MONETARY POLICY ANALYSIS WITH POTENTIALLY MISSPECIFIED MODELS

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NBER Working Paper No. 13099
May 2007
JEL No. C32,E52

ABSTRACT

Policy analysis with potentially misspecified dynamic stochastic general equilibrium (DSGE) models faces two challenges: estimation of parameters that are relevant for policy trade-offs and treatment of estimated deviations from the cross-equation restrictions. This paper develops and explores policy analysis approaches that are either based on a generalized shock structure for the DSGE model or the explicit modelling of deviations from cross-equation restrictions. Using post-1982 U.S. data we first quantify the degree of misspecification in a state-of-the-art DSGE model and then document the performance of different interest-rate feedback rules. We find that many of the policy prescriptions derived from the benchmark DSGE model are robust to the various treatments of misspecifications considered in this paper, but that quantitatively the cost of deviating from such prescriptions varies substantially.

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1   Introduction

Following the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), many central banks are building and estimating dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and using them for policy analysis. Despite the success in improving the empirical performance of DSGE models, misspecification remains a concern, as documented in Del Negro, Schorfheide, Smets, and Wouters (2007, henceforth DSSW). Some of the cross-equation restrictions imposed by these models remain at odds with the data. This paper illustrates the difficulties involved in conducting policy analysis with misspecified models. Two broad issues arise. First, how should the parameters be estimated in the presence of misspecification? Second, how should one treat empirical deviations from model-implied cross-equation restrictions when conducting policy analysis?

In the literature and in the practice of central banks, there exist three different approaches for dealing with misspecification. One approach is to ignore the problem and derive quantitative policy recommendations as if the DSGE model were correctly specified, e.g., Laforte (2003) and Levin, Onatski, Williams, and Williams (2006). A second approach is to manipulate the shock structure of the DSGE model to optimize the fit of the resulting empirical specification. Smets and Wouters (2003) use more shocks than observables to obtain a DSGE model with a fit to Euro area data comparable to that of a Bayesian VAR. Smets and Wouters (2006) generalize the law of motion of some of the exogenous shocks in a DSGE model for U.S. data in which the number of shocks equals the number of observables. The tacit assumption underlying the policy analysis is that these shocks are policy invariant, even if they partially reflect misspecification.

A third approach is to model the deviations from the cross-equation restrictions explicitly in the likelihood. For instance, Ireland (2006), following Sargent (1989), assumes that the observations based on which the DSGE model is estimated are subject to "measurement" errors. While the measurement errors improve the empirical fit, they raise serious identification problems – how can one distinguish measurement error dynamics from the dynamics in the underlying model-based variables - and tie the hands of the researcher in terms of policy analysis: one has to assume that these measurement errors are policy-invariant. Alternatively, Del Negro and Schorfheide (2004) propose a method for incorporating deviations from the VAR representation of the DSGE model, called DSGE-VAR. We will argue in this paper that the DSGE-VAR framework is a versatile tool for policy analysis because it allows us to make different assumptions on how these deviations are affected by policy

changes. The framework thereby enables us to assess the robustness of the DSGE model's policy predictions in view of its empirical deficiencies.

We study how the aforementioned approaches deal with the issues described above, inference and treatment of misspecification. Starting point of our analysis is a state-of-the-art New Keynesian DSGE model in which monetary policy follows an interest-rate feedback rule. The goal is to assess how changes in the feedback rule affect the volatility of output, inflation, and interest rates. We generate three types of parameter estimates. First, we combine the DSGE model likelihood function with a prior distribution for its parameters and compute a posterior distribution as in Schorfheide (2000). Second, we include in our analysis the generalization of the shock structure as a possible method of dealing with misspecification. The final set of estimates is obtained using the DSGE-VAR approach. That is, we approximate the DSGE model with a fourth-order vector autoregression (VAR) and specify a prior distribution for the VAR parameters centered at the model-implied cross-equation restrictions. The resulting estimates of the VAR coefficients are implicitly projected back onto the cross-equation restrictions to obtain a minimum-distance estimator of the DSGE model parameters. While the direct estimator forces the DSGE model to match all the observed fluctuations, the DSGE-VAR estimator (and to some extent the generalized shocks estimator) lets some of the fluctuations be explained by deviations from cross-equation restrictions. This can be appealing if misspecification of the DSGE model's likelihood function is a concern.

While there is a substantive body of literature on how to estimate parameters of DSGE models and how to conduct policy analysis under model uncertainty, our framework allows a unified treatment of some the key issues. It also provides new methods to assess the robustness of policy implications of a benchmark DSGE model in view of evidence of misspecification. The calibration literature initiated by Kydland and Prescott (1982) has traditionally emphasized that it is more important to construct careful measurements of the parameters that determine policy trade-offs using informative data sources, than to obtain a good time series fit of the DSGE model. This sentiment is expressed, for instance, in Kocherlakota (2006). Bayesian analysis allows us to incorporate additional information not captured by the likelihood function through prior distributions. It turns out that the DSGE-VAR analysis has the feature that the greater the evidence for model misspecification, the less weight is placed on the likelihood function to construct the estimates of the DSGE model parameters.

Altig, Christiano, Eichenbaum, and Evans (2002) estimate the parameters of their

DSGE model by matching DSGE model impulse responses to monetary policy and technology shocks with those obtained from an identified VAR. While this impulse response function matching makes parameter estimates more robust to some forms of model misspecification, the resulting empirical specification is not able to explain all the variation of output, inflation, and interest rates that we observe in the data. Hence, policy analysis with such a model would have to assume that the propagation of other unspecified shocks is not altered by changes in the policy rule or that the effects would mirror those of, say, the effects on the technology shock.

Indeed, the second major issue concerns the treatment of misspecification in conducting the policy exercise. Within the generalized-shocks framework we derive policy implication under the standard assumption that the parameters of the shock processes are policy invariant. Within the DSGE-VAR framework we make different assumptions about the policy invariance of the estimated deviations from the model-implied cross-equation restrictions. These assumptions include treating the DSGE-VAR as a backward-looking structural VAR, thereby ignoring effects of policy changes on expectation formation; treating discrepancies in terms of moving average representations as policy invariant; evaluating policies under a priori beliefs about misspecification.[1]

Empirical evidence can help us discriminating between the different approaches to policy analysis. We find strong evidence of DSGE model misspecification. This misspecification affects the estimates of key non-policy parameters – and specifically the persistence of technology shocks – which differ between the DSGE model, the generalized shocks approach, and the DSGE-VAR. This difference drives much of the policy results. We also find that misspecification is not at all severe in the dimension that is most important for policy, the responses to technology shocks. This evidence casts doubt on the polar approaches of ignoring the cross-equation restriction completely and treating the DSGE-VAR as a backward-looking model. Two lessons are robust across all modes of policy analysis considered in this paper. First, deviating from the baseline parameters of the feedback rule is unlikely to provide substantial improvements over the estimated Volcker-Greenspan policy. Second it appears undesirable to decrease the response to inflation, or to increase the reaction to deviations of output from a long-run trend path. Quantitatively, the cost of deviating from these prescriptions varies substantially across approaches.

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[1]Preliminary empirical results for some of the DSGE-VAR analysis based on a simple three-equation New Keynesian model without capital and wage rigidity were reported in the 2005 Proceedings Volume of the Journal of the European Economic Association, Del Negro and Schorfheide (2005).

There is a long-standing recognition that model uncertainty is an important aspect of the assessment of monetary policies, e.g., Brainard (1967), Chow (1975), and Craine (1979). A natural approach in the presence of model uncertainty is to evaluate policy rules within all the model specifications that are under consideration, either following a Bayesian route or a minimax strategy. The literature contains numerous applications of these ideas, e.g., McCallum (1988), Levin, Wieland, and Williams (1999, 2003), Rudebusch (2001, 2002), Onatski and Stock (2002), Onatski and Williams (2003), Brock, Durlauf, and West (2004), Cogley and Sargent (2005), and Hansen and Sargent (2005). All these papers differ with regard to the type of models included in the model set, and the formulation of the decision problem that leads to the choice of a preferred policy.

Our model set is purposefully smaller than that considered in recent papers that study the performance of different interest-rate feedback rules across a variety of econometric models, including models that are currently used by the Board of Governors and the European Central Bank, e.g., Levin, Wieland, and Williams (1999, 2003), Taylor (1999), Coenen (2003), Levin and Williams (2003), Brock, Durlauf, and West (2004), and Adalid, Coenen, McAdam, and Siviero (2005).[2] The benchmark DSGE model is at the center of our analysis. We surround this benchmark model with (i) DSGE models with a more general shock structure, and (ii) structural VARs that allow for deviations from the cross-equation restrictions to improve the empirical fit. We view our framework as a diagnostic tool. If our likelihood-based measure of fit point toward misspecification, we provide tools to parameterize the discrepancies between theory and data, and assess the robustness of the DSGE model's policy implications under different assumptions about the policy invariance of the discrepancies.

The paper is organized as follows. Section 2 describes the econometric framework. The DSGE model is presented in Section 3. This model is based on work by Altig, Christiano, Eichenbaum, and Linde (2002), Smets and Wouters (2003), and Christiano, Eichenbaum, and Evans (2005). Compared to the benchmark New Keynesian models discussed, for instance, in Woodford (2003), our model has been subjected to a number of modifications, all designed to improve its empirical fit. Section 4 describes the data set and discusses our empirical findings, and Section 5 concludes. Detailed analytical derivations, a description of the Markov-Chain-Monte Carlo methods used to implement the Bayesian computations, and the precise specification of the prior distribution for the DSGE model parameters are

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[2]With the exception of Brock, Durlauf, and West (2004) little or no attention is paid to fit and forecasting performance when weighting predictions from various models.

provided in a Technical Appendix that is available from the authors upon request.

2 Setup and Inference

This section describes our analytical framework for monetary policy analysis under potential misspecification. The goal of the analysis is to study the effects of changing coefficients in an interest rate feedback rule such as

$$R_t = \psi_1 \pi_t + \psi_2 \widetilde{y}_t + \sigma_R \epsilon_{1,t} \tag{1}$$

on the variability of some key macroeconomic variables. Here R_t is the nominal interest rate, π_t is the inflation rate, $\tilde{y}_t$ are output deviations from a smooth trend, which we refer to as output gap, and $\epsilon_{1,t}$ is a monetary policy shock with unit variance. We make the simplifying assumption that the public believes the new policy to be in place indefinitely after being announced credibly.

Starting point for the analysis is a DSGE model that describes the optimal behavior of households and firms and determines an equilibrium law of motion for the macroeconomic variables of interest. The following two equations provide a simple version of a New Keynesian DSGE model

$$\widetilde{y}_t - g_t = \mathbb{E}_t[\widetilde{y}_{t+1} - g_{t+1}] - (R_t - \mathbb{E}_t[\pi_{t+1}]), \tag{2}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa(\widetilde{y}_t - z_t - g_t), \tag{3}$$

which we use in this section for the sake of exposition. Equation (2) is obtained from the consumption Euler equation, (3) is derived from the optimal price setting of monopolistically competitive firms and typically referred to as New Keynesian Phillips curve. To complete the model specification one needs to specify a law of motion for the exogenous processes g_t (government spending) and z_t (technology), such as:

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{g,t}, \quad z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}, \tag{4}$$

where the innovations $\epsilon_{g,t}$ and $\epsilon_{z,t}$ are identically and independently distributed standard normal random variates.

We proceed by grouping the parameters into two categories: $\theta_{(p)} = [\psi_1, \psi_2, \sigma_R]'$ is the vector of parameters that describe central bank behavior and the elements of $\theta_{(np)} = [\beta, \kappa, \rho_g, \rho_z, \sigma_g, \sigma_z]'$ characterize preferences and technologies as well as the law of motion of the exogenous processes. We let $\theta = [\theta'_{(p)}, \theta'_{(np)}]'$, $y_{1,t} = R_t$, and $y_{2,t} = [\widetilde{y}_t, \pi_t]$. Equations (1)

to (4) characterize a linear rational expectations system. Its solution leads to a moving-average representation for $y_{2,t}$ in terms of the innovations $\epsilon_t = [\epsilon_{1,t}, \epsilon_{g,t}, \epsilon_{z,t}]'$:

$$y_{2,t} = \sum_{j=0} D_j^*(\theta_{(p)}, \theta_{(np)})\epsilon_{t-j}. \tag{5}$$

The matrices D_j^* embody the cross-equation restrictions imposed by the DSGE model. With (1) and (5) in hand it is straightforward to compute the covariance matrix for interest rates, output, and inflation $\mathbb{V}^*(\theta_{(p)}, \theta_{(np)})$ as well as weighted averages of variances $tr[\mathcal{W}\mathbb{V}^*(\theta_{(p)}, \theta_{(np)})]$ that can serve as performance measures for monetary policy. Here $tr[\cdot]$ denotes the trace operator and $\mathcal{W}$ is a symmetric positive definite weight matrix.

In the absence of misspecification policy analysis is straightforward. Suppose we adopt a Bayesian framework and start from a prior distribution with density $p(\theta_{(p)}, \theta_{(np)})$ for the DSGE model parameters. We then combine the prior with the likelihood function $p(Y|\theta_{(p)}, \theta_{(np)})$ of the DSGE model to obtain a posterior distribution with density $p(\theta_{(p)}, \theta_{(np)}|Y)$. The calculations can be implemented with the methods described in An and Schorfheide (2007). The posterior expected weighted variance differential for two policies $\theta_{(p)}^{(1)}$ and $\theta_{(p)}^{(2)}$ is given by:

$$\int \left(tr\big[\mathcal{W}\mathbb{V}^*\big(\theta_{(p)}^{(1)}, \theta_{(np)}\big)\big] - tr\big[\mathcal{W}\mathbb{V}^*\big(\theta_{(p)}^{(2)}, \theta_{(np)}\big)\big] \right) p\big(\theta_{(np)}|Y\big)d\theta_{(np)}, \tag{6}$$

where $p(\theta_{(np)}|Y)$ is the marginal posterior density of the non-policy parameters.

Unfortunately, much of the empirical evidence points towards misspecification of the restricted moving average terms $D_j^*(\theta_{(p)}, \theta_{(np)})$ in (5). As an alternative to (5) we consider an unrestricted moving-average representation for output and inflation of the form

$$y_{2,t} = \sum_{j=0}^{\infty} \big[D_j^*(\theta_{(p)}, \theta_{(np)}) + D_j^{\Delta}\big]\epsilon_{t-j}. \tag{7}$$

Using a modified version of the DSGE-VAR framework developed in Del Negro and Schorfheide (2004) and DSSW we first re-confirm the empirical evidence that D_j^{Δ} is not zero. Once model misspecification has been detected two challenges arise: (i) How does the presence of misspecification affect the estimate of the non-policy parameters $\theta_{(np)}$? This is an important question because some of these parameters (e.g., the degree of nominal rigidities, or the persistence of shocks) play a key role in the policy analysis. (ii) How do policy changes of $\theta_{(p)}$ translate into changes in D_j^{Δ}? In turn we will discuss two approaches to address these challenges.

The first approach is described in Section 2.1 and builds upon the DSGE-VAR framework. We solve the DSGE model, then create a prior distribution for a VAR that concen-

trates in the neighborhood of the DSGE model implied restrictions yet allows for deviations.[3] We construct a joint posterior distribution for VAR and DSGE model parameters. The VAR likelihood function is penalized by the prior distribution if its parameters strongly deviate from the DSGE model-implied restrictions. Simultaneously, the DSGE model parameters θ are essentially estimated by minimizing the deviations D_j^Δ from the cross-equation restrictions $D_j^*(\theta_{(p)}, \theta_{(np)})$. The DSGE model is treated as a reference model around which the more loosely parameterized VAR is centered. We will subsequently provide reasons why the estimates of $\theta_{(np)}$ obtained under the DSGE-VAR, rather than those obtained under the DSGE model, might be preferable for policy analysis.

Once we have obtained estimates of the VAR and the DSGE dynamics we consider four methods of conducting a policy analysis that differ with respect to the assumptions about the policy-invariance of private agents' behavior and the discrepancies D_j^Δ: (i) One extreme is to perform the policy exercise under the DSGE model. In this case the DSGE-VAR is only a tool to provide alternative estimates of $\theta_{(np)}$. (ii) The other extreme is to ignore the rational expectations responses of the private sector behavior. That is, one treats the DSGE-VAR as a structural (backward looking) VAR and only changes the coefficients of the monetary policy rule. The third and forth approach lie in between these two polar cases. Under (iii) we use $D_j^*(\theta_{(p)}, \theta_{(np)})$ but acknowledge the presence of misspecification. We regard historical estimates of the discrepancy matrices as largely uninformative about the post-intervention misspecification. Performance measures are computed from (7) under the *prior* instead of *posterior* distribution of the D_j^Δ's. Finally, method (iv) uses the *posterior* of the D_j^Δ's and treats the discrepancies as policy invariant. We discuss under what circumstances each of these methods can be appealing.

A second approach to cope with model misspecification is discussed in Section 2.2 and refines a common practice in the empirical work with DSGE models. This practice amounts to relaxing the restrictions placed on the law of motion of the exogenous shocks in (4) by introducing additional shocks into the model and/or by generalizing the AR(1) structure. The modification of the shock structure introduces additional parameters that have to be estimated along with $\theta_{(p)}$ and $\theta_{(np)}$. Although macroeconomists understand that these additional parameters in most cases do not capture micro-founded propagation mechanisms, they are nonetheless often introduced to ameliorate misspecification problems. Once these

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[3]Unlike in our earlier work which applies this prior to the law of motion of both R_t and $y_{2,t}$, we use this prior distribution only for the equations that describe the evolution of $y_{2,t}$. These equations are then combined with the monetary policy rule.

parameters have been estimated, it is common practice to treat them as "structural," i.e. policy invariant and to conduct policy analysis by calculating rational expectations solutions of the DSGE model under the modified shock structure.[4]

2.1 Relaxing Restrictions on the Reduced Form

We begin by modifying the DSGE-VAR framework to obtain estimates of the extent to which the DSGE model implied restrictions are violated. Rather than working with infinite-order moving-average representations along the lines of Equation (7), the DSGE-VAR uses finite-order VAR representations since they are easier to handle at the model estimation step. We then explain how $\theta_{(np)}$ and D_j^{Δ} are identified.

