common forces have changed the hierarchies across industries between 1980 and 2000.

Since the earnings data in 2000 go to higher levels, it is also possible to examine whether Pareto's law, reflected in the distribution, holds. If Pareto's law holds, then the value of α should be the same when calculated using higher pairs of income. Using earning levels of \$120,000 and \$144,000 in 2000, the average α is 2.134. However, using earnings levels that are double those levels, the average α is .437. This result suggests that Pareto's law does not strictly hold as one reaches sufficiently high earnings levels. This result is consistent with the very large increases in earnings shares going to individuals at the very top of the distribution. Another possibility is income recording procedures in the Census.

3.3 Hierarchical Models Generating Pareto's Law

Herbert Simon (1957) and Harold Lydall (1959) provide simple hierarchical models that generate the Pareto distribution. Within a hierarchy, suppose each person has q subordinates (the span of control). Suppose further that each person is paid p times the wage of his or her subordinates. Then comparing numbers and wages of people in successive hierarchical levels,

$$\frac{Log(N_{i+1}/N_i)}{Log(y_{i+1}/y_i)} = \frac{Log(1/q)}{Log(p)} = \frac{-Log(q)}{Log(p)} - \alpha$$

An important consequence of this construction is that a decline in α, consistent with increasing inequality in the upper tail, can be caused either by an increase in pay differentials, p, or a decrease in the span of control, q. In a study of increasing inequality in hierarchies, both sources would need to be investigated. For example, for the hospital industry, α is estimated to be 1.75 in 2000. This could be generated by $q = 2$ and $p = 1.486$, or by $q = 4$ and $p = 2.208$.

Possible explanations for a greater pay ratio include principle-agent problems, determination of pay through games, tournaments or races (Reder, 1969; Lazear and Rosen, 1981), or incentives in general. Possible explanations for a lower span of control q include technological change in organizations, changes in costs of monitoring, information flows (Rosen, 1982; Garicano, 2000), and changes in management tasks (e.g. from production to coordination, outsourcing or product innovation).

Without further information spans of control or pay ratios, it is not immediately possible to determine what has caused Pareto's α to decline in so many industries. However, it is possible that there is evidence of ancillary consequences of changes in span of control or pay ratios. For a given pay ratio, when the span of control is larger, there will be slower promotion or more discharges, fewer ranks for a given firm size, a lower proportion of non-production workers in the hierarchy, and lower expected earnings. The slower promotion or more discharges occur because a smaller proportion of individuals at one level of the hierarchy get promoted to the next higher level, and in the absence of promotion more people may leave a hierarchy. There are fewer ranks because for a given number of workers in the hierarchy at the lowest level, it will take fewer ranks to get down to one individual running the company. With fewer ranks, and smaller numbers of individuals at higher ranks, the total size of the hierarchy, and the proportion of non-production workers, will be smaller. For a given pay ratio, with a smaller likelihood of getting to higher pay levels, the expected earnings of an individual at the lowest level will be lower.

Similarly, when the pay ratio is higher, there will be greater expected earnings in an industry for a given entry level wage. Comparing industries, those with higher average wages (from smaller q or larger p) would have lower starting salaries, greater inequality, and lower unemployment (from fewer discharges). A higher average wage in an industry is therefore consistent with lower expected wages for a new worker. Empirical work comparing inter-industry wage differentials based on average wages for a given skill group (e.g., four years of college education) would then be misleading.

4 Conclusions

This paper has proposed methods of determining whether increases in inequality over the past decades could be explained by changing educational or skill differentials, or by increases in the dispersion of outcomes for individuals with given educational or skill levels. This question is complicated by the endogeneity of educational differentials and residual wage variance in the presence of self-selection. The preliminary examination in this paper suggests that skill or educational differentials by themselves could not explain the magnitude of changes in inequality. This examination needs to be followed up by use of more relevant data.

The paper then considers simple hierarchical models that generate a Pareto upper tail, based on early work by Simon and Lydall. Empirical work shows that Pareto's α has increased substantially within industries. There is also some question as to whether Pareto's law holds at higher incomes.

