Coin Inspection Technology and Commodity Money*

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Abstract

We develop a model of commodity money with uncertainty concerning the quality of coins and study the role played by a coin inspection and certification technology in improving monetary circulation and welfare. We show that this technology reduces the extent of *circulation by weight* (or Gresham's law) to situations in which information on coins is good and that *circulation by weight* and the certification of heavy coins cannot coexist as equilibria. Welfare is higher with certified heavy coins than in *circulation by weight*. The coin inspection technology also restricts *circulation by tale* to situations with good information on coins, yet *circulation by tale* survives for high discount rates regardless of the level of information on coins. Both *circulation by tale* and certification can coexist as equilibria, yet agents are better off when heavy coins are not certified.

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1 Introduction

Recognizability is an important attribute of money. The use of a universally recognizable money reduces the information problem on goods to one side–the buyer's–in contrast to barter trade in which both parties must acquire information on the goods they want. This was noted by Smith (1776) and later by Jevons (1875) who cites *cognizability* as one of the seven important properties of money.[1] This point has been formalized by Williamson and Wright (1994), who show that the use of a universal recognizable fiat money also increases the probability of acquiring high quality goods. In Alchian's (1977) terms, money reduces the cost of reducing ignorance.[2]

But what if money is also difficult to recognize? An example is the commodity money system. In the commodity money system, money was made of precious metal coins whose purchasing power was based on their intrinsic content, through the type of metal used for the coin (gold, silver, etc.), the quantity of metal incorporated in the coin, and its fineness. The problem was that agents could only have a vague idea of the intrinsic content for two main reasons. First, evaluating the intrinsic content of a coin required a set of costly operations that not everybody had access. Second, the intrinsic content was subject to fluctuations, and sources of fluctuations were numerous. From the rudimentary minting technology–basically hammer and pile up to the 16th century (Sargent and Velde 2002)–to debasements by kings, wear, clipping and counterfeiting, the intrinsic content of coins was unstable. British historian Thomas Macaulay, reporting on the consequences of heavy clipping before the 1696 Great Recoinage in Britain, noted that "nothing could be purchased without a dispute. Over every counter there was wrangling from morning to night" (Macaulay, 1855, p. 187).

Velde, Weber and Wright (1999) have recently clarified the impact of this uncertainty on monetary circulation. If sellers can only imperfectly recognize the quality of the coins offered in payment, then trade is impeded in two ways: when not recognized, good (heavy) coins either trade at the same price as light coins, or are hoarded. The authors refer to the first inefficiency as *circulation by tale*. This inefficiency plays on the intensive margin since heavy coins circulate

[1] The other six properties identified by Jevons are : utility and value, portability, indestructibility, homogeneity, divisibility, and stability of value.

[2] Other models in this vein are King and Plosser (1986), Bernhardt and Engineer (1991), and Berentsen and Rocheteau (2004). See also Brunner and Meltzer (1971).

below their full information value when not recognized. The second inefficiency is referred to as *circulation by weight* "because an observer of the economy would distinguish two types of coins, each circulating at its own price". Here the inefficiency plays on the extensive margin as heavy coins do not circulate when not recognized. This second inefficiency matches the informational version of Gresham's law: when coins are difficult to recognize, it is the circulation of light coins that triggers the hoarding of heavy coins ("bad money drives out good money").

In this paper we introduce a coin inspection and certification technology in an economy characterized by imperfectly recognizable coins. Our goal is to study the impact of this technology on the circulation of coins, especially on the activation of Gresham's law, and on welfare. Specifically, we ask the following three questions: Do the by-weight and by-tale equilibria displayed in Velde, Weber and Wright's (1999) survive once agents have access to a coin inspection technology? Under which conditions do agents appraise their coins? Does a coin inspection technology increase welfare?

Historical evidence shows that such a technology was available throughout the commodity money system. The verification process–called assay–involved a series of costly operations. Weight was determined using precise scales, and fineness was estimated using a set of touchstones. The touchstone test involved rubbing a coin on a special stone and comparing the color of the trace left with that of needles of known fineness (Gandall and Sussman, 1997). A more precise assay, called assay by fire, involved melting down a sample of coins to weigh the quantity of pure metal. For obvious reasons, this type of assay was limited to payments involving many coins. Often agents would seek advice from experts known as moneychangers (Bompaire 2007), suggesting that the technology was in part intermediated.[3]

To conduct this study, we amend Velde, Weber and Wright's (1999) model by allowing agents to have access to a technology that tests and certifies their coins for a fee. The economy comprises buyers and sellers who swap roles after trade takes place. Money is in the form of light and heavy gold coins that are imperfectly recognizable by sellers, but that can be certified by

[3] See De Roover (1948), Chevalier (1973), De La Roncière (1973) and Bompaire (1987) for case studies of moneychangers in Medieval Europe and their job as coin experts. There is ample evidence that these coin experts were also active in Ancient Greece and in the Roman Empire (Lothian, 2003), in Bizance (Kaplanis 2003) and the Islamic world (Udovitch, 1975).

buyers via the technology. Because the technology is costly, there will be a trade-off for buyers between the cost of certification, and the gain from using of a fully recognizable money. Our method consists in redefining the equilibria displayed in Velde, Weber and Wright (1999) by taking into account the possibility of agents deviating from their equilibrium strategy–by-tale or by-weight circulation–and paying to certify the coin. This adds a non-deviating constraint to each equilibrium in Velde, Weber and Wright (1999) which typically reduces its scope. We then conduct the mirror exercise, that is, characterize an economy in which heavy coins are always certified and ask under which condition agents deviate and play either the by-weight or by-tale strategy.

In terms of results we show that such a technology limits circulation by weight (or Gresham's law) to regions where information on coins is good. When information is good, agents are better off waiting for someone to recognize their coin rather than pay for certification. But when information is poor, expected gains from trade are small due to the low probability of having one's heavy coin recognized. Agents are then better off certifying and Gresham's law vanishes. The technology also limits circulation by tale to regions where information on coins is good, for the same reason as above. But circulation by tale persists for large discount rates despite the technology. In that case, and regardless of the level of information, gains from trade are not big enough to cover the cost of expertise so that agents are better off not paying for certification. Interestingly, circulation by weight and the certification of good coins cannot coexist as equilibria, yet circulation by tale and the certification of good coins do coexist as equilibria.

With regard to welfare, agents are better off with certified heavy coins than in circulation by weight. This is not surprising. Whether in a by-weight equilibrium or in a certified heavy coins equilibrium, light coins always trade at their full information value since unrecognized coins can only be light coins. As a result light coin holders are not affected by the decision whether to certify heavy coins. If buyers with heavy coins opt for certification, they must be better off that way leaving buyers with light coins indifferent. So not only does this technology dispose of Gresham's law when information on coins is poor, it also strictly increases welfare. Interestingly, welfare is *lower* with certification than with circulation by tale. This comes from risk aversion: when his coin is not recognized, whether it is light or heavy, the buyer purchases an average quantity for sure in circulation by tale. But when heavy coins are certified, all coins are fully recognizable so that buyers get to consume low or high quantities with probabilities equal to the share of light and heavy coins in the money supply. In Von Neuman-Morgenstern terms, certification substitutes a lottery to a sure payment. The concavity of the utility function ensures that agents prefer the sure payment to the lottery, that is circulation by tale to certification. This result has an interesting implication. If the economy settles on a certification equilibrium that overlaps with a by-tale equilibrium, agents would collectively be better off by dropping the coin inspection technology but have no incentive as individuals to do that.