2.1.1 The DSGE-VAR Framework

Let us write Equation (1), which describes the policymaker's behavior, in more general form as:

$$y_{1,t} = x_t'\beta_1(\theta) + y_{2,t}'\beta_2(\theta) + \epsilon_{1,t}\sigma_R, \tag{8}$$

where $y_t = [y_{1,t}, y_{2,t}']'$ and the $k \times 1$ vector $x_t = [y_{t-1}', \ldots, y_{t-p}', 1]'$ is composed of the first p lags of y_t and an intercept. The vector-valued functions $\beta_1(\theta)$ and $\beta_2(\theta)$ interact with x_t and $y_{2,t}$ to generate the policy rule. In our simple example $\beta_1(\theta) = 0$ and $\beta_2(\theta)$ extracts inflation and output from the vector $y_{2,t}$ and multiplies it by the policy rule coefficients ψ_1 and ψ_2. Our notation is general enough to cover the more elaborate monetary policy rules used in the empirical analysis.[5]

We proceed by approximating the DSGE model-implied moving average representation (5) of $y_{2,t}$ with a p-th order autoregression, which we write as

$$y_{2,t}' = x_t'\Psi^*(\theta) + u_{2,t}'. \tag{9}$$

Assuming that under the DSGE model the law of motion for $y_{2,t}$ is covariance stationary for every θ, we define the moment matrices $\Gamma_{XX}(\theta) = E_\theta^D[x_t x_t']$ and $\Gamma_{XY_2}(\theta) = E_\theta^D[x_t y_{2,t}']$.

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[4]A related approach to misspecification amounts to introducing ad-hoc features into the model. Given that these features are model specific, we do not deal explicitly with them in our framework. One can see however that this related approach is in many ways germane to relaxing the shock structure.

[5]Considering forecast-based policy rules in this framework would require significant modifications. However, according to the findings reported by Levin, Wieland, and Williams (2003) forecast-based rules do not provide substantial gains in stabilization performance compared with simple outcome-based rules. Hence, we decided not to pursue these modifications at this point.

In our notation $\mathbb{E}_\theta^D[\cdot]$ denotes an expectation taken under the probability distribution for y_t and x_t generated by the DSGE model conditional on the parameter vector θ. We define the VAR approximation of $y_{2,t}$ through

$$\Psi^*(\theta) = \Gamma_{XX}^{-1}(\theta)\Gamma_{XY_2}(\theta). \tag{10}$$

The equation for the policy instrument (8) can be rewritten by replacing $y_{2,t}$ with expression (9):

$$y_{1,t} = x_t'\beta_1(\theta) + x_t'\Psi^*(\theta)\beta_2(\theta) + u_{1,t}, \tag{11}$$

Let $u_t' = [u_{1,t}, u_{2,t}']$ and define

$$\Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta). \tag{12}$$

If we assume that the u_t's are normally distributed, denoted by $u_t \sim \mathcal{N}(0, \Sigma_*(\theta))$, then Equations (9) to (12) define a restricted VAR(p) for the vector y_t. While the moving-average representation of y_t under the linearized DSGE model does in general not have an exact VAR representation, the restriction functions $\Psi^*(\theta)$ and $\Sigma^*(\theta)$ are defined such that the covariance matrix of y_t, which enters the construction of our policy performance measure (6), is preserved. Let $\mathbb{E}_{\Psi,\Sigma}^{VAR}[\cdot]$ denote expectations under the restricted VAR. It can be verified that

$$\mathbb{E}_{\Psi^*(\theta),\Sigma^*(\theta)}^{VAR}[y_t y_t'] = \mathbb{E}_\theta^D[y_t y_t']. \tag{13}$$

This point is important given that the second moments of the endogenous variables are the objects of interest for this analysis.

To account for potential misspecification we now relax the DSGE model restrictions and introduce discrepancy matrices Ψ^Δ and Σ^Δ such that

$$\begin{aligned}
y_{1,t} &= x_t'\beta_1(\theta) + x_t'\big(\Psi^*(\theta) + \Psi^\Delta\big)\beta_2(\theta) + u_{1,t}, \\
y_{2,t}' &= x_t'\big(\Psi^*(\theta) + \Psi^\Delta\big) + u_{2,t}',
\end{aligned} \tag{14}$$

and $u_t \sim \mathcal{N}(0, \Sigma^*(\theta) + \Sigma^\Delta)$. Our analysis is cast in a Bayesian framework in which initial beliefs about the DSGE model parameter θ and the model misspecification matrices Ψ^Δ and Σ^Δ are summarized in a prior distribution. In contrast to Del Negro and Schorfheide (2004) and DSSW, we assume in (14) that there is no misspecification in the policy rule. Our prior distribution for Ψ^Δ and Σ^Δ is chosen such that conditional on a DSGE model parameter θ

$$\Sigma^\Delta|\theta \quad \sim \quad \mathcal{IW}\left(T^*\Sigma^*(\theta), T^* - k\right) - \Sigma^*(\theta) \tag{15}$$

$$\Psi^\Delta|\Sigma^\Delta, \theta \quad \sim \quad \mathcal{N}\left(0, \frac{1}{T^*}\left[\big(B_2(\theta)\big(\Sigma^*(\theta) + \Sigma^\Delta\big)^{-1}B_2(\theta)'\big) \otimes \Gamma_{XX}(\theta)\right]^{-1}\right),$$

where $\mathcal{IW}$ denotes the inverted Wishart distribution, $B_1(\theta) = [\beta_1(\theta), 0_{k \times (n-1)}]$, and $B_2(\theta) = [\beta_2(\theta), I_{(n-1) \times (n-1)}]$.

A few remarks on the shape of the prior contours for $\Psi^\Delta, \Sigma^\Delta$, and how the prior distributes mass along these contours are in order. First, the distribution of mass is controlled by the hyperparameter T^*, which we will re-parameterize in terms of multiples of the actual sample size T, that is $T^* = \lambda T$. Large values of λ imply that large discrepancies are unlikely to occur and the prior concentrates near the restriction functions $\Psi^*(\theta)$ and $\Sigma^*(\theta)$. We consider values of λ on a finite grid Λ and use a data-driven procedure to determine an appropriate value for this hyperparameter. A natural criterion to select λ in a Bayesian framework is the marginal data density

$$p_\lambda(Y) = \int p(Y|\Psi, \Sigma, \theta) p_\lambda(\Psi, \Sigma, \theta) d(\Psi, \Sigma, \theta). \tag{16}$$

Here $p_\lambda(\Psi, \Sigma, \theta)$ is a joint prior distribution for the VAR coefficient matrices $\Psi = \Psi^*(\theta) + \Psi^\Delta$ and $\Sigma = \Sigma^*(\theta) + \Sigma^\Delta$ and the DSGE model parameters θ. This prior is obtained by combining the prior in (15) with a prior density for θ, denoted by $p(\theta)$:

$$p_\lambda(\Psi, \Sigma, \theta) = p(\theta) p_\lambda(\Sigma|\theta) p_\lambda(\Phi|\Sigma, \theta). \tag{17}$$

We define

$$\hat{\lambda} = \text{argmax}_{\lambda \in \Lambda} \, p_\lambda(Y). \tag{18}$$

As discussed in DSSW, $\hat{\lambda}$ and the marginal likelihood ratio $p_{\lambda = \hat{\lambda}}(Y)/p_{\lambda = \infty}(Y)$ provide an overall measure of fit for the DSGE model. If there is a large discrepancy between the autocovariances implied by the DSGE model and the sample autocovariances, $\hat{\lambda}$ will be small and the marginal likelihood ratio will be large.

Second, the size of the model misspecification is associated with the ease with which it can be detected via likelihood ratios, in the spirit of Hansen and Sargent's (2005) robust control approach, as discussed in detail in DSSW. Third, holding the innovation matrix $\Sigma^*(\theta)$ constant, $\Gamma_{XX}(\theta)$ tends to be large – hence the prior variance of Ψ^Δ small – whenever θ implies that the endogenous variables are highly persistent. We view this as an attractive feature of the prior. Since due to the presence of transversality conditions DSGE model solutions are restricted to be stationary, our prior steers us away from VAR parameterizations that imply non-stationarity and explosiveness. Finally, our prior is also computationally convenient. We use Markov-Chain-Monte Carlo methods described in the Technical Appendix to generate draws from the joint posterior distribution of Ψ, Σ, and θ as well as to evaluate the marginal data density $p_\lambda(Y)$. We refer to empirical model comprised of

the likelihood function associated with the restricted VAR in Equation (14) and the prior distributions $p_\lambda(\Psi, \Sigma|Y)$, given in (15), and $p(\theta)$ as DSGE-VAR(λ).

2.1.2 Parameter Estimation

This subsection provides some intuition for how the parameters of the DSGE model are estimated under the DSGE-VAR procedure. To simplify the exposition, we will condition on the covariance matrix Σ and the hyperparameter λ that controls the magnitude of deviations from the DSGE model restrictions. Conforming with the partitioning of u_t into $u_{1,t}$ and $u_{2,t}$, we partition the covariance matrix into the sub-matrices Σ_{11}, Σ_{12}, Σ_{21}, and Σ_{22}. Moreover, we define $\Sigma_{11.22} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$. The joint posterior density can be expressed as

$$
\begin{aligned}
p(\Psi, &\theta_{(p)}, \theta_{(np)}|Y) \hspace{6cm} (19)\\
\propto\ & |\Sigma_{22}|^{-T/2}\text{etr}\left[\Sigma^{-1}(Y_2 - X\Psi)'(Y_2 - X\Psi)\right]\\
& \times|\Sigma_{11.22}|^{-T/2}\text{etr}\left[\Sigma_{11.22}^{-1}\left(Y_1 - X\beta_1(\theta_{(p)}) - X\Psi\beta_2(\theta_{(p)}) - (Y_2 - X\Psi)\Sigma_{22}^{-1}\Sigma_{21}\right)'\right.\\
& \times\left.\left(Y_1 - X\beta_1(\theta_{(p)}) - X\Psi\beta_2(\theta_{(p)}) - (Y_2 - X\Psi)\Sigma_{22}^{-1}\Sigma_{21}\right)\right]\\
& \times\text{etr}\left[\lambda TB_2\Sigma^{-1}B_2'(\Psi - \Psi^*(\theta_{(p)}, \theta_{(np)}))'\Gamma_{XX}(\Psi - \Psi^*(\theta_{(p)}, \theta_{(np)}))\right]\\
& \times p(\theta_{(p)}, \theta_{(np)})
\end{aligned}
$$

where $\text{etr}[A] = \exp\{-tr[A]/2\}$, $\propto$ denotes proportionality, and the columns of Y and X are composed of y_t' and x_t', respectively.

We can draw several conclusions from the form of the posterior density. First, the policy parameters are essentially identified via exclusion restrictions. The functions $\beta_1(\cdot)$ and $\beta_2(\cdot)$ only depend on the policy parameters, the only DSGE model parameters that enter the monetary policy rule. Conditional on Ψ most of the information about the policy parameters stems from the contribution of Y_1 to the likelihood function (lines 2 and 3 in (19)) as well as the prior $p(\theta_{(p)}, \theta_{(np)})$. The term $(Y_2 - X\Psi)\Sigma_{22}^{-1}\Sigma_{12}$ corrects for the endogeneity of the contemporaneous regressors in the policy rule. Identification is achieved through an exclusion restriction embodied in $\beta_2(\cdot)$. In the above example no lagged endogenous variables enter the policy rule (1). In the empirical application we assume that only lagged interest rates enter the policy rule in addition to contemporaneous measures of inflation and output. These exclusion restrictions are consistent with the specification of the DSGE model and identify the monetary policy shock $\epsilon_{1,t}$ in the DSGE-VAR.

Second, we discuss the estimation of the non-policy parameters. The value of λ – the extent to which we admit the presence of misspecification – plays an important role here. Conditional on Ψ, the shape of the posterior of $\theta_{(np)}$ is determined by $p(\Psi|\theta_{(p)}, \theta_{(np)})$ (line 4 in (19)) and the prior $p(\theta_{(p)}, \theta_{(np)})$. If the hyperparameter λ is small, the latter will dominate. For large values of λ the estimate of $\theta_{(np)}$ has the flavor of a minimum distance estimate and is identified provided that different values of $\theta_{(np)}$ imply different values for $\Psi^*(\cdot)$ (see Smith 1993). The posterior density is high for values of $\theta_{(np)}$ that imply a small discrepancy between Ψ and the restriction function $\Psi^*(\theta_{(p)}, \theta_{(np)})$. In summary, if we assume there is no misspecification ($\lambda = \infty$), we force the $\theta_{(np)}$ to generate all the dynamics observed in the data. As we lower λ, we relax this constraint and let our prior density $p(\theta_{(p)}, \theta_{(np)})$ play a more important role in the formation of the posterior. Given that current macroeconomic practice puts emphasis on the prior information obtained from – say – microeconomic evidence (for example, on the degree of price stickiness), this reliance on the prior can be appealing.

Third, conditional on $\theta_{(p)}$ and $\theta_{(np)}$, the shape of the posterior for Ψ is mostly determined by the contribution of Y_2 to the likelihood function (line 1 in (19)). The prior density $p(\Psi|\theta_{(p)}, \theta_{(s)})$ serves as a penalty function, that penalizes values of Ψ that deviate strongly from the restriction function $\Psi^*(\theta_{(p)}, \theta_{(np)})$. In fact, we show in a slightly different setup in Del Negro and Schorfheide (2004) that the VAR is estimated subject to the restriction $\Psi = \Psi^*(\theta_{(p)}, \theta_{(np)})$, if $\lambda = \infty$.

2.1.3 Policy Analysis

The challenge in the evaluation of monetary policy rules is to predict the private sector's behavioral responses to policy regime changes, which are captured by the coefficient matrix Ψ and the elements of Σ that are unrelated to the monetary policy shock. We will describe four types of analysis that differ according to the assumptions that are being made about the policy-invariance of private agents' behavior and model misspecification.

Forward-looking Analysis: One can decide to use the DSGE-VAR framework only to obtain estimates of $\theta_{(np)}$ that in the presence of misspecification do not force the DSGE model to capture all the dynamics in the data. Conditional on these parameter estimates, the policy analysis is conducted directly with the DSGE model.

Backward-looking Analysis: At the other extreme, one can choose to conduct the exercise using DSGE-VAR as a structural VAR, as for instance in Sims (1999). This amounts

to assuming that the decision rules of firms and households are unaffected by the policy change. The DSGE-VAR developed previously can be interpreted as a structural VAR in which the monetary policy rule is identified through exclusion restrictions:

$$
\begin{aligned}
y_{1,t} &= x_t'\beta_1(\theta_{(p)}) + [x_t'\Psi + u_{2,t}']\beta_2(\theta_{(p)}) + \epsilon_{1,t}\sigma_R \\
y_{2,t}' &= x_t'\Psi + u_{2,t}'.
\end{aligned}
$$

According to the underlying DSGE model, $u_{2,t}$ is a function of the monetary policy shock $\epsilon_{1,t}$ and other structural shocks $\epsilon_{2,t}$. We assume that the shocks $\epsilon_{2,t}$ have unit variance and are uncorrelated with each other and the monetary policy shock. We express $u_{2,t}$ as

$$
u_{2,t}' = \epsilon_{1,t}A_1 + \epsilon_{2,t}'A_2. \tag{20}
$$

Straightforward matrix algebra leads to the following formulas for the effect of the structural shocks on $u_{2,t}'$:

$$
A_1 = \left[\Sigma_{11} - \beta_2'\Sigma_{22}\beta_2 - 2(\Sigma_{12} - \beta_2'\Sigma_{22})\beta_2\right]^{-1}(\Sigma_{12} - \beta_2'\Sigma_{22}) \tag{21}
$$

$$
A_2'A_2 = \Sigma_{22} - A_1'\left[\Sigma_{11} - \beta_2'\Sigma_{22}\beta_2 - 2(\Sigma_{12} - \beta_2'\Sigma_{22})\beta_2\right]A_1. \tag{22}
$$

While this decomposition of the forecast error covariance matrix identifies A_1, it does not uniquely determine the matrix A_2. Let $A_{2,tr}$ be a Cholesky factor of $A_2'A_2$ and $\tilde{u}_{2,t}$ be a vector of innovations with mean zero and unit variance, that is uncorrelated with the monetary policy shock $\epsilon_{1,t}$. Following Sims (1986), we express the private sector equations as follows:

$$
y_{2,t}'(I + \beta_2 A_1) - y_{1,t}A_1 = x_t'(\Psi - \beta_1 A_1) + \tilde{u}_{2,t}'A_{2,tr}. \tag{23}
$$

In the backward-looking analysis we use posterior draws of (θ, Ψ, Σ) to determine β_1, β_2, A_1, and $A_{2,tr}$ and assume that the coefficients in (23) are policy-invariant. The counterfactual law of motion of y_t under a new policy $\tilde{\theta}_{(p)}$ is obtained from

$$
\begin{aligned}
y_{1,t} - y_{2,t}'\beta_2(\tilde{\theta}_{(p)}) &= x_t'\beta_1(\tilde{\theta}_{(p)}) + \epsilon_{1,t}\sigma_R \\
y_{2,t}'(I + \beta_2 A_1) - y_{1,t}A_1 &= x_t'(\Psi - \beta_1 A_1) + \tilde{u}_{2,t}'A_{2,tr}.
\end{aligned} \tag{24}
$$

The first equation represents the new monetary policy rule, whereas the second equation captures the (unchanged) law of motion for the private-sector variables.

