Optimal Monetary Policy in a Financially Fragile Economy

Ufuk Devrim Demirel*

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Abstract

This paper studies optimal monetary policy in an economy where firms use external funds to finance operational costs. It is shown that direct and indirect cost channels for monetary policy arise when firms can default on borrowed funds. The direct cost channel calls for milder contractions in the face of inflationary pressures. The indirect cost channel, on the other hand, encourages policy conservatism and suggests stringent anti-inflationary measures. It is found that a super-inertial interest rate feedback rule can implement the globally optimal plan as the unique rational expectations equilibrium.

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*University of Colorado at Boulder, Department of Economics, 256 UCB, Boulder, CO, 80309, USA, tel: (303) 492 2585, email:demirel@colorado.edu. I would like to thank Pavel Kapinos, Wolfgang Keller, Robert King, Jinill Kim and, especially, Martin Boileau for helpful comments. I have also benefited from comments by session participants at the 2007 conference of the Society for Computational Economics, 2007 Midwest Macro Conference and the 2007 annual meetings of the Eastern Economic Association.

1 Introduction

The recent "credit crunch" and the subsequent reduction of the Federal Funds Rate has spurred yet again the discussion over how monetary policy should be conducted during times of financial distress. The rate cut has its critics: Some view it as "bailing out" overexposed financial circles while penalizing sound investors.[1] Others think that it is appropriate and justified on the basis of economic fundamentals since "tightening credit conditions" have "adverse affects on broader economy".[2] The heated discussion raises a number of questions regarding the nature of the relationship between credit markets and the rest of the economy, and regarding the appropriate monetary responses in the face of worsening financial conditions. How should monetary policy respond when business bankruptcies become more frequent or when the costs associated with financial contracts become more severe? What can the policy maker do to mitigate the undesirable aspects of a fragile relationship between production and financial sectors? The following analysis proposes answers to these questions from a welfare-maximizing viewpoint in the context of an otherwise standard new Keynesian model modified to allow firm borrowing and possible default in the manufacturing sector.

The monetary policy environment differs substantially from the standard new Keynesian setting when firms depend on external funds to finance projects. External finance links the nominal rate of interest directly to firms' marginal cost. In the literature, this relationship is referred to as the cost channel. The following analysis considers an environment where firms can possibly default on borrowed funds. The interest rate faced by firms in the loanable funds market deviate from the risk-free rate insofar as the possibility of default induces a risk premium. The dependence of the cost of borrowing on risk premia generates an indirect cost channel through which monetary policy's leverage over default incentives influence equilibrium marginal cost dynamics. I explore the optimal policy implications of default risk and the wedge it drives between the risk-free rate and the rate faced by producers. Under a reasonable parameterization, I find that the optimizing policy maker chooses to exercise a more stringent control over inflation as the cost of financial monitoring increases. This pattern obtains regardless of the degree of commitment with which policies are conducted.

Christiano and Eichenbaum (1992), Khan et al. (2003) and Christiano et al. (2005) account for a cost channel by studying economies where firms must borrow working capital to pay for their operational costs.[3] This study contributes to this line of research by exploring an indirect cost channel introduced by default risk in addition to the previously studied direct cost channel. The resulting environment involves certain peculiar policy trade-offs, which are relevant to the

[1] See the article by Allan Sloan, Fortune Magazine, September 28th, 2007.

[2] See the FOMC press release on September 18th, 2007.

[3] Barth and Ramey (2001) and Ravenna and Walsh (2006) present empirical evidence for the cost channel.

extent that default is an issue and financial monitoring is costly. Ravenna and Walsh (2006) show that in the presence of a direct cost channel the policy maker always faces a trade-off between inflation and output gap stabilization even when inherently cost-push disturbances are absent. This is because any shock can exhibit cost-push characteristics in the presence of a direct cost channel and can pose a trade-off between inflation and output gap stabilization. In the face of this trade-off, strict inflation targeting is not optimal and the policy maker allows for a certain amount of inflation variability. I show that if an indirect cost channel is present in addition to a direct cost channel the optimal magnitude of inflation fluctuations is considerably smaller. This is due to a steepening in the aggregate supply schedule which makes inflation stabilization less costly in terms of output gap stability. Therefore, considerations of default risk (under costly financial monitoring) yields stricter inflation targeting relative to the previous studies which only account for a direct cost channel.

I adopt the agency costs/financial accelerator framework developed by Bernanke and Gertler (1989) and Bernanke et al. (1998) to incorporate the direct and indirect cost channels and endogenously account for default risk in the optimal policy problem. The implication of this treatment is threefold: First, firms' marginal costs are directly linked to the interest rate as firms are required to borrow to finance operational costs. This calls for sparing use of nominal rates to fight inflation since interest rate hikes themselves have direct inflationary consequences. Second, the modification alters the aggregate supply relationship to make inflation more sensitive to output deviations. In other words, it implies a steeper aggregate supply schedule. This is because firms can raise the external funds needed to support a larger-scale production only if they accept to pay higher interest. Due to increased default likelihood, risk premia rise as firms borrow more to expand production. This renders firms' real marginal cost more sensitive to changes in the output level. The policy maker understands that, in this environment, output movements are associated with relatively larger fluctuations in inflation. Equivalently, a given decline in inflation can be engineered with a relatively smaller output loss. This tempts the policy maker to take a more aggressive stance against inflation. Third, output fluctuations in this environment are relatively more welfare-reducing compared to a standard new Keynesian economy. This encourages the policy maker to adopt a less aggressive anti-inflationary stance in the face of a trade-off between inflation and output deviations. Even though the optimal policy calls for a compromise between these three tendencies, our overall results are mainly driven by the second implication that the opportunity cost of inflation stabilization is lower if firms can default on loans and financial monitoring is costly. In consequence, the optimal monetary policy involves more stringent inflation targeting under an indirect cost channel induced by default risk.

Another contribution is the formulation of an optimal and implementable interest rate feedback rule. As discussed by Woodford (1999), it often proves quite difficult to formulate an interest rate rule that can successfully implement the optimal plan and maintain equilibrium determinacy. I find that the optimal plan in this setting can be implemented by the means of a "super-inertial" interest rate feedback rule which governs the evolution of the risk-free rate. This provides a justification for interest rate smoothing and parallels the findings of Woodford (1999).

The paper is organized as follows: The theoretical model is laid out in section 2. I fully motivate and discuss the welfare criterion and policy constraints used in the optimal policy analysis in section 3 with particular reference to the idiosyncrasies of the trade-offs and restrictions the setting involves. Analytical expressions for optimal targeting rules are derived as well as some numerical results for the optimal implementable interest rate policies. Section 4 concludes.

2 Model

This section lays out the theoretical framework. Most features of the model resembles the standard new Keynesian setting as discussed in Clarida et al. (1999). The economy is populated by five types of agents: Households, manufacturers, retailers, financial intermediaries and a government. Time is discrete. The economy involves aggregate shocks in the retail sector as well as idiosyncratic shocks in the manufacturing sector. Aggregate shocks are revealed at the start of each period. The end of period t and the start of period $t+1$ are infinitesimally close.

2.1 Households

Households are identical and maximize the expectation of a discounted sum of utilities. Household utility depends on consumption and work effort. They also have access to a bond market where financial intermediaries, households and the government trade one-period risk-free bonds. Households maximize

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \kappa \frac{L_t^{1+\chi}}{1+\chi} \right) \tag{1}$$

subject to

$$M_{t+1} \leq M_t + (1+s_e) W_t L_t - P_t C_t - B_t - T_t + R_t^f B_t + \int_j \pi(j)_t^m dj + \int_i \pi(i)_t^r di + \pi_t^f \tag{2}$$

$$P_t C_t \leq M_t + (1+s_e) W_t L_t - B_t \tag{3}$$

where $\kappa > 0$, $\chi > 0$. The variable C denotes a composite consumption good, L is work effort, B denotes the outstanding nominal bond holdings, R^f denotes the gross risk-free nominal interest rate, P stands for the price level and W is the nominal wage. The variables $\pi(j)^m$, $\pi(i)^r$ and π^f denote manufacturer, retailer and financial intermediary profits in nominal terms and M stands for money holdings. Household wage income is subsidized at the rate s_e. Households own the firms and claim their profits each period. They also face a lump-sum tax/transfer scheme denoted by T. Households enter each period with cash holdings M. They receive wage income together with employment subsidies in cash, pay lump-sum taxes (or receive transfers) and make their consumption and bond holding decisions earlier in the period. Consumption expenditures cannot exceed money holdings plus wage income as required by the cash-in-advance constraint (3). Bonds mature at the end of the period. Manufacturer, retailer and financial intermediary dividends are transferred to households also at the end of each period after production takes place and profits are realized.

The composite consumption index that enters the utility function is defined as an aggregation of a continuum of differentiated products indexed by $i \in [0,1]$. More specifically,

$$C_t = \left[\int_0^1 \left(C(i)_t^{1-\frac{1}{\eta}} \right) di \right]^{\frac{\eta}{\eta-1}} \tag{4}$$

where $\eta > 1$ measures the elasticity of substitution between differentiated products. The consumption based price-index for the households can then be found as

$$P_t = \left[\int_0^1 \left(P(i)_t^{1-\eta} \right) di \right]^{\frac{1}{1-\eta}}. \tag{5}$$

Note that (5) gives the minimum expenditure required to assemble one unit of composite consumption. It follows from (4) and (5) that the allocation of individual demand across differentiated goods is governed by the rule

$$C(i)_t = \left(\frac{P(i)_t}{P_t} \right)^{-\eta} C_t. \tag{6}$$

Maximization of (1) subject to (2) and (3) yield the following standard first-order optimality conditions:

$$\frac{1}{C_t} = \beta E_t \left(R_t^f \frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}} \right) \tag{7}$$

$$\frac{\kappa L_t^{\chi}}{C_t^{-1}} = (1 + s_e)\frac{W_t}{P_t} \tag{8}$$

$$P_t C_t = M_t + (1 + s_e)W_t L_t - B_t \tag{9}$$

Equation (7) is the standard intertemporal substitution equation and (8) is the familiar labor supply relation. The cash-in-advance constraint binds as suggested by equation (9) in an equilibrium with positive nominal interest rates.

