Capital Income Taxation and Risk-Sharing

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Abstract

In this paper I examine the role of capital income taxation in providing risk-sharing in an economy characterized by uninsurable idiosyncratic uncertainty and borrowing constraints. I show that the introduction of a capital income tax system in this economy leads to a reduction in precautionary savings since the tax acts as a risk-sharing mechanism among individuals. I find that the capital income tax rate associated with the highest welfare gain is increasing with the earnings risk and with the consequent demand for insurance. I then calibrate the model to US data and find that the long-run capital tax rate providing the optimal level of risk sharing is 34.5 percent.

*The views expressed in this paper are those of the author and should not be interpreted as those of the Congressional Budget Office.

1 Introduction

Many researchers have found that income inequalities are largely explained by differences in luck. In his 1972 book on sources of inequality, Jencks documents that only a small fraction of the variation of an individual income in the US can be explained by differences in socioeconomic characteristics, such as family background, educational attainment or occupational status. Further empirical studies on income determination have confirmed Jencks' findings (Coe, 1977; Lilliard and Willis, 1978; Castaneda, Diaz-Gimenez and Rios-Rull, 2003). These studies suggest that, after controlling for both observed and unobserved characteristics, the movement in an individual's income over time may have a large random component.

In absence of insurance tools, individuals with uncertain incomes are forced to self-insure themselves through the accumulation of a large buffer-stock of assets in periods of good luck. In this environment, if a government introduces a tax on income from capital creates a distortion to asset accumulation decisions but also a risk-sharing mechanism between the ones enjoying an high income period and the ones experiencing a period of bad luck. The risk-sharing derives from the fact that individuals receiving an high income shock will save more and pay more taxes on income from capital compared to the ones facing a bad income shock . This mechanism also represents a way to redistribute income across time for the same individual. Although capital income taxation introduces a wedge between what the firms earns and what savers receive as a return on investment, at the same time it also reduces the needs for savings for precautionary reasons.

This paper analyzes and highlights the insurance role that capital income taxation plays in presence of uncertainty and incomplete markets. It also deals with the trade-off between the efficiency gains of risk-sharing and the distortionary effects associated with capital income taxation.

There has been extensive literature pointing at the distortionary nature of capital income taxation on intertemporal allocation of resources (see, for example, Chamley, 1986; Lucas, 1990; King and Rebelo, 1990; Cooley and Hansen, 1992). When markets are perfectly competitive and complete, capital taxation reduces economic welfare and optimal capital income tax is zero. Recent literature, instead, highlights how uncertainty, asymmetric information or incomplete markets can alter this results and suggest that optimal capital income taxation would be positive (Aiyagari, 1995; Golosov, Tsyvinski and Kocherlakota, 2003; Domenji and Heathcote, 2004; Conesa, Kitao and Krueger, 2007).

This paper deals with the latter themes in an economic set-up a la Bewley (1986) and Aiyagari (1994, 1995). Aiyagari demonstrates that optimal capital tax rate is positive in a dynamic dynastic general equilibrium model in presence of borrowing constraints and idiosyncratic uninsurable risk. In particular, he shows that positive capital income taxation is necessary to bring the interest rate into equilibrium with the time preference parameter.

The contribution of the paper is to show how capital income taxation substitutes for some of the roles played by insurance markets in a similar environment. The purpose of my study is mainly theoretical and illustrative. I build an infinite-horizon dynamic general equilibrium model where agents become (endogenously) heterogeneous by receiving uninsurable, idiosyncratic earning shocks. Furthermore, agents are also borrowing-constrained. Precautionary saving represents the only way for them to protect against bad income shocks.

I compare the stationary equilibrium of this benchmark economy with the one of a parallel economy in which individuals do not face earning risk but are ex-ante heterogeneous. The two economies feature the same stationary distribution of earnings. By confronting the two economies, I first measure the buffer-stock of saving accumulated for insurance purposes in the benchmark economy. I then introduce a government that levy a tax on capital income in the benchmark economy.

I show that higher levels of capital income taxation reduce asset accumulation. In presence of taxation, agents need to accumulate less precautionary savings since capital acts as a risk-sharing mechanism among individuals. Furthermore, I find that the tax rate associated with the highest welfare gains is increasing with the earnings risk and with the consequent demand for insurance. I then calibrate the model to US data and find that the long-run capital tax rate providing the optimal level of risk sharing is 35.4 percent.

The rest of the paper is organized as follows. The second section presents the model economies. The third section sketches out the role of precautionary savings in presence of uncertainty and incomplete markets. The fourth section describes the numerical analysis. Finally the fifth section adds some concluding remarks.

2 The model economies

I study two economies. One economy, considered as benchmark case, is characterized by individuals ex-ante identical facing idiosyncratic shocks to their earning ability. Insurance markets are missing. In the second economy, individuals are born heterogeneous in their earning ability and face probability zero of moving to a different ability state during their lives. In this economy individuals face no uncertainty or their uncertainty is completely insurable. Ex-ante and ex-post heterogeneity in earnings coincide in the stationary equilibrium of the two economies or i.e. the stationary distribution of earnings is identical. In both economies, there is a continuum of infinitely-lived households and a representative firm operating in competitive markets. In the basic setup I do not consider the existence of a government as I just want to show the features of these two hypothetical economies different in their degree of uncertainty and market completeness.

I will spend some time describing in detail the benchmark economy since the economy without uncertainty can be seen as a particular case of the former.

2.1 Households

2.1.1 Economy with idiosyncratic uncertainty (benchmark)

The benchmark economy's population is composed by a continuum of measure one ex-ante identical households living infinitely. Time flows through several calendar dates t and each time period is equivalent to one year.