Acknowledge Misspecification We use the historical sample to estimate the non-policy parameters $\theta_{(np)}$ and the overall degree of misspecification measured by $\hat{\lambda}$. Starting from the forward-looking analysis we do acknowledge misspecification and hence introduce the

matrices Ψ^Δ and Σ^Δ into the policy analysis step. The DSGE-VAR framework is used to predict policy outcomes:

$$
\begin{aligned}
y_{1,t} &= x_t'\beta_1(\tilde{\theta}_{(p)}) + x_t'\big(\Psi^*(\tilde{\theta}_{(p)},\theta_{(np)}) + \Psi^\Delta\big)\beta_2(\tilde{\theta}_{(p)}) + u_{1,t} \quad\quad\quad (25)\\
y_{2,t}' &= x_t'\big(\Psi^*(\tilde{\theta}_{(p)},\theta_{(np)}) + \Psi^\Delta\big) + u_{2,t}',
\end{aligned}
$$

where the covariance matrix of u_t is given by $\Sigma^*(\tilde{\theta}_{(p)},\theta_{(np)}) + \Sigma^\Delta$. In the absence of a firm theory that explains how the discrepancy matrices respond to policy changes, we use the prior distribution (15) to characterize beliefs about post-intervention model misspecification. This analysis reflects the belief that the sample, and hence the posterior of Ψ^Δ and Σ^Δ provides no information misspecification after a new policy has been implemented. This scepticism about the relevance of sample information is shared by the robust control approaches of Giannoni (2002), Onatski and Stock (2002), Onatski and Williams (2003), and Hansen and Sargent (2005). However, instead of using a minimax calculation we rely on a prior probability distribution $p(\Psi^\Delta, \Sigma^\Delta | \theta, \lambda)$ to cope with misspecification uncertainty.

Policy-Invariant Misspecification To characterize the degree of misspecification in an estimated DSGE model, DSSW compare impulse response functions from the DSGE-VAR($\hat{\lambda}$) to those from a DSGE-VAR(∞). If the DSGE model is well specified, $\hat{\lambda}$ is likely to be large and the discrepancy among the impulse response functions is small. If on the other hand, the misspecification is substantial and $\hat{\lambda}$ is small, then the discrepancy between the impulse response functions can be used to diagnose in which dimension the DSGE model is misspecified. We will now assume that the estimated discrepancy, in terms of impulse response functions, is policy invariant.

For the impulse response functions to be interpretable, it is useful to apply an identification scheme that links them to the structural shocks in the underlying DSGE model. Recall that the monetary policy shock has been identified through an exclusion restriction. However, we still have to identify the matrix A_2 in (20). We follow the approach taken in Del Negro and Schorfheide (2004). Let $A_{2,tr}'A_{2,tr} = A_2'A_2$ be the Cholesky decomposition of $A_2'A_2$. The relationship between $A_{2,tr}$ and A_2 is given by $A_2' = A_{2,tr}'\Omega$, where Ω is an orthonormal matrix that is not identifiable based on the estimates of $\beta(\theta)$, Ψ, and Σ. However, we are able to calculate an initial effect of $\epsilon_{2,t}$ on $y_{2,t}$ based on the DSGE model, denoted by $A_2^D(\theta)$. This matrix can be uniquely decomposed into a lower triangular matrix and an orthonormal matrix:

$$
A_2^{D'}(\theta) = A_{2,tr}^{D'}(\theta)\Omega^*(\theta). \quad\quad\quad (26)
$$

To identify A_2 above, we combine $A'_{2,tr}$ with $\Omega^*(\theta)$.[6] Loosely speaking, the rotation matrix is constructed such that in the absence of misspecification the DSGE model's and the DSGE-VAR's impulse responses to $\epsilon_{2,t}$ coincide. To the extent that misspecification is mainly in the dynamics as opposed to the covariance matrix of innovations, the identification procedure can be interpreted as matching, at least qualitatively, the short-run responses of the VAR with those from the DSGE model.

In order to implement the policy analysis, we use posterior draws of (θ, Ψ, Σ) to create two moving average representations for $y_{2,t}$:

$$\sum_{j=0}^{\infty} \widetilde{D}_j^*(\theta)u_{2,t-j} = \sum_{j=0}^{\infty} \widetilde{D}_j^*(\theta)\big(A_1^*(\theta)'\epsilon_{1,t-j} + A_2^*(\theta)'\epsilon_{2,t-j}\big)$$

$$\sum_{j=0}^{\infty} \widetilde{D}_j(\Psi)u_{2,t-j} = \sum_{j=0}^{\infty} \widetilde{D}_j\big(A_1(\theta,\Sigma)'\epsilon_{1,t-j} + A_2(\theta,\Sigma)'\epsilon_{2,t-j}\big).$$

The first representation is calculated from the VAR approximation of the DSGE model $\Psi^*(\theta)$ and $\Sigma^*(\theta)$. The second representation is obtained from the estimated DSGE-VAR specification. The impulse response function discrepancies (DSGE-VAR($\hat{\lambda}$) versus DSGE-VAR(∞)) are given by

$$IRF_j^\Delta = \widetilde{D}_j(\Psi)\big[A_1(\theta,\Sigma)', \ A_2(\theta,\Sigma)'\big] - \widetilde{D}_j^*(\theta)\big[A_1^*(\theta)', \ A_2^*(\theta)'\big].$$

We consider the following post-intervention law of motion for $y_{2,t}$:

$$y_{2,t} = \sum_{j=0}^{\infty} \left[\widetilde{D}_j^*(\tilde{\theta}_{(p)}, \theta_{(np)})\big[A_1^*(\tilde{\theta}_{(p)}, \theta_{(np)})', \ A_2^*(\tilde{\theta}_{(p)}, \theta_{(np)})'\big] + IRF_j^\Delta \right] \begin{bmatrix} \epsilon_{1,t-j} \\ \epsilon_{2,t-j} \end{bmatrix}. \quad (27)$$

If we were endowed with with a credible model of how Ψ^Δ and Σ^Δ vary with policy, we should of course use such model, and the DSGE-VAR analysis would lose its appeal. In the absence of such a model our forward-looking analysis, the backward-looking analysis, and the policy-invariant misspecification analysis take different attitudes toward the Lucas critique in that they assume that certain coefficients in the empirical model are policy invariant. The acknowledge-misspecification analysis is more agnostic in that it places a probability distribution over the post-intervention misspecification matrices.

2.2 Relaxing Restrictions on the Exogenous Shocks

Equation (4) in our example restricts the exogenous shocks to follow AR(1) processes that are uncorrelated with each other at all leads and lags. While this assumption is common

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[6]The calculation is easily implementable in a MCMC analysis. For every draw of $(\theta, \Psi^\Delta, \Sigma^\Delta)$ from their joint posterior distribution we compute $\Omega^*(\theta)$ and A_2.

in the literature on estimated DSGE models, it is also arbitrary. For instance, there is no theory that implies that technology shocks have to follow AR(1) processes. In general, the literature strives to build models in which persistence and co-movements are generate endogenously, through some economic mechanism, rather than exogenously. This pursuit favors specification in which shocks are indeed uncorrelated with each other and have fairly simple dynamics. However, once taken to the data, these specifications often miss important aspects, which has lead researchers to consider more general shock processes.

The generalization of the exogenous shocks takes in most cases the form of additional AR(1) processes. For instance, Smets and Wouters (2003) fit a model with 10 exogenous shocks to seven macroeconomic variables. Several of these shocks have been added in the model building process to overcome specification problems. More recently, a number of authors including Chari, Kehoe, and McGrattan (2007) and Primiceri, Schaumburg, and Tambalotti (2006) have studied the extent to which intra- and intertemporal optimality conditions implied by DSGE models are consistent with the data. If they are not, the resulting "wedges" are represented by stochastic shocks, underlining that the proliferation of exogenous shocks in empirical DSGE models can be thought of as an attempt to overcome specification problems. Alternatively, Smets and Wouters (2006) are using ARMA(1,1) processes to describe the law of motion of price and wage mark-up shocks in their DSGE model. We show that this approach is simply another ways of allowing for deviations D_j^Δ from the cross-equation restrictions $D_j^*(\theta_{(p)}, \theta_{(np)})$ in (7). Unlike in the DSGE-VAR framework, these deviations are introduced in the structural form rather than the reduced form. But when it comes to policy analysis, this approach shares the same conceptual difficulties as the DSGE-VAR: One needs to assume that the D_j^Δ's are policy invariant.

Suppose we generalize (4) in our example as follows

$$
\begin{bmatrix} g_t \\ z_t \end{bmatrix} = \sum_{j=0}^{\infty} \left(\underbrace{\begin{bmatrix} \rho_g^j \sigma_g & 0 \\ 0 & \rho_z^j \sigma_z \end{bmatrix}}_{C_j^*(\theta_{(x)}^*)} + C_j^\Delta \right) \epsilon_{2,t-j}. \tag{28}
$$

and partition the vector of non-policy parameters $\theta_{(np)} = [\theta_{(s)}', \theta_{(x)}^{*'}, \theta_{(x)}^{\Delta'}]'$, where $\theta_{(s)} = [\beta, \kappa]'$, $\theta_{(x)}^* = [\rho_g, \rho_z, \sigma_g, \sigma_z]'$, and θ_x^Δ is composed of the non-redundant elements of the lag polynomial $\sum_{j=0}^{\infty} C_j^\Delta L^j$. It follows from Sims (2002) that the law of motion of $y_{2,t}$ can then be expressed as

$$
y_{2,t} = \sum_{j=0}^{\infty} \left[D_j^*(\theta_{(p)}, \theta_{(s)}, \theta_{(x)}^*) + D_j^\Delta(\theta_{(p)}, \theta_{(s)}, \theta_{(x)}^\Delta) \right] \epsilon_{t-j}. \tag{29}
$$

Hence, this approach generates a representation for the discrepancy matrices D_j^Δ in (7), and links them to the policy parameters $\theta_{(p)}$ under the assumption that $\theta_{(x)}^\Delta$ is policy invariant.

2.2.1 Implementation

As in Section 2.1 it is more convenient to work with vector autoregressive representations when implementing the analysis in practice. In the context of our example we stack the exogenous processes in the vector $\widetilde{z}_t = [g_t, z_t]'$ and consider a general representation of the form

$$\widetilde{z}_t = \Phi_1 \widetilde{z}_{t-1} + \ldots + \Phi_p \widetilde{z}_{t-1} + \widetilde{\epsilon}_t. \tag{30}$$

The innovations $\widetilde{\epsilon}_t$ are not normalized and have a covariance matrix Σ_ϵ. Let $x_t' = [\widetilde{z}_{t-1}, \ldots, \widetilde{z}_{t-p}]'$, $\Phi' = [\Phi_1', \ldots, \Phi_p']$, and write

$$\widetilde{z}_t' = x_t' \Phi + \widetilde{\epsilon}_t'. \tag{31}$$

Notice that (31) mirrors (9). From the restricted moving-average representation $\widetilde{z}_t = \sum_{j=0}^\infty C_j^*(\theta_{(x)}^*) \epsilon_{t-j}$ we can derive the moment matrices $\Gamma_{ZZ}(\theta_{(x)}^*)$, $\Gamma_{XX}(\theta_{(x)}^*)$, and $\Gamma_{XZ}(\theta_{(x)}^*)$. Following the steps in Section 2.1, we can define the restriction functions $\Phi^*(\theta_{(x)}^*)$ and $\Sigma_\epsilon^*(\theta_{(x)}^*)$. Then let $\Phi = \Phi^*(\theta_{(x)}^*) + \Phi^\Delta$, and $\Sigma_\epsilon = \Sigma_\epsilon^*(\theta_{(x)}^*) + \Sigma_\epsilon^\Delta$. Finally, we can use a prior of the form

$$\Sigma_\epsilon^\Delta | \theta_{(x)}^* \quad \sim \quad \mathcal{IW}\left(T^* \Sigma_\epsilon^*(\theta_{(x)}^*), T^* - m\right) - \Sigma_\epsilon^*(\theta_{(x)}^*) \tag{32}$$

$$\Phi^\Delta | \Sigma_\epsilon^\Delta, \theta_{(x)}^* \quad \sim \quad \mathcal{N}\left(0, \frac{1}{T^*}\left[\left(\Sigma_\epsilon^*(\theta_{(x)}^*) + \Sigma_\epsilon^\Delta\right)^{-1} \otimes \Gamma_{XX}(\theta_{(x)}^*)\right]^{-1}\right).$$

The intuition for this prior is very simple. Suppose we are generating a prior for an AR(2) model from an AR(1). Given the parameters of the AR(1) $(\theta_{(x)}^*)$ we can generate artificial observations from an AR(1) model. The expected moments of these observations are summarized in the matrices $\Gamma_{ZZ}(\theta_{(x)}^*)$, $\Gamma_{XX}(\theta_{(x)}^*)$, and $\Gamma_{XZ}(\theta_{(x)}^*)$ and serve as sufficient statistics for the estimation of the parameters of the AR(2) model. The posterior distribution from this (fictional) estimation is given by (32).

Under the generalized shocks, the DSGE model in our example would consist of Equations (1) to (3) and (30). The parameter vector is composed of $\theta_{(p)}$, $\theta_{(s)}$, Φ, and Σ_ϵ. Equation (32) generates a prior distribution for Φ and Σ_ϵ conditional on $\theta_{(x)}^*$, which can be combined with a prior on the hyperparameters $\theta_{(x)}^*$ as in (17). Thus, the joint distribution of data and parameters has the following factorization

$$p(Y | \theta_{(p)}, \theta_{(s)}, \Phi, \Sigma_\epsilon) p(\theta_{(p)}) p(\theta_{(s)}) p_{T^*}(\Phi, \Sigma_\epsilon | \theta_{(x)}^*) p(\theta_{(x)}^*).$$

2.2.2 Parameter Estimation

Under the generalized shock structure the law of motion for the exogenous processes is parameterized in terms of Φ and Σ_ϵ instead of just $\theta^*_{(x)}$. Nevertheless, we can estimate the DSGE model with standard Bayesian methods. However, the less restrictive the specification for the exogenous shock processes, the more difficult it becomes to disentangle the taste-and-technology parameters $\theta_{(s)}$ from the parameters that determine the evolution of $\widetilde{z}_t$. From an econometric perspective, the likelihood function may flatten as we generalize the shock structure. It is well known that in a Bayesian framework, prior distributions will not be updated along directions in the parameter space in which the likelihood function is flat (see Poirier, 1998).

2.2.3 Policy Analysis

If one is willing to assume that the generalized shocks are structural, in the sense that they are invariant to changes in economic policies, analyzing the effect of changing $\theta_{(p)}$ remains straightforward:

$$D_j^\Delta(\theta^{(1)}_{(p)}, \theta_{(s)}, \theta^\Delta_{(x)}) - D_j^\Delta(\theta^{(2)}_{(p)}, \theta_{(s)}, \theta^\Delta_{(x)})$$

can be calculated by solving the DSGE model under the generalized shock structure for policy parameter settings $\theta^{(1)}_{(p)}$ and $\theta^{(2)}_{(p)}$. However, to the extent that the lag polynomial $\sum_{j=0}^{\infty} C_j^\Delta L^j$ has essentially been added to compensate for model misspecification, its policy invariance is not self-evident.

3   Model

For the empirical analysis we will use a model that is more sophisticated than the one used in Section 2. In addition to responding to inflation and output, the central bank also engages in interest rate smoothing:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_1 \pi_t + \psi_2 \widetilde{y}_t) + \sigma_R \epsilon_{1,t}. \tag{33}$$

As before, R_t and π_t are the nominal interest rate and the inflation rate, respectively. The output gap $\widetilde{y}_t$ represents output deviations from a smooth trend path. This notion is broadly consistent with the measure of potential output calculated by the Congressional Budget Office (CBO) and used historically in monetary policy making. While much of the theoretical literature defines potential output as the level of output that would prevail in

the absence of nominal rigidities, we want (33) to closely resemble the specifications in the empirical literature on interest rate feedback rules (e.g. Taylor, 1993).

The remainder of the model is based on work by Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2003) and is identical to the specification in DSSW with one exception. Since $\widetilde{y}_t$ captures deviation from a long-run trend path we model technology shocks as a stationary process rather than a unit-root process. For brevity we only present the log-linearized equilibrium conditions and refer the reader to the above referenced papers for the derivation of these conditions from assumptions on preferences and technologies. All variables that appear subsequently are expressed as log-deviations from the steady state.

The economy is populated by a continuum of firms that combine capital and labor to produce differentiated intermediate goods. These firms have access to the same Cobb-Douglas production function with capital elasticity α and total factor productivity z_t. The intermediate goods producers hire labor and rent capital in competitive markets and hence face identical real wages, w_t, and rental rates for capital, r_t^k. Cost minimization implies that all firms produce with the same capital-labor ratio

$$k_t - L_t = w_t - r_t^k \tag{34}$$

and have marginal costs

$$mc_t = (1 - \alpha)w_t + \alpha r_t^k - (1 - \alpha)z_t. \tag{35}$$

The intermediate goods producers sell their output to perfectly competitive final good producers, which aggregate the inputs according to a CES function. Profit maximization of the final good producers implies that

$$\widetilde{y}_t(j) - \widetilde{y}_t = -\left(1 + \frac{1}{\lambda_f e^{\widetilde{\lambda}_{f,t}}}\right)(p_t(j) - p_t). \tag{36}$$

Here $\widetilde{y}_t(j) - \widetilde{y}_t$ and $p_t(j) - p_t$ are quantity and price for good j relative to quantity and price of the final good. The price p_t of the final good is determined from a zero-profit condition for the final good producers.

We assume that the price elasticity of the intermediate goods is time-varying. Since this price elasticity affects the mark-up that intermediate goods producers can charge over marginal costs, we refer to $\lambda_{f,t}$ as mark-up shock. Following Calvo (1983), we assume that in every period a fraction of the intermediate goods producers ζ_p is unable to re-optimize their prices. These firms adjust their prices mechanically according to the steady state inflation

π^*.[7] All other firms choose prices to maximize the expected discounted sum of future profits, which leads to the following equilibrium relationship, known as New Keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \frac{(1 - \zeta_p\beta)(1 - \zeta_p)}{\zeta_p}mc_t + \frac{1}{\zeta_p}\lambda_{f,t} \tag{37}$$

where β is the discount rate.[8] Our assumption on the behavior of firms that are unable to re-optimize their prices implies the absence of price dispersion in the steady state. As a consequence, we obtain a log-linearized aggregate production function of the form

$$\widetilde{y}_t = (1 - \alpha)L_t + \alpha k_t + (1 - \alpha)z_t. \tag{38}$$

Equations (35), (34), and (38) imply that the labor share lsh_t equals marginal costs in terms of log-deviations: $lsh_t = mc_t$.