Our paper shares similarities with Kim (1996). In his monetary economy with frictions, agents can invest in an inspection technology that reveals information on the quality of the goods traded. Here we conduct the reverse exercise by allowing agents to pay to know the quality of the medium of exchange rather than of the goods purchased. Related to our paper, there also exists a literature on commodity money in the presence of asymmetric information on goods (Cuadras-Morato, 1994; Li, 1995; Haegler, 1997). These papers are mainly about emergence of one commodity as money, while we focus on how to fix the recognizability problem of commodity money ex post, that is, once a commodity has been chosen to play the role of money.[4] Finally, there exists a literature on qualitative intermediaries with search frictions. Li (2002), especially, studies the endogenous emergence of intermediaries selling information on the quality of goods in an environment with trading frictions and their impact on the incentive to produce either low or high quality goods. Our paper differs is two dimensions. First we model the informational device as a technology used directly by agents rather than via intermediaries. Doing this makes the model a simple extension of Velde, Weber and Wright (1999) and avoids dealing with the origin and distribution of these intermediaries. Also we are able to provide a complete partition of the set of equilibria displayed in Velde, Weber and Wright (1999), and this would not have been possible if the informational device was modelled explicitly as agents.

[4] Other models of commodity money yet without an asymmetric information problem are Burdett, Trejos and Wright (2001), Sargent and Wallace (1983) and Sargent and Smith (1995).

The second difference with Li (2002) is that we focus on verifying coins quality rather than goods quality as she and Kim (1996) do. To our knowledge, this makes our paper the first attempt to model the impact of a currency certification technology on the circulation of money and welfare.

The paper is organized as follows. In section 2 we present the environment. Section 3 studies how the coin inspection technology impacts on the equilibria displayed in Velde, Weber and Wright (1999). Section 4 shows that an equilibrium with certified heavy coins exists for low levels of information on coins and normal discount rates. Section 5 contains our welfare results and section 6 concludes.

2 The Environment

The environment is Velde, Weber and Wright (1999), hereafter VWW, to which we add a costly coin inspection technology. There is a $[0,1]$ continuum of infinitely lived agents indexed by k and there are $I \geqslant 3$ types of goods. A type $k \in I$ agent consumes good k and produces good $k+1$. Consuming q units of his consumption good yields $u(q) = q^{\eta}$ with $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$. Producing q units of his production good costs $c(q)$ which is assumed linear for simplicity so that $c(q) = q$. Further there is a unique $\hat{q}$ such that $u(\hat{q}) = \hat{q}$. Agents discount the future at rate $r > 0$.

Since agents are specialized and there is no record-keeping device, money is essential (Wallace 2001). Money is in the form of gold coins coming in light (L) and heavy (H) weight. Each agent can hold at most one coin.[5] We let M_i be the measure of agents endowed with coins of type $i = \{L, H\}$ and $M = M_H + M_L$ represents the fraction of buyers so that $1 - M$ represents the fraction of sellers, also called producers. Each coin yields to its owner a flow of utility γ_i per period proportional to its weight (or intrinsic content) so that $\gamma_H > \gamma_L$. This utility flow could be interpreted either as the utility one gets from possession of the metal *per se* or as a shortcoming of a more complicated story that goes as follows. With some exogenous probability, a buyer holding a coin i meets some foreign trader who trades money solely on the

[5] Making money divisible would make comparison with VWW less straightforward without adding much to our story. For a model of divisible commodity money and imperfect recognizability of coins see Dutu, Nosal and Rocheteau (2007).

basis of its intrinsic content. γ_i reflects the expected return of meeting such a foreign trader per period. Finally, it is assumed that there is no legal exchange rate between the light and the heavy coin imposed by the monarch, or, if there is one, the monarch has no power to enforce it.

Agents meet bilaterally according to an anonymous random matching Poisson process with arrival rate α. Thus $\frac{\alpha}{I}(1-M)$ is the probability per unit of time of a single coincidence of wants, i.e. a buyer meets a seller who produces his consumption good. In any such meeting, the buyer may or may not offer to trade his unit of money for some output. It is assumed that terms of trade are formed via bargaining in which the buyer has all the bargaining power. When he chooses to make an offer, his offer leaves the seller indifferent between accepting and refusing. If the buyer decides to trade, agents swap their inventories so that the buyer becomes a seller and the seller becomes a buyer with the seller's coin.

In terms of information, the buyer always knows the true quality of his coin while the seller, when presented with a coin, learns its true quality via a common knowledge signal that is informative with probability θ and uninformative with probability $1-\theta$. That is, the informational structure is the same as in Williamson and Wright (1994) but applied to coins rather than goods. In a single coincidence of wants meeting, when the signal is informative and if the two parties agree to trade, a type $i \in \{L, H\}$ coin is exchanged against q_i units of the seller's production good (which is also the buyer's consumption good). When the signal is uninformative, and the parties agree to trade, the unrecognized coin is traded against a quantity $\bar{q}$. This quantity is a weighted average of q_L and q_H with the weights coming from the probabilities of receiving either a light or a heavy coin (of which more below). Barter is simply assumed away.

To circumvent the information problem with coins, a buyer has the option to rent a coin assaying technology for a periodic fee of δ. This technology enables a buyer to expertise and certify the quality of the coin *in front of the seller*, and is valid for one trading period only. Renting the technology fully reveals the quality of the coin, in contrast to Kim (1996) in which the good inspection technology increases the probability with which the quality is revealed to the buyer. It should be noted that sellers have no incentive to rent the technology because of the extreme hold-up problem. As will be clear shortly, buyers holding light coins have no incentive to rent the technology either since they actually benefit from the information problem by trading their coins above their full information value in by-tale equilibria.

In the end, the sequence of events per period goes as follows: at the beginning of the trading period, each buyer (who knows the quality of his coin) decides whether to rent the coin-testing technology or not. Then he searches for a seller. If he decides to rent the technology and finds a seller producing his consumption good, he uses the technology to show the quality of his coin to the seller and then makes an offer. If the parties agree to trade, they swap inventories so that the seller becomes a buyer and the buyer a seller. Finally the new seller returns the technology. If the buyer decides not to rent the technology, he trades heavy coins either by weight or by tale depending on parameters.

3 Equilibria without Assaying

In this section we conduct the following exercise. Given the monetary equilibria displayed in VWW (circulation by weight, circulation by tale and single currency), we characterize for each of these three equilibria the conditions under which agents have no incentive to deviate from their equilibrium strategy by renting the coin inspection technology. We start with a brief presentation of VWW's model, that is an economy in which no coin inspection technology is available.

As in VWW, we note λ_{ij} the probability (endogenously determined) that a buyer with a coin of type $i \in \{L, H\}$ wants to trade with a seller of type $j \in \{K, U\}$ where K means that the weight of the coin is known to the seller (the signal is informative), and U means that the weight of the coin is unknown.

Noting $\beta = \frac{\alpha}{I}(1-M)$, the Bellman equation for a buyer with a light coin is

$$V_L = \frac{1}{1+r}\left\{ \begin{array}{c} \gamma_L + \beta\theta \max\limits_{\lambda_{LK}} \left[\lambda_{LK}\left[u(q_L)+V_0\right]+(1-\lambda_{LK})\,V_L\right] \\ +\beta\,(1-\theta) \max\limits_{\lambda_{LU}} \left[\lambda_{LU}\left[u(\bar{q})+V_0\right]+(1-\lambda_{LU})\,V_L\right] \end{array} \right\}. \tag{1}$$

Multiplying by $(1+r)$ and rearranging yields the flow version of the Bellman equation,

$$rV_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK}\left[u(q_L)+V_0-V_L\right] + \beta\,(1-\theta) \max_{\lambda_{LU}} \lambda_{LU}\left[u(\bar{q})+V_0-V_L\right]. \tag{2}$$

Equation (2) gives the flow return to a buyer holding a light coin, rV_L. It has three components. The first part gives the periodic return on holding the light coin, γ_L. The second part corresponds to the probability that he meets a producer and there is a single coincidence of wants, β, multiplied by the probability that the seller recognizes the light coin, θ, times the net gain from trading the light coin against q_L, which is equal to consuming q_L and switching from buyer with a light coin to producer, that is $u(q_L) + V_0 - V_L$, times the probability that he decides to trade with him, λ_{LK}. The last part has a similar interpretation with the difference that because the coin is not recognized it is not traded for q_L but for an average quantity $\bar{q}$ defined in equation (5) below.