At each date t, the household receives a shock to its own labor earning ability e_t, taking values in the space $E = \{e_1, ..., e_i\}$, and cannot insure against it. The incompleteness in the insurance markets makes it possible for the idiosyncratic risk to transform the households in ex-post heterogeneous. The household's earning ability or productivity follows a first-order Markov process, with transition probabilities between two states e_i, e_j in the space E given by $\pi_{i,j}(e_{t+1} = e_j \mid e_t = e_i)$. The probability measure or distribution of household on E at each time t is represented by μ_t, with $\mu_t(E) = pr(e_t \in E) \geq 0$. If the initial measure of households with respect to their earning ability is represented by a vector $\mu_{t=0}$, then the measure at some date t would be $\mu_t = \mu_{t=0}\Pi^t$, where Π represents the transition probability matrix, whose elements are the $\pi_{i,j}$.

At each date t, after observing the realization of e_t the household decides how much to consume c_t, how much labor to supply h_t, and next period's asset holding a_{t+1}, in the form of a single risk-free savings instrument. In the choice of the optimal assets to carry over the next period the household also faces borrowing constraints, in terms of the minimum assets he is allowed to hold. Hence, although agents live forever, sequences of bad shocks will lead to periods of binding borrowing constraints.

Let A be the asset space assumed to be non-negative, $A \in \mathbb{R}_+$. The household's state space is therefore determined by $(A \times E)$ and the household's states are represented by the vector $s_t = (a_t, e_t)$. Let $x_t(s)$ be the measure of households across both the individual assets and the earning abilities at time t, and $X_t(s)$ be the corresponding cumulative measure such that $\int_{A\times E} dX_t(s) = 1$.

Since the economy does not experience any aggregate uncertainty, the households have perfect foresight of the aggregate return on capital r_t and on the aggregate wage rate w_t, although they do not know their own future earnings (which is determined by e_t, the earning ability state). The aggregate states of the economy relevant for the individual vector of decisions rules $d_t = (c_t, h_t, a_{t+1})$ are $S_t = x_t(s)$. The optimization problem of the household can now be defined recursively as:

$$V(s_t, S_t) = \max_{c_t, h_t, a_{t+1}} u(c_t, h_t) + \beta E_{e_{t+1}|e_t}[V(s_{t+1}, S_{t+1})] \tag{1}$$

subject to

$$a_{t+1} = (1 + r_t)a_t + w_t e_t h_t - c_t \tag{2}$$

$$A \in \mathbb{R}_+ \tag{3}$$

where β is the time preference parameter. The utility function $u(c_t, h_t)$ expressing the individual preferences over the consumption level c_t and the leisure

$\ell_t = (1 - h_t)$ is specified as a time-separable isoelastic Cobb-Douglas[1]:

$$u(c_t, h_t) = \frac{[c_t^{\alpha}(1 - h_t)^{(1-\alpha)}]^{(1-\gamma)}}{1 - \gamma} \tag{4}$$

γ is the coefficient of relative risk aversion and α is the share of consumption in the household's preferences. Given the household optimization problem, the law of motion of the measure $x_t(s)$ is determined by:

$$x_{t+1}(s) = \int_{A \times E} I_{[a_{t+1}=a_{t+1}(s_t,S_t)]} \pi_{t,t+1}(e_{t+1} \mid e_t) dX(s_t) \tag{5}$$

where $I_{[a_{t+1}=a_{t+1}(s_t,S_t)]}$ is an indicator function taking value of 1 if the decision variable $a_{t+1} = a_{t+1}(s_t, S_t)$.

2.1.2 Economy with no uncertainty

The second economy is similarly populated by a continuum of households living infinitely. Households in this economy, though, receive a certain earning ability when they are born to the model e, taking values in the same ability space $E = \{e_1, ..., e_i\}$ as in the benchmark economy. Households are distributed across the ability states according with the measure $\widehat{\mu}$.

At each time t, the household faces no uncertainty regarding their ability to work or i.e. a probability to move to a different state $\pi_{i,j}(e_{t+1} = e_j \mid e_t = e_i) = 0$ therefore the measure $\widehat{\mu}$ is invariant Similarly to the benchmark economy, the household decides how much to consume c_t, how much labor to supply h_t, and next period's asset holding a_{t+1}, taking values in $A \in \mathbb{R}$ in this case. The household is faced with the same problem as in (1) and (2), but under certainty. The aggregate states of the economy are represented by $S_t = S_t(\widehat{\mu})$.

At each time t, it's not necessary to keep track of the distribution of assets in the previous period so the measure $x_t(s)$ is just a function of $\widehat{\mu}$.

2.2 Production

The production side is identical in the two economies under study. It takes place in a representative firm operating with a Cobb-Douglas constant-return-to-scale technology. At each date t, the firm uses aggregate capital K_t and aggregate labor L_t as inputs to produce a single output Y_t through the production function $F(K_t, L_t)$. Taking the latter into account, the firm chooses the optimal level of the inputs to maximize a stream of profits:

$$\max_{L,K} \sum_{t=0}^{\infty} \prod_{n=0}^{t} (1 + r_n)^{-1} [F(K_t, L_t) - w_t L_t] \tag{6}$$

subject to

$$K_{t+1} = I_t + (1 - \delta) K_t \tag{7}$$

[1] The utility specification makes preferences consistent with the analysis of Aiyagari (1994, 1995).

where (7) represents the law of motion of capital and $F(K_t, L_t)$ determines the production technology as follows:

$$F(K_t, L_t) = \Psi_t K_t^{\theta} L_t^{(1-\theta)} \tag{8}$$

The parameter θ represents the share of the capital input in the production process, while Ψ_t is the total factor productivity (tfp).

2.3 Equilibrium definition

2.3.1 Benchmark economy

A recursive equilibrium for the benchmark economy is a value function $\{V(s_s, S_s)\}_{s=t}^{\infty}$, and a vector of decision rules $\{c_s, h_s, a_{s+1}\}_{s=t}^{\infty}$ for the household optimization problem, a probability measure μ_0 and $\{\mu_s(E)\}_{s=t}^{\infty}$ for the initial and time path of the mass of the population in each earning ability state $e_s \in E$, the return on capital $\{r_s\}_{s=t}^{\infty}$ and the wage rate $\{w_s\}_{s=t}^{\infty}$, a measure of households across both the individual wealth and earning ability $\{x(s_s)\}_{s=t}^{\infty}$, and a vector of aggregate variables $\{K_s, L_s\}_{s=t}^{\infty}$ such that:

1. $\forall t$ the decision rules $\{c_t, h_t, a_{t+1}\}$ solve the household's optimization problem described by (1)-(2), given r_t and w_t, and the sequence $\{\mu_s(E)\}_{s=0}^{t}$.