There is a continuum of households with identical preferences, which are separable in consumption, leisure, and real money balances. Households' preferences display (internal) habit formation in consumption, that is, period t utility is a function of $\ln(C_t - hC_{t-1})$. Households supply monopolistically differentiated labor services. These services are aggregated according to a CES function that leads to a demand elasticity $1 + 1/\lambda_w$ (see Equation (36)). The composite labor services are then supplied to the intermediate goods producers at real wage w_t. To introduce nominal wage rigidity, we assume that in each period a fraction ζ_w of households is unable to re-optimize their wages. These households adjust their $t - 1$ nominal wage by π^*e^γ, where γ represents the average growth rate of the economy. All other households re-optimize their wages. First-order conditions imply that

$$\begin{aligned}
\left(1 + \nu_l\frac{1 + \lambda_w}{\lambda_w}\right)&\tilde{w}_t + \left(1 + \zeta_w\beta\nu_l\frac{1 + \lambda_w}{\lambda_w}\right)w_t \\
= \quad & \zeta_w\beta\left(1 + \nu_l\frac{1 + \lambda_w}{\lambda_w}\right)E_t\left[\tilde{w}_{t+1} + w_{t+1} + \pi_{t+1}\right] \\
& + (1 - \zeta_w\beta)\left(\nu_lL_t - \xi_t + \frac{e^\gamma(e^\gamma - h)}{(e^{2\gamma} + \beta h^2)}b_t + \frac{1}{1 - \zeta_w\beta}\phi_t\right),
\end{aligned} \tag{39}$$

where $\tilde{w}_t$ is the optimal real wage relative to the real wage for aggregate labor services, w_t, and ν_l would be the inverse Frisch labor supply elasticity in a model without wage

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[7]An alternative assumption is what Eichenbaum and Fisher (2003) refer to as "dynamic indexation," where these firms' prices grow at the previous period's inflation. In Del Negro and Schorfheide (2006), we discuss the extent to which a model with dynamic indexation is roughly observationally equivalent to one with autocorrelated mark-up shock, using a similar framework and the same set of observables. Of the two alternatives, here we use the one with autocorrelated mark-up shocks. It is beyond the scope of this paper to investigate the implications of this choice for policy questions.

[8]We used the following re-parameterization: $\lambda_{f,t} = (1 - \zeta_p\beta)(1 - \zeta_p)\lambda_f/(1 + \lambda_f)\widetilde{\lambda}_{f,t}$.

rigidity ($\zeta_w = 0$) and differentiated labor. Moreover, ϕ_t is a preference shock that affects the intratemporal substitution between consumption and leisure and b_t is a discount rate shock that shifts the intertemporal substitution. The real wage paid by intermediate goods producers evolves according to

$$w_t = w_{t-1} - \pi_t + \frac{1 - \zeta_w}{\zeta_w} \tilde{w}_t. \tag{40}$$

Households are able to insure the idiosyncratic wage adjustment shocks with state contingent claims. As a consequence they all share the same marginal utility of consumption ξ_t, which is given by the expression:

$$
\begin{aligned}
(e^\gamma - h\beta)(e^\gamma - h)\xi_t &= -(e^{2\gamma} + \beta h^2)c_t + \beta h e^\gamma \mathbb{E}_t[c_{t+1}] + h e^\gamma c_{t-1} \\
&\quad + (e^{2\gamma} + \beta h^2)b_t - \beta h e^{-\gamma}(e^{2\gamma} + \beta h^2)\mathbb{E}_t[b_{t+1}],
\end{aligned}
\tag{41}
$$

where c_t is consumption. In addition to state-contingent claims households accumulate three types of assets: one-period nominal bonds that yield the return R_t, capital $\bar{k}_t$, and real money balances. Since preferences for real money balances are assumed to be additively separable and monetary policy is conducted through a nominal interest rate feedback rule, money is block exogenous and we will not use the households' money demand equation in our empirical analysis.

The first order condition with respect to bond holdings delivers the standard Euler equation:

$$\xi_t = \mathbb{E}_t[\xi_{t+1}] + R_t - \mathbb{E}_t[\pi_{t+1}]. \tag{42}$$

Capital accumulates according to the following law of motion:

$$\bar{k}_t = (2 - e^\gamma - \delta)\bar{k}_{t-1} + (e^\gamma + \delta - 1)\left[i_t + S''e^{2\gamma}(1 + \beta)\mu_t\right], \tag{43}$$

where i_t is investment, δ is the depreciation rate of capital, and μ_t is a stochastic disturbance to the price of installed capital relative to consumption. Investment in our model is subject to adjustment costs, and S'' denotes the second derivative of the investment adjustment cost function at steady state. Optimal investment satisfies the following first-order condition:

$$i_t = \frac{1}{1 + \beta}i_{t-1} + \frac{\beta}{1 + \beta}\mathbb{E}_t[i_{t+1}] + \frac{1}{(1 + \beta)S''e^{2\gamma}}(\xi_t^k - \xi_t) + \mu_t, \tag{44}$$

where ξ_t^k is the value of installed capital and evolves according to:

$$\xi_t^k - \xi_t = \beta e^{-\gamma}(1 - \delta)\mathbb{E}_t\left[\xi_{t+1}^k - \xi_{t+1}\right] + \mathbb{E}_t\left[(1 - (1 - \delta)\beta e^{-\gamma})r_{t+1}^k - (R_t - \pi_{t+1})\right]. \tag{45}$$

Capital utilization u_t in our model is variable and r_t^k in the previous equation represents the rental rate of effective capital k_t, which is given by

$$k_t = u_t + \bar{k}_{t-1}. \tag{46}$$

The optimal degree of utilization is determined by

$$u_t = \frac{r_*^k}{a'} r_t^k. \tag{47}$$

Here a' is the derivative of the per-unit-of-capital cost function $a(u_t)$ evaluated at the steady state utilization rate.

The aggregate resource constraint is given by:

$$\widetilde{y}_t = (1 + g_*) \left[\frac{c_*}{y_*} c_t + \frac{i_*}{y_*} \left(i_t + \frac{r_*^k}{e^\gamma - 1 + \delta} u_t \right) \right] + g_t. \tag{48}$$

Here c_*/y_* and i_*/y_* are the steady state consumption-output and investment-output ratios, respectively, and $g_*/(1+g_*)$ corresponds to the government share of aggregate output. The process g_t can be interpreted as exogenous government spending shock. It is assumed that fiscal policy is passive in the sense that the government uses lump-sum taxes to satisfy its period budget constraint. In addition to the monetary policy shock $\epsilon_{1,t}$ the DSGE model has six exogenous shocks z_t, g_t, ϕ_t, $\lambda_{f,t}$, b_t, and μ_t. In the benchmark specification we assume that b_t and μ_t are equal to zero and that the other four shocks follow AR(1) processes. We use the method in Sims (2002) to solve the DSGE model.[9] We collect all the DSGE model parameters in the vector θ and stack the normalized innovations of the structural shocks in the vector ϵ_t.

4   Empirical Results

The goal of our empirical analysis is to study the effects of changes in the coefficients of the monetary policy rule, $\theta_{(p)}$, on the dynamics of the output gap, inflation, and nominal interest rates. In addition to these three key macroeconomic variables we include the labor share and hours worked in our estimation sample because these series can provide additional information about the degree of price and wage rigidity. Our interest and inflation rates are measured as annualized percentages. Within the model, $\widetilde{y}_t$ denotes the percentage deviation of output from its trend path $\gamma(t)y_*$. We interpret the potential output series published

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[9]The simplified model in Section 2 is obtained by setting $\alpha = 0$, $h = 0$, $\nu_l = 0$, $\zeta_w = 0$, $\rho_R = 0$, $1/a' = 0$, $S'' = 0$, $\gamma = 0$, $k_t = 0$, $b_t = 0$, $\mu_t = 0$, $\phi_t = 0$, $\lambda_{f,t} = 0$.

by the Congressional Budget Office (CBO) as a measure of $\gamma(t)y_*$. Hence, the output gap, computed as log difference of real and potential GDP provides us with a measure of $\widetilde{y}_t$. We scale the output gap, labor share, and log hours worked by a factor of 100 to obtain percentages. Further details about the data are provided in the Appendix. The empirical results reported subsequently are based on a quarterly sample from 1982:Q4 to 2005:Q4. Following Clarida, Gali, and Gertler (2000) the beginning of the sample is chosen to exclude the high inflation episode of the 1970s as well as Volcker's disinflation.

The relationships between the deviations from steady state that appear in the model description of Section 3 and the observables y_t are given by the following measurement equation:

$$
y_{1,t} = r_*^a + 400\gamma + \pi_*^a + 4R_t, \quad y_{2,t} = \begin{bmatrix} \pi_*^a + 4\pi_t \\ \widetilde{y}_t \\ 100\ln(1-\alpha)/(1+\lambda_f) + lsh_t \\ L_t \end{bmatrix}. \quad (49)
$$

Here, we have partitioned y_t such that $y_{1,t}$ corresponds to the policymaker's instrument (the interest rate), and $y_{2,t}$ is a vector that includes the remaining four observables. The steady state (net) real interest rate in our model is given by $r_*^a + 400\gamma$. The parameter r_*^a is related to the discount rate β according to $\beta = 1/(1 + r_*^a/400)$, and $\pi_*^a = 400\pi^*$ denotes steady state annualized inflation.

The remainder of this section is organized as follows. As a benchmark we use the DSGE model of Section 3 with four exogenous shocks that follow independent AR(1) processes (technology z_t, government spending g_t, labor supply ϕ_t, mark-up $\lambda_{f,t}$) and the serially uncorrelated monetary policy shock $\epsilon_{1,t}$. First, we will present evidence of misspecification in the benchmark DSGE model by comparing its fit to the fit of a DSGE-VAR that relaxes the DSGE model implied restrictions. Discrepancies in the dynamics of DSGE model and DSGE-VAR are used to motivate generalizations of the shock structure in the theoretical model. Overall, we will consider three alternative empirical models that are meant to capture some of the misspecification present in the benchmark DSGE model. These are: (i) a DSGE-VAR with four lags that relaxes the cross-equation restrictions implied by the rational expectations solution of the DSGE model; and two versions of the DSGE model in which we generalize the exogenous shock structure prior to solving the model, namely, (ii) a version in which the AR(1) government spending shock is replaced by an AR(2) process (AR(2)-in-g_t); (iii) a version that contains two additional AR(1) shocks: an investment-specific technology shock μ_t and a shock to the discount factor b_t (7-shocks).

Second, we document how the treatment of misspecification will affect the estimation of preference and technology parameters in the underlying DSGE model. In particular, we compare the estimates obtained from the four empirical specifications and study how differences in parameter estimates translate into differences in policy predictions with the underlying DSGE model. Finally, we conduct the policy analysis using the benchmark DSGE model, the AR(2)-in-g_t and the 7-shocks DSGE models, and the DSGE-VAR (backward-looking analysis, policy-invariant misspecification, and acknowledge misspecification). We show that the policy implications of the DSGE model are by and large robust to the treatment of misspecification.

4.1   Assessing Misspecification

The first step in our empirical analysis is the specification of a prior distribution for the parameters of the DSGE model. Columns 2 and 3 of Table 1 contain prior means and standard deviations. The prior distribution for the policy parameters ψ_1 and ψ_2 is approximately centered at Taylor's (1993) values, whereas the smoothing parameter lies in the range from 0.18 to 0.83. The prior for the Calvo parameters ζ_p and ζ_w, which characterize the nominal rigidities in prices and wages, respectively, are centered at 0.6 with a standard deviation of 0.15. This is a fairly diffuse distribution that encompasses findings in micro-level studies of price adjustments such as Bils and Klenow (2004).[10] The priors for the autocorrelation and standard deviation of the shocks processes are chosen with two criteria in mind. First, we want to be close to previous studies in the literature, such as Smets and Wouters (2003), DSSW, and Levin, Onatski, Williams, and Williams (2005). Second, we want to make sure that the second moments (especially volatility and autocorrelation) of the endogenous variables are roughly in line with the evidence from the pre-sample (1955:Q3 and 1982:Q3). Further details about the prior are provided in the Technical Appendix.

We proceed by computing log marginal likelihood values for our four empirical model specifications. The marginal likelihoods provide an overall measure of relative fit that trades-off in-sample fit with model complexity. Log marginal likelihoods and posterior odds relative to the DSGE-VAR (assuming that all four specifications receive equal prior probability)

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[10]We have also estimated the same model under a prior for the Calvo parameters centered at the higher value of 0.75, with standard deviation of 0.1. Interestingly, the fit of the model under the two priors, one largely agnostic and one that assumes a high degree of nominal rigidities, is roughly comparable. In the interest of space we show only the results from the agnostic prior, which achieves slightly better fit. But most of the results are robust under the alternative prior view of the world.

are reported in Table 2. The fit of the DSGE-VAR crucially depends on the choice of hyperparameter λ. For $\lambda = \infty$ we are dogmatically imposing all the restrictions of a VAR(4) approximation of the DSGE model, whereas for $\lambda = 0$ these restrictions are ignored. We have estimated the DSGE-VAR model for the grid of λ values $\{0.5, 0.75, 1, 1.5, 1, 2, 5\}$. Consistent with the results in DSSW we find that the marginal likelihood of the DSGE-VAR as a function of λ has an inverted U-shape with a peak reached for $\lambda = 0.75$. The marginal likelihood of the DSGE-VAR reported in Table 2 as well as all DSGE-VAR results presented subsequently are therefore based on $\lambda = 0.75$.[11] The log marginal likelihood differential between the DSGE benchmark model and the DSGE-VAR is 109. We conclude that allowing for deviations from the restricted moving average representation associated with the DSGE benchmark DSGE model – we generically denoted these deviations by D_j^Δ in Equation (7) – leads to a substantial improvement in the marginal likelihood. A generalization of the shock structure also leads to better fit: the marginal likelihood differentials relative to the DSGE-VAR shrink to 79 (AR(2)-in-g_t) and 64 (7-shocks), respectively.

To gain insights into the misspecification of the DSGE model restrictions we examine the moving-average representation generated by the DSGE-VAR. More specifically, we use Equations (24) and (26) to compute DSGE-VAR impulse responses to technology, government spending, mark-up, labor supply, and monetary policy shock innovations. For exposition purposes we focus on those variables that are most important for the policy exercise: the interest rate, inflation, and the output gap. The following impulse-response functions are in principle of interest: the benchmark DSGE model, the VAR approximation of the DSGE model, that is, DSGE-VAR($\lambda = \infty$), and the DSGE-VAR($\lambda = 0.75$), which provides the best fit. A comparison of DSGE and DSGE-VAR($\lambda = \infty$) documents the approximation error induced by potential lack of invertibility and truncation of VAR lags (see Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007)). We find that for our model and identification procedure the VAR approximation error is not a concern.[12] Hence, we will focus on a comparison of impulse-responses obtained from the DSGE-VAR($\lambda = 0.75$) and the DSGE-VAR($\lambda = \infty$) using the posterior draws of the DSGE model parameters θ associated with the estimated DSGE-VAR($\lambda = 0.75$). Figure 1 displays the posterior mean impulse responses and illustrates the effect of the misspecification matrices Ψ^Δ and Σ^Δ in (14).

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[11]Instead of conditioning on the value of λ that maximizes the marginal likelihood function, we could average all of our results with respect to the posterior probabilities of λ, which are proportional to the marginal likelihood values. However, since the marginal likelihood function is sharply peaked and the model predictions of interest tend to be smooth functions of λ, we believe that our simplification does not distort the empirical results.

[12]A Figure with the relevant impulse response function comparison is provided in the Technical Appendix.

Consistent with results reported in DSSW we find that the misspecification of the propagation mechanism for the technology shock is fairly small. The propagation of mark-up and labor supply shocks is also by and large not affected by the discrepancy matrices. Most of the misspecification is concentrated in the response to government spending/demand shocks (g_t). The DSGE-VAR($\lambda = 0.75$) responses are hump-shaped and much more persistent than the DSGE-VAR($\lambda = \infty$)'s.

The DSGE-VAR analysis provides some justification for the generalizations of the exogenous shock structure in the benchmark DSGE model that we are considering. Since it is the response to a government spending shock that appears to be most severely misspecified, we replace the AR(1) process for g_t with an AR(2) process using the hierarchical prior described in Section 4.4:

$$g_t = \tilde{\rho}_{g,1}(1 - \tilde{\rho}_{g,2})g_{t-1} + \tilde{\rho}_{g,2}g_{t-1} + \tilde{\sigma}_g\epsilon_t.$$

The parameterization of the AR(2) process in terms of partial autocorrelations $\tilde{\rho}_{g,1}$ and $\tilde{\rho}_{g,2}$ guarantees that the process is stationary for $\tilde{\rho}_j \in (-1, 1)$ (see Barndorff-Nielson and Schou (1973)). We shrink the coefficients of this AR(2) process toward an AR(1) representation with autocorrelation ξ_g and innovation variance ω_g (see Table 1). Overall, our prior takes the form $p(\tilde{\rho}_{g,1}, \tilde{\rho}_{g,2}, \tilde{\sigma}_g | \xi_g, \omega_g)p(\xi_g, \omega_g)$, where $p(\tilde{\rho}_{g,1}, \tilde{\rho}_{g,2}, \tilde{\sigma}_g | \xi_g, \omega_g)$ is given in (32) and we set $T^* = 70$.[13] Figure 2 overlays the impulse responses for the AR(2)-in-g_t DSGE model (black lines) with those from the benchmark specification (gray lines). A comparison with Figure 1 indicates that the responses to a demand shock in the AR(2)-in-g_t model are indeed much closer to those of the DSGE-VAR than in the baseline specification. The responses of interest rates and the output gap are hump-shaped, whereas the response of inflation is more persistent under than under the baseline specification. The responses to the other shocks, on the other hand, appear essentially unaffected and closely resemble those of the benchmark DSGE model.

As an alternative to the DSGE model with AR(2) government spending shock we consider a version of the model with investment-specific technology shocks and shocks to the stochastic discount factor. Fisher (2006) documents that investment-specific technology shocks are an important source of business cycle fluctuations and Justiano and Primiceri (2006) argue that a reduction in the volatility of this shock can account for much of the great moderation observed since the mid 1980s. Numerous studies document that the asset pricing implications of models as the one used in this paper are at odds with the data.