Similarly, the flow Bellman equation for a buyer holding a heavy coin is given by

$$rV_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u(q_H) + V_0 - V_H\right] + \beta\left(1-\theta\right) \max_{\lambda_{HU}} \lambda_{HU} \left[u(\bar{q}) + V_0 - V_H\right] \tag{3}$$

From the take-it-or-leave-it bargaining protocol, the informed seller is indifferent between not producing or producing q_i for the buyer and becoming a buyer with a coin of type i. Therefore the offers made by buyers satisfy

$$V_0 = -q_i + V_i \text{ for } i \in \{L, H\}. \tag{4}$$

Similarly the uninformed seller is indifferent between not producing and producing and trading $\bar{q}$ against the unknown coin so that

$$V_0 = -\bar{q} + \pi V_H + \left(1-\pi\right) V_L \tag{5}$$

where π is the probability that the buyer has a heavy coin given that he wants to trade,

$$\pi = \frac{\lambda_{HU} M_H}{\lambda_{HU} M_H + \lambda_{LU} M_L}.$$

Because sellers never get any utility from trade, we have $V_0 = 0$ so that $V_H = q_H$ and $V_L = q_L$. Once we insert these values into (2) and (3) we obtain

$$rq_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK} \left[u(q_L) - q_L\right] + \beta\left(1-\theta\right) \max_{\lambda_{LU}} \lambda_{LU} \left[u(\bar{q}) - q_L\right] \tag{6}$$

$$rq_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u(q_H) - q_H\right] + \beta\left(1-\theta\right) \max_{\lambda_{HU}} \lambda_{HU} \left[u(\bar{q}) - q_H\right] \tag{7}$$

with $\bar{q} = \pi q_H + (1-\pi) q_L$.

Finally, the λ_{ij} satisfy the following incentive conditions: for $i \in \{L, H\}$,

$$\lambda_{iK} = \begin{cases} 1 & \text{if } u(q_i) - q_i \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

$$\lambda_{iU} = \begin{cases} 1 & \text{if } u(\bar{q}) - q_i \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Definition 1 *A symmetric monetary equilibrium with no coin inspection technology is a vector of quantities $q = (q_L, q_H)$ and strategies $\Psi = (\lambda_{LK}, \lambda_{HK}, \lambda_{LU}, \lambda_{HU})$ such that: (i) : $\lambda_{LK} = 1$ and $(\lambda_{HK}, \lambda_{LU}, \lambda_{HU}) \in [0,1]^3$; (ii) : q satisfies (6) and (7).*

This is VWW's economy. In this economy, there exist three types of pure-strategy monetary equilibria: (i) both coins circulate by weight, (ii) both coins circulate by tale, and (iii) only light coins circulate (single currency equilibrium).

To show how introducing a coin-testing technology impacts on circulation and welfare, we will consider each of these three equilibria and offer an agent the opportunity to deviate from his strategy and rent the technology. This will induce a non-deviating condition for each equilibrium that will restrict its application. For instance, a by-weight equilibrium will now be an equilibrium in which light coins always circulate, heavy coins circulate when they are recognized *and* no buyer holding a heavy coin deviates by renting the technology and certifying his coin.

3.1 By-weight equilibrium

In circulation by weight, a heavy coin trades only if it is recognized by the seller. Therefore light coins always circulate whether recognized or not (and at the same price q_L) since unrecognized coins can only be light coins. Then $\lambda_{LK} = \lambda_{LU} = 1$. From (8) the light coins circulate in informed meetings if $\lambda_{LK} = 1$ equivalent to $u(q_L) \geq q_L$, and the heavy coin circulates in informed meetings if $\lambda_{HK} = 1$ equivalent to $u(q_H) \geq q_H$. These two conditions imply $r > \gamma_H$.[6]

[6] Inserting $\lambda_{HK} = \lambda_{HU} = 0$ into (7) shows that the return to keeping home the heavy coin is $rq_H = \gamma_H$ so that $q_H = \gamma_H / r$. There is an incentive to deviate and trade the heavy coin if $u(q_H) - q_H \geq 0$, which is equivalent to $q_H \leq \hat{q}$, or $r\hat{q} \geq \gamma_H$. But since $u(q_H) = q_H^\eta$ and $c(q_H) = q_H$, we have $\hat{q} = 1$ so that $u(q_H) - q_H \geq 0$ implies $r \geq \gamma_H$. Finally because $\gamma_H > \gamma_L$ the condition $r \geq \gamma_H$ is sufficient for both light and heavy coins to circulate when recognized. Basically what this condition says is that the discount rate cannot be too small for the heavy coin to circulate otherwise agents hoard their coin and enjoy γ_H per period.

From (9) heavy coins do not circulate when not recognized if $\lambda_{HU} = 0$ which from (9) is equivalent to $u(\bar{q}) = u(q_L) \leq q_H$ since $\bar{q} = q_L$ when unrecognized heavy coins are hoarded. Inserting these values into (6) and (7), a by-weight equilibrium is a list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta [u(q_L) - q_L] \tag{10}$$

$$rq_H = \gamma_H + \beta\theta [u(q_H) - q_H] \tag{11}$$

that satisfy[7]

$$r \geq \gamma_H \tag{12}$$

$$q_H \geq u(q_L). \tag{13}$$

Set to equality, equation (13) together with (10)-(11) define the by-weight frontier (BWF) in VWW. The by-weight equilibrium exists for all points in the parameter space (r, θ) to the right of $r = \gamma_H$ and to the left of the BWF (see Fig. 1).

Now suppose that every buyer plays the by-weight equilibrium, and one buyer contemplates deviating and certifying his coin. If he does not deviate, he gets $rV_H = rq_H$ given by (11). If he deviates, he pays δ to rent the technology, certifies the coin in front of the seller and makes a take-it-or-leave-it (deviating) offer $\tilde{q}_H$ to the seller such that

$$-\tilde{q}_H + V_H = V_0. \tag{14}$$

That is, with this offer the seller is indifferent between producing this quantity $\tilde{q}_H$ and becoming a holder of an uncertified heavy coin, V_H, or staying as a producer, V_0. Since $V_0 = 0$ from the the take-it-or-leave-it protocol, from (14) we have

$$V_H = \tilde{q}_H = q_H. \tag{15}$$

That is, the deviating buyer asks the seller for exactly the same quantity as if he was not deviating.

[7] Note that in general q_L and q_H are different across equilibria. In this paper, unless specified, q_L and q_H will implicitly refer to the equilibrium we are considering.

Noting $\tilde{\lambda}_H$ the deviating buyer's strategy whether to trade the certified heavy coin or not, with $\tilde{\lambda}_H = 1$ if $u(\tilde{q}_H) \geq \tilde{q}_H$, the Bellman equation for the deviator is given by

$$\tilde{V}_H = \frac{1}{1+r}\left\{-\delta + \gamma_H + \beta \max_{\tilde{\lambda}_H}\left[\tilde{\lambda}_H \left\{u(\tilde{q}_H) + V_0\right\} + \left(1 - \tilde{\lambda}_H\right) V_H\right] + (1-\beta) V_H\right\}. \tag{16}$$

Note that with probability $\beta\left(1 - \tilde{\lambda}_H\right) + (1-\beta)$ he does not trade, returns the technology and moves back to holding an uncertified heavy coin. Using $\tilde{q}_H = q_H$ and $V_0 = 0$, the flow version (assuming the buyer wants to trade the certified heavy coin, $\tilde{\lambda}_H = 1$) is

$$r\tilde{V}_H = \gamma_H - \delta + \beta\left[u(q_H) - \tilde{V}_H\right] + (1-\beta)\left[V_H - \tilde{V}_H\right]. \tag{17}$$

Equation (17) says that the net gain from deviating and renting the technology is equal to the periodic return on the heavy coin minus the rent, plus the net gains from trading heavy coins in single coincidence of wants meetings, plus the net gain from swapping from deviator back to holding an uncertified heavy coin in all other circumstances.