2. $\forall t$ the firm solves the optimization problem described by (6)-(8), given $x(s_t)$. From the solution of the firm's problem, the return on capital and the wage rate satisfy:

$$r_t = \theta \Psi_t K_t^{\theta-1} L_t^{1-\theta} \tag{9}$$

$$w_t = (1-\theta) \Psi_t K_t^{\theta} L_t^{-\theta} \tag{10}$$

3. $\forall t$ given the conditions (9)-(10) the factor markets clear:

$$K_t + B_t = \int_{A \times E} a_t dX(s_t) \tag{11}$$

$$L_t = \int_{A \times E} e_t h_t(s_t, S_t) dX(s_t). \tag{12}$$

4. $\forall t$ the goods market clears:

$$C_t + G + K_{t+1} - (1-\delta) K_t = F(K_t, L_t) \tag{13}$$

$$C_t = \int_{A \times E} c_t(s_t, S_t) dX(s_t) \tag{14}$$

The equilibrium definitions highlight the fact that households do not make any portfolio choice between shares of the capital and government debt. Households are indifferent between capital and government debt because there is no aggregate uncertainty in the form of aggregate productivity shocks. The return on capital and from the government debt service is certain and in equilibrium the two returns must be equal.

The economy is in a steady state recursive equilibrium if the aggregate states of the economy are constant over time which implies that $S_{t+1} = S_t$. Household's variables, though, are subject to changes due to idiosyncratic uncertainty but the measure $\mu_t(E)$ converges to an unique invariant μ^* such as $\mu = \mu\Pi$. The μ^* represents the ergodic distribution around the earning ability states.

2.3.2 Economy with no uncertainty

The equilibrium in the economy with no uncertainty is very similar to the one described above except that households's decision rules at point 1. are computed given the measure $\widehat{\mu}$ that does not change over time. This measure is set to coincide with the ergodic measure μ^* in the benchmark economy. A steady state equilibrium for this economy is similarly defined by the condition that the aggregate state S_t does not change over time.

3 Precautionary savings and insurance

The two economies under analysis are characterized by the same invariant aggregate distribution of households around the earning ability states in the stationary equilibrium. They differ, instead, since in the benchmark economy households face uncertainty and they cannot buy insurance contracts to hedge against it. In presence of this market incompleteness and borrowing constraints, they would accumulate a buffer-stock of assets to insure future welfare. As a result, the asset accumulation in the two economies would differ for the presence of the precautionary motive for saving in the benchmark economy.

To study how this difference stems from optimal insurance behavior, I start from the first order conditions for the households optimization problem in (1)-(3).

$$\frac{\partial u(c_t, h_t)}{\partial c_t} = \lambda_t \tag{15}$$

$$-\frac{\partial u(c_t, h_t)}{\partial h_t} = \lambda_t w_t e_t \tag{16}$$

$$\lambda_t = \beta(1 + r_{t+1})E_t(\lambda_{t+1}) + \eta_t \tag{17}$$

where λ represents the lagrange multiplier attached to the constraint (2) and η the lagrange multiplier attached to the borrowing constraint (3) and expresses the marginal utility of borrowing. Combining (15)-(17) we can obtain the following Euler equation:

$$\frac{\partial u(c_t, h_t)}{\partial c_t} = \beta(1 + r_{t+1})E_t\left[\frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}}\right] + \eta_t \tag{18}$$

In absence of insurance markets, this equation governs the level of asset-holdings that attains the optimal insurance in face of idiosyncratic uncertainty. The condition states that household (in the benchmark economy) renounces optimally to some consumption at time t in exchange for uncertain consumption at time

$t+1$ and the possibility of being borrowing-constrained. The same equation (18) holds for the household's problem in both economies, although the expectation term on the right-hand side does not appear and markets are perfect ($\eta_t = 0$) in the economy where agents do not face any uncertainty.

I focus on the analysis of the two economies in their stationary equilibrium since these are the ones that differ substantially. As I mentioned, the aggregate states of the benchmark economy do not change but households are still subject to idiosyncratic uncertainty. Equation (18) holds pretty much unchanged[2] in the stationary equilibrium of the benchmark economy. In the economy where households do not face uncertainty instead the steady state Euler equation becomes:

$$1 = \beta(1+r) \tag{19}$$

Equation (19) also implies the well known condition that in a stationary competitive equilibrium if markets are complete interest rate is $r = 1/\beta - 1$, dictating also the condition for the efficient asset accumulation. To study the optimal asset accumulation in the two economies, I therefore compare the following:

$$\frac{\partial u(c,h)}{\partial c} = \beta(1+r)E\left[\frac{\partial u(c,h)}{\partial c}\right] + \eta \tag{20}$$

with equation (19). I abstract for simplicity from the possibility for the household to be borrowing-constrained and study the difference between the two equations due to future earning risk and missing insurance markets. If we assume β to be the same in the two economies, the two crucial terms in the analysis are $\frac{\partial u(c,h)}{\partial c}$ and $E\left[\frac{\partial u(c,h)}{\partial c}\right]$ since in the case where agents face no uncertainty these two expressions coincide. For simplicity, let's first assume that labor is supplied inelastically, i.e. $h = 1$ at each moment. Substituting for the steady state budget constraint (2) in $u(c)$, we have that, if $\frac{\partial^3 u(c)}{\partial c^3} > 0$, Jensen inequality implies that

$$E\left[\frac{\partial u(ra+we)}{\partial a}\right] > \frac{\partial u[E(ra+we)]}{\partial a} \equiv \frac{\partial u[ra+wE(e)]}{\partial a} \tag{21}$$

Since the assumed specification for the utility function implies $\frac{\partial^3 u(c)}{\partial c^3} > 0$ and $E(e) = e$ in the economy with no uncertainty, households are forced to optimally increase asset-holdings in periods of good earning shocks to protect their welfare in periods of bad shocks (below the mean earning values). In other words they would try to use savings to establish an equality in equation (21) and get full insurance against future earning risks. As a result, it follows that aggregate asset accumulation also differs between the two economies and that the interest rate $r < 1/\beta - 1$. This inequality holds even more strongly when individuals are faced with the possibility to be borrowing-constrained and the accumulation of assets can result even higher at each earning state.