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[13] We chose this value of T^* as it implies roughly the same prior weight as in the DSGE-VAR.

The shock b_t exogenously modifies the model implied stochastic discount factor and hence can be viewed as a device that corrects misspecification in the consumption Euler equation. Figure 3 compares the impulse responses for this 7-shocks model (black lines) with those from the baseline specification (gray lines). The impulse responses to the two additional shocks are quite persistent, as confirmed by the estimates for the corresponding autocorrelation parameters ρ_b and ρ_μ in Table 1. The additional shocks capture some of the dynamics previously captured by demand shocks. In particular, the responses of interest rates and the output gap to an investment-specific technology shock are hump-shaped, and the response to inflation is quite persistent. These impulse responses very much resemble the DSGE-VAR responses to a demand shock, much more so than the demand shock impulse responses in the baseline model (at least in terms of the variables considered here). In other words, introducing additional shocks captures some of the misspecification present in the benchmark specification of the DSGE model.

4.2   Misspecification and Parameter Estimates

We will now examine to what extent the estimates of the DSGE model parameters differ across the four empirical models. Posterior means and standard deviations are reported in Table 1. We begin with the coefficients of the monetary policy rule. The posterior means of ψ_1, the central bank's reaction to inflation deviations from steady state range from 2.49 (AR(2)-in-g_t DSGE) to 3.06 (7-shocks DSGE). Since the posterior standard deviations are about 0.35, there is considerable overlap of the credible intervals associated with these estimates. Posterior means of ψ_2 range from 0.07 to 0.12 with standard deviations of approximately 0.05, indicating a modest response of the Federal Reserve to output gap movements. Finally, we find a fairly high degree of interest rate smoothing, with posterior mean estimates of ρ_R between 0.81 and 0.85. By and large, our policy rule estimates are in line with the numbers reported in DSSW, who find $\hat{\psi}_1 = 2.21$, $\hat{\psi}_2 = 0.07$, and $\hat{\rho}_R = 0.82$. The estimates in DSSW are based on a slightly larger sample, starting in 1974:Q2, a broader set of observables that includes consumption and investment, and a different measure of output, namely real GDP growth rates. Finally, our estimates are also broadly consistent with numbers reported in Smets and Wouters (2006) for the 1984:Q1 to 2004:Q4 sample, who find a somewhat smaller response to inflation: $\hat{\psi}_1 = 1.77$. We conclude that the policy rule estimates are fairly insensitive to the adjustments that were made to the benchmark DSGE model to account for misspecification. Based on our estimates we use the values $\psi_1 = 2.75$, $\psi_2 = 0.062$, and $\rho_R = 0.8$ as historical reference points in the evaluation of

monetary policy rules.

Important for the effects of monetary policy and the propagation of monetary policy shocks are the parameters ζ_p and ζ_w, which determine the degree of nominal rigidity in the DSGE model. For price stickiness the estimates range from 0.67 (7-shocks DSGE) to 0.76 (AR(2)-in-g_t DSGE) with standard deviations of about 0.05 and considerable overlap in the posterior densities. Interestingly, there is more divergence in the estimated wage stickiness with posterior mean estimates between 0.34 (AR(2)-in-g_t DSGE) and 0.77 (7-shocks DSGE). As a reference point, we also provide estimates from DSSW: $\hat{\zeta}_p = 0.83$ and $\hat{\zeta}_w = 0.89$; Smets and Wouters (2006, post 1983 sample): $\hat{\zeta}_p = 0.73$ and $\hat{\zeta}_w = 0.74$; Levin, Onatski, Williams, and Williams (2006): $\hat{\zeta}_p = 0.82$ and $\hat{\zeta}_w = 0.80$. In general, the analysis differs with respect to sample period and specification of the Phillips curve. Nevertheless, the dispersion suggests that the estimates are quite sensitive to auxiliary assumptions.

As was apparent from the impulse response functions discussed in the previous subsection, the estimated exogenous processes are highly serially correlated. Technology shocks are particularly persistent, with ρ_z tightly estimated around 0.97 in the three versions of the DSGE model. To provide a comparison, Smets and Wouters (2006) estimate ρ_z to be 0.97 for the 1966:Q1 to 1979:Q2 sample, 0.94 for the post 1983 sample, and 0.95 for the combined sample. Demand shocks appear to be strongly autocorrelated in the benchmark DSGE model ($\hat{\rho}_g = 0.91$) and in the AR(2)-in-g_t model, for which the estimated first-order partial autocorrelation of g_t is 0.97. Labor supply and mark-up shocks are less persistent with estimates ranging from 0.66 to 0.8 and 0.3 to 0.75, respectively.

Interestingly, the persistence of technology shocks under the DSGE-VAR is only about 0.89 and the standard deviation of the innovation is 0.42 as opposed to 0.72 in the benchmark DSGE model. Under DSGE-VAR model misspecification is partly captured by the deviations Ψ^Δ and Σ^Δ from the cross-equation restrictions. This leads to smaller forecast errors and ultimately to smaller shock volatility estimates. If the DSGE model restrictions are not relaxed, then the misspecification has to be absorbed by some of the structural shocks, which may result in highly persistent and fairly volatile processes. For instance, while in the 7-shock DSGE model the estimated autocorrelations for the government spending, the labor supply, and the mark-up shock are not as high as in the benchmark DSGE model, the additional investment-specific technology shock and the discount factor shock appear to be highly serially correlated: $\hat{\rho}_\mu = 0.91$ and $\hat{\rho}_b = 0.94$.

In general, the posterior estimates for the non-policy parameters $\theta_{(np)}$ obtained from the DSGE-VAR lie between the prior and the benchmark DSGE posterior. This finding

is consistent with Equation (19) and the theory presented in Section 2.1.2. For moderate values of λ, indicating the presence of misspecification, less weight is placed on the likelihood function and more weight on the prior distribution when determining the posterior.

4.3   Policy Implications of Estimates

With the parameter estimates in hand, we will now explore how the volatility of the output gap, inflation, and interest rates is affected by changes in the coefficients ψ_1, ψ_2, and ρ_R of the monetary policy rule (33). In this subsection we will compute unconditional variances with the benchmark DSGE model using (i) the estimates of the non-policy parameters $\theta_{(np)}$ obtained when fitting the state-space representation of the benchmark DSGE models to the data (columns 4 and 5 of Table 1), and (ii) the estimates of $\theta_{(np)}$ obtained from the DSGE-VAR analysis conditional on $\lambda = 0.75$ (columns 6 and 7 of Table 1). For brevity we will refer to (i) as the direct DSGE model estimates and to (ii) as the DSGE-VAR estimates.

We consider a two-dimensional grid for the policy rule coefficients: ψ_1 takes nine values ranging from 1.001 to 3 in intervals of 0.25; ψ_2 takes six different values, computed taking the Taylor's (1993) value $\psi_2^T = 0.125$ as a reference, namely 0, $\frac{1}{2}\psi_2^T = 0.062$, $\psi_2^T = 0.125$, $\frac{3}{2}\psi_2^T = 0.188$, $2\psi_2^T = 0.250$, $3\psi_2^T = 0.375$. We set the interest rate smoothing coefficient $\rho_R = 0.8$. We will report variance differentials relative to the baseline policy rule $\psi_1 = 2.75$, $\psi_2 = 0.062$, $\rho_R = 0.8$. These values are chosen based on the parameter estimates reported in Section 4.2 and roughly correspond to the historical Volcker-Greenspan policy rule.

As in Section 2, we use $\mathbb{V}(\theta_{(p)}, \theta_s, \Psi^\Delta, \Sigma^\Delta)$ to generically denote the covariance matrix of the output gap, inflation, and interest rates associate with an empirical model. Mainly for expositional convenience we summarize the covariance matrix $\mathbb{V}(\cdot)$ through the (loss) function

$$\min \{tr[\mathcal{W}\mathbb{V}(\theta_{(p)}, \theta_s, \Psi^\Delta, \Sigma^\Delta)], \mathcal{B}\},$$

where the upper bound $\mathcal{B}$ ensures that the posterior expected value of the variance is well defined when averaging over θ_s, Ψ^Δ, and Σ^Δ, regardless of the shape of the posterior distribution. The upper bound $\mathcal{B}$ is set to 100. This value is substantially larger than the sample variances of the output gap, inflation, and interest rates, which are approximately 4.1, 1, and 6.5, respectively, in our estimation sample. The weighting matrix for this summary measure is diagonal with entries 1/4 (annualized interest rates), 1 (annualized inflation rates), and 1/4 (output gap, percentage deviations from potential output). Since misspecification is a serious concern in our subsequent analysis, we decided not to use the expected utility of

the representative household in the underlying DSGE model as a measure of policy performance. While not welfare-based, our performance measure is of interest to many central bankers, who are generally concerned with the stabilization of output and inflation fluctuations. Moreover, it is widely used for the comparison of policy rules across broad classes of models (see for instance Taylor's (1999) volume).

To understand how changes in the policy rule affect the volatility of the output gap, inflation, and interest rates it is instructive to explore how the propagation of the structural shocks is altered. Using the benchmark DSGE model and its direct parameter estimates for the non-policy parameters $\theta_{(np)}$ we compute impulse response functions for three different values of the response to inflation in the policy rule, ψ_1: 2.75, 2, and 1.25, while fixing $\psi_2 = 0.0625$. Posterior mean responses are plotted in Figure 4. The lines' darkness is proportional to the magnitude of the response. Since the estimated ψ_1 is approximately 2.75, the dark impulse responses in Figure 4 are essentially identical to the posterior estimates of the impulse responses for the DSGE model. The propagation of the technology shock is most sensitive to changes in the central bank's reaction to inflation. Since the estimated autocorrelation of the technology shock is near unity, a decrease in ψ_1 from 2.75 to 1.25 results in a large and prolonged response of inflation and the interest rate.

Figure 5 shows two surfaces summarizing the posterior expected differentials of the weighted variances as a function of the responses to inflation (ψ_1) and the output gap (ψ_2). The black surface is based on the direct estimates of the parameters $\theta_{(np)}$ whereas the gray surface is based on the DSGE-VAR estimates. For both surfaces the variance differentials are computed from the state space representation of the benchmark DSGE model. The loss differential shown by the black surface reflect the impulse responses in Figure 4: As ψ_1 decreases from its historical value the variance of inflation and the interest rate increase. The increase is first gradual, but then quite dramatic as ψ_1 approaches 1. Under the DSGE-VAR posterior distribution (gray) the loss differential also rises as ψ_1 declines, but the increase is not nearly as stark.

The difference in the surfaces is due to one element of the $\theta_{(np)}$ vector: the persistence of the technology shock ρ_z. Recall from Section 4.2 that the direct estimate of ρ_z is 0.97, whereas the DSGE-VAR estimate is 0.89. Indeed, if we recompute the loss surface for the DSGE-VAR estimates subject to the restriction that $\rho_z = 0.97$ and $\sigma_z = 0.72$ the two loss surfaces are almost identical. Hence, the differences in the remaining non-policy parameters matter very little in explaining the different shape of the loss differentials in Figure 5.

To understand how ρ_z affects the volatility of output and inflation, it is instructive to

consider the simplified version of the DSGE model introduced in Section 2. If we restrict $\psi_1 = 1/\beta$, it becomes straightforward to calculate impulse response functions for output and inflation analytically based on Equations (1) to (4):

$$\frac{\partial \widetilde{y}_{t+h}}{\partial \epsilon_{z,t}} = \frac{\kappa/\beta}{\kappa/\beta + \psi_2 + 1 - \rho_z} \rho_z^h \sigma_z, \quad \frac{\partial \pi_{t+h}}{\partial \epsilon_{z,t}} = -\frac{\psi_2 + 1 - \rho_z}{(1/\beta + (\psi_2 + 1 - \rho_z)/\kappa)(1 - \beta\rho_z)} \rho_z^h \sigma_z. \tag{50}$$

In the simple model, for $\psi_2 > 0$ real marginal costs fall below steady state in response to a positive technology shock:

$$\frac{\partial mc_{t+h}}{\partial \epsilon_{z,t}} = -\frac{\psi_2 + 1 - \rho_z}{\kappa/\beta + 1 - \rho_z} \rho_z^h \sigma_z.$$

The autocorrelation of the technology shock has two effects. Since the impulse responses decay at the rate ρ_z^h, the more persistent the technology shock, the longer it takes for marginal costs, output, and inflation to revert back to their steady state levels. Second, ρ_z affects the magnitude of the fall in real marginal costs. For values of $\psi_2 > \kappa/\beta$ an increase in ρ_z raises the initial response of real marginal costs to a technology shock. Since inflation is given by the sum of discounted future marginal costs, its response is amplified. We see this mechanism at work in Figure 5. For a fixed value of ψ_1 the loss increases much more rapidly as a function of ψ_2 under the benchmark DSGE model estimates (high $\hat{\rho}_z$) than under the DSGE-VAR parameter estimates (moderate $\hat{\rho}_z$).

As the central bank increases its reaction to output gap movements, the response of $\widetilde{y}_{t+h}$ is dampened. Since marginal costs in this simple model are given by the difference between output and the technology shock, the volatility of marginal costs and inflation increases. For many reasonable weight functions, the rise in inflation fluctuations outweighs the reduction in output volatility and the overall loss increases as a function of ψ_2, which explains the shape of loss surface in Figure 5. In short, a strong response to output is undesirable under the DSGE model – more so if the technology shock is highly persistent.

The theoretical literature (e.g., Woodford (2003)) emphasizes that the central bank should not respond to output but rather to deviations of output from the level that would prevail in the absence of nominal rigidities. In the simple model of Section 2, this flexible price output is given by $\widetilde{y}_t^{(fp)} = z_t + g_t$ and the flexible price output gap equals marginal costs. If we change the monetary policy rule (1) to

$$R_t = \psi_1 \pi_t + \psi_2 mc_t + \sigma_R \epsilon_{1,t},$$

we can show that in the simple model an increase in ψ_2 leads to more stable marginal costs and inflation. In our estimated DSGE model marginal costs correspond to the labor

share, which we include as observable variable in the estimation. However, the flexible price output gap is not simply given by marginal costs. Moreover, once misspecification of the DSGE model is a concern, the concept of flexible price output is not well defined anymore. We recomputed the loss surfaces depicted in Figure 5 under a policy rule in which the central bank responds to the labor share instead of our measure of output. Since inflation in the larger DSGE model is also the expected sum of discounted future marginal costs, a stabilization of marginal costs leads to a reduction of inflation volatility. Thus, according to the estimated DSGE model, a response to the labor share instead of output does neither lead to a performance deterioration, nor does it generate any improvements over a policy that strongly responds to inflation. This conclusion holds for both the DSGE model and the DSGE-VAR based parameter estimates, as well as for the case where misspecification is taken into account.

4.4   Relaxing the Restrictions on the Exogenous Shocks

The policy analysis in Section 4.3 was based on the benchmark DSGE model and ignored the model misspecification documented in Section 4.2. Subsequently, we will incorporate concern about model misspecification into the policy analysis. We begin by studying the policy implications for the two versions of the DSGE model with a generalized exogenous shock structure.

It is common in the literature on policy analysis with DSGE models to assume that the exogenous shocks are policy invariant. This assumption is plausible in so far the shocks truly capture fundamental shifts in preferences and technologies. If, on the other hands, the shocks partly capture model misspecification, their policy-invariance is not self-evident. In computing the subsequent results, we assume that the (generalized) shocks are indeed policy invariant.

Figure 6 depicts the loss surfaces associated with the AR(2)-in-g_t model (black) and the 7-shocks model (gray). To compute the policy performance measure we use the posterior parameter distributions associated with the two models, summarized in columns (7,8) and (9,10) of Table 1. Both loss functions have roughly the same shape as under the benchmark specification: the loss increases quite rapidly as ψ_1 decreases or ψ_2 increases. Thus, addressing the misspecification in the benchmark model by relaxing the restrictions on the process for the exogenous shocks results in empirical specifications that fit the data better but have qualitatively similar policy implications.

The main reason for the similarity of the loss surfaces is that neither for the AR(2)-in-g_t nor for the 7-shocks specification, the generalization of the shock structure significantly affects the estimated persistence ρ_z of the technology shock. Under the benchmark DSGE model $\hat{\rho}_z = 0.97$, whereas for the two alternative specifications $\hat{\rho}_z = 0.96$. As we argued in Section 4.3, it is the persistence of the technology shock that drives the policy implications of the DSGE model. One can interpret this finding as stating that the 'true' technology process is indeed highly persistent or, alternatively, that this persistence is a symptom of model misspecification. In the latter case one should interpret the policy recommendations from all three DSGE models (benchmark, AR(2)-in-g_t, and 7-shocks) with caution.

4.5 Relaxing Restrictions on the VAR Representation

We now proceed by directly relaxing the restrictions that the benchmark DSGE model imposes on the (approximate) vector autoregressive representation (9) for our observables. To account for model misspecification, the DSGE-VAR approach introduces discrepancy matrices Ψ^Δ and Σ^Δ into the law of motion (14). We established in Section 4.2 that a deviation from the restriction functions $\Psi^*(\theta)$ and $\Sigma^*(\theta)$ improves the log marginal likelihood by 109 points. To conduct policy analysis with the DSGE-VAR we have to make assumptions about the post-intervention values of the discrepancy matrices and will in turn implement the approaches discussed in Section 2.1.3.

Sims (1999) uses a structural VAR framework to study whether a modern interest-rate feedback rule could have prevented the great depression. He estimates the VAR on pre World War II data and replaces the actual policy rule by a hypothetical one. Similarly, Rudebusch and Svenson (1999) fit a small scale backward looking model to output, inflation, and interest rate data and assess the performance of different, hypothetical interest rate feedback rules in the context of the estimated model. In the context of the DSGE-VAR framework this *backward-looking* analysis amounts to treating the estimated empirical model as a structural VAR and conducting policy analysis by changing the interest-rate feedback rule under the assumption that the decision rules of the private sector remain unchanged (see Equation (24)).