Table 2: Pareto's α

Industry	1980	2000	2000 High Limits	Ratios
105	2.82212	1.91748	0.6626	0.679446657
226	2.95327	3.05073	0.16539	1.033000708
246	2.50155	1.83495	0.53938	0.733525214
307	3.18016	1.98712	0.93445	0.624849064
309	2.29752	1.66214	0.40482	0.723449633
336	2.74348	1.94877	0.32158	0.710327759
346	2.58415	1.84632	0.44681	0.714478649
357	3.47507	2.85166	0.37788	0.820605053
358	2.9587	2.01744	0.53941	0.681867036
367	2.8926	2.62365	0.50521	0.907021365
376	2.75324	3.01689	0.28561	1.095759905

377	3.63407	2.83878	0.3429	0.781157215
386	2.70072	2.46415	0.36004	0.91240484
399	2.65412	2.19117	0.5366	0.825573071
406	3.24357	1.83081	0.5876	0.564442882
439	2.6265	2.28983	0.89925	0.871818009
448	2.24625	1.77701	0.34867	0.791100723
456	2.81217	2.18399	0.81225	0.776620901
459	2.39536	2.09267	0.31024	0.873634861
467	2.97405	2.35884	0.14703	0.793140667
469	3.09721	2.81652	0.34119	0.909373275
506	3.20877	2.54947	0.21196	0.794531861
516	2.2059	1.8297	0.50239	0.829457364
526	3.20154	2.5682	0.65394	0.802176453
556	3.97275	2.46653	0.62122	0.620862123
568	2.43574	2.58995	0.26405	1.063311355
578	3.44134	2.51371	0.3403	0.730445117
586	4.27591	3.14034	0.61691	0.734426122
597	2.29752	1.7914	0.37788	0.779710296
606	2.82647	2.64232	0.47722	0.934848061
609	2.42629	1.89773	0.34233	0.782152999
616	2.73791	2.02969	0.42332	0.741328239
617	3.0225	2.16998	0.43896	0.717942101
626	2.60405	2.16848	0.46469	0.832733626
627	2.92344	2.02778	0.33719	0.693628055
636	2.29843	1.66683	0.51977	0.725203726
646	3.26497	2.15381	0.50743	0.659672218
656	2.85317	1.72699	0.19465	0.605288153
658	2.86464	1.65066	0.35689	0.576219001
659	2.47703	2.44791	0.35072	0.988243986
667	2.68874	2.17447	0.66315	0.808731971
668	2.85454	2.02183	0.28476	0.708285748
669	3.12738	2.55605	0.46758	0.817313534
679	2.58546	1.73566	0.55366	0.671315743
687	2.51456	2.41934	0.50519	0.96213254
698	2.93503	2.18726	0.5203	0.745225773
699	3.56291	2.08754	0.47304	0.585908709
716	2.78663	1.98785	0.28537	0.713352688
726	2.10816	1.24017	0.24358	0.588271289
736	2.98936	1.96768	0.41447	0.658227848
746	2.58174	1.82798	0.40256	0.708041863
806	2.89332	2.08022	0.22405	0.718973359
807	2.37065	2.16183	0.35617	0.911914454
808	2.77013	2.2783	0.24781	0.822452376
816	2.85775	1.9576	0.54824	0.685014434
817	2.04298	2.17662	0.44388	1.065414248
826	4.12452	0.70397	0.63671	0.170679255

836	2.34825	1.47341	0.52563	0.627450229
846	2.66839	1.50607	0.43776	0.564411499
849	2.67954	1.86982	0.29417	0.697813804
856	2.77801	2.1385	0.24458	0.769795645
857	2.28811	1.6558	0.08636	0.72365402
859	2.03375	2.10216	0.51101	1.033637369
868	1.98808	1.44396	0.54236	0.7263088
869	2.55215	1.74833	0.4973	0.685042023
879	2.12325	1.57017	0.36078	0.73951254
888	3.28029	2.73237	0.41034	0.832965988
896	3.17445	1.91927	0.31624	0.604599222
897	2.59878	2.47499	0.28907	0.95236611
898	3.23733	2.68565	0.43558	0.829587963
899	2.40275	1.42263	0.24398	0.59208407
906	1.53906	1.64677	0.72385	1.069984276
916	3.42582	3.16664	0.39886	0.92434512
936	4.08854	3.7239	0.39758	0.910814129
Averages	2.809	2.14158	0.4305	0.7711

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