2.2 Production

Production activities are performed in a two-sector environment populated by manufacturer and retailer firms. Manufacturing firms produce homogenous intermediate inputs for the retail sector. Monopolistically competitive retailers differentiate these homogenous products and set the price level for each differentiated product following the standard Calvo (1983) treatment.

2.2.1 Manufacturers

Manufacturing sector is populated by a continuum of ex-ante identical firms indexed by $j \in [0, 1]$. These firms hire labor in a perfectly competitive labor market and manufacture intermediate goods using a constant-returns-to-scale production technology. It is assumed that the factor productivity of each manufacturing firm is subject to idiosyncratic uncertainty. More specifically, the output of the j^{th} manufacturing firm at time t is given by

$$Y(j)_{m,t} = A(j)_t H(j)_t \tag{10}$$

where $A(j)$ is an idiosyncratic productivity shock which is assumed to be distributed independently and identically over time and across firms on the support $[0, \bar{A}]$ where $\bar{A} > 0$. The variable $H(j)$ is defined as a CES aggregate of labor hired by firm j and a fixed factor of production accessible to all manufacturers in a similar way:

$$H(j)_t = [\xi L(j)_t^{v} + (1 - \xi)N^{v}]^{\frac{1}{v}} \tag{11}$$

where $0 < \xi < 1$, $L(j)$ stands for the amount of labor hired by firm j and N denotes a fixed factor of production. In addition to land, the fixed factor N can be interpreted as entrepreneurial endowments or unpaid public resources that complement private production activities. As briefly mentioned in the previous section, manufacturers must pay for their operational costs up-front, before the actual production takes place.[4] Hence, they must borrow the funds needed to pay for the wage bill from financial intermediaries at the start of each period. Financial intermediaries have the option to buy or sell bonds directly from or to the central bank at the risk-free rate R^f in the bond market.

After the idiosyncratic uncertainty resolves and ex-post heterogeneity among firms materializes, subsequent to production, a manufacturing firm may choose to default on its debt in which case all the proceeds of its manufacturing-related activities are confiscated by its external financiers. Along the lines of Townsend (1979), it is assumed that, upon a default decision, financial intermediaries can observe the real state of a manufacturing firm only if they agree to pay a monitoring cost defined as a fraction, μ, of the firm's productive inputs.

Following Bernanke et al. (1998), the contractual arrangement is specified as follows: At the start of each period, after aggregate uncertainty resolves, ex ante identical manufacturing firms submit credit applications to the financial intermediaries to acquire the external funds needed to pay for manufacturing inputs stating the amount they want to borrow. The proposed size of the loan for the manufacturer j at the start of period t is denoted by $D(j)_t$ and is determined by its wage bill, that is

$$D(j)_t = W_t L(j)_t. \tag{12}$$

Taking into account the possibility of default, financial intermediaries then decide whether to accept the credit application. Risk-neutral intermediaries find the terms of the contract acceptable as long as it promises to deliver a non-negative expected return. Given the creditor behavior and outside option, manufacturers then determine a threshold productivity level, $A_t^*(j)$, below which they choose to default. If their productivity level turns out being higher than this threshold level, they choose to repay the loan at the rate determined by the debt contract. The problem faced by the j^{th} manufacturer is then defined as maximization of

$$\int_{A_t^*(j)}^{\overline{A}} \left[Q_t A_t(j) H(j)_t - R_t D_t(j)\right] \, dF(A_t) \tag{13}$$

subject to

$$\int_0^{A_t^*(j)} \left[Q_t A_t(j) H(j)_t - \mu Q_t H(j)_t\right] \, dF(A_t) + \int_{A_t^*(j)}^{\overline{A}} R_t D_t(j) \, dF(A_t) \geq R_t^f D_t(j) \tag{14}$$

and (12) by choosing $A_t^*(j)$ and $L(j)_t$, and taking as given the contract rate, R_t,intermediate

[4]See Christiano and Eichenbaum (1992), Ravenna and Walsh (2006) and Neumeyer and Perri (2005) for similar treatments.

goods price, Q_t, and the monitoring cost parameter, μ. Expression (13) is the expected profit for the manufacturer and (14) can be regarded as a participation constraint for the financial intermediary. It states that the expected return for the intermediary must be greater than or equal to the cost of the loan to the intermediary. The break-even point, A_t^*, is defined as the level of productivity at which the manufacturer is indifferent between defaulting and repaying the principle and interest. That is, $Q_t A_t^*(j) H(j)_t = R_t D_t(j)$. Given this relationship, the problem can be rewritten conveniently by dropping the firm index, reorganizing (13), (14) and using (12) as follows:

$$\max_{A_t^*, L_t} \; Q_t H_t \phi(A_t^*) \quad \text{subject to} \quad Q_t H_t \gamma(A_t^*) \geq R_t^f W_t L_t$$

where

$$\phi(A_t^*) = \int_{A_t^*}^{\overline{A}} (A_t - A_t^*) \; dF(A_t) \;\; \text{and} \;\; \gamma(A_t^*) = \int_0^{A_t^*} (A_t - \mu) \; dF(A_t) + \int_{A_t^*}^{\overline{A}} A_t^* dF(A_t)$$

The first-order-conditions for this problem yield:

$$Q_t H_{L,t} = \frac{R_t^f W_t}{\Phi(A_t^*)} \tag{15}$$

$$H_t Q_t \gamma(A_t^*) = R_t^f W_t L_t \tag{16}$$

where $H_{L,t} = \partial H_t / \partial L_t$ and

$$\Phi(A_t^*) = \frac{\phi'(A_t^*)\gamma(A_t^*) - \gamma'(A_t^*)\phi(A_t^*)}{\phi(A_t^*)} \tag{17}$$

where $\phi'(A_t^*) = \partial \phi_t / \partial A_t^*$. The labor demand equation (15) equalizes the value of the marginal product of labor to the cost of hiring an additional unit of labor. Since each manufacturer must borrow to hire labor and can possibly default on borrowed funds, this cost includes the risk-free interest rate as well as a risk premium embedded in the expression $\Phi(A_t^*)$. As the threshold, A_t^*, increases, so does the probability of default, making it costlier for the manufacturer to borrow and hire labor. The financial intermediaries, on the other hand, can diversify all the idiosyncratic risk which assigns the risk-free interest rate as their relevant opportunity cost. The manufacturers are then able to borrow until the participation constraint for financial intermediaries binds as conveyed by equation (16). Consolidating (15) and (16) we obtain

$$\frac{\gamma(A_t^*)}{\Phi(A_t^*)} = \frac{H_{L,t} L_t}{H_t} = \frac{\xi L_t^{\upsilon}}{\xi L_t^{\upsilon} + (1 - \xi) N^{\upsilon}}. \tag{18}$$

Equation (18) establishes a relationship between risk premia and the aggregate work effort. Under the distributional assumption we adopt later, this relationship is a monotonically increasing one implying an upward-sloping supply schedule for loans. Increased borrowing to expand production, therefore, entails higher risk premia which translates into increased marginal costs in equilibrium. As will become clear in the next section, this feature introduces a dimension along which the policy trade-offs considerably differ from those that would be relevant in an environment free of all capital market imperfections.

2.2.2 Retailers

In order to incorporate the sticky price framework into the picture, it is assumed that the economy is also populated by monopolistically competitive retailers each of which sells a differentiated product indexed by i. Each retailer purchases intermediate goods from the manufacturers and transforms them into differentiated final products using a constant-returns-to-scale transformation technology:

$$Y(i)_t = Z_t Y(i)_{m,t}$$

The variable $Y(i)_m$ denotes the manufactured intermediate input for the retailer i, the variable Z is the retail sector productivity parameter and $\log Z_t = \rho \log Z_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma_z^2)$, $0 \leq \rho < 1$. The retailers set the price level for each differentiated good as in Calvo (1983), that is, each retailer can change its price level during a given period only with probability $(1 - \theta)$. They face a downward sloping demand curve in the final goods market and take the price of the intermediate good as given since the intermediate goods market is perfectly competitive. Retailers also receive an ad valorem production subsidy $(1 + s_p)$. Each period, a measure $(1 - \theta)$ of randomly selected retailer firms set their prices, $P(i)_s$, to maximize,

$$E_s \sum_{t=s}^{\infty} \theta^{t-s} \lambda_{s,t} \left\{(1 + s_p) P(i)_s Y(i)_t - MC(i)_t Y(i)_t\right\} \tag{19}$$

subject to the demand function implied by (6). The variable $\lambda_{s,t}$ denotes the stochastic discount factor with which retailers value time-t random nominal income. It is defined as

$$\lambda_{s,t} = \beta^{t-s} \frac{C_s}{C_t} \frac{P_s}{P_t}.$$

$MC(i)$ stands for the nominal marginal cost for retailer i and is given by

$$MC(i)_t = \frac{Q_t}{Z_t}.$$

where Q denotes the intermediate good price. First-order-conditions for the retailer problem imply that retailer i sets its price level to satisfy:

$$E_s \sum_{t=s}^{\infty} \theta^{t-s} \lambda_{s,t} \left\{ Y(i)_t \left((1+s_p)P(i)_s - \frac{\eta}{\eta - 1} \frac{Q_t}{Z_t} \right) \right\} = 0 \tag{20}$$

Equation (20) states that the expected discounted marginal cost of the firm must be equalized to its expected discounted marginal revenue over the period during which the price level is not allowed to adjust. Note that the real marginal cost for retailers is given by the relative price of intermediate goods and final retail goods divided by the productivity parameter, Z_t. Given (20), the overall price level can be found to evolve according to the following rule:

$$P_t = [\theta(P_{t-1})^{1-\eta} + (1-\theta)(P_s)^{1-\eta}]^{\frac{1}{1-\eta}} \tag{21}$$

2.3 Financial Intermediaries

Financial intermediaries are identical and competitive. They participate in the bond market and, at the same time, provide loans to the manufacturers. They acquire the funds they need for risky manufacturer loans in the bond market where they face the gross risk-free nominal interest rate, R_t^f, which defines the opportunity cost of the funds supplied to the manufacturers. Financial intermediaries hold perfectly diversified portfolios of risky manufacturer loans. This allows them to eliminate all the idiosyncratic risk involved with these risky debt contracts. Let R_t^D denote the gross rate of return on a diversified portfolio of manufacturer loans, D_t^s. The financial intermediary problem then can be stated as

$$\max_{B_t^I, D_t^s} R_t^f B_t^I + R_t^D D_t^s - B_t^I - D_t^s \tag{22}$$

subject to

$$B_t^I \leq 0 \tag{23}$$

$$0 \leq D_t^s \leq -B_t^I. \tag{24}$$

At the beginning of each period, financial intermediaries acquire the funds they need to finance manufacturer loans by selling bonds. Therefore, their beginning-of-period bond position, B_t^I, must be non-positive. Maximization of (22) subject to (23) and (24) reveals that an equilibrium with positive loans must satisfy

$$R_t^f = R_t^D \quad \text{and} \quad D_t^s = -B_t^I.$$

But we already know from manufacturer optimization that in equilibrium, $R_t^D = Q_t H_t \gamma(A_t^*)/D_t$, where $D_t = W_t L_t$. Therefore,

$$R_t^f = R_t^D = \frac{Q_t H_t \gamma(A_t^*)}{W_t L_t}$$

which is consistent with the manufacturer optimality condition (16).