Again, the simple model of Section 2 can be used to shed light on the analysis. Suppose that $g_t = 0$, $\epsilon_{1,t} = 0$, and all fluctuations are due to the technology shock. Moreover, past policy is given by $\psi_1^* = 1/\beta$ and $\psi_2^* = 0$. According to (50), the conditional expectations $\mathbb{E}_t[\widetilde{y}_{t+1}]$ and $\mathbb{E}_t[\pi_{t+1}]$ under the historical policy are given by $\rho_z \widetilde{y}_t$ and $\rho_z \pi_t$, respectively.

If we plug the conditional expectations into (2) and (3) and quasi-difference (3) we obtain the following backward-looking system

$$R_t - \psi_1 \pi_t - \psi_2 \widetilde{y}_t \;=\; 0 \tag{51}$$
$$(1 - \rho_z)\widetilde{y}_t - \rho_z \pi_t + R_t \;=\; 0$$
$$-\kappa \widetilde{y}_t + (1 - \beta \rho_z)\pi_t \;=\; -\rho_z \kappa \widetilde{y}_{t-1} + \rho_z(1 - \beta \rho_z)\pi_{t-1} - \kappa \sigma_z \epsilon_{z,t},$$

which is a special case of our general representation (24). It is straightforward to show that inflation and output evolve according to

$$\pi_t \;=\; \rho_z \pi_{t-1} - \kappa \frac{1 - \rho_z + \psi_2}{(1 - \beta \rho_z)(1 - \rho_z + \psi_2) + (\psi_1 \beta - \rho_z \beta)\kappa/\beta} \sigma_z \epsilon_{z,t} \tag{52}$$
$$\widetilde{y}_t \;=\; -\frac{\psi_1 - \rho_z}{1 - \rho_z + \psi_2}\pi_t.$$

If we keep ψ_1 at the historical value $1/\beta$, then (52) is identical to the rational expectations solution for a wide range of values of ψ_2 because the conditional expectations of output and inflation are independent of ψ_2. For other values of ψ_1, we deduce from (52) that the backward-looking system inherits a key feature of the rational expectations system: a stronger response to inflation tends to reduce the volatility of inflation.

Our empirical analysis is of course more complex. The vector autoregressive law of motion of the endogenous variables is altered by Ψ^Δ and Σ^Δ (see Equation (24)). The backward-looking analysis is essentially based on two assumptions: (i) private agents' expectations of future variables as functions of current and past observables are not affected by changes in the monetary policy rule; and (ii) the discrepancy matrices are policy invariant. The expected loss differentials as a function of ψ_1 and ψ_2 based on the backward-looking analysis are shown in Figure 7 (dark surface). We also plot the loss-differential obtained under the benchmark DSGE model (gray surface), using the posterior parameter estimates of the DSGE-VAR($\lambda = 0.75$).[14] In line with the implications of the simplified model (51), the outcomes under the backward-looking analysis resemble the forward-looking analysis with the benchmark DSGE model, at least qualitatively. Small values of ψ_1 tend to generate more volatility and strong responses to output tend to destabilize the economy. Quantitatively, the loss differentials are much larger for high values of ψ_2. This is because the system becomes explosive in that region, which is a common issue with backward looking analysis.

The backward-looking approach is appealing if the degree of DSGE model misspecification (captured by Ψ^Δ and Σ^Δ) is so large that the DSGE model structure (captured by

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[14]The gray surfaces in Figure 7 is identical to the gray surface in Figure 5.

$\Psi^*(\theta_{(p)}, \theta_{(np)})$ and $\Sigma^*(\theta_{(p)}, \theta_{(np)}))$ is unable to explain the dynamics in the data. In this case, one might call into question information coming from the DSGE model, that is, how the $\Psi^*(\theta_{(p)}, \theta_{(np)})$ and $\Sigma^*(\theta_{(p)}, \theta_{(np)})$ matrices change with policy, and decide to completely ignore it in carrying out the policy analysis. Is the backward-looking approach justified in the context of the estimated DSGE-VAR? Arguably, the answer is no. According to Figure 1 the DSGE model captures the dynamic responses to a technology shock quite well. We have also shown that much of the shape of the loss surface is due to the contribution of the technology shock to our weighted average of variances. Hence the rationale for ignoring the impact of the policy parameters on $\Psi^*(\theta_{(p)}, \theta_{(np)})$ and $\Sigma^*(\theta_{(p)}, \theta_{(np)})$ is not very strong, at least in those dimensions where the model fits well. For this reason we consider two alternative approaches for dealing with model misspecifications.

The first approach – *policy-invariant misspecification* – amounts to assuming that the misspecification matrices are invariant to the policy parameters. As described in (27), we compute impulse response functions from the DSGE-VAR for $\lambda = 0.75$ and $\lambda = \infty$. These impulse responses deliver us discrepancies IRF_j^Δ that capture deviations of the estimated from the restricted moving average representation. The IRF_j^Δ's are displayed in Figure 1 as the discrepancies between the DSGE-VAR impulse responses for $\lambda = 0.75$ and $\lambda = \infty$. In terms of the notation developed in Section 2.1 we will essentially let $\widetilde{D}_j^*(\theta_{(p)}, \theta_{(np)})$ vary with the policy parameters $\theta_{(p)}$ and assume that the discrepancies IRF_j^Δ are policy invariant. Hence, if according to the DSGE-VAR analysis the response to a particular shock, for instance the technology shock, is well captured by the underlying DSGE model, then we recover the policy prediction of the DSGE model. If the DSGE-VAR analysis implies that the propagation of a particular shock is poorly captured by the DSGE model, e.g., the demand shock, then our policy predictions are potentially different from those of the DSGE model.

The gray surface in Figure 8 shows the expected loss differential under the assumption that moving-average discrepancies are policy invariant. One can readily see that these loss differentials are almost identical to the DSGE model's loss differentials, shown in Figure 7. According to (27) we can write

$$\mathbb{E}[y_{2,t}y_{2,t}'] = \sum_{j=0}^{\infty} \left[\widetilde{D}_j^* \widetilde{D}_j^{*'} + \widetilde{D}_j^* IRF_j^{\Delta'} + IRF_j^\Delta \widetilde{D}_j^{*'} + IRF_j^\Delta IRF_j^{\Delta'} \right].$$

While the last term affects the overall variance, it does not alter variance differentials across policies. Large values of IRF_j^Δ only matter if they interact with model-implied moving average coefficient matrices $\widetilde{D}_j^*$ that are sensitive to changes in monetary policy. We deduce

from Figures 1 and 4 that the discrepancies IRF_j^Δ are large for the demand shock, but the response to the demand shock is according to the DSGE model not very sensitive to changes in ψ_1 and ψ_2. Hence, overall the presence of misspecification does not change the policy implications under the assumption that the moving-average discrepancies IRF_j^Δ are policy-invariant.

The second approach – *acknowledge misspecification* – is closer to the robust control literature in that the policy-maker refuses to estimate the misspecification matrices using past data. The data are only used to assess the overall magnitude of the discrepancies, as our analysis is conditional on the value of λ that maximizes the marginal likelihood function associated with the DSGE-VAR. The smaller the estimated λ, the more diffuse the prior covariance matrix for Ψ^Δ and Σ^Δ. We generate draws draws from the DSGE-VAR based posterior distribution of $\theta_{(np)}$ and the prior distribution of $\Psi^\Delta, \Sigma^\Delta$ conditional on $\theta_{(np)}$ and the counterfactual policy parameter $\tilde{\theta}_{(p)}$ to compute expected values for our performance measure. The dark surface in Figure 8 shows the expected loss differential. A comparison with Figure 5 (gray surface) indicates that the shape of the loss surface under the acknowledge-misspecification analysis closely resembles the loss surface associated with the benchmark DSGE model. The reason for this similarity is that under the prior distribution the discrepancy matrices have essentially mean zero. However, the uncertainty surrounding the outcomes is quite different in the two cases, as we now proceed to show.

A robust control analysis can typically be represented as a Nash equilibrium between a policy maker and an evil adversary who chooses model misspecifications to harm the policy maker. Bayesian analysis, on the other hand emphasizes the calculation of expected losses and place less weight on extreme forms of model misspecification. So far, we have essentially conducted policy analysis under the assumption of risk-neutrality. That is, we focused on expected variance differentials, ignoring the uncertainty associated with these differentials. Figure 9 presents pointwise 90% credible intervals (dotted) for the weighted variance differentials as a function of ψ_1. The solid gray and black lines correspond to the expected values that have been depicted in Figure 8. While the mean differentials obtained from the acknowledge misspecification analysis are similar to those from the analysis that ignores model misspecification, the uncertainty is much larger in the former case. Hence, a risk-averse policy maker has an additional rationale for avoiding a weak response to inflation. She also has a much smaller incentive to increase the inflation response beyond the baseline value of 2.75 because the expected gains in performance are outweighed by the uncertainty once potential misspecification is taken into account.

5 Conclusion

The presence of misspecification in DSGE models raises two challenges for policymakers. The first challenge is recovering the structural (non-policy) parameters. Direct estimation of the DSGE model is generally conducted under the assumption that the DSGE is the data generating process, e.g. that there are no serious misspecification issues. When this assumption is violated, the parameter estimates can be misleading. In the case considered here, a key non-policy parameter is the persistence of technology shocks ρ_z. DSGE model estimation delivers an estimate of ρ_z close to one. If we believe that technology shocks are in reality extremely persistent, direct estimation of the DSGE model is fine and we can proceed with the policy analysis. If on the other hand we suspect that this estimate of ρ_z results from misspecification, we may be suspicious of the policy implications. These implications are that if the reaction to inflation in the policy rule drops below 1.5, and at the same time the reaction to the output gap rises much above the historical value, the outcomes in terms of the volatility of inflation and the interest rate are simply disastrous.

DSGE-VAR provides the policymakers with an alternative set of estimates. Under DSGE-VAR, the DSGE is treated as a reference model around which the more loosely parameterized VAR is centered. While the non-policy parameters are still estimated as to minimize deviations from the cross-equation restrictions, the penalty attached to these deviations (λ) is not infinity. As a consequence, the DSGE-VAR parameter estimates are more influenced by the prior distribution than in the case of direct DSGE model estimation. This is an advantage of the DSGE-VAR procedure to the extent that the underlying prior distribution for the DSGE model parameters has been specified in a careful manner, drawing information about key structural parameters from a larger set of observations that are excluded from the likelihood function. Thus, in the presence of misspecification the determination of the structural parameters in the DSGE model to some extent resembles parameterization strategies favored by the calibration literature, which emphasizes the careful use of data sources that provide prima facie evidence on the model parameters.

A popular approach to DSGE model estimation in presence of misspecification is to relax the cross-equation restrictions by adding free parameters to the model. This can be done by adding more shocks, or by enriching the dynamics of the existing shocks. We pursue both approaches and find that they indeed improve fit. They do not however affect the estimates of ρ_z, that remain very close to 1. This could be seen as indirect evidence that neither approach fully addresses the misspecification issue. In any case, this estimate

of ρ_z implies that the policy prescriptions remain very close to those of the DSGE model.

The second challenge is to address misspecification in the policy analysis. If we are confident that the DSGE model at hand, in spite of being misspecified, captures the relevant policy trade-offs (see for instance the example in Kocherlakota (2006)), then misspecification might not be a concern. If one option is to ignore misspecification completely, an alternative option is to ignore the cross-equation restrictions, and conduct policy analysis with a backward-looking model. Our empirical analysis with the DSGE-VAR framework casts some doubts on both extremes. On the one hand, we document that misspecification is present and likely affects the key policy trade-offs. On the other hand, we find that in dimensions that are important for policy analysis, such as the propagation of technology shocks, the misspecification does not seem to be a concern.

If we decide to explicitly model misspecification, either in the structural (adding free parameters) or in the vector autoregressive (DSGE-VAR) form, the key question is how misspecification interacts with policy. The structural approach treats the additional free parameters as policy invariant. Our DSGE-VAR approach treats misspecification matrices either as policy invariant (policy-invariant misspecification analysis) or uses a prior distribution for the post-intervention misspecification matrices that is centered at zero (acknowledge-misspecification analysis). The approaches considered in this paper capture different attitudes toward the Lucas critique. The structural approach makes sense only if the exogenous dynamics are truly exogenous. The DSGE-VAR/policy-invariant misspecification analysis is appealing only if one believes that the discrepancy matrices capture adjustments to the dynamics that are insensitive to policy interventions. The DSGE-VAR/acknowledge misspecification approach is more agnostic, as it refuses to learn from past data about the misspecification matrices, but shares the view that these matrices are not something we can model explicitly. To some extent, this view can be justified by the following argument: If we knew how to change the model to address misspecification, we should have done it already.

We have documented the challenges of performing policy analysis with a state-of-the-art, albeit misspecified DSGE model and developed a framework that allows researchers and policy makers to explore the sensitivity of policy predictions under a variety of assumptions about the policy invariance of discrepancies between theory and data. Two lessons are robust across all modes of policy analysis considered in this paper. First, deviating from the baseline parameters of the feedback rule is unlikely to provide substantial improvements over the estimated Volcker-Greenspan policy. Second, it appears undesirable to decrease the response to inflation, or increasing the reaction to deviations of output from a long-run

trend path.

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A   Appendix

We obtain all other series from Haver Analytics (Haver mnemonics are in italics). The nominal rate corresponds to the annualized effective federal funds rate (*FED*), in percent. Inflation is computed using quarter-to-quarter log-differences of the chained-price GDP deflator (*JGDP*), scaled by 400 to obtain annualized percentages. The output gap is defined as the log difference of real GDP (nominal *GDP* divided by the chained-price deflator) and the CBO's real potential output (*GDPPOTH*). The log differences are scaled by 100 to convert them to percentages. Our measure of hours worked is computed by taking total hours worked reported in the National Income and Product Accounts (NIPA), which is at annual frequency, and interpolating it using growth rates computed from hours of all persons in the non-farm business sector (*LXNFH*). We divide hours worked by *LN16N* to convert them into per capita terms. We then take the log of the series multiplied by 100 so that all figures can be interpreted as percentage changes in hours worked. The labor share is computed by dividing total compensation of employees (*YCOMP*) obtained from the NIPA by nominal GDP. We then take the log of the labor share multiplied by 100.

Table 1: Prior and Posterior Moments

| Name | Prior | | DSGE | | DSGE-VAR | | Generalized Shocks | | | |
| | | | Benchmark | | $\lambda = 0.75$ | | AR(2)-in-g_t | | 7-Shocks | |
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Policy Rule Parameters									
ψ_1	1.56	(0.36)	2.71	(0.35)	2.75	(0.37)	2.49	(0.44)	3.06	(0.35)
ψ_2	0.20	(0.10)	0.09	(0.04)	0.13	(0.05)	0.12	(0.07)	0.07	(0.03)
ρ_r	0.50	(0.20)	0.81	(0.02)	0.85	(0.02)	0.84	(0.02)	0.85	(0.02)
	Nominal Rigidities									
ζ_p	0.60	(0.15)	0.76	(0.04)	0.71	(0.05)	0.77	(0.05)	0.67	(0.04)
ζ_w	0.60	(0.15)	0.40	(0.14)	0.45	(0.09)	0.77	(0.08)	0.34	(0.08)
	Preference Parameters									
h	0.70	(0.05)	0.80	(0.04)	0.72	(0.05)	0.66	(0.06)	0.62	(0.05)
ν_l	2.00	(0.75)	1.29	(0.48)	1.82	(0.61)	2.05	(0.72)	1.37	(0.37)
	Technology Parameters									
α	0.35	(0.05)	0.34	(0.00)	0.34	(0.01)	0.34	(0.00)	0.34	(0.00)
S''	4.00	(1.50)	11.19	(2.23)	5.04	(1.59)	7.31	(2.11)	2.69	(0.83)
a'	0.30	(0.08)	0.28	(0.08)	0.30	(0.07)	0.30	(0.07)	0.30	(0.08)
	Steady State Parameters									
r_*^a	1.00	(0.40)	1.23	(0.28)	0.99	(0.35)	1.16	(0.32)	0.85	(0.30)
π_*^a	4.01	(2.00)	2.63	(0.18)	2.51	(0.27)	2.58	(0.27)	2.43	(0.25)
g^*	0.25	(0.10)	0.19	(0.08)	0.22	(0.09)	0.19	(0.08)	0.28	(0.11)

Table 1: Prior and Posterior Moments (continued)

Name	Prior		DSGE Benchmark		DSGE-VAR $\lambda = 0.75$		Generalized Shocks g_t-in-AR(2)		7-Shocks	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Shock Autocorrelations										
ρ_z	0.75	(0.10)	0.97	(0.02)	0.89	(0.05)	0.96	(0.03)	0.96	(0.02)
ρ_g	0.75	(0.10)	0.91	(0.02)	0.88	(0.05)			0.84	(0.08)
$\tilde{\rho}_{g,1}$	(implicit)						0.97	(0.01)		
$\tilde{\rho}_{g,2}$	(implicit)						-0.62	(0.06)		
ρ_{λ_f}	0.75	(0.10)	0.72	(0.07)	0.66	(0.11)	0.78	(0.07)	0.80	(0.06)
ρ_ϕ	0.75	(0.10)	0.75	(0.10)	0.58	(0.10)	0.30	(0.14)	0.53	(0.16)
ρ_μ	0.75	(0.10)							0.91	(0.03)
ρ_b	0.75	(0.10)							0.94	(0.04)
Shock Standard Deviations										
σ_R	0.25	(0.13)	0.16	(0.01)	0.12	(0.01)	0.15	(0.01)	0.16	(0.01)
σ_z	0.38	(0.20)	0.72	(0.05)	0.42	(0.05)	0.75	(0.06)	0.79	(0.06)
σ_g	0.63	(0.32)	0.65	(0.06)	0.32	(0.04)			0.29	(0.04)
$\tilde{\sigma}_g$	(implicit)						0.77	(0.11)		
σ_{λ_f}	0.19	(0.10)	0.09	(0.01)	0.09	(0.01)	0.08	(0.01)	0.10	(0.01)
σ_ϕ	3.76	(1.97)	3.11	(1.72)	3.16	(1.14)	19.90	(7.66)	2.45	(0.58)
σ_μ	0.95	(0.57)							0.53	(0.11)
σ_b	0.95	(0.57)							0.56	(0.11)
Hyperparameters for AR(2) Shocks										
ξ_g	0.75	(0.10)					0.90	(0.04)		
ω_g	0.63	(0.32)					0.63	(0.09)		

Notes: We report means and standard deviations (in parentheses). The parameters ρ_b, ρ_μ, σ_b, and σ_μ only enter the DSGE model with 7-shocks. The DSGE model with AR(2) government spending shocks is parameterized as $g_t = \tilde{\rho}_{g,1}(1 - \tilde{\rho}_{g,2})g_{t-1} + \tilde{\rho}_{g,2}g_{t-1} + \tilde{\sigma}_g \epsilon_t$ and we use a hierarchical prior of the form: $p(\tilde{\rho}_{g,1}, \tilde{\rho}_{g,2}, \tilde{\sigma}_g | \xi_g, \omega_g)p(\xi_g, \omega_g)$. In the table we report $p(\xi_g, \omega_g)$. The following parameters were fixed: $\delta = 0.025$, $\gamma = 1.5/400$, $\lambda_f = 0.15$, and $\lambda_w = 0.3$.