[2] Indeed, the fact that the economy is in a stationary equilibrium implies that we practically drop the time subscript in the Euler equation.

Another way to analyze the role of precautionary asset accumulation in the benchmark economy is to use the functional form assumed for the utility function and then manipulate equation (20). Taking the exponential and log of (20) and then using the fact that, given any variable z, $E[\exp(z)] = \exp[E(z)] + \frac{1}{2}Var(z)$, the equation becomes:

$$\log[\beta(1+r)]+\gamma\{\log(ra+we)-E[\log(ra+we)]\}+\frac{1}{2}\gamma^2 Var[\log(ra+we)] = 0 \quad (22)$$

The optimal asset accumulation decision in the uncertain environment takes into account two terms that are absent in the decision problem under certainty: the possibility of being surprised $\log(ra+we)-E[\log(ra-we)]$ and the level of risk associated to the earning process $Var[\log(ra-we)]$. Both term are augmented by the parameter that governs the risk aversion and the curvature of the utility function, γ.

Aggregating over the distribution of the households' states we obtain:

$$\log[\beta(1+r)] + \Phi + \Sigma = 0 \quad (23)$$

$$\Phi = \gamma \int \{\log(ra+we) - E[\log(ra+we)]\} dX(s)$$

$$\Sigma = \int \frac{1}{2}\gamma^2 Var[\log(ra+we)] dX(s)$$

where it is visible the contribution of precautionary savings to the aggregate capital accumulation. The two terms Φ and Σ would be zero in an environment with no uncertainty and equation (23) would assume (after taking the exponential on both sides) the same form as (19). The higher the variance of the earnings the larger will be the difference between the optimal capital accumulation in the two economies since the higher will be the two terms that account for uncertainty (i.e. insurance needs). As a consequence, if insurance contracts were available, their demand would increase int he variance of the earnings shocks.

If labor is supplied elastically, households have an additional decision variable to adjust in the face of uncertainty. Optimal asset-holdings can will then be modified by labor supply choice. I will deal with this issue more extensively in the numerical analysis.

4 Numerical analysis

I solve the model and use it to study numerically the optimal decision rules characterizing the two economies and how they differ with the presence or absence of idiosyncratic uncertainty and borrowing constraints.

4.1 Solution method

I solve for a initial steady state equilibrium for both economies in $t = 0$. I study the economic properties of the two economies in the steady state and the difference between the decision rules in presence of uncertainty and missing insurance markets. Then I introduce a tax on capital income and I compare the two economies again.

For both economies, I first compute a general dynamic equilibrium and then find the stationary equilibrium by assuming that the aggregate states of the economies are time independent.

I first start computing the equilibrium for the benchmark economy. To compute an equilibrium, I use an inner loop to solve for the households stochastic optimization problem and then aggregate the individual optimal decision rules to obtain the aggregate variables. I discretize the state space $A \times E$, using $g \times 7$ grid points to determine the assets and earning space, as a function of the scale and the standard deviation of the earning ability shock σ: $E = \{e_1, ..., e_7\}$ and $A = \{a_1, ..., a_g\}$. The discretization of the space allows a solution of the household Euler equations and to find the value function over the entire assets space for the different types of households using a Newton-Raphson method.

With an outer loop, the algorithm searches for convergence in the return on capital and the wage rate that, given the aggregate capital and labor, would satisfy all the conditions for the recursive equilibrium. The solution method is described in further details in Appendix A1.

The equilibrium in the economy without idiosyncratic uncertainty is computed similarly but assuming $\widehat{\mu} = \mu^*$ over the same earning ability space $E = \{e_1, ..., e_7\}$ as in the benchmark economy. With an inner loop, I compute optimal individual decision rules using the same Newton-Raphson method but I do not need to discretize the asset space to keep track of the asset distribution. With an outer loop I aggregate and check for prices convergence.

4.2 Parametrization

In order to pursue a numerical analysis of the stationary equilibria I need to pick parameters in both the economies. Since the time period in the model is one year, all the parametrized values are in yearly terms. The parameters are mainly taken from the standard literature as the exercise is mainly illustrative; I though perform an experiment calibrating the economies to relevant facts of the US economy.

The following Table 1 reports the value assigned to some of the parameters characterizing the steady state in the benchmark economy. The economy with no uncertainty is parametrized to the same values.

Table 1

Parameter values in the initial steady state equilibrium

		inelastic labor	*elastic labor*
Capital share in the production function	θ	0.3	0.3
Depreciation rate	δ	0.05	0.05
Time preference parameter	β	0.92	0.93
Share of consumption in the utility function	α	1.0	0.48
Relative risk-aversion parameter	γ	2.0	2.0
Total factor productivity	Ψ	0.87	0.89

The parameters for the specification of the Cobb-Douglas technology and the depreciation of physical capital are standard. The capital's share θ in the production function is set to 0.3 and the depreciation rate δ is set to 0.05. Given the specified technology, the capital-to-output ratio is targeted to 3.36, as in Cooley and Prescott (1995) and Domeij and Heathcote (2004). To reproduce this fact, the time preference parameter β is set to 0.93.