In the end there is no incentive to deviate if the payoff to holding an uncertified heavy coin is larger than the payoff to deviating and shopping with a certified heavy coin, that is

$$V_H > \tilde{V}_H \tag{18}$$

which, using (11) and (17), gives

$$\delta > \beta(1-\theta)\left[u(q_H) - q_H\right] + V_H - \tilde{V}_H \tag{19}$$

This inequality says that for a heavy coin holder *not* to deviate from playing by-weight, the cost of expertise needs to be larger than the benefit, which is the gain from trade coming from the circulation of previously hoarded unrecognized heavy coins, $\beta(1-\theta)\left[u(q_H) - q_H\right]$, plus the net gain from shifting from deviator back to playing by-weight, $V_H - \tilde{V}_H$.

To obtain the new frontier (called *CF1a*) altering VWW's by-weight equilibrium, we just need to insert the indifference condition between deviating or not, $V_H = \tilde{V}_H$, into (19) and set it to equality. This gives

$$\delta = \beta(1-\theta)\left[u(q_H) - q_H\right]. \tag{20}$$

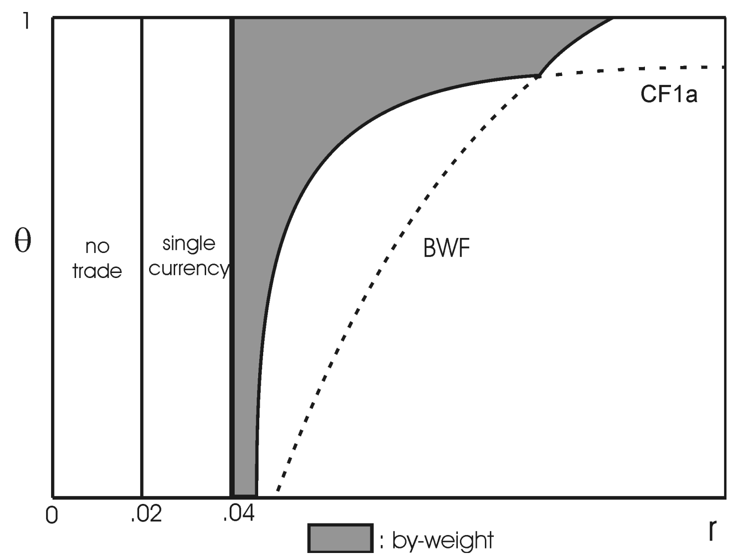

Figure 1: By-weight equilibrium with coin inspection technology

The parameters r and θ that are solutions to (11) and satisfy (20) for a given δ define an additional frontier for circulation by weight to be an equilibrium, noted $CF1a$. It is represented on Fig. 1. On this frontier agents are indifferent between certifying heavy coins or trading them only when recognized by sellers.

Proposition 1 *The coin inspection technology limits circulation by weight (or Gresham's law) to regions where information on coins is good.*

Proof. See Appendix. ■

Compared to VWW, the by-weight equilibrium zone shrinks to the North-West. By offering an alternative to the hoarding of good coins, the coin inspection technology restricts the by-weight equilibrium zone (and therefore the informational version of Gresham's law) to regions where information on coins is good. In that case, buyers with heavy coins should wait for another seller who is likely to recognize the coin rather than pay for certification. As soon as information deteriorates, however, the by-weight equilibrium vanishes and we will see in the next section that it becomes attractive for buyers to certify the heavy coin. Finally, the new frontier shifts up as the cost of expertise decreases.

3.2 By-tale equilibrium

With circulation by tale, unrecognized heavy coins trade at the same price $\bar{q}$ as unrecognized light coins. From (8), the two coins circulate in informed meetings if $u(q_L) \geq q_L$ and $u(q_H) \geq q_H$, which again simplify into $r > \gamma_H$. From (9), heavy coins circulate when not recognized if $u(\bar{q}) \geq q_H$. Inserting the corresponding $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = \lambda_{HU} = 1$ into (6) and (7), a by-tale equilibrium is a list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta\theta\left[u(q_L) - q_L\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_L\right] \tag{21}$$

$$rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_H\right] \tag{22}$$

that satisfy the two conditions

$$r \geq \gamma_H \tag{23}$$

$$u(\bar{q}) \geq q_H. \tag{24}$$

Set to equality, equation (24) together with (21)-(22) define the by-tale frontier (BTF) in VWW. The by-tale equilibrium exists for all points in the parameter space (r, θ) to the right of $r = \gamma_H$ and to the right of the BTF (see Fig. 2).

Now suppose that every buyer plays the by-tale equilibrium and one contemplates deviating and renting the technology. If he does not deviate, he obtains $rV_H = rq_H$ given by (22). If he deviates, he pays δ to rent the equipment and makes a deviating offer $\tilde{q}_H$ to the seller that also satisfies (14) so that $V_H = \tilde{q}_H = q_H$. The continuation payoff to the deviating buyer with a heavy coin $\tilde{V}_H$ is again given by (17) so that there is no incentive to deviate from the by-tale equilibrium if $V_H > \tilde{V}_H$ which using (17) and (22) transforms into

$$\delta > \beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right] + V_H - \tilde{V}_H. \tag{25}$$

This inequality says that for a heavy coin holder not to deviate from playing the by-tale strategy, the cost of certification needs to be greater than the increase in gains from trade due to the full recognizability of heavy coins $\beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right]$, plus the net gain from shifting from deviator back to playing by tale, $V_H - \tilde{V}_H$. Inserting the indifference condition $V_H = \tilde{V}_H$ into

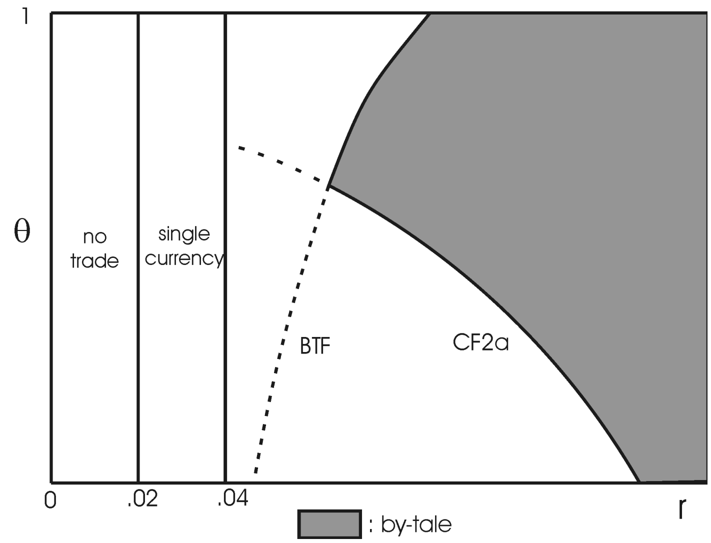

Figure 2: By-tale equilibrium with coin inspection technology

(25) and setting it to equality yields

$$\delta = \beta \left(1 - \theta\right) \left[u \left(q_H\right) - u(\bar{q})\right] \tag{26}$$

The parameters r and θ that are solutions to (21)-(22) and that satisfy (26) for a given δ define an additional frontier for by-tale circulation to be an equilibrium, noted $CF2a$. It is represented on Fig. 2.

Proposition 2 *The coin inspection technology limits circulation by tale to regions where information on coins is good and/or the discount rate is high.*

Proof. See Appendix. ■

From Fig. 2, the possibility to certify heavy coins restricts circulation by tale to regions where information on coins is abundant for the same reason as in the by-weight equilibrium. Interestingly, circulation by tale survives the coin assaying technology for high discount rates regardless of θ. In that case, coins buy so little and gains from trading heavy coins are so small that it does not pay to certify the coin. Agents are better off just trading unrecognized heavy coins at the same price as light coins, saving the cost of the technology. Finally, the frontier shifts up as the cost of expertise decreases.