Table 2: Log Marginal Data Densities and Posterior Odds

Specification	$\ln p(Y)$	Post Odds
DSGE Model, Benchmark Specification	-525.22	4.6E-48
DSGE-VAR ($\lambda = \hat{\lambda} = 0.75$)	-416.23	1.00
DSGE Model, Generalized Shocks: AR(2)-in-g_t	-495.66	3.2E-35
DSGE Model, Generalized Shocks: 7-shocks	-479.92	2.2E-28

Notes: The difference of log marginal data densities can be interpreted as log posterior odds under the assumption of that the two specifications have equal prior probabilities. We report odds relative to the DSGE-VAR in the third column of the table.

Figure 1: IMPULSE RESPONSES: DSGE-VAR($\lambda = 0.75$) VERSUS DSGE-VAR(∞)
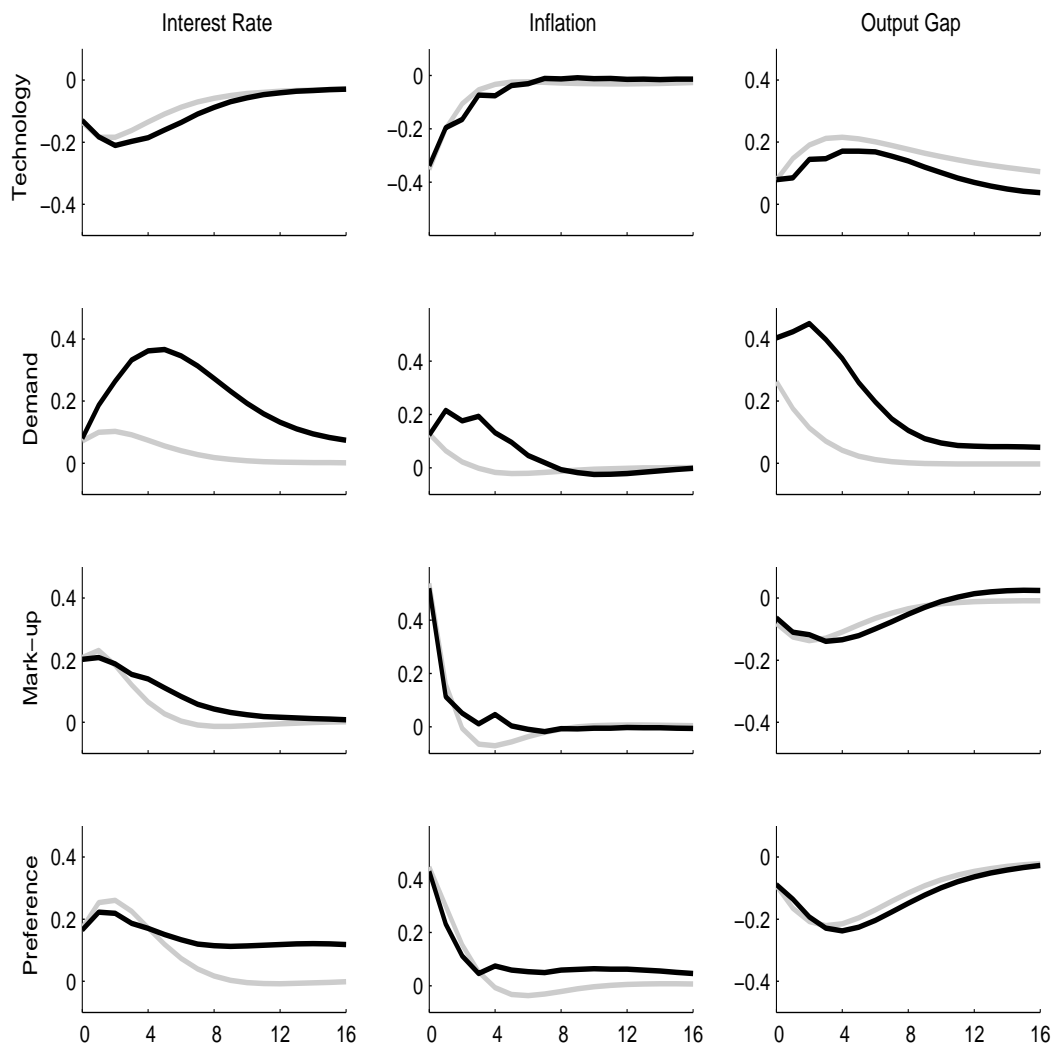
Notes: The figure depicts impulse responses from the DSGE-VAR($\lambda = 0.75$) (black) and the DSGE-VAR(∞) (gray) based on the DSGE-VAR($\lambda = 0.75$) posterior estimates summarized in columns (5,6) of Table 1.

Figure 2: IMPULSE RESPONSES: BENCHMARK DSGE VERSUS AR(2)-IN-g_t MODEL
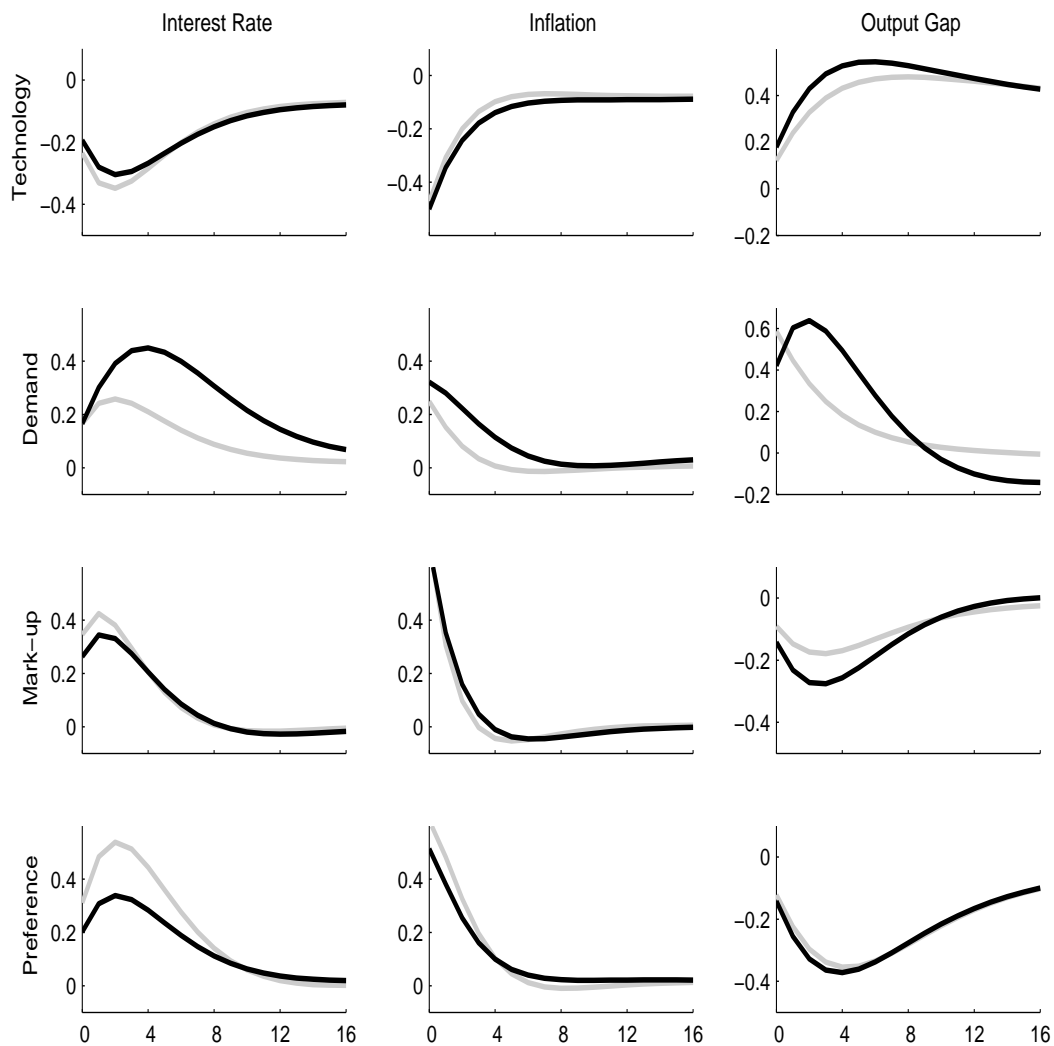
Notes: The figure depicts impulse responses from the benchmark DSGE model (gray) and the AR(2)-in-g_t model (black) using the respective posterior estimates summarized in columns (3,4) and (7,8) of Table 1.

Figure 3: IMPULSE RESPONSES: BENCHMARK DSGE VERSUS 7-SHOCKS MODEL
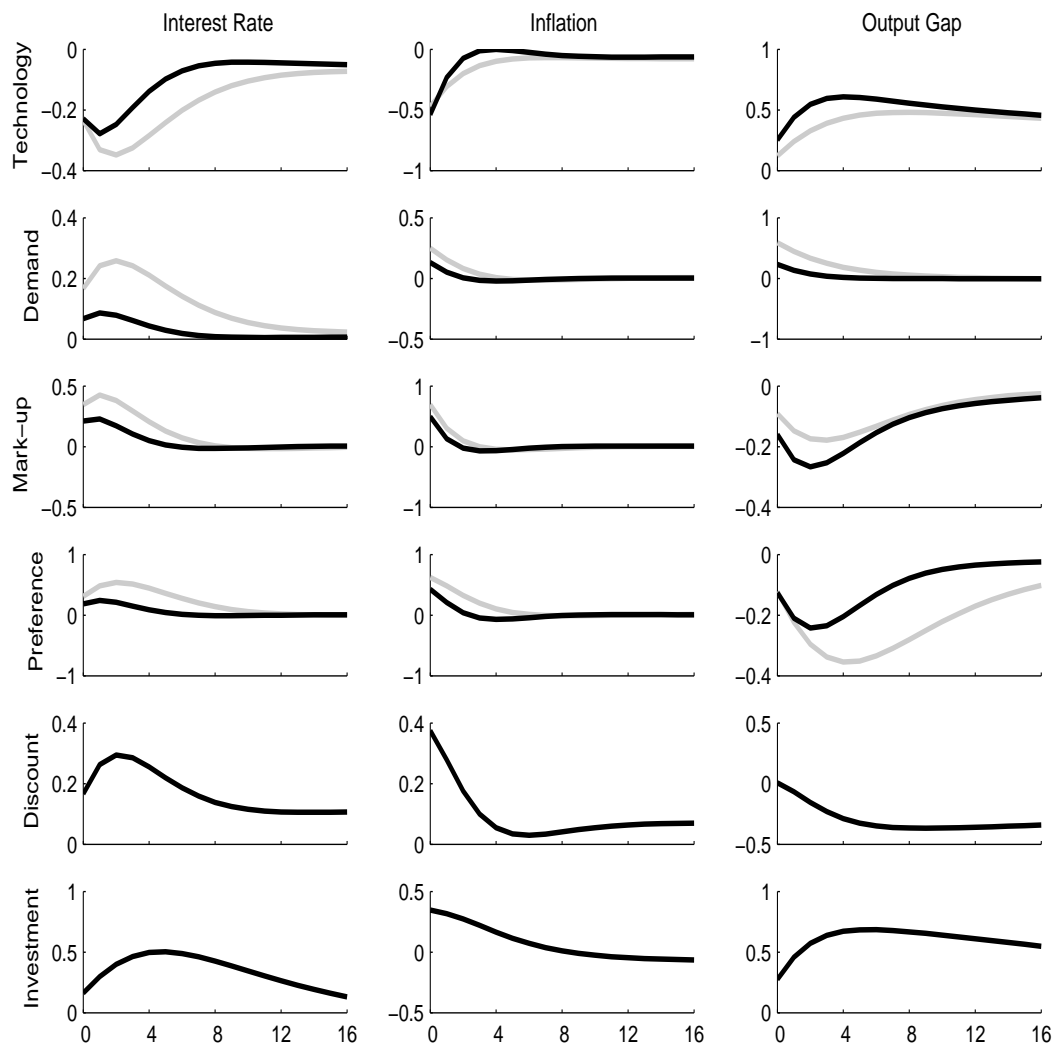
Notes: The figure depicts impulse responses from the benchmark DSGE model (gray) and the 7-shocks model (black) using the respective posterior estimates summarized in columns (3,4) and (7,8) of Table 1.

Figure 4: BENCHMARK DSGE MODEL IMPULSE RESPONSES AS FUNCTION OF ψ_1
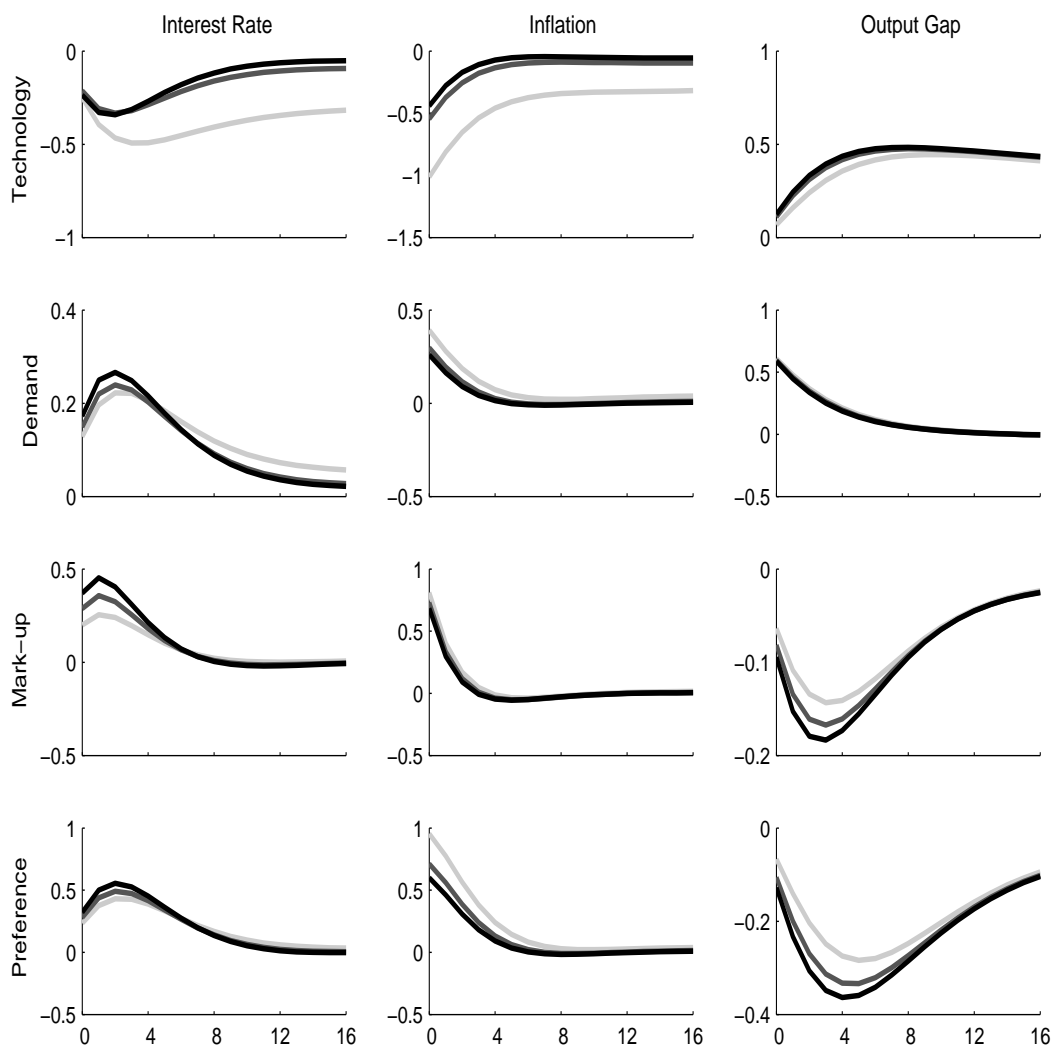
Notes: The figure plots the posterior mean of the DSGE model impulse responses computed for three different values of the response to inflation in the policy rule, ψ_1: 2.75 (black), 2 (dark gray), and 1.25 (light gray). The remaining policy parameters ψ_2 and ρ_R are kept at the baseline values of 0.0625 and 0.8, respectively. For all impulse responses we use the posterior estimates of the non-policy parameters $\theta_{(np)}$, summarized in columns (3,4) of Table 1.