2.4 Government

The government is benevolent and conducts fiscal and monetary policies to maximize household utility. As mentioned above, the fiscal toolkit includes both distortionary and lump-sum tax/subsidy tools. Lump-sum taxes allow the government to clear the steady-state distortions caused by agency costs and monopolistic competition. It is assumed that the production subsidy is set to eliminate the distortion introduced by monopolistic competition in the long run, that is $(1+s_p) = \eta/(\eta-1)$. The wage income subsidy, on the other hand, is set to equalize the marginal rate of substitution between consumption and leisure to the marginal product of labor in the non-stochastic steady state.[5] These subsidy policies lead to an optimal "undistorted" non-stochastic steady-state .

The central bank is in charge of monetary policy on behalf of the government. It actively participates in the bond market where it conducts open market operations to implement an interest rate policy which governs the rate of return on one-period risk-free nominal bonds. The operational losses or profits of the central bank is then transferred to the consolidated government budget. The government budget constraint is then given by

$$s_e W_t L_t + \int_i s_p P(i)_t Y(i)_t di \leq T_t + M_{t+1}^s - M_t^s + R_t^f B_t^G - B_t^G \tag{25}$$

where B^G denotes the central bank's net bond position. The government budget constraint (25) states that subsidy payments cannot exceed the summation of tax revenue, seigniorage and central bank profits.

2.5 Equilibrium

The equilibrium allocations must solve household, manufacturer, retailer and financial intermediary problems and must satisfy the market-clearing conditions. Define the set

$$\Theta_t = \{j \in [0,1] : A(j)_t < A_t^*(j)\}.$$

[5] See appendix for the derivation of the implied subsidy rate.

Then the market-clearing conditions for the retail goods, intermediate goods, labor, loan, bond and money markets can respectively be stated as

$$Y(i)_t = C(i)_t \tag{26}$$

$$\int_{j\in\Theta_t} H(j)_t (A(j)_t - \mu)\ dj + \int_{j\notin\Theta_t} H(j)_t A(j)_t\ dj \ = \int_i Y(i)_{m,t} di \tag{27}$$

$$L_t = \int_j L(j)_t dj \tag{28}$$

$$D_t^s = \int_j D(j)_t dj \tag{29}$$

$$B_t^I + \ B_t^G + B_t = 0 \tag{30}$$

$$M_t^s = M_t \tag{31}$$

for all i and t. The left-hand side of the intermediate goods market-clearing condition (27) represents the aggregate supply of intermediate manufactured products and takes into account the fraction that is destroyed during the monitoring process of bankrupt manufacturers. The right-hand side simply denotes the total demand by retailers for intermediate products. Note that the optimality condition $D_t^s = -B_t^I$ and the clearing conditions for loan and bond markets (29), (30) imply $B_t + B_t^G = \int_j D(j)_t dj$. That is, in equilibrium, all household and government savings are converted into manufacturer loans through the financial intermediaries.

3 Optimal Monetary Policy

The setting outlined thus far provides a tractable analytical framework for the analysis of optimal monetary policy. This section lays out the optimal policy problem and presents the solutions under full discretion and full commitment. Following Rotemberg and Woodford (1998) and Benigno and Woodford (2003), (2006) the optimal behavior of the policy maker is assessed adopting a linear-quadratic approach, which involves, a quadratic approximation to the life-time expected household utility and first-order linear approximations to the structural equations implied by the rational expectations equilibrium. This approach, under certain conditions to be discussed below, yields an accurate first-order approximation to the exact non-linear optimal policy. The accuracy and plausibility of the linear-quadratic approach to optimal policy problems has been investigated in a variety of contexts in a series of studies by Michael Woodford. Benigno and Woodford (2006) discuss the general conditions under which a linear-quadratic policy evaluation yields an accurate first-order approximation to the exact optimal policy. Below we shall discuss the modifications to the standard linear-quadratic approach needed to have this particular policy problem comply with the conditionality discussed in Benigno and Woodford (2006).

3.1 Welfare Measure

We start the analysis with an explicit account of the policy objective in the outlined environment with particular reference to the implications of the incorporated financial market imperfections. A second-order Taylor series expansion for the utility function (1) around a non-stochastic steady-state yields

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - (L^{1+\chi}) l_t - \frac{L^{1+\chi}}{2} (1+\chi) l_t^2 \right) + O\left(\|\varepsilon\|^3\right) + t.i.p. \tag{32}$$

where $c_t = \log C_t - \log C$, $l_t = \log L_t - \log L$, $\|\varepsilon\|$ is an upper bound on the magnitude of the stochastic disturbances, $O\left(\|\varepsilon\|^3\right)$ denotes third or higher order terms and $t.i.p.$ stands for "terms independent of policy".[6] To obtain analytical expressions, we shall assume from here on that the idiosyncratic productivity shock in the manufacturing sector is uniformly distributed on the support $[0, \overline{A}]$ with $0 < \overline{A} < \infty$. A standard linear-quadratic policy evaluation involves first-order linearization of the equations describing the rational expectations equilibrium and maximizing (32) subject to these linear constraints. This, however, would be what Benigno and Woodford (2006) refers to as a "naive" linear-quadratic approximation and in this environment is unable to deliver an accurate first-order approximation to the exact non-linear optimal policy. This is because the quadratic welfare measure (32) includes first-order terms (c_t and l_t) which depend on certain second-order terms neglected by approximating the structural equations of the model only up to first-order. One cannot accurately evaluate the exact optimal policy even up to first-order without taking into account these neglected second-order terms.[7] To confront this problem, I adopt the approach favored by Benigno and Woodford (2006) and express the first order terms c_t and l_t in terms of second and higher order terms thereby eliminating all first-order terms from the welfare measure. This entails second order approximations to certain structural equations in the model which would otherwise be first-order approximated under the

[6] Here we adopt the notation and terminology of Woodford (2002).

[7] If these second order terms are large enough they may lead to spurious welfare outcomes as demostrated by Kim and Kim (2003).

"naive" linear-quadratic approach. Note that (27) can be written as

$$\int_i \left(\frac{P(i)_t}{P_t}\right)^{-\eta} \frac{Y_t}{Z_t} di = H_t \psi(A_t^*) \tag{33}$$

where

$$\psi(A_t^*) = \int_0^{A_t^*} (A_t - \mu) dF(A_t) + \int_{A_t^*}^{\overline{A}} A_t dF(A_t). \tag{34}$$

Woodford (2002) shows that

$$\sum_{t=0}^{\infty} \beta^t \log \int_i (P(i)_t/P_t)^{-\eta} \, di = \sum_{t=0}^{\infty} \beta^t (\eta/2\delta) \pi_t^2 + O(\|\varepsilon\|^3) + t.i.p. \tag{35}$$

where $\delta = (1-\theta)(1-\beta\theta)/\theta$ and π_t denotes the log-deviation of the gross inflation rate from unity. As explained in the appendix in detail, second-order expanding (11), (18), (34) and using the intermediate goods market clearing condition (33) and equation (35), the welfare measure (32) can be rewritten as

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{L^{1+\chi}}{\omega_1}\right) \left[y_t \left(\frac{\omega_1}{L^{1+\chi}} - 1\right) - \Delta(y_t - z_t)^2 - \frac{\eta}{2\delta}\pi_t^2 \right] + O\left(\|\varepsilon\|^3\right) + t.i.p. \tag{36}$$

where $\omega_1 > 0$ and $\Delta > 0$ are derived as functions of the model parameters in the appendix. Expression (36) suggests that there still is a linear term in the welfare measure unless $\omega_1 = L^{1+\chi}$. Note that in this economy, availability of lump-sum taxes together with proportional subsidies renders a distortion-free non-stochastic steady-state attainable. It is shown in the appendix that if the production subsidy is set to eliminate the steady-state distortion due to monopolistic competition and the employment subsidy is set to equalize the marginal product of labor (MPL) to the marginal rate of substitution between consumption and leisure (MRS) in the steady-sate then the condition $\omega_1 = L^{1+\chi}$ holds. It is assumed that fiscal policy sets the production and employment subsidies once-and-for-all to offset the steady-state wedges between the equilibrium price level and the equilibrium marginal cost and between the equilibrium MPL and the equilibrium MRS while adjusting lump-sum taxes so as to ensure that the government budget constraint (25) is satisfied for all t. Then (36) reduces to the following expression which is comprised entirely of second or higher order terms and of certain terms independent of policy:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(-\frac{\eta}{2\delta}\pi_t^2 - \Delta x_t^2\right) + O\left(\|\varepsilon\|^3\right) + t.i.p. \tag{37}$$

where the output gap variable x_t denotes the difference between the log-deviation of actual output, y_t, and the log-deviation of the natural level of output, $y_t^n = z_t$.[8] Expression (37) provides us with the following appropriate welfare measure which involves purely quadratic terms:

$$\overline{W}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(-\frac{\eta}{2\delta}\pi_t^2 - \Delta x_t^2 \right) \tag{38}$$