The share of consumption α in the utility function is 0.48 when labor is supplied elastically and households value leisure and the coefficient of relative risk aversion γ is 2.0 as in most of the macroeconomic literature. Given the latter, the α is chosen as to make the average working hours of a household to be the 40% of the maximum available time as in Nishiyama and Smetters (2005).

4.2.1 Earning ability process

Households' earning abilities and their stochastic properties are a key feature of the benchmark economy since they will generate agents' ex-post heterogeneity in asset-holding decision rules. The probability measure of household across the earning abilities states $\mu(E)$ and the matrix that defines the transition probabilities between two states, Π, are crucial to the introduction of heterogeneity in the distribution of capital and labor income. It is also necessary to define the persistence ρ and variance σ^2 of the earning ability shock and derive the condition that ensures that μ_t converges to a unique ergodic distribution μ^*, independent on the initial measure μ_0. The process involves finding the eigenvector μ associated with the unit eigenvalue of the matrix Π, such that $\mu = \mu\Pi$.

The earning ability state space is discretized using seven grid points, $E = \{e_1, ..., e_7\}$ and a Markov chain with seven states is used to approximate a first-order autoregressive process for the logarithm of the earning ability shock e_t, as in Aiyagari (1994). The autoregressive process approximated is:

$$\log(e_t) = \rho \log(e_{t-1}) + \sigma(1-\rho^2)^{\frac{1}{2}} \epsilon_t \tag{24}$$

where ρ represents the serial correlation, σ represents the coefficient of variation, and ϵ_t the innovation of the earning ability shock e_t. The algorithm to approximate the continuous representation of the precess follows Tauchen (1986). The algorithm is implemented with a serial correlation $\rho = 0.9$ and a coefficient of variation $\sigma = 0.6$, which implies a standard deviation of the residual of autoregressive representation of the earning ability process (24), $\sigma(1-\rho^2)^{\frac{1}{2}} = 0.26$. Both values of the serial correlation and the standard deviation of the ability shock are in line with the range of values found in many empirical studies of data from PSID (Card, 1991; Flodén and Lindé, 2001; Storesletten, Telmer, and Yaron, 2001).

4.3 Precautionary savings and insurance: numerical results

I use the parametrized version of the two economies to study numerically how uncertainty and incomplete markets modify individual saving behaviors, in the way I showed theoretically in the Section 3. Household's decision rules and macroeconomic aggregates differ in the two economies, in particular at the individual level. As Aiyagari (1994, 1995) points out, an economy where individuals face idiosyncratic uncertainty and borrowing constraints without having the possibility to buy insurance contracts would accumulate inefficient asset stocks. I therefore compare the welfare levels of each ability class (defined by equation 1) in the benchmark economy and in the economy with no uncertainty. I compare behaviors of households born with an identical ability to work, facing idiosyncratic shocks and borrowing constraints without the possibility to buy insurance with an economy where households are perfectly insured.

I first analyze the relevant economic variables in the two economies when households supply work inelastically in Table 2. The table presents the percentage differences between decision rules and aggregate variables in the benchmark economy and where markets are perfect (i.e. with no uncertainty).

Table 2

Difference(%) between benchmark economy and economy with no uncertainty

Inelastic labor supply

Aggregates variables			
Capital (K)	18.41		
Consumption (C)	12.03		
Individual decision rules and welfare (by earnings ability class)			
	Assets	*Consumption*	*Welfare*
e_1	4.94	-10.28	-18.20
e_2	10.02	-5.00	-13.57
e_3	17.65	-1.68	-7.96
e_4	20.27	9.06	-1.43
e_5	35.24	20.49	-0.49
e_6	44.81	32.64	-0.18
e_7	57.28	44.39	-0.04

Households' behavior differ substantially in an economy with no possibility to insure. In particular, we notice that asset accumulation is generally higher in the benchmark economy across all the earning classes because of the effect of uncertainty and borrowing constraints. The latter are particularly binding for low earning ability classes, so we can infer that this would explain most of the difference in the asset accumulation between the two economies. Low earning classes are forced to retain more asset and enjoy less consumption due to the impossibility to to borrow when constraints are binding. Further, not only asset accumulation is higher across all classes and in the aggregate but is even higher among high earning ability classes.

As I mentioned in Section 3, individuals in presence of uncertainty and no possibility to trade on insurance markets will find it optimal to accumulate higher stocks of capital compared to low earning ability classes, since uncertainty for them means falling in some lower earning ability state. They also enjoy higher consumption compared to the low income classes.

Looking into differences in welfare between the two economies, we can notice that all classes are worse off in the benchmark economy. They accumulate more assets but are forced to reduce levels of consumption compared with the same income classes in an economy with perfect certainty. The high ability classes, though, are able to save more and to enjoy higher level of welfare, very close to the one that individuals belonging to the same earning ability class would enjoy in presence of no uncertainty (or i.e. perfect insurance markets).

I now turn to the analysis of the same economies when households value leisure and labor is supplied elastically. Table 3 reports the same relevant variables but adds the labor supply dimension to the individual decision rules space.

Table 3

Difference(%) between benchmark economy and economy with no uncertainty

Elastic labor

Aggregates variables				
Capital (K)	22.21			
Labor (L)	-3.22			
Consumption (C)	9.51			
Welfare	-3.21			
Individual decision rules and welfare (by earnings ability class)				
	Assets	*Labor*	*Consumption*	*Welfare*
e_1	5.17	-11.29	-27.63	-9.08
e_2	10.24	-10.39	-20.69	-7.75
e_3	17.45	-9.13	-12.39	-6.06
e_4	24.07	-4.23	0.68	-3.95
e_5	36.09	0.94	16.95	-1.10
e_6	50.94	5.25	36.21	-0.08
e_7	68.84	7.29	59.64	-0.01

The possibility to use labor supply choice in the face uncertainty when enjoying higher ability states allows households to accumulate even more assets in good states. As a result, high income classes enjoy higher welfare. Lower income classes enjoy more leisure but relatively much less consumption and their welfare levels are lower.