Figure 5: Comparative Performance of Policy Rules: Benchmark DSGE versus DSGE-VAR($\lambda = 0.75$) Parameter Estimates
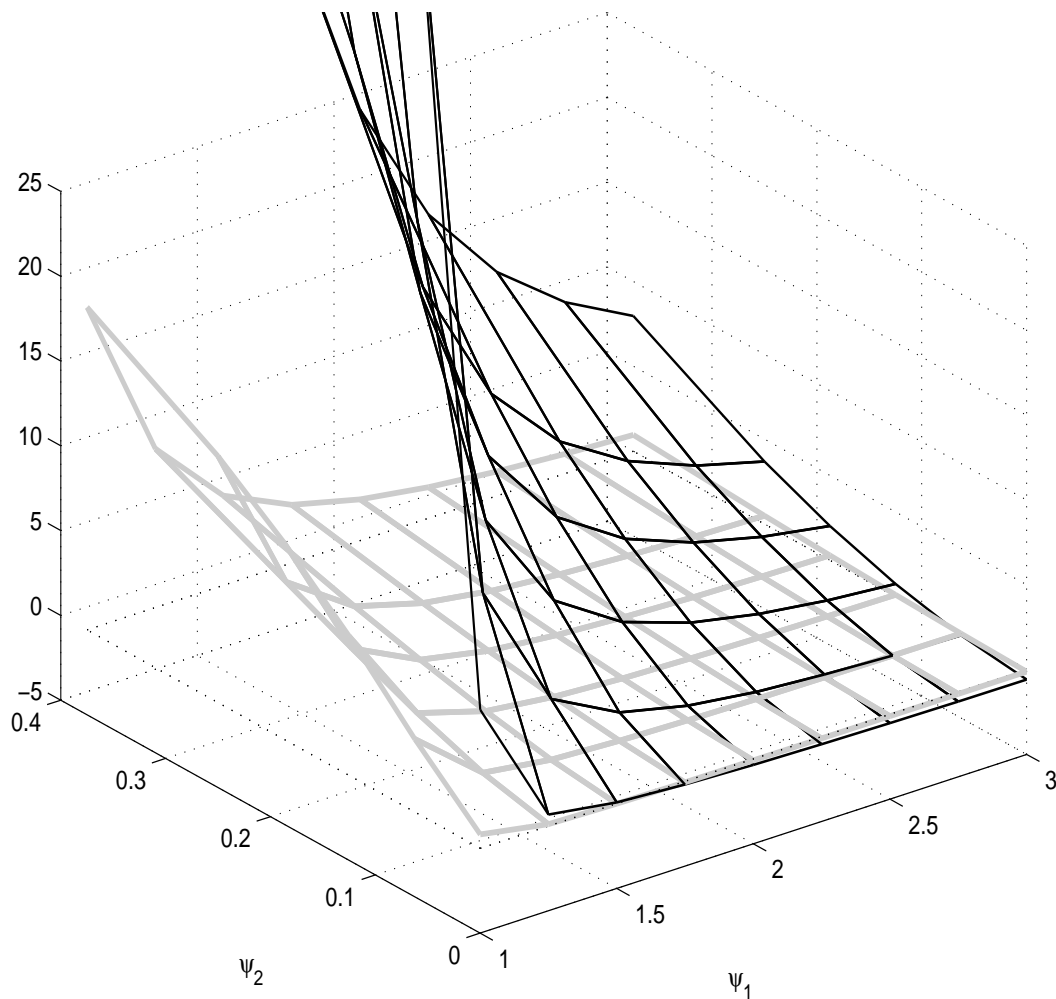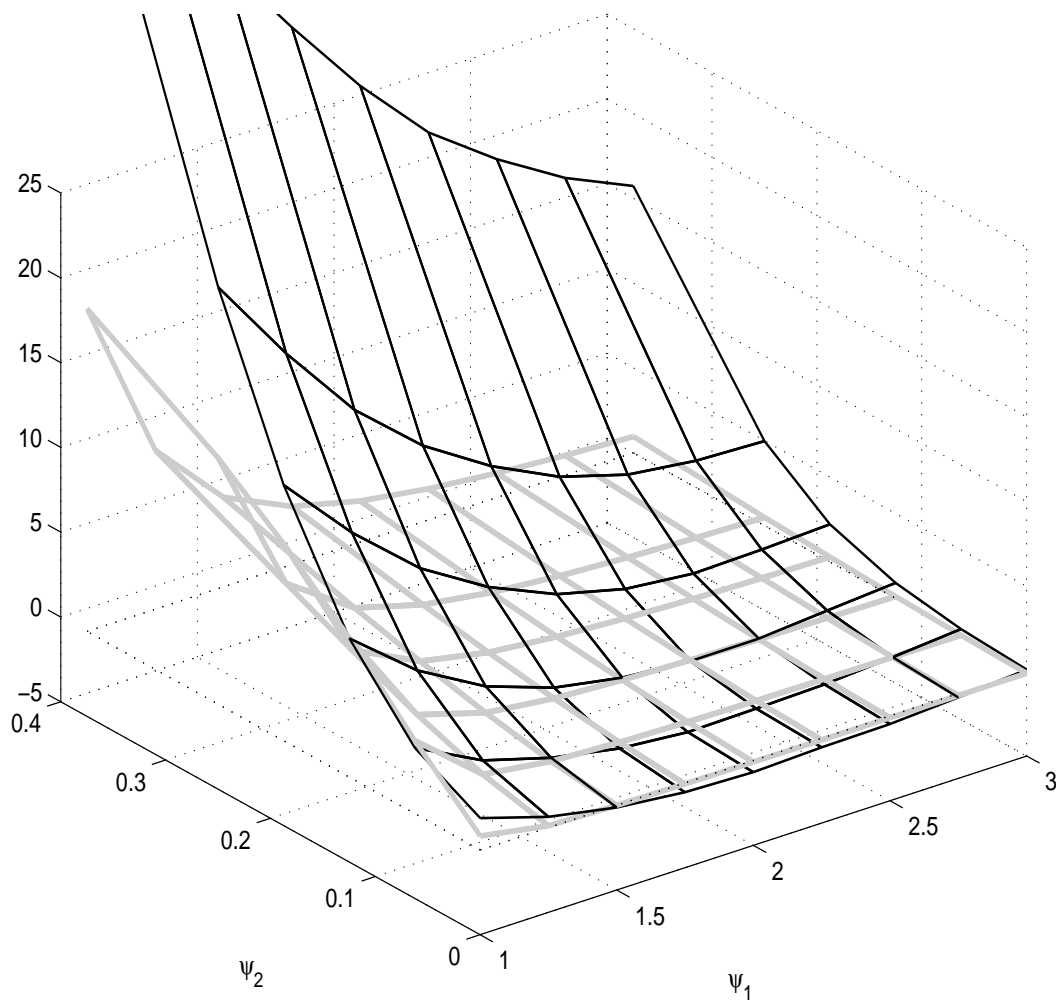

Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.75$, $\psi_2 = 0.0625$. The remaining policy parameter ρ_R is kept at the baseline value of 0.8. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed using DSGE-VAR posterior (gray) and DSGE model (black) posterior estimates of the non-policy parameters $\theta_{(np)}$, summarized in columns (3,4) and (5,6) of Table 1.

Figure 6: Comparative Performance of Policy Rules: AR(2)-in-g_t versus 7-Shocks DSGE Model
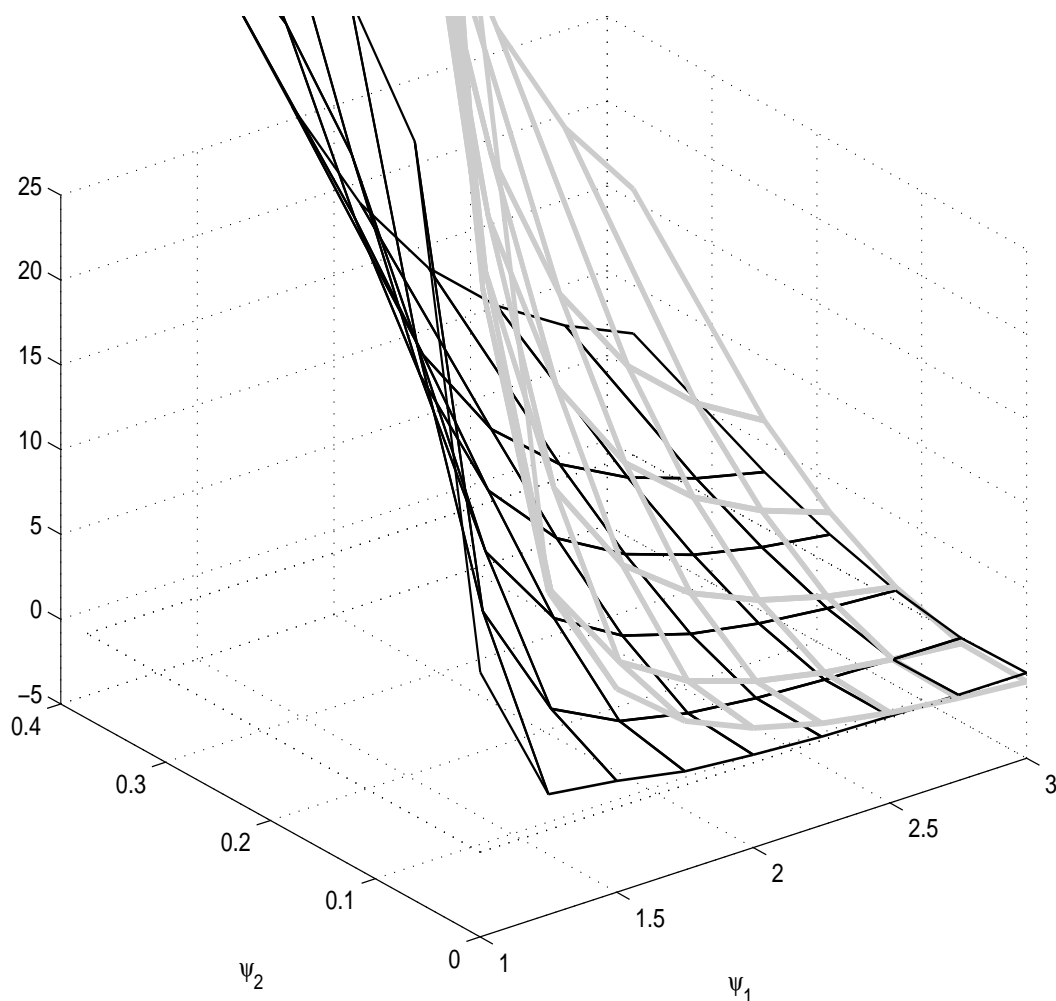
Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.75$, $\psi_2 = 0.0625$. The remaining policy parameter ρ_R is kept at the baseline value of 0.8. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed for the AR(2)-in-g_t (black) and 7-shocks (gray) model using the respective posterior estimates, summarized in columns (7,8) and (9,10) of Table 1.

Figure 7: Comparative Performance of Policy Rules: Benchmark DSGE versus DSGE-VAR/Backward-Looking Analysis
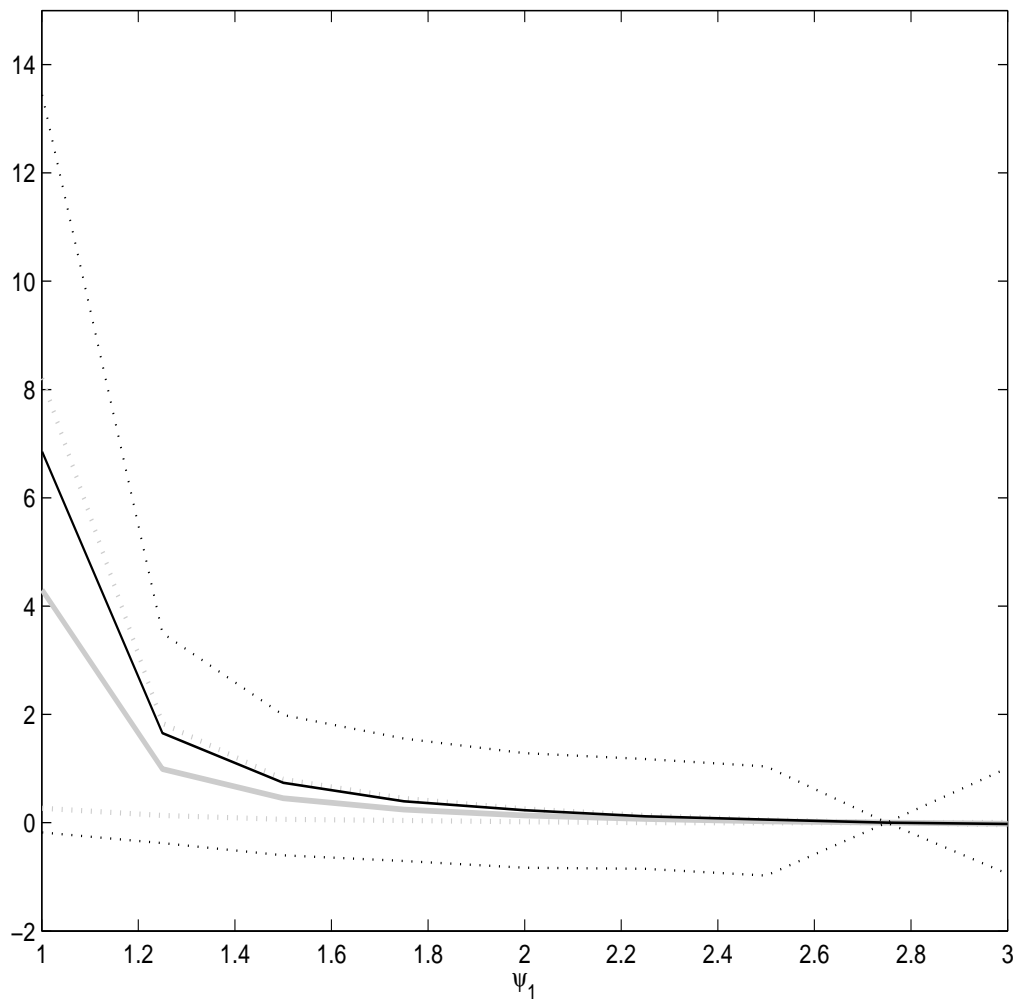

Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.75$, $\psi_2 = 0.0625$. The remaining policy parameter ρ_R is kept at the historical values of 0.8. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed under the DSGE-VAR/Backward-Looking analysis (black) and the DSGE model (gray). For the latter we use the DSGE-VAR posterior estimates of the non-policy parameters $\theta_{(np)}$, summarized in columns (5,6) of Table 1.

Figure 8: Comparative Performance of Policy Rules: DSGE-VAR/Acknowledge Misspecification and DSGE-VAR/Policy-Invariant Misspecification Analysis
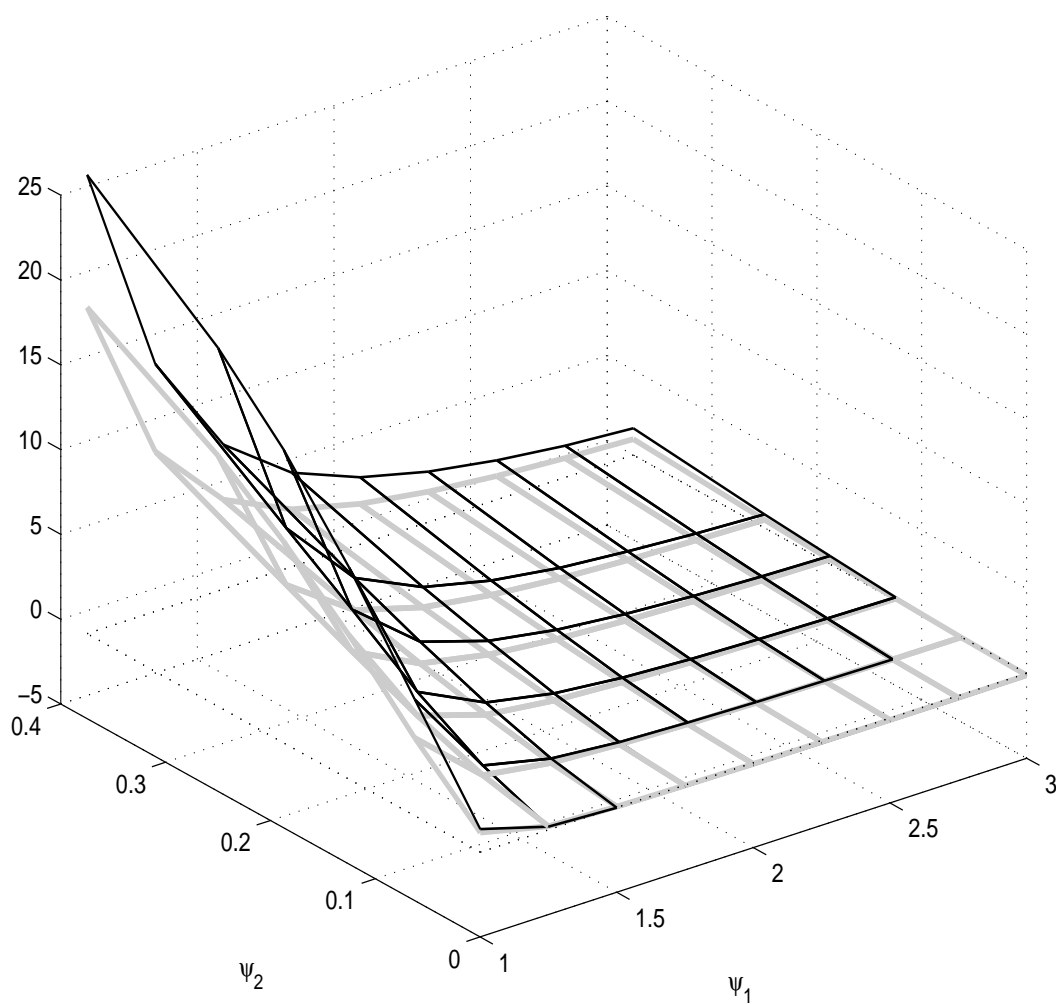
Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.75$, $\psi_2 = 0.0625$. The remaining policy parameter ρ_R is kept at the historical values of 0.8. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed under the DSGE-VAR/Acknowledge Misspecification (black) and DSGE-VAR/Policy-Invariant Misspecification (gray) analysis.

Figure 9: PERFORMANCE UNCERTAINTY: BENCHMARK DSGE VERSUS DSGE-VAR/ACKNOWLEDGE MISSPECIFICATION ANALYSIS

Notes: Posterior expected variance differentials as a function of ψ_1 relative to baseline policy rule $\psi_1 = 2.75$. The remaining policy parameters ψ_2 and ρ_R are kept at their historical values of .0625 and .8, respectively. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed under the DSGE-VAR/Acknowledge Misspecification approach (black) and the DSGE model (gray). For the latter we use the DSGE-VAR posterior estimates of the non-policy parameters θ_{np}. Dash-and-dotted lines represent 90% posterior bands.