3.3 Single-currency equilibrium

In a single currency equilibrium, light coins circulate and heavy coins are hoarded whether recognized or not. From (8), the light coin circulates if $u(q_L) \geq q_L$ and the heavy one does not if $u(q_H) < q_H$ or $q_H > \hat{q}$, which simplifies into $\gamma_L \leq r \leq \gamma_H$. A single currency equilibrium in VWW is then a list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta \left[u(q_L) - q_L \right] \tag{27}$$

$$rq_H = \gamma_H \tag{28}$$

such that $r \in [\gamma_L, \gamma_H]$. A key element of this equilibrium is the low discount rate. It makes it more attractive for agents to hoard the heavy coin and enjoy its high return than trading it. That is, the quantity that has to be produced by the seller to compensate the buyer is so large that the disutility of producing this quantity overcomes the utility of consuming it so that net gains from trade are negative.

Suppose now that in this configuration a buyer hoarding his heavy coin contemplates deviating by certifying his coin and trading it. Again, from the buyer-takes-all protocol, he makes a deviating offer $\tilde{q}_H$ such that a seller is indifferent between producing this quantity and becoming a non-deviating buyer hoarding his heavy coin, and staying as a producer, that is, $-\tilde{q}_H + V_H = V_0$. Since $V_0 = 0$, again $V_H = \tilde{q}_H = q_H$. He will not deviate if $V_H > \tilde{V}_H$ which using (28) and (17) yields

$$\delta > \beta \left[u(q_H) - \tilde{V}_H \right] + (1 - \beta) \left[V_H - \tilde{V}_H \right]. \tag{29}$$

The frontier is given by the indifference condition $V_H = \tilde{V}_H$ yielding $\delta = \beta \left[u(q_H) - q_H \right]$ which is never satisfied since $u(q_H) - q_H < 0$ in a single currency equilibrium. We conclude that there is never an incentive to deviate from the single currency equilibrium and certify. The intuition is simple: a deviator makes the same offer as the non-deviator, yet the non-deviating offer is not an equilibrium offer. Why certify a coin that I will not trade?[8]

We summarize the above discussions in the following proposition.

[8] Since Berentsen and Rocheteau (2002), it is well known that the indivisibility of money generates inefficient terms of trade. We have a clear illustration here as divisible coins or lotteries would make trade possible with low discount rates even with buyers making take-it-or-leave-it offers.

Proposition 3 *In a commodity money economy in which a coin inspection technology is available,*
(i) A by-weight equilibrium is characterized by a couple (q_L, q_H) *given by (10)-(11) and a value function* $\tilde{V}_H$ *given by (17) satisfying (12), (13) and (19);*
(ii) A by-tale equilibrium is characterized by a couple (q_L, q_H) *given by (21)-(22) and a value function* $\tilde{V}_H$ *given by (17) satisfying (23), (24) and (25);*
(iii) A single currency equilibrium is characterized by a couple (q_L, q_H) *given by (27) and (28) and satisfying* $\gamma_L \leq r \leq \gamma_H$.

4 Equilibria with Assaying

In this section we characterize the equilibria in which heavy coins are certified and buyers do not deviate from certification to play either the by-weight, by-tale or single-currency strategy. Let q_{HC} be the quantity traded against a certified heavy coin and V_{HC} be the Bellman equation for the buyer with a certified heavy coin. The decision by the buyer whether to trade the certified heavy coin is λ_{HC} with $\lambda_{HC} = 1$ if

$$u(q_{HC}) \geq q_{HC}. \tag{30}$$

Because all unrecognized coins can only be light coins, the payoff to holding and trading the light coin is identical to the by-weight case in the previous section. Light coins circulate at full information value, that is $\lambda_{LK} = \lambda_{LU} = 1$, and the payoff is given by

$$rq_L = \gamma_L + \beta \left[u(q_L) - q_L\right]. \tag{31}$$

For a holder of a certified heavy coin we have (assuming it circulates)

$$rq_{HC} = \gamma_H + \beta \left[u(q_{HC}) - q_{HC}\right] - \delta. \tag{32}$$

Equations (31) and (32) describe a full information monetary economy with a periodic return on light coins equal to γ_L, and a periodic return on heavy coins equal to $\gamma_H - \delta$. From (8) the circulation of light coins requires $r \geq \gamma_L$. Things are slightly different for certified heavy coins as (30) and (32) require

$$r > \gamma_H - \delta, \tag{33}$$

but we will see shortly that it is dominated by another constraint.

Let us note $\tilde{\lambda}_{HK}$ the probability of trading the recognized heavy coin when deviating from certification–that is not renting the technology–and let $\tilde{q}_{HC}$ be the quantity purchased in that case. Also, let us note $\tilde{\lambda}_{HU}$ the probability of trading the unrecognized heavy coin when deviating from certification. The Bellman equation for the deviator, the flow version of which is explained after equation (36), is given by

$$\tilde{V}_{HC} = \frac{1}{1+r}\left\{ \begin{array}{c} \gamma_H + \beta\theta \max_{\tilde{\lambda}_{HK}} \left[\tilde{\lambda}_{HK}\left\{u(\tilde{q}_{HC}) + V_0\right\} + \left(1-\tilde{\lambda}_{HK}\right)V_{HC}\right] \\ +\beta\left(1-\theta\right)\max_{\tilde{\lambda}_{HU}} \left[\tilde{\lambda}_{HU}\left\{u(q_L) + V_0\right\} + \left(1-\tilde{\lambda}_{HU}\right)V_{HC}\right] \\ +(1-\beta)V_{HC} \end{array} \right\}. \tag{34}$$

In order to simplify this expression, first note that whether in the by-weight or by-tale case, a deviator in an informed meeting makes an offer $\tilde{q}_{HC}$ that leaves the seller indifferent so that

$$-\tilde{q}_{HC} + V_{HC} = V_0 \tag{35}$$

from which we obtain $\tilde{q}_{HC} = q_{HC}$. Then inserting $V_0 = 0$ and $V_{HC} = q_{HC}$, and multiplying by $1+r$ the flow version of (34) is

$$\begin{aligned} r\tilde{V}_{HC} \quad &= \quad \gamma_H + \beta\theta \max_{\tilde{\lambda}_{HK}} \left\{\tilde{\lambda}_{HK}\left[u(q_{HC}) - \tilde{V}_{HC}\right] + \left(1-\tilde{\lambda}_{HK}\right)\left[V_{HC} - \tilde{V}_{HC}\right]\right\} \\ &\quad +\beta\left(1-\theta\right)\max_{\tilde{\lambda}_{HU}} \left\{\tilde{\lambda}_{HU}\left[u(q_L) - \tilde{V}_{HC}\right] + \left(1-\tilde{\lambda}_{HU}\right)\left[V_{HC} - \tilde{V}_{HC}\right]\right\} \\ &\quad +(1-\beta)\left[V_{HC} - \tilde{V}_{HC}\right]. \end{aligned} \tag{36}$$

With probability $\beta\theta$ the deviating buyer has to decide whether to trade the recognized heavy coin against q_{HC}. With probability $\beta\left(1-\theta\right)$ she has to decide whether to trade the unrecognized heavy coin, which will be treated as a light coin since in an economy with active certification all unrecognized coins are inferred to be light by sellers. Finally, if there is no single-coincidence-of-wants meeting, which happens with probability $(1-\beta)$, the deviator shifts back to holding a certified heavy coin. In the end there is no incentive to deviate from certification and play either BW, BT or SC if

$$V_{HC} > \tilde{V}_{HC}. \tag{37}$$

4.1 Deviation to by-weight

Assume first that a buyer with a heavy coin deviates and plays the by-weight strategy ($\tilde{\lambda}_{HK} = 1$ and $\tilde{\lambda}_{HU} = 0$). Inserting these values into (36) his payoff is

$$r\tilde{V}_{HC} = \gamma_H + \beta\theta \left[u(q_{HC}) - \tilde{V}_{HC}\right] + (1 - \beta\theta) \left[V_{HC} - \tilde{V}_{HC}\right]. \tag{38}$$

Proceeding as in the previous section, the indifference condition is given by $V_{HC} = \tilde{V}_{HC}$ so that the frontier, noted $CF1b$, is characterized by

$$\delta = \beta (1 - \theta) [u (q_{HC}) - q_{HC}]. \tag{39}$$

This is very much like (20) defining $CF1a$ in the previous section, yet equilibrium q_{HC} here is different since it is given by (32) whereas q_H in (20) is given by (11). However,

Lemma 1 *$CF1a$ and $CF1b$ are the same.*

Proof. See Appendix. ■

The values of parameters (r, θ) that leave agents indifferent between shifting from circulation by weight to certification also leave them indifferent between deviating from certification to circulation by weight. This means that the two frontiers are one and the same and that the two equilibria cannot coexist (see Fig. 3). We delay the explanation for this Lemma and the following to subsection 4.4 in which we comment on the results.