The quadratic welfare measure (38) identifies the policy objective as inflation and output gap stabilization. Inflation fluctuations become more welfare-reducing as the elasticity of substitution between differentiated final products, η, increases exacerbating the substitution effects resulting from fluctuations in relative prices. Likewise, slower price adjustment worsens the welfare loss from inflation as it implies a smaller value for the parameter δ. The welfare measure also suggests that the agency cost problem and the resulting frictions in the financial market effects the preferences of the policy maker as it, in part, determines the magnitude of the welfare losses associated with output gap fluctuations, Δ. As exhibited in Figure 1, when other parameters are assigned values so as to match certain key characteristics of the data, the relative weight placed on output gap stabilization is positively linked to the monitoring cost parameter, μ, which invokes our first remark:[9]

Figure 1

Remark 1 *Manufacturer borrowing and accompanying agency cost problems alter the preferences of the policy maker in favor of output gap stabilization.*

As the fraction of productive inputs destroyed during financial monitoring becomes larger, equilibrium employment must increase (decrease) more in percentage terms to have output rise (fall) by a given amount. Thus, output fluctuations are associated with larger fluctuations in equilibrium labor under costlier financial monitoring. Given the concavity of the utility function in labor, this makes output fluctuations more welfare-reducing as the monitoring cost parameter increases.

3.2 Policy Constraints

Having identified the implications on preferences, we next proceed to assess the implications on policy constraints. The borrowing requirement and the agency cost problem also alter the

[8] The natural level of output, Y_t^n, is the output level which obtains under flexible prices. It is shown in the appendix that the log-deviation of the natural level of output, y_t^n, is given by the log-deviation of the productivity parameter, z_t.

[9] See appendix for the baseline parameterization.

constraints faced by the policy maker who seeks to maximize (38) subject to the restrictions imposed by the rational expectations equilibrium. One can log-linearize the first-order conditions of the household, manufacturer, retailer and financial intermediary problems together with the market clearing conditions (26)-(31) to form a set of linear policy constraints as follows:

$$E_t\left(x_{t+1}\right)+E_t(\pi_{t+1})=x_t+r_t^f-r_t^n+O\left(\|\varepsilon\|^2\right) \tag{39}$$

$$\beta E_t(\pi_{t+1})=\pi_t-(\delta\Omega)x_t-\delta r_t^f+O\left(\|\varepsilon\|^2\right) \tag{40}$$

where, as derived in the appendix, $\Omega>1$ is a function of the structural parameters and r_t^n denotes the log-deviation of the natural (Wicksellian) rate of interest from its non-stochastic steady-state level. It is defined as $E_t y_{t+1}^n-y_t^n=(\rho-1)z_t$ where y_t^n denotes the log-deviation of the natural level of output. The natural interest rate is the real rate of return which would obtain in the equilibrium of this economy's flexible-price counterpart. The variable r_t^f denotes the log-deviation of the risk-free rate. The expression (39) is the standard log-linearized intertemporal substitution equation and (40) represents a supply schedule which links inflation positively to the output gap and to the risk-free interest rate. The presence of the risk-free rate and the dependence of Ω on the parameters related to the financial market such as the monitoring cost parameter, μ, differentiate (40) from a standard new Keynesian, forward-looking price-setting constraint. The parameter Ω, which partly determines the slope of the aggregate supply schedule embodies the indirect affect of default likelihood in the manufacturing sector on equilibrium real marginal costs. The log-deviation of the risk-free rate in (40), on the other hand, represents the direct cost channel.[10] In justification of our second remark, Figure 2 attests that the slope parameter Ω is also positively related to μ.

Figure 2

Remark 2 *The indirect cost channel steepens the aggregate supply schedule while the direct cost channel acts as a supply shifter.*

The role of the risk-free interest rate as a supply shifter is evident from the price-setting constraint (40). As shown by Ravenna and Walsh (2006), in the presence of a direct cost channel, all types of shocks, including monetary disturbances, exhibit cost-push characteristics rendering a short-run trade-off between output and inflation stabilization relevant through the direct dependence of the real marginal cost on the risk-free interest rate. The link between default likelihood and the cost of borrowing consolidates this feature introducing an additional channel

[10]Recall that the real marginal cost in this setting is given by Q_t/Z_tP_t.

through which shocks can exhibit cost-push characteristics. Furthermore, the steepening of the aggregate supply schedule as suggested by costlier financial monitoring implies that output deviations are more inflationary relative to an economy free of agency cost issues. At the same time, as a flipside to that coin, inflation can be controlled with a relatively smaller output loss. In other words, the required output gap response to accomplish a given correction in inflation becomes smaller .

To summarize, the considered financial market imperfections have the following consequences: Increase in the relative welfare cost of output gap fluctuations; reduction in the "opportunity cost" of inflation stabilization; direct inflationary (deflationary) consequences of interest rate hikes (cuts). The optimal policy calls for a middle ground between the competing motivations introduced by these channels.

A first-order approximation to the exact optimal policy is found by maximizing the welfare measure (38) subject to the log-linearized structural equations (39) and (40). Below we shall consider two alternative scenarios regarding the degree of commitment on the part of the policy maker and assess the optimal targeting rules for each scenario. Then we shall proceed to the design of an interest rate policy which can successfully implement the globally optimal plan.

3.3 Sub-optimal Policy: The Case of Full Discretion

The assumption of full discretion often proves a fruitful approach in interpreting actual monetary policy practices. Since the pioneering works of Kydland and Prescott (1977) and Calvo (1978), implications of the lack of commitment has been extensively researched with particular emphasis on the welfare implications which are found to be inferior compared to those that obtain when policies can be implemented with full commitment.

The distinguishing feature of the case of discretionary policy is that, since the policy maker cannot credibly bind itself to a particular contingent sequence of policy variables, it tends to re-optimize every period without any leverage over output and inflation expectations. Thus, policy under full discretion seeks to maximize (38) subject to (39) and (40) by taking the expectations that appear in (39) and (40) as exogenous. The optimal targeting rule under discretionary policy then can be found as

$$x_t = -\frac{\eta(\Omega - 1)}{2\Delta}\pi_t \tag{41}$$

for all t. Note that unless certain parameters are not assigned unreasonable values, $\Omega > 1$ and $-\frac{\eta(\Omega-1)}{2\Delta} < 0$. The inverse relationship between inflation and the output gap, as declared by equation (41), suggests that the policy maker curbs the aggregate demand by raising the risk-free rate whenever the gross inflation rate is above the desired level, which is unity. The parameters determining the magnitude of the optimal output gap response in the face of inflationary pressures provides an overview of the interplay between competing motivations regarding the conduct of monetary policy. Note that the anti-inflationary response is more aggressive when the welfare loss due to inflation variability, as partly determined by the parameter η, is greater. On the other hand, it becomes smaller as the welfare loss due to output-gap variability, Δ, increases. Recall that the parameter Ω depends positively on the value of μ. This relationship between the monitoring cost parameter, μ, and the slope of the Phillips curve implies that the monetary authority, realizing that inflation can be controlled with a relatively smaller output loss, is inclined to respond to inflation more aggressively for larger values of μ. However, at the same time, output gap fluctuations become more welfare-reducing as the value of μ increases which prompts the policy maker to adopt a less aggressive stance against inflation. This involves parsimonious use of output gap responses to curb inflation. Figure 3 shows that the overall relationship between the response parameter $\Upsilon = \frac{\eta(\Omega-1)}{2\Delta}$ and the parameter μ is negative, thus, discretionary policy involves stricter inflation targeting for higher levels of agency costs.

Figure 3

A discretionary sub-optimum is defined as a set of contingent sequences $\left\{x_t, \pi_t, r_t^f\right\}_{t=0}^{\infty}$ that satisfy (39), (40) and (41) for any given bounded stochastic sequence $\{r_t^n\}_{t=0}^{\infty}$. For future reference, note that the system can be solved to obtain reduced form representations for x_t, π_t and r_t^f as

$$x_t = \varphi_x r_t^n \qquad \pi_t = \varphi_\pi r_t^n \qquad r_t^f = \varphi_f r_t^n \tag{42}$$

where parameters $\{\varphi_x, \varphi_\pi, \varphi_f\}$ are derived in the appendix. The reduced form solutions (42) reveal that, as is a standard result under discretionary policy, the endogenous variables do not exhibit any autocorrelation structure beyond the one exogenously imposed by the stochastic process $\{r_t^n\}_{t=0}^{\infty}$. It will be discovered in the following section that this is not the case under the globally optimal policy.

3.4 Global Optimum: The Case of Full Commitment

We next derive the solution for the globally optimal policy under commitment. The fundamental difference of this case with full discretion is that now the policy maker can successfully influence private sector expectations as a commitment technology is assumed to be available which renders the policy maker credible. The policy problem for this case can be described as maximization of (38) subject to (39) and (40). This time, however, the expectations are not taken as given.