4.4 Introduction of a capital income tax

I now turn to possibility for capital income taxation to play the role of substituting for missing insurance markets. As highlighted by the seminal work of Chamley (1986), in a general equilibrium model with infinitely lived agents and no uncertainty, capital income taxation plays only the role of distorting intertemporal allocation of resources. In presence of uncertainty and missing insurance markets can play a role of social insurance mechanism.

I assume that in the benchmark economy a government taxes linearly income from capital at the marginal rate τ_c and rebates the tax revenue $T(a_t)$ back to the households in the form of a lump-sum transfer TR. In this way. the government cares about balancing the budget at each period:

$$\begin{aligned} TR_t &= \int_{A\times E} T(a_t) dX(s_t) \\ T(a_t) &= \tau_c r a_t \end{aligned} \tag{25}$$

The household problems is therefore modified and the Euler equation becomes:

$$\frac{\partial u(c,h)}{\partial c} = \beta[1 + r(1-\tau_c)]E\left[\frac{\partial u(c,h)}{\partial c}\right] + \eta \tag{26}$$

The equation points at the fact that now the return on capital is taxed at the rate τ_c. Therefore, in taking the decision on the optimal asset accumulation the household should account for the fact that by accumulating more assets it can incur in higher taxes on income from capital. In this way, the taxation system acts at the same time as a distortionary mechanism and a risk-sharing device. As seen int he previous section, individuals in high ability states are able to accumulate more assets and as a result of the introduction of an income tax now they will also pay more taxes. The opposite is valid for the households in low ability states. Furthermore, the lump-sum transfers mitigates the conditions of households with binding borrowing constraints. In this way, the all system substitutes for missing insurance markets.

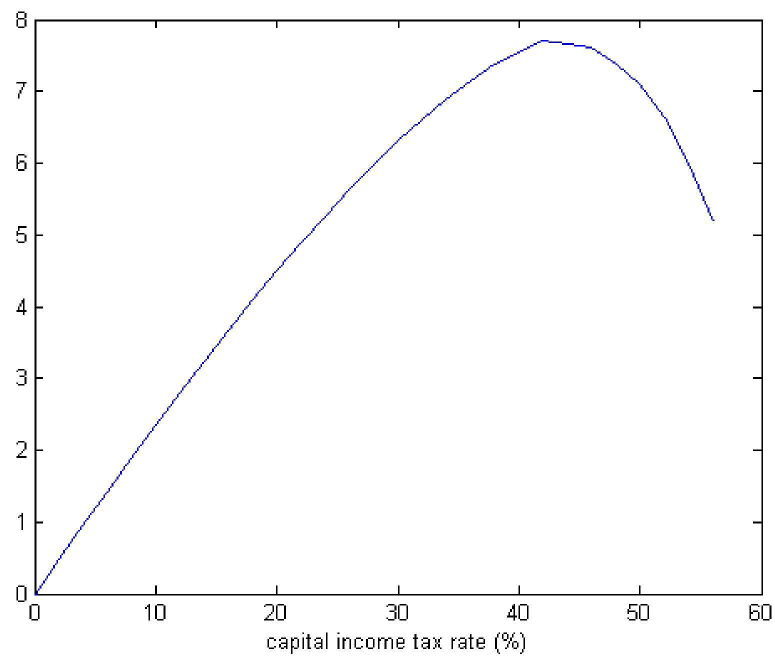
To study how welfare can be ameliorated by the introduction of a capital income tax, I compute the welfare gains in the benchmark economy brought by the introduction of different levels of marginal tax rates τ_c.

A welfare gain is the percentage increase in household's wealth or consumption equivalent Δ that leaves the household indifferent between the introduction of the tax and the case with no taxes, analyzed in the previous sections. The aggregate welfare gain accounts for the distribution of the welfare gains among households belonging to different income classes as follows:

$$\begin{aligned} &\int_{A\times E} \sum_{t=0}^{\infty} \beta^t u(c_t(s_t, S_t; \tau_c), h_t(s_t, S_t; \tau_c)) dX(s_t) \\ &= \int_{A\times E} \sum_{t=0}^{\infty} \beta^t u((1+\Delta)c_t(s_t, S_t), (1+\Delta)\ell_t(s_t, S_t)) dX(s_t) \end{aligned} \tag{27}$$

Figure 1 displays how aggregate welfare gains increase in the benchmark economy as effect of the introduction of higher marginal tax rates up to a maximum point. After that point, the increase in efficiency provided by higher capital taxation as substitute for missing insurance markets starts being offset by the loss in terms of distortionary effects on resources allocation.

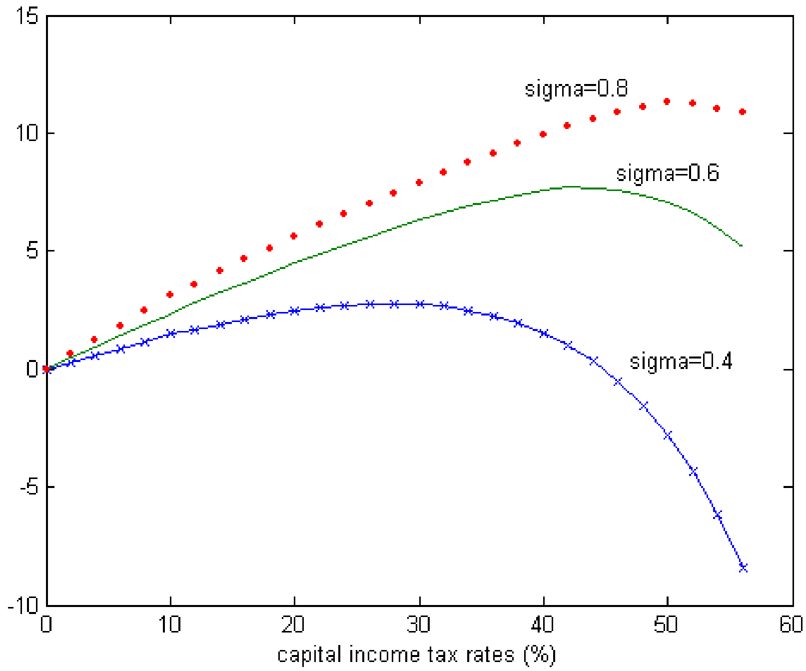
Figure 1: Welfare gains (%) from the introduction of capital income tax



It is optimal then to introduce a capital income tax up to the level at which this maximizes the aggregate welfare gains. In the case of the benchmark economy under study the optimal level of tax rate would be set at $\tau_c = 41\%$.