4.2 Deviation to by-tale

Assume now that the buyer deviates and plays the by-tale strategy. Using the same method, the payoff to the deviator is

$$\begin{aligned} r\tilde{V}_{HC} &= \gamma_H + \beta\theta \left[u(q_{HC}) - \tilde{V}_{HC}\right] + \beta (1 - \theta) \left[u (q_L) - \tilde{V}_{HC}\right] \\ &\quad + (1 - \beta) \left[V_{HC} - \tilde{V}_{HC}\right] \end{aligned} \tag{40}$$

so that $CF2b$ is given by

$$\delta = \beta (1 - \theta) [u (q_{HC}) - u(q_L)] \tag{41}$$

which is clearly different from equation (26) defining $CF2a$.

Lemma 2 *CF2b stands above CF2a.*

Proof. See Appendix. ■

Fig. 3 shows that the two frontiers $CF2a$ and $CF2b$ are different leaving space for circulation by tale and certification to coexist.

4.3 Deviation to single-currency

The last thing we need to do in order to characterize the certification equilibrium is to find the constraint that makes sure agents do not deviate from certification to play the single-currency equilibrium. Substituting the single-currency equilibrium deviating strategy values for $\tilde{\lambda}_{HK}$ and $\tilde{\lambda}_{HU}$ (that is $\tilde{\lambda}_{HK} = \tilde{\lambda}_{HU} = 0$) into the equation for a deviator (36) enables to express $\tilde{V}_{HC}$ as a function of V_{HC}

$$\tilde{V}_{HC} = \frac{\gamma_H + V_{HC}}{1+r}. \tag{42}$$

Recalling that $V_{HC} = q_{HC}$, inserting (42) into the non-deviating condition (37) yields

$$rq_{HC} > \gamma_H. \tag{43}$$

Doing the same exercise with the non-deviating condition from certification to by-weight, that is use (38) to express $\tilde{V}_{HC}$ as a function of V_{HC} and then insert it into (37) yields

$$rq_{HC} > \gamma_H + \beta\theta \left[u\left(q_{HC}\right) - q_{HC}\right] \tag{44}$$

equivalent to

$$rq_{HC} > \gamma_H + \mu\left(\theta\right) \tag{45}$$

with $\mu\left(\theta\right) > 0$.[9] That is, the constraint (43) such that agents do no deviate from certification to single-currency is dominated by the constraint (45) such that agents do not deviate from certification to by-weight. Put it another way, if agents have no incentive to deviate and play by-weight, they have no incentive to deviate and play single-currency. Finally, note that (33) is also dominated by (44).

[9] Setting (44) to equality and substituting rq_{HC} by its value given by (32) yields (39).

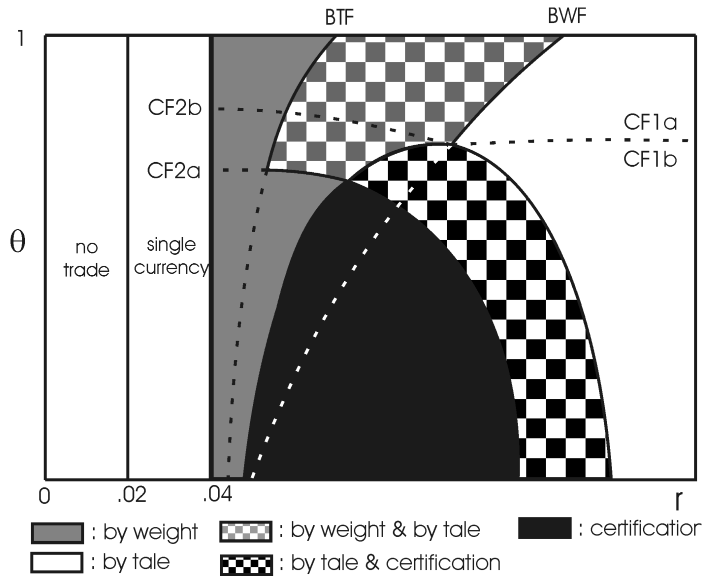

Figure 3: All equilibria when a coin inspection technology is available.

4.4 Comments

The above results can be summarized in a proposition.

Proposition 4 *An equilibrium with certification is a list* (q_L, q_H) *given by (31) and (32) and parameter values that lie below* $CF1a \equiv CF1b$ *and* $CF2b$*. Circulation by weight and certification never coexist. Circulation by tale and certification can coexist as for circulation by weight and circulation by tale.*

Interestingly, we obtain a complete partition of the set of equilibria in VWW. The major changes brought by the coin inspection technology are the following. First, circulation by weight and circulation by tale still exist when a coin testing technology is available, yet their equilibrium regions are reduced. These two types of equilibrium can still coexist also, but for high levels of information only. Second, a new type of equilibrium shows up, which is the certification of heavy coins. When information on coins is poor and when the discount rate has intermediate values, the certification of heavy coins is an equilibrium. Finally, in the region where certification is an equilibrium, circulation by tale and certification coexist for the highest discount rates.

The reason why certification and circulation by tale coexist is the following. When considering deviating from certification to playing by-tale, buyers realize that their unrecognized heavy coin is going to be treated as a light coin by sellers since all other unrecognized yet circulating coins are necessarily light coins in an equilibrium with certification. But when considering deviating from by-tale to certification, which seems to be the exact symmetric decision but is not, they realize the unrecognized heavy coin is going to be treated as an unknown coin by sellers in circulation by tale. Since an unknown coin is priced more in a by-tale equilibrium than when deviating from certification to by-tale, two symmetric deviations that should yield the same payoff in absolute value actually do not because sellers interpret differently an unrecognized heavy coin in a by-tale economy and in an economy with certification. This means that *different* parameters leave agents indifferent between deviating from by-tale to certification or deviating from certification to by-tale. The frontiers between these two equilibria and their corresponding deviating threats are then different so that the two equilibria overlap.

Things are different for circulation by weight. Whether deviating from certification to playing by-weight or deviating from by-weight to certification, gains from trade on unrecognized heavy coins are zero in both cases since unrecognized heavy coins do not circulate. Here the two symmetric deviations yield the same payoffs in absolute values so that the frontiers between these two equilibria and their corresponding deviating threats merge. Circulation by weight and certification cannot coexist as equilibria. See Fig. 3.

5 Welfare

The welfare function in each equilibrium is the weighted average of lifetime utilities across agent types. The following propositions summarize how the technology affects agents' welfare.

Proposition 5 *The introduction of a coin inspection technology that triggers a transition from a by-weight equilibrium to certification is always welfare improving.*

The proof is straightforward. When coins trade by weight, unrecognized coins can only be light coins so that light coins circulate at their full information value, regardless of the quality of information on coins. Therefore the decision by heavy coin holders whether to rent the technology will only impact buyers holding heavy coins. If they opt for certification, they must be better off and buyers with light coins are indifferent. These observations imply that welfare is higher with certification than in circulation by weight.