A Lagrangian for this problem can be formulated as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{\eta}{2\delta}\pi_t^2 - \Delta x_t^2 + \lambda_t \left(x_{t+1} + \pi_{t+1} - x_t - r_t^f + r_t^n \right) + \mu_t \left(\beta\pi_{t+1} - \pi_t + (\delta\Omega)x_t + \delta r_t^f \right) \right\}$$

where λ_t and μ_t denote the Lagrange multipliers for the constraints (39) and (40) at time t. This setup yields the following first-order optimality conditions:

$$-(2\Delta)x_t - \lambda_t + \beta^{-1}\lambda_{t-1} + (\delta\Omega)\mu_t = 0 \quad (43)$$

$$(-\eta/\delta)\pi_t + \beta^{-1}\lambda_{t-1} - \mu_t + \mu_{t-1} = 0 \quad (44)$$

$$-\lambda_t + \delta\mu_t = 0 \quad (45)$$

for $t \geq 1$ and

$$\lambda_{-1} = \mu_{-1} = 0. \quad (46)$$

A global optimum is defined as a set of contingent bounded sequences $\left\{ x_t, \pi_t, \lambda_t, \mu_t, r_t^f \right\}_{t=0}^{\infty}$ that satisfy (43)-(45) as well as (39) and (40) for $t \geq 1$ and consistent with the initial conditions (46) for any given bounded stochastic sequence $\{r_t^n\}_{t=0}^{\infty}$. Note that the lagged Lagrange multipliers in (43)-(45) signify the policy maker's leverage over private sector expectations. They represent the value of prior commitments to the planner and their endogenous evolution guides the commitment plan into the future. However, as is generally the case, the optimal commitment plan suffers from time-inconsistency. That is, if for any reason the commitment device becomes unavailable at a future date $T > 0$, the resulting discretionary practice will entail setting $\lambda_{T-1} = \mu_{T-1} = 0$ which will, in general, be at odds with the optimal commitment plan chosen at the initial period.

As explained in the appendix, equations (43)-(45) and (39)-(40) can be reorganized and shown to admit a dynamic representation of the form

$$\begin{bmatrix} E_t z_{t+1} \\ \mu_t \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} z_t \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} -\delta \\ 0 \end{bmatrix} r_t^n \quad (47)$$

where $z_t = (\beta + \delta)\pi_t + \delta x_t$. The coefficients $\{r_{ij}\}_{i,j=1}^2$ are derived as functions of the model parameters in the appendix. Note that for the existence of a unique solution, only one of the two eigenvalues of the matrix premultiplying the vector $[z_t, \mu_{t-1}]'$ must lie outside the unit circle. Employing the methods developed by Blanchard and Kahn (1980), it can be shown that a bounded solution to the system of equations (47), provided that one exists, is

$$z_t = -(c_{21}^{-1}c_{22})\mu_{t-1} - c_{21}^{-1}\sum_{k=0}^{\infty} J_2^{-k-1}(c_{21}\gamma_1 + c_{22}\gamma_2)E_t r_{t+k}^n \tag{48}$$

where, J_2 is the eigenvalue of the matrix $M(i,j) = r_{ij}$, which lies outside the unit circle, the vector $[c_{21}, c_{22}]$ represents the right eigenvector corresponding to the unstable eigenvalue, J_2, and $[\gamma_1, \gamma_2]' = [-\delta, 0]'$. Since $\|J_2\| > 1$, the infinite sum in (48) converges for any bounded stochastic sequence $\{r_t^n\}_{t=0}^{\infty}$. Equation (48) can be reexpressed as

$$z_t = \psi_\mu^z \mu_{t-1} + \psi_r^z r_t^n \tag{49}$$

where $\psi_\mu^z = -(c_{21}^{-1}c_{22})$ and $\psi_r^z = -c_{21}^{-1}\sum_{k=0}^{\infty} J_2^{-k-1}(c_{21}\gamma_1 + c_{22}\gamma_2)\rho^k$. We already know from (47) that $\mu_t = r_{21}z_t + r_{22}\mu_{t-1}$ and from (44)-(45) that $(\eta/\delta)\pi_t = (1+\beta^{-1}\delta)\mu_{t-1} - \mu_t$. Consolidating these two equations and using (49) we find π_t and μ_t as

$$\pi_t = \psi_\mu^\pi \mu_{t-1} + \psi_r^\pi r_t^n \tag{50}$$

$$\mu_t = \psi_\mu^\mu \mu_{t-1} + \psi_r^\mu r_t^n \tag{51}$$

Likewise, solutions for the optimal output and interest rate deviations are

$$x_t = \psi_\mu^x \mu_{t-1} + \psi_r^x r_t^n \tag{52}$$

$$r_t^f = \psi_\mu^f \mu_{t-1} + \psi_r^f r_t^n \tag{53}$$

The derivation of the coefficients $\{\psi_i^j\}_{i=\mu,r}^{j=\pi,x,f,\mu}$ is deferred to the appendix. Unlike the solutions that obtain under full discretion (42), the presence of the lagged Lagrange multiplier μ_{t-1} in (50)-(53) suggests an additional autocorrelation structure for the endogenous variables which augments the one that is exogenously imposed by the stochastic process $\{r_t^n\}_{t=0}^{\infty}$. As pointed out by Woodford (1999), as a direct outcome of this lagged dependence, the optimal interest rate policy under commitment cannot be defined in an entirely forward-looking fashion. In consequence, the optimal interest rate policy exhibits a certain degree of inertia. This provides a rationale for interest rate smoothing behavior commonly observed in actual policy practices. Mainly these principles, among other considerations, will guide us in our search for optimal-implementable interest rate rules in the following section.

3.5 Optimal-Implementable Interest Rate Feedback Rules

This section formulates a simple interest rate feedback rule which is consistent with the globally optimal plan and, at the same time, deliver a unique rational expectations equilibrium. Determinacy properties of alternative monetary policy rules have been the central theme of a voluminous literature pioneered by Sargent and Wallace (1975). A common result that emerges from a variety of model specifications is that the success of an interest rate rule in terms of maintaining equilibrium determinacy hinges upon sufficiently strong feedback from endogenous variables. An interest rate rule formulated to react only to exogenous shocks, for instance, is unable to pin down a unique equilibrium in the standard new Keynesian setting. If, alternatively, we restrict our attention to Taylor-type rules which admit feedback from expected inflation as well as exogenous variables, a standard requirement for determinacy is that in response to a one percent increase in expected inflation, interest rate must be raised by more than one percent.[11]

It should be mentioned at this point that an optimal interest rate feedback rule can be formulated in a number of alternative ways with potentially different determinacy outcomes. Put differently, there exists a variety of forms with which a rule can be consistently formulated to accord with the global optimum, yet, consistency with the optimal plan alone does not guarantee equilibrium determinacy. Only some of these alternative formulations are able to support the optimal plan as the unique rational expectations equilibrium. To see this, using (51) and (53), let's obtain a reduced form representation for the optimal interest rate policy:

$$r_t^f = A(L) r_t^n \tag{54}$$

where

$$A(L) = \frac{\psi_\mu^f \psi_r^\mu L}{1 - \psi_\mu^\mu L} + \psi_r^f.$$

Provided that the natural rate of interest is observable to the policy maker, (54) can be interpreted as an interest rate feedback rule. This rule, as it is derived from (49)-(53), is consistent with the global optimum. However, the rule is not implementable in a practical sense since it cannot deliver equilibrium determinacy as it supports, at the same time, a multiplicity of other equilibria in addition to the globally optimal plan. This can be confirmed by observing that if the rule is in place, the system of equations given by (39) and (40) has an eigenvalue inside the unit circle. Therefore, even though the rule is consistent with the globally optimal plan, there is no guarantee under this rule that the global optimum will be sustained as the unique outcome as other equilibria may emerge at any time in a self-fulfilling speculative fashion. As discussed by Woodford (1999) in detail, the remedy is provided by sufficiently strong feedback

[11] See Bernanke and Woodford (1997) and Clarida et al. (1999).

from endogenous sources. The endogenous feedback approach is instrumental in terms of providing the economy with an expectational anchor. It is also preferable for the policy maker since exogenous variables such as the natural level of output may not be easy to observe as their identification requires a highly stylized interpretation of the economy's behavior in a hypothetical flexible-price environment. At this point, an optimal rule formulated only as a function of observable endogenous variables provides many practical advantages. Thus, let's consider linear feedback rules of the form

$$r_t^f = \lambda_1 r_{t-1}^f + \lambda_2 \pi_t + \lambda_3 \pi_{t-1}. \tag{55}$$

Note that this formulation is useful as it does not require any knowledge of exogenous disturbances. It only admits feedback from contemporaneous and past realizations of π_t and from r_{t-1}^f with λ_1 being the "inertia" parameter. Furthermore, $\{\lambda_1, \lambda_2, \lambda_3\}$ can be assigned values to have the rule support the globally optimal plan. Using (50)-(53), the optimal values for the response parameters can be found as

$$\lambda_1^* = \frac{\psi_\mu^\mu \psi_r^\pi - \psi_\mu^\pi \psi_r^\mu}{\psi_r^\pi} \tag{56}$$

$$\lambda_2^* = \frac{\psi_r^f}{\psi_r^\pi} \tag{57}$$

$$\lambda_3^* = \frac{\psi_\mu^f \psi_r^\mu - \psi_r^f \psi_\mu^\mu}{\psi_r^\pi}. \tag{58}$$

To compute impulse responses and obtain numerical expressions for $\{\lambda_1^*, \lambda_2^*, \lambda_3^*\}$, we must assign values to the model parameters. The baseline parameter values are picked to match the non-stochastic steady-state values of certain key variables with the long-term averages obtained from the U.S. data. Each period in the model is meant to represent six months. The discount rate, β, is chosen so that the steady-state risk-free rate is 2.245%, the average yield of a 6-month U.S. T-bill between 1986-2007. The price adjustment probability, $1-\theta$, is set so that final goods prices adjust on average in a year. The elasticity of substitution in the final goods sector, η, is chosen to have the mark-up rate around 12%. Even though we shall evaluate the economy's behavior under different degrees of financial fragility measured by the monitoring cost parameter, μ, for the benchmark case we set $\mu = 0.28$. Carlstrom and Fuerst (1997) specify the empirically plausible range for the monitoring cost parameter as 20%-36%. I pick the mid-point of this interval. The quarterly bankruptcy rate is reported as 0.974% for 1984-1990 by Fisher (1999). The upper-bound of the uniform distribution, $\overline{A}$, and the labor share parameter, ξ, are picked so that the implied 6-month bankruptcy rate in the manufacturing sector is roughly 2%.