The demand for insurance increases with the risk associated to the earning ability shocks, as mentioned in (22). I thus compute welfare gains associated with the introduction of a capital income tax in presence of different levels of the coefficient of variation of the earning ability process, σ. Figure 2 plots welfare gains when $\sigma = 0.6$ (standard deviation of the innovation term 0.26), as in Figure 1, against welfare gains associated with higher or lower levels of σ, respectively $\sigma = 0.4$ (standard deviation of the innovation term 0.17) and $\sigma = 0.8$ (standard deviation of the innovation term 0.35).

Figure 2: Welfare gains (%) from the introduction of capital income tax by different earning risks



I find that the level of marginal tax rate increases with the increase in the volatility of the earning shocks and with a consequent higher demand for insurance.

I thus turn to the analysis of how households' economic behavior change in the benchmark economy after the introduction of the optimal capital income tax, $\tau_c = 41\%$. Table 4 compares differences in the two economies under exam in presence or absence of capital income taxation.

Table 4

Difference(%) between benchmark economy and economy with no uncertainty

Macroeconomic aggregates						
	a. no capital income tax			*b.* with capital income tax (τ_c= 41%)		
Capital (K)	29.21			0.91		
(by earnings ability class)						
	Assets		Consumption		Labor	
	a.	*b.*	*a.*	*b.*	*a.*	*b.*
e_1	5.17	-19.50	-27.63	-28.31	-11.29	-26.75
e_2	10.24	-16.67	-20.69	-18.90	-10.39	-18.83
e_3	17.45	-13.12	-12.39	-11.50	-9.13	-14.51
e_4	24.07	-9.80	0.68	-0.41	-4.23	-7.88
e_5	36.09	3.64	16.95	8.67	0.94	0.06
e_6	50.94	24.45	36.21	25.26	5.25	4.41
e_7	68.84	41.76	59.64	35.24	7.29	7.23

The introduction of a capital income tax (case *b.* in the Table) produces a reduction in asset accumulation across all the income classes in the benchmark economy.

Among the low income classes, the asset decrease at a level below the one in the economy with no uncertainty, so differences assume negative values. Consumption levels though remain almost unchanged for these classes. Labor supply falls and differences with the economy with no earnings uncertainty become even more negative indicating that households in low ability states have incentives to work less (wage rate w per earning ability decreases).

For the high income classes the optimal asset holdings fall but consumption falls by even more. Labor supply remains almost unchanged.

In presence of uncertainty and missing insurance markets, capital income taxation can discourage asset accumulation for each earnings class and serve as a. insurance device. Households faced with a low earning shock will need to have less resource to pay for taxes. At the same time, lump-sum transfers redistribute income across income classes so that in periods of bad luck households actually need less assets for insurance purposes and can enjoy more leisure.

As a result, the introduction of capital income taxation system modifies welfare among the different income classes. Table 5, in fact, compares difference in welfare between economies with and without uncertainty when we introduce capital income taxation.

Table 5

Difference (%) in the welfare between benchmark economy and economy with no uncertainty

(by earnings class)	*a.* no capital income tax	*b.* with capital income tax (τ_c= 41%)
e_1	-9.08	-2.12
e_2	-7.75	-2.35
e_3	-6.06	-2.44
e_4	-3.95	-1.12
e_5	-1.10	-0.89
e_6	-0.08	-1.33
e_7	0.02	-2.25
aggregate	-3.71	-1.55

While the introduction of capital income taxation produces a welfare improvement for low earnings classes high earnings classes are worse off. The tax produces a risk-sharing effect and the lump-sum rebate a further redistribution among income classes. The aggregate welfare appears closer to the one of an economy with no uncertainty or i.e. with perfect insurance markets and no borrowing constraints. A similar result appear in Varian (1980).

In Appendix A2 I show the welfare gains associated to the introduction of a capital income tax in the benchmark calibrated to US facts. I find that the tax rate that maximizes the aggregate welfare gain is $\tau_c = 34.5$ percent

5 Concluding remarks

In this paper I examine the role of capital income taxation in providing risk-sharing in an economy characterized by uninsurable idiosyncratic uncertainty and borrowing constraints. To this aim, I compare two economies that differ for presence and absence of uncertainty and complete markets. The economy where households face idiosyncratic uncertainty and borrowing constraints is characterized by higher individuals and aggregate asset accumulation.

I show that the introduction of a capital income taxation in this economy leads to a reduction in precautionary savings since the tax acts as a risk-sharing mechanism among individuals. I find that the capital income tax rate associated with the highest welfare gain is increasing with the earnings risk and with the consequent demand for insurance. I then calibrate the model to US data and find that the long-run capital tax rate providing the optimal level of risk sharing is 34.5 percent.

Appendix

A1. Solution method

To solve for an equilibrium I discretize the household state space $s \in A \times E$ using g grid points for the asset space, $A = \{a_1, ..., a_g\}$, and 7 grid points for the earning space $E = \{e_1, ..., e_7\}$. Consistent with the individual state space the aggregate state space of the economy will be $S = (x(s), \cdot)$.