Proposition 6 *The introduction of a coin inspection technology that triggers a transition from a by-tale equilibrium to certification is always welfare worsening. When the two equilibria coexist, welfare is also lower with certification.*

Proof. See Appendix ■

Now that heavy coins are certified, the uncertainty on coins falls since sellers induce that unrecognized coins are light coins. Buyers with heavy coins are better off, but buyers with light coins worse off since they can no longer pass on their unrecognized light coins as unknown coins as they do in circulation by tale. In the end some lose and some win, but aggregate welfare increases because of risk aversion. When his coin is not recognized, whether it is light or heavy, the buyer purchases an average quantity for sure in circulation by tale. But when heavy coin holders certify, all coins are fully recognizable and buyers get to consume low or high quantities with probabilities equal to the share of light and heavy coins in the money supply. In Von Neuman-Morgenstern terms, certification substitutes a lottery to a sure payment. The concavity of the utility function ensures that agents prefer the sure payment to the lottery, $u[E(q)] > E[u(q)]$, that is circulation by tale to certification. These observations imply that agents are always better off in a by-tale equilibrium than with certification. It also implies that, assuming the economy settles on an equilibrium with certification that overlaps with a by-tale equilibrium, agents would collectively be better off by dropping the coin inspection technology, but have no incentive as individuals to do that.

6 Conclusion

We constructed a model of commodity money where agents have access to a coin inspection technology that enables to circumvent imperfect information regarding the intrinsic content of coins. Both this problem and the solution we examine are amply documented. We derived conditions under which agents certify their heavy coin or keep trading it either by weight or by tale. The coin inspection technology is welfare improving if it moves the economy from circulation by weight to certification. It is welfare worsening if it moves the economy from circulation by tale to certification. Finally it has no effect if certification is not an equilibrium for the parameters we consider. Interestingly, this technology allows for equilibria where both coins are in full circulation and trade at different prices–equilibria that do not exist in Velde, Weber Wright (1999). In a recent work, Dutu, Nosal and Rocheteau (2007) have shown that signaling via the terms of trade (rather than through certification as we do here) is also possible and welfare improving. However, this comes at the cost of lowering the heavy coins' velocity and the quantity of output traded against heavy coins. Here, once the cost of signaling is paid, this fully cleans the economy of the informational friction and coins can circulate at their full information value and full velocity. Although our paper is applied to the commodity money system, the framework can potentially be extended to study contemporary issues regarding counterfeited money, such as the impact of counterfeit detectors on the production and circulation of counterfeited money for instance. We leave this for future research.

Appendix

A1. Proof of Proposition 1

To derive the shape of the *CF1a* frontier, rewrite (20) such that

$$\delta + \beta\theta \left[u\left(q_H\right) - q_H \right] = \beta \left[u\left(q_H\right) - q_H \right]. \tag{46}$$

On the LHS of Fig. 4, we plot both sides of equation (46) as functions of q_H for a given θ^*. The intersection between the two curves gives the q_H^* that satisfy (46) for this θ^*. On RHS we plot the relationship between equilibrium q_H and r for the same θ^*. Inserting q_H^* from the LHS gives the discount rate(s) r^* that satisfy (11) for q_H^* and θ^*.

As θ decreases from 1 to 0, $\delta + \beta\theta \left[u\left(q_H\right) - q_H \right]$ gets flatter and $q(r)$ becomes steeper while $\beta \left[u\left(q_H\right) - q_H \right]$ is unchanged. Starting from $\theta = 0$ it is clear that there are two intersections between $\delta + \beta\theta \left[u\left(q_H\right) - q_H \right]$ and $\beta \left[u\left(q_H\right) - q_H \right]$ so that there are two r, one small one big, compatible with agents being indifferent between certification and by-weight. Also, inspection of (46) shows that when $\theta = 0$ the corresponding r^* is bigger than γ_H since $\delta < \gamma_H - \gamma_L < \gamma_H$ by assumption.[10] As θ increases the distance between these two r decreases and for $\theta > \bar{\theta}$ there are no more r such that $\delta = \beta\left(1-\theta\right)\left[u\left(q_H\right) - q_H \right]$. Especially at $\theta = 1$, the LHS of (46) is always larger than the RHS.

The shape of *CF1a* follows from the above discussion. See Fig. 5. Note that only the low values of r for this frontier appear on Fig. 1 and Fig. 3 in the text since they are the only ones that matter for equilibrium by weight (there is no equilibrium by weight for large value of the discount rate). Finally *CF1a* shifts upward as δ decreases.

A2. Proof of Proposition 2

To derive the shape of the *CF2a* frontier, rewrite (26) such that

$$\delta + \beta\theta \left[u\left(q_H\right) - u(\bar{q}) \right] = \beta \left[u\left(q_H\right) - u(\bar{q}) \right]. \tag{47}$$

On the LHS of Fig. 6, we plot both sides of equation (47) as functions of q_H for a given θ^*. The intersection between the two curves gives the q_H^* that satisfy (47) for this θ^*. On RHS we

[10] Whether *CF1a* starts to the right or left of BWF and BTF on the horizontal axis depends on the value of δ. The bigger δ, the further it starts to the right.