Table 1 exhibits the optimal weights $\{\lambda_1^*, \lambda_2^*, \lambda_3^*\}$ for three different values of the monitoring cost parameter, μ. Note that the optimal policy calls for an active anti-inflationary stance. Furthermore, the responses to contemporaneous and past inflation become more positive as the monitoring cost parameter, μ, increases. As previously mentioned, a rise in the monitoring costs induces a decline in the opportunity cost of inflation stabilization led by the implied steepening of the Phillips curve. At the same time, output gap fluctuations become more welfare-reducing. It appears that, under the considered parameterization this effect is dominated by the former, thus, increased monitoring costs generate more pronounced rate hikes to enable better control over inflation. Also notice that the optimal interest rate rules are "super-inertial" in all cases. That is, the inertia parameter for the nominal rate, λ_1, is greater than unity. As pointed out by Woodford (1999), this does not necessarily imply explosive interest rate dynamics as an equilibrium outcome. To the contrary, the (credible) threat of unbounded fluctuations in the nominal rate in the face of persistent inflationary (or deflationary) pressures serves to stabilize inflation which, in turn, renders unnecessary such non-stationary interest rate dynamics. In consequence, neither the nominal rate nor inflation is excessively volatile in equilibrium.

Table 1

Provided that the response parameters are set to satisfy (56)-(58), the implied interest rate responses will be consistent with the globally optimal plan. But can the rule (55) maintain equilibrium determinacy when (56)-(58) hold? A numerical investigation reveals that it can under the benchmark parameterization. When the rule is in effect, both of the eigenvalues of the system (39)-(40) lie outside the unit circle. Thus, (55) and (56)-(58) describe an optimal-implementable interest rate rule for this economy.

Figure 4 presents the impulse responses of the output gap, inflation and the risk-free rate to a 1% positive shock to the natural rate of interest under the optimal interest rate policy. First, note that the rise in inflation and the fall in the output gap display the cost-push trade-off the shock involves in the presence of the cost channels. In the absence of agency costs, as the required output gap response to bring about a given inflation correction is substantial, the policy maker tolerates a larger inflation deviation. In consequence, inflation responds more when $\mu = 0$. The rise in the risk-free rate in this case is relatively small to allow gross inflation to deviate more from unity. The aggressive response of the risk-free rate when $\mu = 0.36$ exacerbates the fall in the output level, consequently, the initial jump of the output-gap is more negative. Overall, the response patterns reflect the desire of the optimizing policy maker to further restrict inflation fluctuations as financial monitoring becomes costlier.

Figure 4

4 Conclusion

This paper studied the optimal monetary policy implications of financial market imperfections by introducing firm borrowing and associated agency cost problems to a standard new Keynesian economy. It is found that the presence of a financial accelerator in the sense discussed by Bernanke et al. (1998) calls for revisions in the new Keynesian policy framework as it has strong implications on the welfare criterion and policy constraints: The welfare losses associated with output gap fluctuations are more pronounced relative to a standard new Keynesian economy. It is also found that the sensitivity of firms' marginal cost to the risk premia they face in the financial market implies a steeper aggregate supply schedule. Both of these alterations follow from the relationship between default likelihood and equilibrium marginal cost dynamics which invokes what we have identified as an indirect cost channel for monetary policy.

A standard result in the monetary policy literature is that in the absence of cost-push disturbances, the policy maker does not face a trade-off between inflation and output gap stabilization. Thus, the optimal inflation variability is zero. Ravenna and Walsh (2006) show that the direct cost channel, introduced by external finance in the production sector, generates a short-run inflation-output trade-off for any shock that can possibly hit the economy. This implies that optimal monetary policy will allow for greater inflation variability in the presence of a cost channel. Contrary to this result, I show that in the presence of an indirect cost channel, driven by the likelihood of default on loans in the production sector, inflation variability is considerably smaller under the optimal policy. This is due to the fact that the required output gap response to bring about a given level of inflation correction (in other words the opportunity cost of inflation stabilization) is smaller when an indirect cost channel is present.

The approach presented so far can be extended towards a number of directions. One possibility involves allowing firms to accumulate net worth over time. This modification is likely to permit various shocks to have more persistent effects on default risk and firms' marginal cost and may improve the quantitative results. Another reasonable direction involves an extensive joint analysis of optimal monetary and fiscal policies. As I have been primarily concerned with optimal monetary policy issues I have abstracted away from the complications resulting from the unavailability of non-distortionary tax/transfer tools. Obviously, the lack of lump-sum taxes lies at the heart of a meaningful fiscal policy discussion. However, since a distortion-free steady-state cannot be attainable if lump-sum measures are not available, a proper linear-quadratic evaluation of the optimal fiscal policy problem must take into account the first-order welfare effects of inflation and output deviations. This entails some additional second order approximations to certain structural equations in order to evaluate the optimal fiscal policy up to first order accurately as discussed in detail by Benigno and Woodford (2003).

In conclusion, the interaction of firms' marginal cost and interest rates through direct and indirect cost channels suggests additional monetary transmission mechanisms which an optimizing policy maker must take into account. Further investigation of the reasons underlying financial fragility is likely to improve substantially the design and implementation of monetary policies especially during times of financial distress.

Appendix

A.1 Derivation of the Quadratic Welfare Measure and the Weight Parameter Δ

We proceed along the lines of Woodford (2002) and Benigno and Woodford (2003) to derive the quadratic welfare measure used in the optimal policy analysis. Throughout the derivation we shall frequently use the Taylor series expansion

$$\frac{X_t - X}{X} = x_t + \frac{1}{2}x_t^2 + O(\|\varepsilon\|^3)$$

where X denotes the non-stochastic steady-state of the variable X_t and x_t stands for the log-deviation of X_t from its steady-state level, that is, $x_t = \log X_t - \log X$.

First, let's express the utility derived from consumption as follows:

$$\log C_t = \log C + c_t \tag{a.1.1}$$

where $\log C$ denotes the steady-state consumption level and c_t is the log-deviation of consumption from its non-stochastic steady-state level. Similarly, the disutility of work effort can be expressed as:

$$\kappa\frac{L_t^{1+\chi}}{1+\chi} = \kappa\frac{L^{1+\chi}}{1+\chi} + L^{1+\chi}\left(l_t + \frac{1}{2}(1+\chi)l_t^2\right) + O\left(\|\varepsilon\|^3\right). \tag{a.1.2}$$

Now, using (a.1.1) and (a.1.2) household utility can be written as

$$U_t = \overline{U} + E_s\sum_{t=s}^{\infty}\beta^{t-s}\left(c_t - (L^{1+\chi})l_t - \frac{L^{1+\chi}}{2}(1+\chi)l_t^2\right) + O\left(\|\varepsilon\|^3\right) + t.i.p. \tag{a.1.3}$$

The intermediate goods market clearing condition implies

$$\int_i \left(\frac{P(i)_t}{P_t}\right)^{-\eta}\frac{Y_t}{Z_t}di = H_t\psi(A_t^*) \tag{a.1.4}$$

where $\psi(A_t^*) = \int_0^{A_t^*}(A_t - \mu)\; dF(A_t) + \int_{A_t^*}^{\overline{A}} A_t\; dF(A_t)$. Then, it follows from (a.1.4) that

$$h_t + \widetilde{\psi}_t = y_t - z_t + d_t \tag{a.1.5}$$

where $d_t = \log\int_i \left(\frac{P(i)_t}{P_t}\right)^{-\eta} di$, $\widetilde{\psi}_t = \log\psi(A_t^*) - \log\psi(A^*)$ and other lower case letters denote the log-deviations of the corresponding variables from their non-stochastic steady-state levels. Let $s_t = A_t^*/\overline{A}$ denote the aggregate bankruptcy ratio. Assuming that idiosyncratic productivity shocks are uniformly distributed, that is, $A(j)_t \sim U[0, \overline{A}]$ for all j, equation (18) gives

$$\frac{\xi L_t^{\upsilon}}{(1-\xi)N^{\upsilon}} = -\frac{\phi'(A_t^*)\gamma(A_t^*)}{\phi(A_t^*)\gamma'(A_t^*)} = \frac{s_t(K-s_t)}{(1-s_t)((K/2)-s_t)}$$

where $K = 2 - \frac{2\mu}{\overline{A}}$. A second-order approximation to the above equation then yields

$$\upsilon l_t = \alpha_1 \widetilde{s}_t + \alpha_2 \widetilde{s}_t^{\,2} + O\left(\|\varepsilon\|^3\right) \tag{a.1.6}$$

where $\widetilde{s}_t = \log s_t - \log s$,

$$\begin{aligned} \alpha_1 &= 1 + \frac{s}{1-s} - \frac{s}{K-s} + \frac{s}{(K/2)-s}, \\ \alpha_2 &= \frac{1}{2}\left[\alpha_1 - 1 + \left(\frac{s}{1-s}\right)^2 - \left(\frac{s}{K-s}\right)^2 + \left(\frac{s}{(K/2)-s}\right)^2\right], \end{aligned}$$

and $s = A^*/\overline{A}$. Under the assumption of uniform distribution, $\psi(A_t^*)$ can be found as

$$\psi(A_t^*) = \frac{\overline{A}}{2} - \mu s_t$$

which can similarly be second-order expanded to obtain

$$\widetilde{\psi}_t = \alpha_3 \widetilde{s}_t + \alpha_4 \widetilde{s}_t^{\,2} + O\left(\|\varepsilon\|^3\right) \tag{a.1.7}$$

where $\alpha_3 = -\mu s/(\frac{\overline{A}}{2} - \mu s)$ and $\alpha_4 = (\alpha_3/2)(1-\alpha_3)$. Note that (a.1.6) implies $\widetilde{s}_t = (\upsilon/\alpha_1)l_t + O\left(\|\varepsilon\|^2\right)$. Plugging this expression back in (a.1.6) we have

$$\widetilde{s}_t = \left(\frac{\upsilon}{\alpha_1}\right) l_t - \left(\frac{\alpha_2 \upsilon^2}{\alpha_1^3}\right) l_t^2 + O\left(\|\varepsilon\|^3\right). \tag{a.1.8}$$

Using (a.1.5), (a.1.7) and (a.1.8) we find

$$h_t + \left(\frac{\alpha_3 \upsilon}{\alpha_1}\right) l_t + \left(\frac{\upsilon^2(\alpha_1\alpha_4 - \alpha_3\alpha_2)}{\alpha_1^3}\right) l_t^2 + O\left(\|\varepsilon\|^3\right) = y_t - z_t + d_t. \tag{a.1.9}$$