Steady state equilibrium: the initial steady state is characterized by a time-invariant $S = (x(s), \cdot)$. Given this schedule the algorithm uses an inner loop to compute the individual optimal behavior, as follows:

1. Set the initial values for the capital-to-labor ratio, $K/L^0_{t=0}$, and given the production function specification and equilibrium conditions compute the return on capital $r^0_{t=0}$ and the wage rate $w^0_{t=0}$

2. Given $r^0_{t=0}$ and $w^0_{t=0}$ find the optimal household decision rules $d_t(s, S_{t=0})$ for all points in the state space $s \in A \times E$ as follows. Guess an initial value for the next period asset holdings $a^0_{t+1}(s, S_{t=0})$ and compute the optimal consumption and working hours using first order conditions:

$$\begin{aligned} c^0_{t=0}(s, S_{t=0}) &\in (0, c^{\max}(a^0_{t+1})] \\ h^0_{t=0}(s, S_{t=0}) &\in [0, 1] \end{aligned}$$

3. Compute the numerical derivative of the value function with respect to the asset holding $V_a(s, S_{t=0})$, and the value function $V(s, S_{t=0})$.

4. Plug optimal decision rules $c^0_{t=0}$, $h^0_{t=0}$ (found for each possible individual state) in the Euler equation (for consumption) along with the $V_a(s, S_{t=0})$. Check whether the Euler equation holds with a small error[3] and thus stop. If the error is not small enough instead update the guess a^0_{t+1} and repeat the process from step 2 through 4 again. The algorithm here uses a bisection search to update and find the optimal next period asset holding, given the individual states.

5. Compute the measure of households on the asset and earning state space $x(s, S_{t=0})$ through linear interpolation using the decision rules found with steps 2 through 4.

6. An outer loop at this point computes the aggregate variables, the new $r^1_{t=0}$ and $w^1_{t=0}$ consistent with the measure $x(s, S_{t=0})$.

[3] The convergence criterion for the Euler equation to converge is set to a tolerance degree of 10^{-5}.

7. Compare the $r^1_{t=0}$, $w^1_{t=0}$ with $r^0_{t=0}$, $w^0_{t=0}$ if the difference is sufficiently small[4] then stop. Or otherwise update the guesses and start from step 1 again.

[4] The tolerance in this case is set as follows:

$$\max\left\{\left|K/L^1 - K/L^0\right|, \left|G^1 - G^0\right|\right\} < 10^{-5}$$

A2. Optimal capital income tax rate: calibration to US facts

Using the benchmark economy model I compute the capital income tax rate that is associated with the higher welfare gains when the economy is calibrated to US facts. In order to represent the US economy I need to modify the government budget constraint. I assume at the beginning that government only taxes income from labor at the rate τ_l and then I will introduce a tax on capital τ_c. Government can use the revenue from taxes $T(a_t, h_t(s_t, S_t))$ to make lump-sum transfers TR to the households and to fund its own expenditure G, which can be considered as a waste from the household point of view. Government can also accumulate public debt over time B so that the government budget constraint is modified as follows:

$$B_{t+1} = (1 + r_t)B_t + G_t + TR_t - \int_{A\times E} T(a_t, h_t(s_t, S_t))dX(s_t) \quad (28)$$

The following Table A2 summarizes the relevant parameter values and US economic facts in the stationary equilibrium of this economy:

Table A2

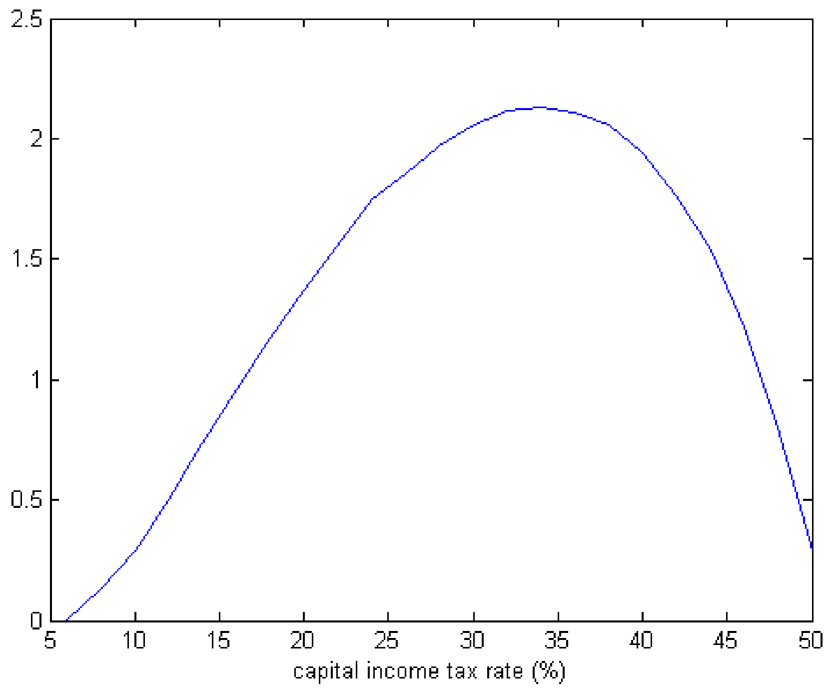
Parameter values in the initial steady state equilibrium

		Facts	*parameters*
Capital share in the prod. function	θ		0.3
Depreciation rate	δ		0.05
Time preference parameter	β	K/Y=3.36	0.94
Share of cons. in the utility function	α	avg hours worked 40% avail. time	0.59
Relative risk-aversion parameter	γ		2.0
Total factor productivity	Ψ	w=1	0.88
Government policy facts			
Government expenditure	G	$G/Y = 0.089$	
Lump-sum transfers	TR	$TR/Y = 0.9$	
Government debt	B	$B/Y = 0.37$ (debit held by the public)	
Labor income taxation	τ_l	0.267	

Government data: source NIPA and CBO Economic Outlook 2007

The parameters characterizing the earning process assume the same value described in Section 3: $\sigma = 0.6$ $\rho = 0.9$. Using the benchmark economy calibrated to this relevant facts, I find the the optimal capital income tax is $\tau_c = 34.5\%$ not different from the findings of Domeij and Heathcote (2004) and Conesa, Kitao and Krueger (2007).

Figure A2: Welfare gains (%) from the introduction of capital income tax



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