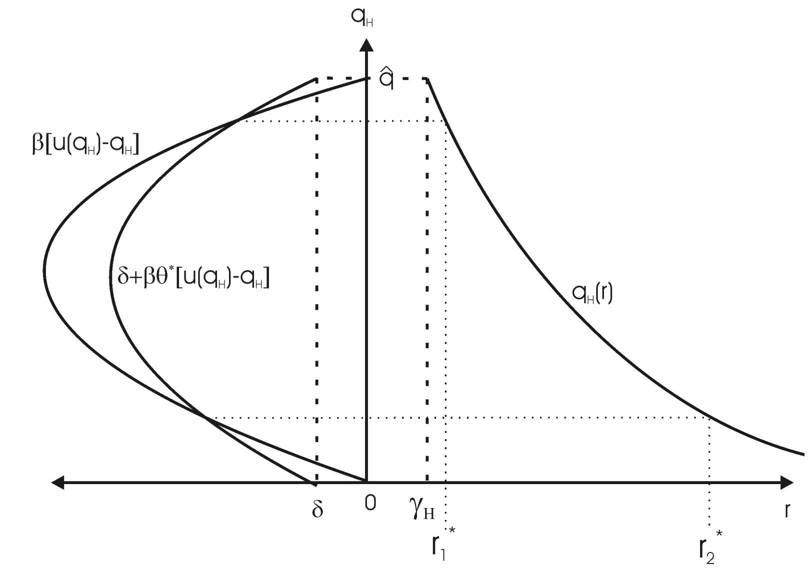

Figure 4: Building the *CF1a* frontier

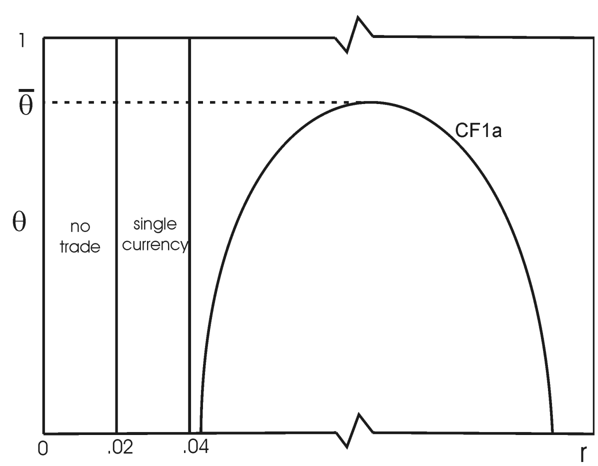

Figure 5: The *CF1a* frontier in full

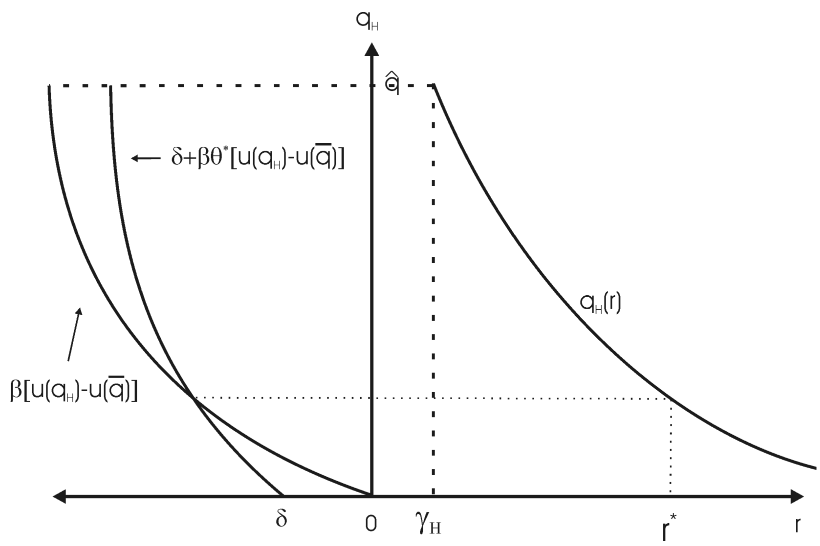

Figure 6: Building the $CF2a$ frontier

plot the relationship between equilibrium q_H and r for the same θ^*. Inserting q_H from the LHS gives the discount rate(s) r^* that satisfy (21)-(22) for q_H^* and θ^*.

Finding the shape of both sides of (47) is slightly more complicated here since $\bar{q}$ changes as we change the value for q_H. First note that as r tends to infinity, q_L, q_H and $\bar{q}$ tend to zero so that $u(q_H) - u(\bar{q})$ tends to zero as well. Second, as r decreases from ∞ to γ_H the distance between q_H and $\bar{q}$ increases so that $u(q_H) - u(\bar{q})$ increases. Finally, both $\delta + \beta\theta\left[u\left(q_H\right) - u(\bar{q})\right]$ and $\beta\left[u\left(q_H\right) - u(\bar{q})\right]$ rotate anti-clockwise when θ increases from 0 to 1, yet the first one rotates faster, and $q(r)$ becomes flatter.[11]

Starting from $\theta = 0$ it can be seen that $\delta + \beta\theta\left[u\left(q_H\right) - u(\bar{q})\right]$ and $\beta\left[u\left(q_H\right) - u(\bar{q})\right]$ intersect once so that there is only one r that leaves agents indifferent between circulation by tale and certification. As in the previous proof, there exists a threshold value for θ above which there is no intersection so that no r leaves agents indifferent between circulation by tale and certification. Again this can be seen by considering the case $\theta = 1$ where the LHS of (47) is larger than the RHS. The shape of $CF2a$ follows from the above discussion. Finally $CF2a$ shifts upward as δ decreases.

[11] $\theta = 1$ corresponds to the full information economy in which $q_H - q_L$ and then $q_H - \bar{q}$ are maximum.

A.3. Proof of Lemma 1

In general, equilibrium values q_L and q_H are not the same across equilibria. We note q_i^{bw}, q_i^{bt} and q_i^c the equilibrium quantities traded for a coin of type i in a by-weight, by-tale and certification equilibrium respectively. They are given by (10)-(11), (21)-(22), and (31)-(32).

From (20), on $CF1a$ we have

$$\beta\theta\left[u\left(q_H^{bw}\right)-q_H^{bw}\right]=\beta\left[u\left(q_H^{bw}\right)-q_H^{bw}\right]-\delta. \tag{48}$$

Inserting this into (11) gives $rq_H^{bw}=\gamma_H+\beta\left[u(q_H^{bw})-q_H^{bw}\right]-\delta$ identical to (32). Then $q_H^{bw}=q_H^c$ on $CF1a$ and we can substitute q_H^{bw} for q_H^c into (20) which yields (39) characterizing $CF1b$.

A.4. Proof of Lemma 2

The proof follows that of Proposition 2. Substituting $u(\bar{q})$ by $u(q_L)$ in (47), the $CF2b$ frontier equation is given by

$$\delta+\beta\theta\left[u\left(q_H\right)-u(q_L)\right]=\beta\left[u\left(q_H\right)-u(q_L)\right], \tag{49}$$

both sides of which stand to the left of their equivalent in (47) since $u\left(q_H\right)-u(q_L)>u\left(q_H\right)-u(\bar{q})$. Then, for the same θ the corresponding discount rate r is higher when agents expect q_L instead of $\bar{q}$.

A.5 Proof of Proposition 6

In each equilibrium, welfare is given by

$$rW_{bw}=M_L\left\{\beta\left[u(q_L^{bw})-q_L^{bw}\right]+\gamma_L\right\}+M_H\left\{\beta\theta\left[u(q_H^{bw})-q_H^{bw}\right]+\gamma_H\right\}, \tag{50}$$

$$\begin{aligned} rW_{bt} &= M_L\left\{\beta\theta\left[u(q_L^{bt})-q_L^{bt}\right]+\beta\left(1-\theta\right)\left[u(\bar{q}^{bt})-q_L^{bt}\right]+\gamma_L\right\} \\ &\quad +M_H\left\{\beta\theta\left[u(q_H^{bt})-q_H^{bt}\right]+\beta\left(1-\theta\right)\left[u(\bar{q}^{bt})-q_H^{bt}\right]+\gamma_H\right\} \end{aligned} \tag{51}$$

and

$$rW_c=M_L\left\{\beta\left[u(q_L^c)-q_L^c\right]+\gamma_L\right\}+M_H\left\{\beta\left[u(q_H^c)-q_H^c\right]+\gamma_H-\delta\right\}. \tag{52}$$

Grouping all the terms in θ and $(1-\theta)$ in (51), we obtain

$$\begin{aligned} rW_{bt} &= \theta\left\{M_L\beta\left[u(q_L^{bt})-q_L^{bt}\right]+M_H\beta\left[u(q_H^{bt})-q_H^{bt}\right]\right\} \\ &\quad +\left(1-\theta\right)\beta\left\{\left(M_L+M_H\right)u(\bar{q}^{bt})-\left(M_Lq_L^{bt}+M_Hq_H^{bt}\right)\right\}+M_L\gamma_L+M_H\gamma_H. \end{aligned} \tag{53}$$

From the concavity of u, we have $u(\bar{q}^{bt}) = u\left[\pi q_H^{bt} + (1-\pi)\, q_L^{bt}\right] > \pi u(q_H^{bt}) + (1-\pi)\, u\left(q_L^{bt}\right).$ Using the definition of π we get $(M_L + M_H)\, u(\bar{q}^{bt}) > M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right)$ so that $rW_{bt} > rW\left(\theta\right)$ whatever θ with

$$\begin{aligned} rW\left(\theta\right) &= \theta\left\{M_L\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\beta\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} \\ &\quad + (1-\theta)\,\beta\left\{M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\} + M_L\gamma_L + M_H\gamma_H \end{aligned} \tag{54}$$

which simplifies into

$$rW\left(\theta\right) = M_L\left\{\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + \gamma_L\right\} + M_H\left\{\beta\left[u(q_H^{bt}) - q_H^{bt}\right] + \gamma_H\right\}. \tag{55}$$

Note now that, when $\theta = 1$, from (21)-(22) and (31) we have $q_L^{bt} = q_L^c$ and from (21)-(22) and (32) we have $\beta\left[u(q_H^{bt}) - q_H^{bt}\right] + \gamma_H > \beta\left[u(q_H^c) - q_H^c\right] + \gamma_H - \delta$ so that $rW\left(\theta = 1\right) > rW_c$. Since $rW_{bt} > rW\ \left(\theta = 1\right)$, we conclude $rW_{bt} > rW_c$.

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