Now, from $H_t = [\xi L_t^{\upsilon} + (1-\xi)N^{\upsilon}]^{\frac{1}{\upsilon}}$ it follows that

$$h_t = (s_l)\, l_t + \frac{\upsilon}{2}\left[s_l(1-s_l)\right] l_t^2 + O\left(\|\varepsilon\|^3\right)$$

where $s_l = \xi L^{\upsilon}/[\xi L^{\upsilon} + (1-\xi)N^{\upsilon}]$. This expression can be plugged into (a.1.9) to get

$$(\omega_1)l_t + (\omega_2)l_t^2 + O\left(\|\varepsilon\|^3\right) = y_t - z_t + d_t. \tag{a.1.10}$$

where

$$\begin{aligned} \omega_1 &= s_l + \frac{\alpha_3 \upsilon}{\alpha_1}, \\ \omega_2 &= \frac{\upsilon^2(\alpha_1\alpha_4 - \alpha_3\alpha_2)}{\alpha_1^3} + \frac{\upsilon}{2} s_l(1-s_l). \end{aligned}$$

Note that (a.1.10) implies $l_t = (1/\omega_1)(y_t - z_t) + O\left(\|\varepsilon\|^2\right)$. Plugging this expression in (a.1.10) we find

$$l_t = \left(\frac{1}{\omega_1}\right)(y_t - z_t) - \left(\frac{\omega_2}{\omega_1^3}\right)(y_t - z_t)^2 + \left(\frac{1}{\omega_1}\right) d_t + O\left(\|\varepsilon\|^3\right). \tag{a.1.11}$$

Incorporating (a.1.11) and the goods market clearing condition $c_t = y_t$ into (a.1.3) we obtain

$$\frac{\omega_1 U_t}{L^{1+\chi}} = E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[y_t \left(\frac{\omega_1}{L^{1+\chi}} - 1 \right) - \Delta (y_t - z_t)^2 - d_t \right] + O\left(\|\varepsilon\|^3\right) + t.i.p. \tag{a.1.12}$$

where

$$\Delta = -\frac{2\omega_2 - (1+\chi)\omega_1}{\omega_1^2}.$$

As discussed in the text, the policy maker, in an attempt to restore the efficient level of equilibrium work effort, sets the labor income subsidy once-and-for-all to equalize the marginal product of labor to the marginal rate of substitution between consumption and leisure in the steady-state. The implied employment subsidy rate must satisfy

$$1 + s_e = \left(\frac{R_t^f}{\gamma(A^*)} \right) \left(s_l \psi(A^*) + \psi_{A^*} A_L^* L \right)$$

where $\psi_{A^*} = \partial \psi / \partial A^*$ and $A_L^* = \partial A^* / \partial L$. This subsidy policy then implies

$$L^{1+\chi} = s_l + \frac{\alpha_3 \upsilon}{\alpha_1} = \omega_1.$$

Consequently, the first-order term, y_t, in (a.1.12) disappears from the welfare criterion together with some second-order terms which are neglected by approximating the structural equations only up to first order. As explained in Benigno and Woodford (2006), this ensures that our linear-quadratic approach yields an accurate first-order approximation to the exact non-linear optimal policy. Furthermore, Benigno and Woodford (2003) show that

$$\sum_{t=s}^{\infty} \beta^{t-s} d_t = \frac{\theta\eta}{(1-\theta)(1-\beta\theta)} \sum_{t=s}^{\infty} \beta^{t-s} \frac{\pi_t^2}{2} + O\left(\|\varepsilon\|^3\right) + t.i.p. \qquad \text{(a.1.13)}$$

Finally, substituting (a.1.13) into (a.1.12) and using the result $L^{1+\chi} = \omega_1$ we obtain the following welfare criterion:

$$U_t = E_s \sum_{t=s}^{\infty} \beta^{t-s} \left(-\Delta(y_t - z_t)^2 - \frac{\eta}{2\delta}\pi_t^2\right) + O\left(\|\varepsilon\|^3\right) + t.i.p. \qquad \text{(a.1.14)}$$

Note that (a.1.14) consists only of second and higher order terms. As it is shown in the second part of the appendix, in a perfectly flexible price environment the log-deviation of output from its non-stochastic steady-state, y_t^n, must be equal to z_t. Therefore, we can rewrite (a.1.14) as

$$U_t = E_s \sum_{t=s}^{\infty} \beta^{t-s} \left(-\frac{\eta}{2\delta}\pi_t^2 - \Delta x_t^2\right) + O\left(\|\varepsilon\|^3\right) + t.i.p.$$

where $x_t = y_t - y_t^n$, which brings us to (38) in the text.

A.2 Derivation of the Linear Policy Constraints and the Slope Parameter Ω

First order expanding the intertemporal substitution equation (7) and the price setting equation (20) together with (21) we find

$$-c_t = r_t^f - E_t\pi_{t+1} - E_t c_{t+1} + O(\|\varepsilon\|^2) \qquad \text{(a.2.1)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \delta(q_t - z_t) + O(\|\varepsilon\|^2). \qquad \text{(a.2.2)}$$

Similarly, linearizing (8) , (15) and (17) we obtain

$$\chi l_t + c_t = w_t + O(\|\varepsilon\|^2) \qquad \text{(a.2.3)}$$

$$q_t = r_t^f + w_t - h_{l,t} - \widetilde{\Phi}_t + O(\|\varepsilon\|^2) \qquad \text{(a.2.4)}$$

$$\widetilde{\Phi}_t = \left(\frac{-\mu s}{2\Phi}\right) \widetilde{s}_t + O(\|\varepsilon\|^2) \qquad \text{(a.2.5)}$$

where $q_t = \log(Q_t/P_t) - \log(Q/P)$, $w_t = \log(W_t/P_t) - \log(W/P)$, $h_{l,t} = \log(H_{L,t}) - \log(H_L)$ and $\widetilde{\Phi}_t = \log(\Phi_t) - \log(\Phi)$. We know from (a.1.6) and (a.1.11) that $\widetilde{s}_t = (\upsilon/\alpha_1) l_t + O(\|\varepsilon\|^2)$ and $l_t = (1/\omega_1)(y_t - z_t) + O(\|\varepsilon\|^2)$. Furthermore, from the definition (11) it can be found that $h_{l,t} = (1-\upsilon)(s_l - 1) l_t + O(\|\varepsilon\|^2)$. These equations together with (a.2.3)-(a.2.5) give

$$q_t = r_t^f + \Omega y_t - (\Omega - 1) z_t + O(\|\varepsilon\|^2) \qquad \text{(a.2.6)}$$

where

$$\Omega = \left(\frac{1}{\omega_1}\right)\left(\frac{-\mu s}{2\Phi} - (1-\upsilon)(s_l - 1) + \chi\right) + 1.$$

Equation (a.2.6) and the goods market clearing condition, $c_t = y_t$, can be substituted into (a.2.1) and (a.2.2) to have

$$-y_t = r_t^f - E_t\pi_{t+1} - E_t y_{t+1} + O(\|\varepsilon\|^2) \tag{a.2.7}$$

$$\pi_t = \beta E_t\pi_{t+1} + \delta\Omega(y_t - z_t) + \delta r_t^f + O(\|\varepsilon\|^2). \tag{a.2.8}$$

Let q_t^n denote the log-deviation of Q_t/P_t from its non-stochastic steady-state in the flexible-price counterpart of the economy. Note that if final goods prices are allowed to adjust freely, we have $Q_t/Z_t = P_t$ for all t. Therefore, it follows from (a.2.6) that $q_t^n = r_t^{fn} + \Omega y_t^n - (\Omega - 1)z_t = z_t$ where r_t^{fn} denotes the log-deviation of the risk-free rate under flexible prices. Provided that $r_t^{fn} = 0$, we have $y_t^n = z_t$. Plugging this expression back into (a.2.8) and defining $r_t^n = E_t y_{t+1}^n - y_t^n$, we obtain the linear policy constraints (39) and (40).

A.3 Derivation of the Dynamic System (47)

Consolidating (39) and (40) we obtain

$$E_t z_{t+1} = \pi_t + \delta(\Omega - 1)x_t - \delta r_t^n \tag{a.3.1}$$

where $z_{t+1} = (\beta + \delta)\pi_{t+1} + \delta x_{t+1}$. Also note that from (44) and (45) we find $(\eta/\delta)\pi_t = (1 + \beta^{-1}\delta)\mu_{t-1} - \mu_t$. Plugging this expression and (45) into (43) and reorganizing, we get

$$\pi_t = \left(\frac{\delta^2[(\Omega - 1)(\beta + \delta) + 1]}{\beta[\eta\delta(\Omega - 1) - 2\Delta(\beta + \delta)]}\right)\mu_{t-1} - \left(\frac{2\Delta}{\eta\delta(\Omega - 1) - 2\Delta(\beta + \delta)}\right) z_t. \tag{a.3.2}$$

From the definition $z_t = (\beta + \delta)\pi_t + \delta x_t$ and (a.3.1) we find that

$$E_t z_{t+1} = (1 - \Omega)z_t + [1 - (1 - \Omega)(\beta + \delta)]\pi_t - \delta r_t^n. \tag{a.3.3}$$

Substituting (a.3.2) into (a.3.3) we obtain

$$E_t z_{t+1} = r_{11} z_t + r_{12}\mu_{t-1} - \delta r_t^n \tag{a.3.4}$$

where

$$r_{11} = \frac{-[\eta\delta(1 - \Omega)^2 + 2\Delta]}{\eta\delta(\Omega - 1) - 2\Delta(\beta + \delta)} \quad \text{and} \quad r_{12} = \frac{\delta^2[1 - (1 - \Omega)(\beta + \delta)]^2}{\beta[\eta\delta(\Omega - 1) - 2\Delta(\beta + \delta)]}.$$

Likewise, substituting (a.3.2) into the expression $(\eta/\delta)\pi_t = (1+\beta^{-1}\delta)\mu_{t-1} - \mu_t$ we find

$$\mu_t = r_{21} z_t + r_{22}\mu_{t-1} \tag{a.3.5}$$

where

$$r_{21} = \frac{2\Delta\eta}{\delta\left[\eta\delta(\Omega-1) - 2\Delta(\beta+\delta)\right]} \quad \text{and} \quad r_{22} = \frac{-[\eta\delta + 2\Delta(\beta+\delta)^2]}{\beta[\eta\delta(\Omega-1) - 2\Delta(\beta+\delta)]}.$$

Equations (a.3.4) and (a.3.5) constitute the dynamic system (47) in the text.

A.4 Derivation of $\{\psi_i^j\}_{i=\mu,r}^{j=\pi,x,f,\mu}$ and $\{\varphi_x, \varphi_\pi, \varphi_f\}$

We know from (49) that $z_t = \psi_\mu^z \mu_{t-1} + \psi_r^z r_t^n$. Using this expression and (a.3.5) we find

$$\mu_t = \psi_\mu^\mu \mu_{t-1} + \psi_\mu^r r_t^n \tag{a.4.1}$$

where

$$\psi_\mu^\mu = r_{21}\psi_\mu^z + r_{22} \quad \text{and} \quad \psi_\mu^r = r_{21}\psi_r^z.$$

Equations (44)-(45) imply $(\eta/\delta)\pi_t = (1+\beta^{-1}\delta)\mu_{t-1} - \mu_t$. Using this expression and (a.4.1) we get

$$\pi_t = \psi_\mu^\pi \mu_{t-1} + \psi_r^\pi r_t^n \tag{a.4.2}$$

where

$$\psi_\mu^\pi = \frac{\delta(\beta+\delta) - \beta\delta(r_{21}\psi_\mu^z + r_{22})}{\eta\beta} \quad \text{and} \quad \psi_r^\pi = -\frac{\delta r_{21}\psi_r^z}{\eta}.$$

From the definition of z_t we know that $x_t = \delta^{-1}(z_t - (\beta+\delta)\pi_t)$. Plugging (49) and (a.4.2) into this expression we obtain

$$x_t = \psi_\mu^x \mu_{t-1} + \psi_r^x r_t^n \tag{a.4.3}$$

where

$$\psi_\mu^x = \frac{\psi_\mu^z}{\delta} - \frac{(\beta+\delta)^2 - \beta(\beta+\delta)(r_{21}\psi_\mu^z + r_{22})}{\eta\beta} \quad \text{and} \quad \psi_r^x = \frac{\psi_r^z}{\delta} + \frac{(\beta+\delta) r_{21}\psi_r^z}{\eta}.$$

Now, from (40) we find $r_t^f = \delta^{-1}\pi_t - (\beta/\delta)E_t\pi_{t+1} - \Omega x_t$. Using this expression, (a.4.2) and (a.4.3) we finally have

$$r_t^f = \psi_\mu^f \mu_{t-1} + \psi_r^f r_t^n$$

where

$$\psi_\mu^f = a_{11} + a_{12} + a_{13} \quad \text{and} \quad \psi_r^f = a_{21} + a_{22} + a_{23}$$

with

$$a_{11} = \delta^{-1}\psi_\mu^\pi \qquad a_{12} = -(\beta/\delta)(r_{21}\psi_\mu^z + r_{22})\psi_\mu^\pi \qquad a_{13} = -\Omega\psi_\mu^x$$
$$a_{21} = \delta^{-1}\psi_r^\pi - (\beta/\delta)r_{21}\psi_r^z\psi_\mu^\pi \qquad a_{22} = -(\beta/\delta)\rho\psi_r^\pi \qquad a_{23} = -\Omega\psi_r^x.$$

The solution for the optimal policy problem under full discretion gives $x_t = -\Theta\pi_t$ where $\Theta = \frac{\eta(\Omega-1)}{2\Delta}$. Using the intertemporal substitution equation (39), one can express the risk-free rate as $r_t^f = (1-\Theta)E_t\pi_{t+1} + \Theta\pi_t + r_t^n$. Substituting this expression and the optimality condition into the Phillips curve (40) yields

$$\pi_t = \left[\frac{\beta + \delta(1-\Theta)}{1 + \delta\Theta(\Omega-1)}\right] E_t\pi_{t+1} + \left[\frac{\delta}{1 + \delta\Theta(\Omega-1)}\right] r_t^n. \qquad \text{(a.4.5)}$$

Iterating forward (a.4.5), we obtain

$$\pi_t = \lim_{i\to\infty}\left[\Xi^i E_t\pi_{t+i} + \left(\frac{\delta}{1+\delta\Theta(\Omega-1)}\right)\sum_{k=0}^{i-1}\Xi^k E_t r_{t+k}^n\right] \qquad \text{(a.4.6)}$$

where $\Xi = \frac{\beta+\delta(1-\Theta)}{1+\delta\Theta(\Omega-1)}$. Provided that $|\Xi| < 1$, the variables π_t, x_t and r_t^f are uniquely defined and can be found as

$$\pi_t = \varphi_\pi r_t^n, \quad x_t = \varphi_x r_t^n, \quad r_t^f = \varphi_f r_t^n$$

where

$$\varphi_\pi = \frac{\delta}{(1+\delta\Theta(\Omega-1))(1-\rho\Xi)}, \ \varphi_x = -\Theta\varphi_\pi, \ \varphi_f = [(1-\Theta)\rho + \Theta]\varphi_\pi + 1.$$

A.5 Tables

Baseline Parameters	
Monitoring Cost Parameter - μ	.28
Labor Supply Elasticity Parameter - χ	.02
Labor Supply Level Parameter - κ	1
Elasticity of Substitution Between Final Goods - η	8
Discount Factor - β	.977
Degree of Price Stickiness - θ	.5
Fixed Factor - N	.58
Labor Share - ξ	.4
Elasticity of Factor Substitution - v	1
Uniform Distribution Upper Bound - $\overline{A}$	1.82
Autoregressive Parameter - ρ	.95

Note: Each period represents six months.

	λ_1^*	λ_2^*	λ_3^*
$\mu = 0$	1.523	3.171	1.585
$\mu = 0.15$	1.523	3.791	1.772
$\mu = 0.28$	1.523	5.944	1.824

Table 1: Optimal Weights

A.6 Figures

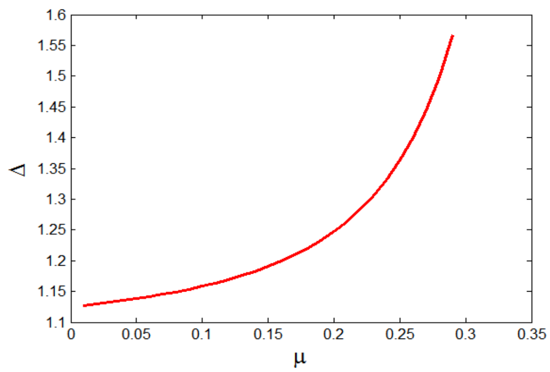

Figure 1

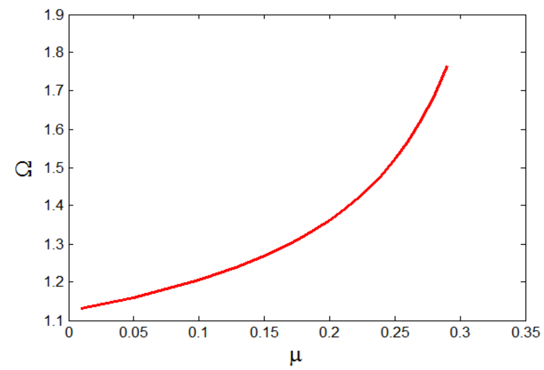

Figure 2

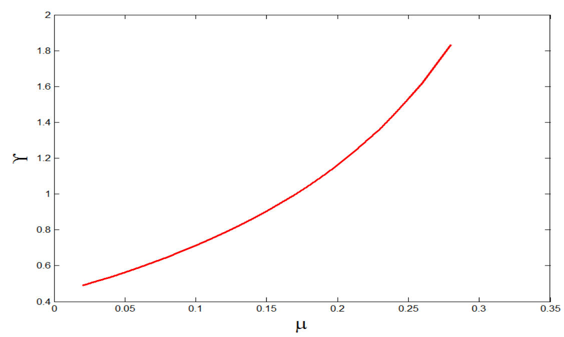

Figure 3

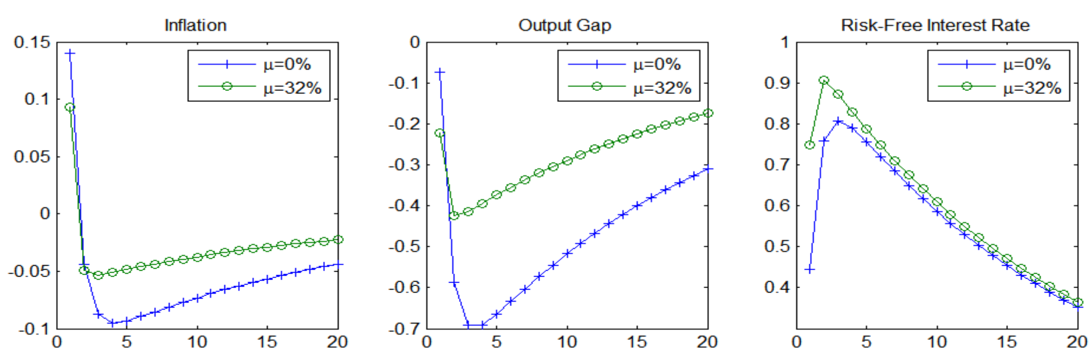

Figure 4

Figure and Table Captions

Figure 1: The relationship between the weight parameter (Δ) and μ.

Figure 2: The relationship between the slope parameter (Ω) and μ.

Figure 3: Anti-Inflationary response (Υ) under discretion as a function of μ.

Figure 4: Impulse responses to a one percent positive shock to the natural rate of interest.

Table 1: Optimal inertia parameter (λ_1^*) and optimal responses to contemporaneous and past inflation (λ_2^*,λ_3^*).

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